Skeletonized Wave-Equation of Surface Wave Dispersion Inversion
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SUMMARY

We present the theory for wave equation inversion of dispersion curves, where the misfit function is the sum of the squared differences between the wavenumbers along the predicted and observed dispersion curves. Similar to wave-equation traveltime inversion, the complicated surface-wave arrivals in traces are skeletonized as simpler data, namely the picked dispersion curves in the \((k_s, \omega)\) domain. Solutions to the elastic wave equation and an iterative optimization method are then used to invert these curves for 2D or 3D velocity models. This procedure, denoted as wave equation dispersion inversion (WD), does not require the assumption of a layered model and is less prone to the cycle skipping problems of full waveform inversion (FWI). The synthetic and field data examples demonstrate that WD can accurately reconstruct the S-wave velocity distribution in laterally heterogeneous media.

INTRODUCTION

Inverting surface waves for the S-wave velocity model fall into two categories: 1) the classical method of inverting dispersion curves (Evison et al., 1959; Park et al., 1998; Xia et al., 2004) for a 1D layered medium, and 2) waveform inversion (Groos et al., 2014; Solano et al., 2014; Dou and Ajo-Franklin, 2014) for 2D and 3D media. The classical method accurately inverts for a 1D S-wave velocity model, but becomes less accurate with increasing lateral heterogeneity in the subsurface velocity model. The 1D assumption is not satisfied for some practical applications, so partial remedies are spatial interpolation of 1D velocity models (Tian et al., 2003) and laterally constrained inversion (Socco et al., 2014; Bergamo et al., 2012).

In comparison, full waveform inversion (FWI) can theoretically account for any lateral heterogeneity, but it is computationally expensive and can easily get stuck in local minima associated with the objective function (Tarantola, 1984). To avoid getting stuck in a local minimum, the initial model should be smooth and time-damping strategies can be used at the early iterations (Brossier et al., 2008; Romdhane et al., 2011; Sheng et al., 2006; Sears et al., 2008). However, there are no proven strategies for avoiding local minima in the context of FWI with surface waves.

A partial surface wave FWI method is that of Pérez Solano et al. who used the magnitude spectra of surface waves as the input data (Solano et al., 2014). Results with synthetic data showed this to be a robust and efficient method for reconstructing the S-velocity model at the near surface. Another surface-wave inversion strategy is proposed by (Yuan et al., 2015), who developed a wavelet multi-scale adjoint method which combined surface waves and body waves. Synthetic tests showed that this approach can avoid cycle skipping for some models. The role of attenuation in FWI with surface waves was studied by (Groos et al., 2014). They concluded that the estimation of a priori quality factors is critical for inverting seismic waves in the near-surface zone. Pan et al. proposed to invert the Love-waves in the time domain to reconstruct the S-wave velocity model at the near surface (Pan et al., 2016).

To avoid the assumption of a layered medium and also mitigate FWI’s sensitivity to local minima, we present a skeletonized inversion method that inverts the dispersion curves of surface waves for 2D or 3D velocity models (Li and Schuster, 2016). The picked dispersion curves are skeletonized data (Luo and Schuster, 1991b) that tend to make the objective function simpler, and hence this new method, denoted as wave equation dispersion inversion (WD), enjoys better convergence properties than FWI. This is similar to wave equation traveltime inversion (Luo and Schuster, 1991a), except picked dispersion curves rather than picked traveltimes are the input data.

The WD procedure is more robust than FWI because it replaces complicated surface-wave arrivals with simple dispersion curves in the wavenumber \(k_s - \omega\) or phase-velocity \(C(\omega) - \omega\) domains in Figure 1. The WD method presented in this paper is the adjoint-state method presented by (Zhang et al., 2015), who used a difference approximation to the gradient rather than an adjoint operation. Hence, our WD method is more than an order-of-magnitude faster for complicated models.

THEORY

The input data are z-component shot gathers excited by a vertical-component force at \(s = (x_0, 0)\) on the surface and recorded at \(g = (x_g, 0)\); and the skeletonized data consist of the picked dispersion curve \(k(\omega)_{ske}\) shown as the red dashed line in Figure 1. For a simplified exposition, we assume a single shot gather and the fundamental curve \(k(\omega)_{obs}\) associated with the Rayleigh waves, but WD is valid for any order or any number of dispersion curves. For higher-order dispersion wave, deeper S-velocity information can be inverted.
WD inversion

There are 4 steps in the WD method.

1. Skeletonized data. A shot gather is recorded in the $x-t$ domain and is Fourier transformed in time to give $D(g, \omega)_{\text{obs}}$ for the shot at $(x_s, 0)$ and geophone at $(x_t, 0)$. A spatial Fourier transform in the $x_t$ variable is then applied to $D(g, \omega)_{\text{obs}}$ to give the spectrum $\hat{D}(k, \omega)_{\text{obs}}$ in the $(k, \omega)$ domain, of which the dispersion curve $\kappa'(\omega)_{\text{obs}}$ is picked for the fundamental mode. The dependency of $D(g, \omega)_{\text{obs}}, \hat{D}(k, \omega)_{\text{obs}},$ and $\kappa'(\omega)_{\text{obs}}$ on the shot position $s$ is silent. A finite-difference method is used to solve the elastic wave equation for a specified starting model to get the predicted spectrum $\hat{D}(k, \omega)$. The goal is to find the S-velocity model that predicts the picked dispersion curve $\kappa'(\omega)_{\text{obs}}$.

2. Objective function. The dispersion misfit function $\varepsilon$ is defined as the sum of squared dispersion residuals:

$$\varepsilon = \frac{1}{2} \sum_{\omega} (\kappa'(\omega) - \kappa'(\omega)_{\text{obs}})^2,$$

where $\kappa'(\omega)$ is the predicted surface-wave wavenumber obtained by a 2D finite-difference solution to the elastic wave equation for a vertical point source at $(x_s, 0)$.

3. Gradient. A gradient optimization method is used to determine the S-slowness model $s(x)$ that minimizes $\varepsilon$, where the gradient is given by

$$\frac{\partial \varepsilon}{\partial s(x)} = \sum_{\omega} Re\left[\frac{\partial \hat{D}(\kappa'(\omega)_{\text{obs}}, \omega)}{\partial s(x)} \hat{D}(\kappa'(\omega)_{\text{obs}}, \omega)^*\right],$$

$$= \frac{1}{2\pi} \sum_{\omega} Re\left[\int \frac{\partial \hat{D}(x_s, \omega)}{\partial s(x)} \hat{D}(\kappa'(\omega)_{\text{obs}}, \omega)^* d_{x_s} \hat{D}(\kappa'(\omega)_{\text{obs}}, \omega)^*\right].$$

The Fréchet derivative $\frac{\partial \hat{D}(x_s, \omega)}{\partial s(x)}$ can be expressed as the Born approximation

$$\frac{\partial \hat{D}(x_s, \omega)}{\partial s(x)} = -2s(x)W(\omega)G(g|x)G(x|s),$$

where $W(\omega)$ is the source-wavelet spectrum and $G(g|x)$ is the harmonic solution to the elastic wave equation for a vertical force at the point $x$ and a vertical-component particle-velocity recording at $g$. This Green’s function is for the mode of the fundamental Rayleigh wave.

Substituting equation 3 into equation 2 gives

$$\frac{\partial \varepsilon}{\partial s(x)} = \frac{s(x)}{\pi} \sum_{\omega} Re\left[\frac{W(\omega)G(g|x)}{\sqrt{-2s(x)}} \hat{D}(\kappa'(\omega)_{\text{obs}}, \omega)^* d_{x_s} \hat{D}(\kappa'(\omega)_{\text{obs}}, \omega)^*\right].$$

4. Conjugate gradient method, The optimal shear-slowness model $s(x)$ is obtained using the iterative conjugate gradient formula:

$$s(x)^{(k+1)} = s(x)^{(k)} - \alpha \frac{\partial \varepsilon}{\partial s(x)}.$$
There is a rough but imprecise correspondence between the to-
profiles are displayed as the S-velocity tomogram in Figure 4d.
file is generated at each shot position. The ensemble of 1D
v
shot gather, frequency is converted to depth, and the
phase velocity is computed as a function of frequency for each
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To convert the frequency variable to depth, we multiply the
by 100 receivers spaced at the same interval as the receivers.
On the surface for a 10 Hz Ricker wavelet. Each shot is recorded
equation for 100 shot gathers, with the shots evenly distributed
in an open-pit mine.
The objective of this synthetic test is to determine how well
WD inversion
a) b) c)
Figure 4: S-velocity 2D wave equation dispersion inversion.
a) Vs true model; b) initial model; c) 2D WD tomogram after
15 iterations; d) 1D tomogram.

curve in e) and f) closely match the true values illustrated in b)
and c), respectively.
The normalized RMS residual plotted against iteration number
shows rapid convergence in Figure 3a, and the misfit gradient
for all of the shot gathers in Figure 3b suggests that the shallow
velocities down to about 3-6 m are accurately reconstructed.
This is consistent with Figure 3c, where the reconstructed S-
velocity profile (blue curve) over the center of the model is a
good approximation to the actual velocity profile (red curve).

Low-Velocity Mineral Model
The objective of this synthetic test is to determine how well
WD can detect the blue low-velocity anomalies in Figure 4a.
These anomalies are based on realistic mineral deposits seen
in an open-pit mine.
The input data are computed by solving the 2D elastic wave
equation for 100 shot gathers, with the shots evenly distributed
on the surface for a 10 Hz Ricker wavelet. Each shot is recorded
by 100 receivers spaced at the same interval as the receivers.
To convert the frequency variable to depth, we multiply the
average S-velocity by 1/3 and divide by the frequency. The
phase velocity is computed as a function of frequency for each
shot gather, frequency is converted to depth, and the \( v_s(x) \)
profile is generated at each shot position. The ensemble of 1D
profiles are displayed as the S-velocity tomogram in Figure 4d.
There is a rough but imprecise correspondence between the to-
mogram and the actual model in Figure 4a.

To generate a more accurate tomogram, the 2D WD method is
used to invert the data. In this case, only 25 shot gathers are
employed with a 8 m shot interval. Figure 4b is the initial gra-
dient model, and the fundamental dispersion curve is picked
for each shot gather and inverted by the WD method. After
15 iterations, the reconstructed model is shown in Figure 4c.
This tomogram shows much better correspondence to the ac-
tual model than does the 1D tomogram. The predicted and ob-
served dispersion curves are plotted against iteration number
in Figure 5. After 15 iterations, the normalized misfit residual
decreased to 0.3 and shows an acceptable fit to the data.

Qademah Fault Controlled Noise Source Seismic Data
A controlled noise source (CNS) seismic survey is conducted
across the Qademah fault, a normal fault near the KAUST
campus. The geophone line consists of 60 receivers at a 10
m spacing and a noise-making truck is driven around the sur-
vey line for 2 hours. The resulting seismic noise is recorded at
each of the traces. Then, the traces are broken up into small
windows, and each window of arrivals is correlated with the
corresponding window of arrivals in other traces to give a vir-
tual CSG (Hanafy et al., 2015). Stacking the virtual CSGs for
the same source position gives the virtual shot gather.

A common offset gather (COG) is shown in Figure 6a with the
source-receiver offset of 50 m. The dashed lines in Figure 6a
indicate the location of the Qademah fault, which is consistent
with the lateral velocity decrease in the P-velocity tomogram
in Figure 6b. The P-velocity tomogram is computed by inver-
ting the first-arrival traveltimes. The shot gather is transformed
into the f-v domain by a Radon transform and the maximum
energy values of the dispersion curve are picked. Figure 6c
shows the S-velocity tomogram obtained from the traditional
1D inversion of dispersion curves. The tomogram roughly es-
timates the position of the Qademah fault according to the low
S-wave velocity structure. Then, the 2D WD method is ap-
plied to the picked dispersion curves to give the S-velocity to-
mogram in Figure 6d, where there is a low-velocity zone on
the downthrown side of the fault. This is consistent with the
P-velocity tomogram in Figure 6b and the COG profile in Fig-
ure 6a for 150 m < x < 300 m. As the surface waves enter
the fault zone there is strong dispersion in the surface-wave
arrivals.

It is difficult to assess the accuracy of the 2D WD tomogram,
but it appears to have much more complexity than the simpler
1D tomogram in Figure 6c. In fact, the 1D tomogram appears
WD inversion

to be too simple to fully explain the complexity of events in the Figure 6a COG.

SUMMARY

We present the theory for wave equation inversion of dispersion curves, where the dispersion misfit function is the difference between the wave-numbers along the predicted and observed dispersion curves. The S-wave velocity model is updated by migrating the weighted data, where the weight is proportional to the dispersion residual. It largely overcomes the expense of finding the Fréchet derivative by a finite-difference approximation. Numerical simulations suggest that WD inversion is effective for selected 2D velocity models where the dispersion curves can be readily identified. The corresponding 2D tomograms are more accurate than the ones inverted by assuming a local 1D velocity model over each common shot gather. Tests on both synthetic and field data suggest that reasonable velocity models can be inverted to reveal the presence of faults.

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Figure 6: Results from Qademah data for a) common offset gather, b) P-velocity tomogram inverted from 1st-arrival traveltimes, c) S-velocity tomogram inverted by the standard 1D inversion of dispersion curves, and d) S-velocity tomogram computed by 2D WD inversion of dispersion curves.
REFERENCES
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