Preconditioned Inexact Newton for Nonlinear Sparse Electromagnetic Imaging

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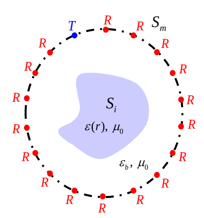
Abstract

Newton-type algorithms have been extensively studied in nonlinear microwave imaging due to their quadratic convergence rate and ability to recover images with high contrast values. In the past, Newton methods have been implemented in conjunction with smoothness promoting optimization/regularization schemes. However, this type of regularization schemes are known to perform poorly when applied in imagining domains with sparse content or sharp variations. In this work, an inexact Newton algorithm is formulated and implemented in conjunction with a linear sparse optimization scheme. A novel preconditioning technique is proposed to increase the convergence rate of the optimization problem. Numerical results demonstrate that the proposed framework produces sharper and more accurate images when applied in sparse/sparsified domains.

Formulation

2D Electromagnetic Equations:



R = Receiver,
T=Transmitter,
$\varepsilon(r)$ =permitivity
S_i=investigation domains
S_m= measurement domains
r= 2D position vector

TMz sources:

$$E_z^{\text{inc}}(r) = -j\omega\mu_0 I_e G_{2D}(r, r_T)$$

Contrast-Sourece Equations (EFIE):

For a given i^{th} excitation:

$$J_i(r) - \chi(r)E_i^{\text{inc}}(r) - k_b^2\chi(r)\int_{S^{\text{inv}}} J_i(r')G(r,r')dr' = T_i^1(\chi, J_i) = 0, \quad r \in S^{\text{inv}}$$

$$E_i^{\text{scat}}(r) = k_b^2 \int_{S^{\text{inv}}} J_i(r')G(r,r')dr' = T_i^2(J_i), \quad r \in S^{\text{m}}$$

Definition of the nonlinear operator:

$$T(z) = \begin{pmatrix} T_1^1(\chi, J_1) \\ T_2^1(\chi, J_2) \\ . \\ . \\ . \\ T_{N^T}^1(\chi, J_{N^T}) \\ T_1^2(J_1) \\ T_2^2(J_2) \\ . \\ . \\ . \\ T_{N_T}^2(J_{N^T}) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ . \\ . \\ . \\ 0 \\ E_1^{\text{scat}}(r) \\ E_2^{\text{scat}}(r) \\ . \\ . \\ . \\ E_{N^T}^{\text{scat}}(r) \end{pmatrix} = y, \quad f(z) = T(z) - y = 0$$

Inexact Newton:

$$\partial_z f(z_k)(h_k) = -f(z_k) \quad \text{solve for } h_k$$
$$z_{k+1} = z_k + h_k.$$

Frechet derivative matrix:

$$\partial_z T(z) = \begin{bmatrix} \partial_\chi T_1^1\big|_z & \partial_{J_1} T_1^1\big|_z & 0 & 0 & .... & 0 \\ \partial_\chi T_2^1\big|_z & 0 & \partial_{J_2} T_2^1\big|_z & 0 & .... & 0 \\ & & . & & & \\ & & . & & & \\ & & . & & & \\ \partial_\chi T_{N^T}^1\big|_z & 0 & 0 & 0 & .... & \partial_{J_{N_T}} T_{N^T}^1\big|_z \\ 0 & \partial_{J_1} T_1^2\big|_z & 0 & 0 & .... & 0 \\ 0 & 0 & \partial_{J_2} T_2^2\big|_z & 0 & .... & 0 \\ & & . & & & \\ & & . & & & \\ & & . & & & \\ 0 & 0 & 0 & 0 & .... & \partial_{J_{N^T}} T_{N^T}^2\big|_z \end{bmatrix}$$

where:

$$\partial_\chi T_i^1\big|_z (h_x) = -h_\chi(r)[E_i^{\text{inc}}(r) + k_b^2 \int_{S^{\text{inv}}} J_i(r')G(r,r')dr'], \quad r \in S^{\text{inv}}$$

$$\partial_{J_i} T_i^1\big|_z (h_{J_i}) = h_{J_i}(r) - k_b^2\chi(r)\int_{S^{\text{inv}}} h_{J_i}(r')G(r,r')dr', r \in S^{\text{inv}}$$

$$\partial_{J_i} T_i^2\big|_z (h_{J_i}) = k_b^2 \int_{S^{\text{inv}}} h_{J_i}(r')G(r,r')dr', \quad r \in S^{\text{m}}$$

Discretization:

Under N^{th} cell discretization the variables become:

$$\bar{\chi} = [\chi_1, \chi_2, ..., \chi_N]^t$$
$$\bar{J}_i = [J_{i,1}, J_{i,2}, ..., J_{i,N}]^t$$
$$\bar{E}_i^{\text{inc}} = [E_{i,1}^{\text{inc}}, E_{i,2}^{\text{inc}}, ..., E_{i,N}^{\text{inc}}]^t$$
$$\bar{h}_\chi = [h_{\chi_1}, h_{\chi_2}, ..., h_{\chi_N}]^t$$
$$\bar{h}_{J_i} = [h_{J_{i,1}}, h_{J_{i,2}}, ..., h_{J_{i,N}}]^t$$

The Frechet derivatives become:

$$\partial_\chi \bar{T}_i^1\big|_z (\bar{h}_x) = D\{-\bar{E}_i^{\text{inc}} - \bar{G}^{\text{inv}}\bar{J}_i\}\bar{h}_x$$
$$\partial_{J_i} \bar{T}_i^1\big|_z (\bar{h}_{J_i}) = [\bar{I} - D\{\bar{\chi}\}\bar{G}^{\text{inv}}]\bar{h}_{J_i}$$
$$\partial_{J_i} \bar{T}_i^2\big|_z (\bar{h}_{J_i}) = \bar{G}^{\text{m}}\bar{h}_{J_i}$$

where $D\{.\}$ is a function that transforms the input vector into a diagonal matrix. The block matrices are the integration of the green function computed as:

$$\{\bar{G}^{\text{m,inv}}\}_{mn} = k_b^2 \int_{S^{\text{inv}} \in S_n} G(\mathbf{r}_m, \mathbf{r}')d\mathbf{r}', \ \mathbf{r}_m \in S^{\text{m}}$$
$$\{\bar{G}^{\text{m,inv}}\}_{mn} = k_b^2 \int_{S^{\text{inv}} \in S_n} G(\mathbf{r}_m, \mathbf{r}')d\mathbf{r}', \ \mathbf{r}_m \in S^{\text{inv}}$$

Preconditioning:

to tackle the variables unit mismatch in the solution, preconditioning is required:

$$\bar{L}_k = \left(\sqrt{D\{T\{\partial_z \bar{T}^{\text{n}}(\bar{z}_k)\partial_z \bar{T}^{\text{n}*}(\bar{z}_k)\}\}}\right)^{-1}$$
$$\bar{R}_k = \left(\sqrt{D\{T\{\partial_z \bar{T}^{\text{n}*}(\bar{z}_k)\partial_z \bar{T}^{\text{n}}(\bar{z}_k)\}\}}\right)^{-1}$$

Sparse Preconditioned Inexact Newton:

In the following, a pseudo-code for the complete modified-preconditioned inexact algorithm with sparse optimization is written:

initialize $\delta, \gamma_1, \bar{z}_0 = 0$

for $k = 1, ..., N_{it}^{\text{Newton}}$

compute $\partial_z \bar{F}(\bar{z}_k) = \partial_z \bar{T}(\bar{z}_k)$

compute $\bar{F}(\bar{z}_k) = \bar{T}(\bar{z}_k) - \bar{y}$

normalization $\bar{F}^{\text{n}}(\bar{z}_k) = \bar{F}(\bar{z}_k) / \sigma_1^2$

$\partial_z \bar{F}^{\text{n}}(\bar{z}_k) = \partial_z \bar{F}(\bar{z}_k) / \sigma_1^2$

Preconditioning $\bar{F}^{\text{np}}(\bar{z}_k) = \bar{L}_k \bar{F}^{\text{n}}(\bar{z}_k)$

$\partial_z \bar{F}^{\text{np}}(\bar{z}_k) = \bar{L}_k \partial_z \bar{F}^{\text{n}}(\bar{z}_k)\bar{R}_k$

$\bar{h}_k = \arg\min_{\bar{h}} \frac{1}{2}\left\|-\bar{F}^{\text{np}}(\bar{z}_k) - \partial_z \bar{F}^{\text{np}}(\bar{z}_k)\bar{h}\right\|_2^2 + \gamma_k \left\|\bar{h}\right\|_{0,1}$

$\bar{z}_{k+1} = Thr^{\hat{m}}(\bar{z}_k + \bar{h}_k)$

$\gamma_{k+1} = \delta\gamma_k$

If $\left\|\bar{z}_{k+1} - \bar{z}_k\right\|_2 / \left\|\bar{z}_{k+1}\right\|_2 < \varepsilon^{\text{Newton}}$, **break**

end

Two-Step Iterative Shrinkage Thresholding (TWIST):

In the following, a pseudo-code for the TWIST algorithm to solve the sparse linear optimization problem.

Initialize σ_m, γ

set $\rho = (1 - \sqrt{\xi_1 / \xi_m}) / (1 + \sqrt{\xi_1 / \xi_m})$,

$\alpha = \rho^2 + 1$,

$\varsigma = 2\alpha / (\xi_1 + \xi_m)$.

$\bar{h}_{(1)}^{(k)} = T^{\gamma_k}(\bar{h}^{(k-1)} - \partial_z \bar{F}^{\text{np}*}(\bar{z}_k)[\bar{F}^{\text{np}}(\bar{z}_k) + \partial_z \bar{F}^{\text{np}}(\bar{z}_k)\bar{h}^{(k-1)}])$

For $i = 1, ..., N_{it}^{\text{TWIST}}$

$\bar{h}_{(i+1)}^{(k)} = (\alpha - \varsigma)\bar{h}_{(i)}^{(k)} + (1 - \alpha)\bar{h}_{(i-1)}^{(k)}$
$+ \varsigma Thr^{\gamma_k}(\bar{h}_{(i)}^{(k)} - \partial_z \bar{F}^{\text{np}*}(\bar{z}_k)[\bar{F}^{\text{np}}(\bar{z}_k)$
$+ \partial_z \bar{F}^{\text{np}}(\bar{z}_k)\bar{h}_{(i)}^{(k)}])$

If $\left\|\bar{h}_{(i+1)}^{(k)} - \bar{h}_{(i)}^{(k)}\right\|_2 / \left\|\bar{h}_{(i+1)}^{(k)}\right\|_2 < \varepsilon^{\text{Sparse}}$, **break**

end

set $\bar{h}_k = \bar{h}_{(i+1)}^{(k)}$

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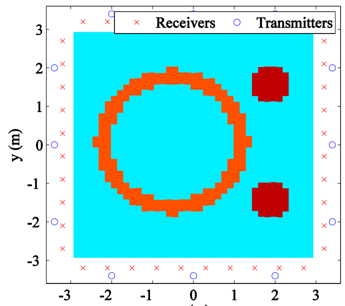
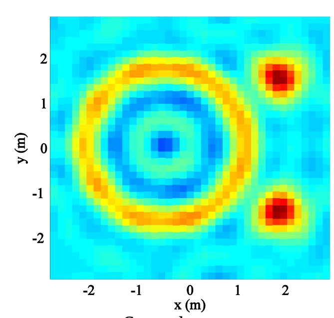
Numerical Results

Austria example:

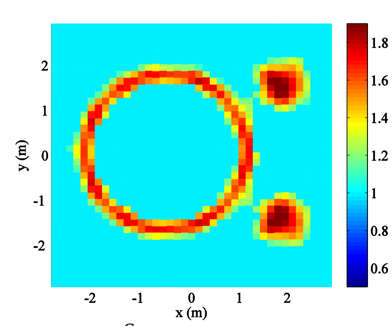
40 receivers, 12 transmitters, Sparseness level: 15%, N =1600, Dimensions: 6m×6m, f =150MHz. For smooth reconstruction, truncated Lnadweber is used, truncated at 50 iterations. TWIST iterations has been also truncated at 50 iterations within each Newton iterations. TWIST parameters are $\delta = 0.7$, $\gamma = 2.5$, $\sigma_m = 0.45$.



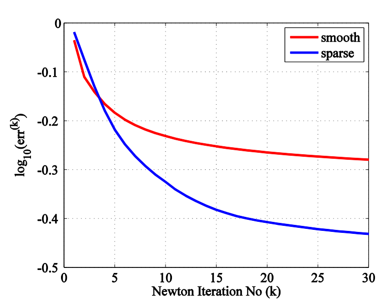
Austria profile



Smooth recovery



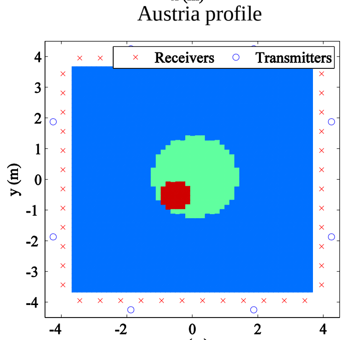
Sparse recovery



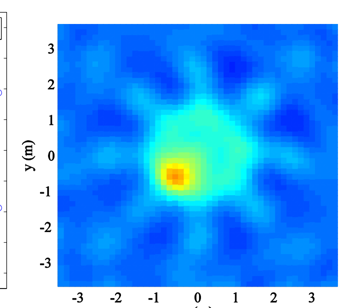
Relative norm error

Layered example:

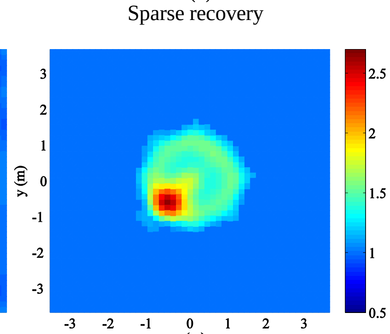
48 receivers, 8 transmitters, Sparseness level: 9.9%, $N = 2500$, Dimensions: 7.5m×7.5m, f =125MHz. For smooth reconstruction, truncated Lnadweber is used, truncated at 50 iterations. TWIST iterations has been also truncated at 50 iterations within each Newton iterations. TWIST parameters are $\delta = 0.6$, $\gamma = 2$, $\sigma_m = 0.45$.



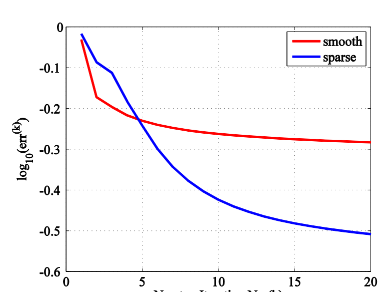
Layered scatterer



Smooth recovery



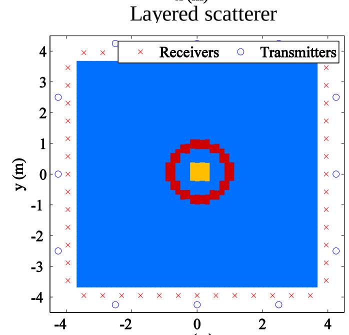
Sparse recovery



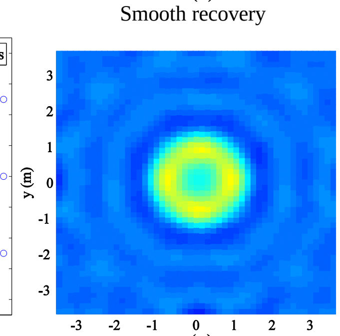
Relative norm error

Circular example:

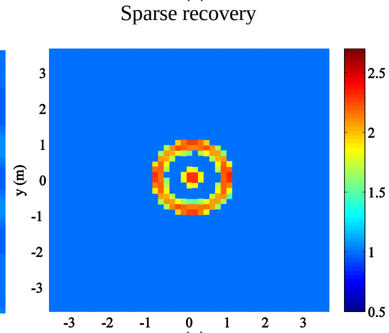
52 receivers, 12 transmitters, Sparseness level: 3.36%, $N = 2500$, Dimensions: 7.5m×7.5m, f = 125MHz. For smooth reconstruction, truncated Lnadweber is used, truncated at 50 iterations. TWIST iterations has been also truncated at 50 iterations within each Newton iterations. TWIST parameters are $\delta = 0.85$, $\gamma = 3$, $\sigma_m = 0.45$.



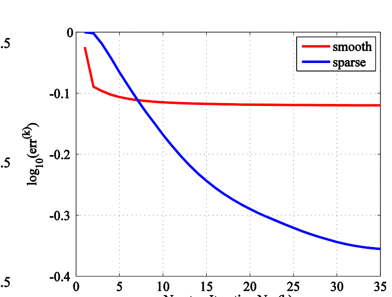
Circular scatterer



Smooth recovery



Sparse recovery



Relative norm error

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