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# Numerical Simulation of Magnetic Nanoparticles Injection into Two–phase Flow in a Porous Medium

Mohamed F. El-Amin[1,2]*, Ahmed M. Saad[2], Shuyu Sun[2], and Amgad Salama[2]

[1] College of Engineering, Effat University, Jeddah 21478, Kingdom of Saudi Arabia
[2] King Abdullah University of Science and Technology (KAUST), Thuwal 23955–6900, Kingdom of Saudi Arabia
`momousa@effatuniversity.edu.sa`

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## Abstract

In this paper, the problem of magnetic nanoparticles injection into a water–oil two–phase flow under an external permanent magnetic field is investigated. The mathematical model of the problem under consideration has been developed. We treat the water-nanoparticles suspension as a miscible mixture while it is immiscible with the oil phase. The magnetized phase pressure includes an additional pressure term with the conventional thermodynamic pressure. The countercurrent imbibition flow problem is taken as an example. Physical variables including water–nanoparticles suspension saturation, nanoparticles concentration, and pore wall/throat deposited nanoparticles are investigated under the influence of the magnetic field.

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*Keywords:* IMPES, Two-phase flow, Porous media, Magnetic field, Nanoparticles

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## 1 Introduction

Nanotechnology has been used in different areas of the oil and gas industry including exploration, drilling, production to reservoir monitoring and refining. The industry now looks at nanotechnology as a resort facing the new challenges such as unconventional heavy oil and shale gas reservoirs. The conventional Enhanced Oil Recovery (EOR) methods have several problems from high cost to low oil recovery and operations problems especially in thermal and chemical methods. The tiny nature of nanoparticles results in some useful characteristics such as increased surface area, at the nano scale size does matter when it comes to how molecules react to and bond with each other. So, for example, nanoparticles can be used in EOR, because they are small enough to pass through pore throats in typical reservoirs, and they can be retained by the rock. Ju et al. [5] have calibrated a model for nanoparticles transport in two-phase flow in porous media based on the formulation of the colloid model of fine particles transport in two-phase flow in porous media. El-Amin et al. [6, 4, 11] have presented modeling and simulations of nanoparticles transport associated with two-phase flow in porous media. In

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*Corresponding Author

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the last few years, a number of publications has been considered Nano-ferrofluids in oil and gas recovery or environmental applications (e.g. Refs. [1, 2, 3, 7]). In the current work, we developed a mathematical model to describe the magnetic nanoparticles–water suspension that imbibes into a water-oil two–phase flow in porous media under magnetic field effect. Countercurrent imbibition benchmark problem is taken as an example. Physical variables are investigated under the influences of magnetic field.

## 2 Modeling and Mathematical Formulation

Consider suspension of magnetic nanoparticles injected in a water-oil two-phase flow under an external magnetic field. In the followings, we describe the mathematical modeling of the problem under consideration [8, 10]. The external magnetic force acts as a body force on the nanoparticles suspension per unit volume which can be expressed as [2], $\mathbf{F}_{mag} = \mu_0 M \frac{\partial H}{\partial z}$, $\mu_0$ is the magnetic permeability, $M$ is the magnetization and $H$ is the magnetic field strength. The magnetization $M$ is a function of $H$, approximated by, $M = a_1 \; tan^{-1}(b_1 H)$. The parameters $a_1$ and $b_1$ depend on the particular type of the ferromagnetic material. The magnetic field strength in 1D was given in Refs. [2, 13]. In the current model, we treat the water-nanoparticles suspension as a miscible mixture while it is immiscible with the oil phase. For the immiscible conceptualization, density and viscosity are considered properties of the respective phases, and no mixing relations are required. Therefore, in the current model we consider immiscible two-phase (water-oil) model and the water phase itself is a miscible mixture. So, the magnetization of this system is, $M(S_w, c) = M(S_w = 1, c = 1)S_w c$. Oldenburg et al. [2] have assumed that the volumes of pure water and ferrofluid are additive, and define for the mixture density as, $\frac{1}{\rho_w} = \frac{1-c}{\rho_{w,p}} + \frac{c}{\rho_f}$. $\rho_{w,p}$ is the density of water component and $\rho_f$ is the density of ferrofluid component. Viscosity effects in miscible ferrofluid–water mixtures are incorporated through the correlation, $\mu_w = \mu_{w,p}(1 + \mu_1 c)$. Given $\phi$[-] as the porosity, $\rho_\alpha$[kg·m$^{-3}$] is the density of phase $\alpha$ , $S_\alpha$[–] is the saturation of phase $\alpha$ and $\mathbf{u}_\alpha$[m·s$^{-1}$] is the velocity of phase $\alpha$. $w$ stands for the nanoparticles-water suspension phase, and $o$ stands for the oil phase. $K$[m$^2$] is the permeability, $k_{r\alpha}$[–] is the relative permeability of phase–$\alpha$, $p_\alpha$[Pa] is the pressure of phase–$\alpha$, $g$[m·s$^{-2}$] is the gravitational acceleration, and $\mu_\alpha$[Pa·s] is the viscosity of phase–$\alpha$. The fluid saturations for the two-phase flow of water and oil are related by, $S_w + S_o = 1$. In the countercurrent imbibition, the sum of the velocities of the wetting and non–wetting phases is zero, $u_t = u_w + u_o = 0$. The capillary pressure is defined as, $p_c = p_o - p_w$. The magnetized phase pressure will have additional pressure term with the conventional thermodynamic pressure. The resulting pressure is called composite pressure [1], which can be given as, $p_w^* = p_w + (p_m + p_s + p_n)$, $p_w$ is the ferrofluid phase dynamic pressure, $p_m$ is the fluid magnetic pressure, $p_s$ is the magneto-strictive pressure, and $p_n$ is the magnetic normal pressure, which is neglected in this study. Eliminating $\partial p_o/\partial z$, we may have,

$$\frac{\partial p_w^*}{\partial z} = -f_w \frac{\partial p_c^*}{\partial z} + (\rho_w f_w + \rho_o f_o)g + f_w \mu_0 M(S_w, c)\frac{\partial H}{\partial z}, \tag{1}$$

The velocity of water phase becomes,

$$u_w = K\lambda_w f_o \left(\frac{\partial p_c^*}{\partial z} - \Delta\rho g + \mu_0 M(S_w, c)\frac{\partial H}{\partial z}\right) \tag{2}$$

Therefore, the saturation equation for the water phase becomes,

$$\phi\frac{\partial S_w}{\partial t} + \frac{\partial}{\partial z}\left[K\lambda_w f_o \left(\frac{\partial p_c^*}{\partial z} - \Delta\rho g + \mu_0 M(S_w, c)\frac{\partial H}{\partial z}\right)\right] = 0 \tag{3}$$

where $\lambda_w = k_{rw}/\mu_w$ and $\lambda_o = k_{ro}/\mu_o$ are mobility ratios of water and oil phases, respectively. $\lambda_t = \lambda_w + \lambda_o$ is the total mobility. $f_w = \lambda_w/\lambda_t$ and $f_o = \lambda_o/\lambda_t$ are the flow fraction of water and oil phases, respectively. $\Delta\rho = \rho_w - \rho_o$. The capillary pressure is a function of the normalized saturation which can be given as, $p_c = p_d \ln S$, $p_d$ the entry pressure for the imbibition. Moreover, the relative permeabilities are $k_{rw} = k^0_{rw} S^{a_2}$, $k_{ro} = k^0_{ro}(1-S)^{b_2}$, $S = \frac{S_w - S_{iw}}{1 - S_{ro} - S_{iw}}$, $0 < S < 1$, and $k^0_{rw} = k_{rw}(S=1)$ and $k^0_{ro} = k_{ro}(S=0)$ are the endpoint relative permeability of the water and oil phase, respectively. $a_2$ and $b_2$ are positive numbers. $S_{iw}$ is the irreducible water saturation and $S_{ro}$ is the residual oil saturation.

The transport equation of the nanoparticles–water suspension in the water–phase can be written as,

$$\phi \frac{\partial (S_w c)}{\partial t} - \frac{\partial c_{s1}}{\partial t} - \frac{\partial c_{s2}}{\partial t} + \frac{\partial}{\partial z}\left( u_w c - \phi S_w D \frac{\partial c}{\partial z} \right) = 0, \tag{4}$$

where $c[\mathrm{m^3 \cdot m^{-3}}]$ is the concentration of nanoparticles in the water phase. $c_{s1}[\mathrm{m^3 \cdot m^{-3}}]$ is the volume concentration of the nanoparticles in contact with the water phase available on the pore surfaces per unit bulk volume of the porous medium. $c_{s2}[\mathrm{m^3 \cdot m^{-3}}]$ is the volume of the nanoparticles entrapped in pore throats from the water phase per unit bulk volume of porous medium due to plugging and bridging. $D[\mathrm{m^2 \cdot s^{-1}}]$ is the molecular diffusion coefficient. The modified model for the surface deposition is used in this study [5, 4]. The surface deposition only particle retention occurs while above it retention and entrainment of the nanoparticles take place simultaneously, which can be modeled as,

$$\frac{\partial c_{s1}}{\partial t} = \begin{cases} \gamma_d |u_w| c, & u_w \le u_c \\ \gamma_d |u_w| c - \gamma_e |u_w - u_c| c_{s1}, & u_w > u_c \end{cases} \tag{5}$$

Also, the rate of entrapment of the nanoparticles in the water–phase is given by,

$$\frac{\partial c_{s2}}{\partial t} = \gamma_{pt} |u_w| c, \tag{6}$$

where $\gamma_d[\mathrm{m^{-1}}]$ is the rate coefficients for surface retention of the nanoparticles in the water phase. $\gamma_e[\mathrm{m^{-1}}]$ is the rate coefficients for entrainment of the nanoparticles. $u_c$ is the critical velocity for the water phase. $\gamma_{pt}[\mathrm{m^{-1}}]$ is the pore throat blocking constants.

## 3 Results and Discussion

The above highly nonlinear parabolic partial differential equation is solved numerically using an efficient algorithm [12]. The spatial discretization is handled by Galerkin method, while an adaptive time step is used with the time integration. The above governing equations (3), (4), (5) and (6) are solved along with their initial and boundary conditions [9]. The parameters values that used in the computations are provided in [5, 9]. The magnet field is located to the right of the core, with respect to the core. It is interesting to notice from Fig. 1 (upper left) that as the effect of the magnet increases the saturation of nanoparticles–water suspension increases. This can be explained through an important fact that is the magnetic force is proportional to magnetic field strength, so fluid that is closer to the magnet is more strongly magnetized and pulled more strongly toward the magnet. From Fig. 1 (upper right), it can be seen that after long time of imbibition, the nanoparticles concentration decreases slightly under the effect of the magnetic field. Fig. 1 (middle left) shows the concentration of deposited nanoparticles on

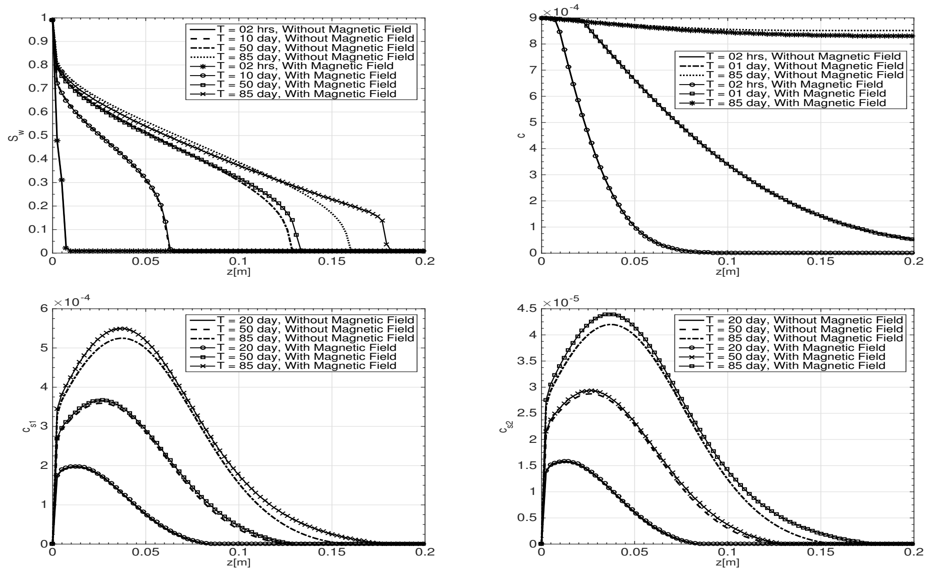

Figure 1: Saturations (upper left), nanoparticles concentration (upper right), deposited nanoparticles concentration (middle left), entrapped nanoparticles concentration (middle right).

the pore-wall as plotted against the core length with and without magnetic field effect for various values imbibition times. Also, Fig. 1 (middle right) shows the concentration of entrapped nanoparticles on the pore-throat which is plotted against the core length with and without magnetic field effect for various values imbibition times. As expected the deposited nanoparticles have an opposite behavior of the nanoparticles in the water, i.e., when the nanoparticles concentration in the water decreases, the deposited nanoparticles concentration increases.

In our work [13], we presented the general two-dimensional case of the current problem. The magnet is supposed to be placed at the right upper side of the domain. Distributions of the water saturation without and with the effect of the magnetic field are shown in Fig. 2. One may notice from this figure that the magnetic field has a clear effect on the distribution of the water saturation due to the attraction of magnetic nanoparticles toward the magnet which is located at the upper right side of the domain.

# 4 Conclusions

This paper was devoted to study the magnetic field effects on the magnetic nanoparticles injected into a two–phase water–oil system in porous media. The countercurrent imbibition in a small core is considered as a benchmark problem. Effects of the magnetic field on the physical variables such as saturation and nanoparticles concentrations have been examined. Simulation results are introduced for both locations of the magnet. The saturation of nanoparticles–water suspension increases. The nanoparticles concentration decreases slightly under the effect of the magnetic field. The deposited nanoparticles concentration increases.

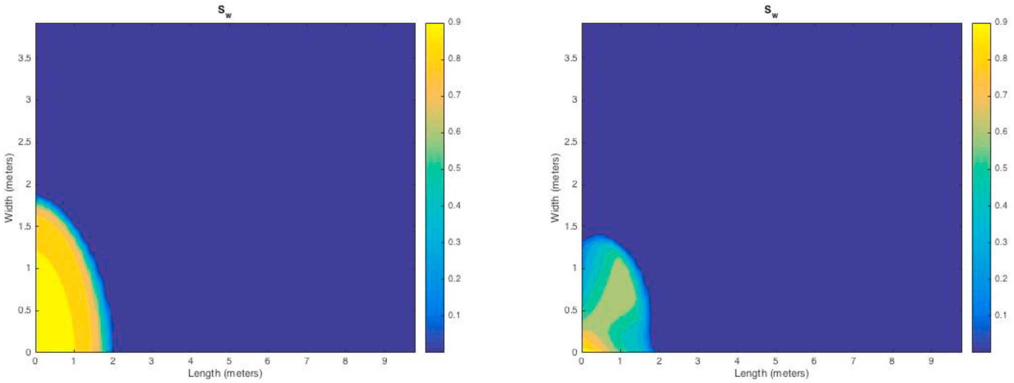

Figure 2: Saturation without magnetic field (left) and under the effect of magnetic field (right).

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