On the DC Loop Modes in the Solution of the Time Domain Electric Field Integral Equation

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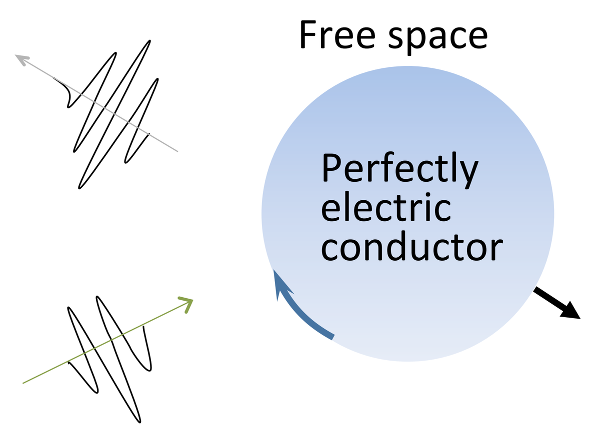
PROBLEM

When marching-on-in-time (MOT) method is applied to solve the time domain electric field integral equation, DC loop modes are always observed in the solution. DC loop modes, in theory, should not be observed in the MOT solution since they do not satisfy the relaxed initial conditions; and their appearance is attributed to numerical errors.

FORMULATION



ε_0 :	Permittivity	$\hat{\mathbf{n}}$:	Normal unit vector
μ_0 :	Permeability	$\mathbf{r}$:	Position vector
c :	Speed of light	t :	Time
$\mathbf{J}$:	Electric current	R :	Distance
$\mathbf{E}^i$:	Incident field	S :	Surface
$\mathbf{E}^s$:	Scattered field		

Total electric field $\mathbf{E}^i + \mathbf{E}^s$ satisfies the boundary condition on S , yielding the Time Domain Electric Field Integral Equation (TD-EFIE) [1]:

$$-\hat{\mathbf{n}}(\mathbf{r})\times\hat{\mathbf{n}}(\mathbf{r})\times\mathbf{E}^i(\mathbf{r},t) = \hat{\mathbf{n}}(\mathbf{r})\times\hat{\mathbf{n}}(\mathbf{r})\times\mathbf{E}^s(\mathbf{r},t)$$

$$= \hat{\mathbf{n}}(\mathbf{r})\times\hat{\mathbf{n}}(\mathbf{r})\times\left\{\frac{-\mu_0}{4\pi}\int_S \frac{ds'}{R}\frac{\partial \mathbf{J}(\mathbf{r}',t')}{\partial t'}\bigg|_{t'=t-R/c} + \frac{1}{4\pi\varepsilon_0}\nabla\int_S\frac{ds'}{R}\int_{-\infty}^{t-R/c} dt'\nabla'\cdot\mathbf{J}(\mathbf{r}',t')\right\}.$$

The marching-on-in-time (MOT) solution can be obtained:

$$\bar{\mathbf{Z}}^0\mathbf{J}^k = \mathbf{V}^k - \sum_{l=1}^{k-1}\bar{\mathbf{Z}}^{k-l}\mathbf{J}^l$$

where

$$\left[\bar{\mathbf{Z}}^l\right]_{mn} = \frac{-\mu_0}{4\pi}\int_S ds\mathbf{S}_m(\mathbf{r})\cdot\int_S\frac{ds'}{R}\mathbf{S}_n(\mathbf{r})\frac{dT_l(t')}{dt'}\bigg|_{t'=-R/c} - \frac{1}{4\pi\varepsilon_0}\int_S ds\left[\nabla\cdot\mathbf{S}_m(\mathbf{r})\right]\int_S\frac{ds'}{R}\left[\nabla'\cdot\mathbf{S}_n(\mathbf{r}')\right]\int_{-\infty}^{-R/c}T_l(t')dt'$$

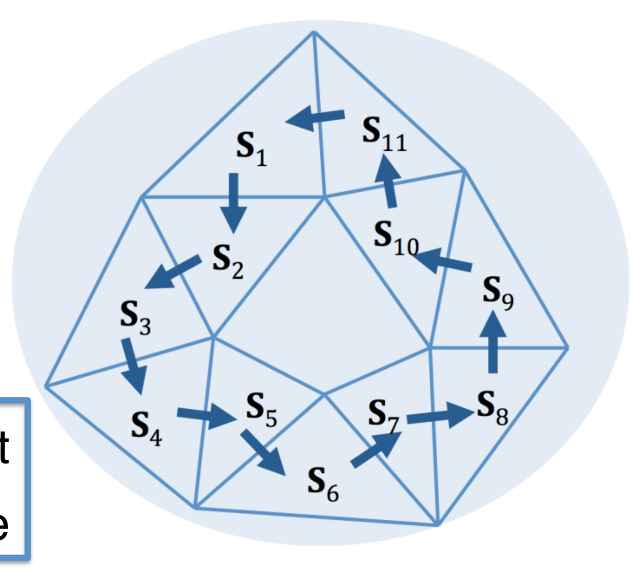
$\mathbf{S}(\mathbf{r})$:	RWG function [2]	$\mathbf{J}^k$:	coefficients of RWG at time step k
$T(t)$:	Temporal function	$\mathbf{V}^k$:	coefficients of tested $\mathbf{E}^i$ at time step k

DC LOOP MODES

DC loop modes belong to the null space of the discretized TD-EFIE [3].

Example : A loop is comprised of 11 RWGs

$$\mathbf{J}_0(\mathbf{r}',t') = \sum_{n=1}^{11}\sum_l a_n\mathbf{S}_n(\mathbf{r}')T_l(t').$$



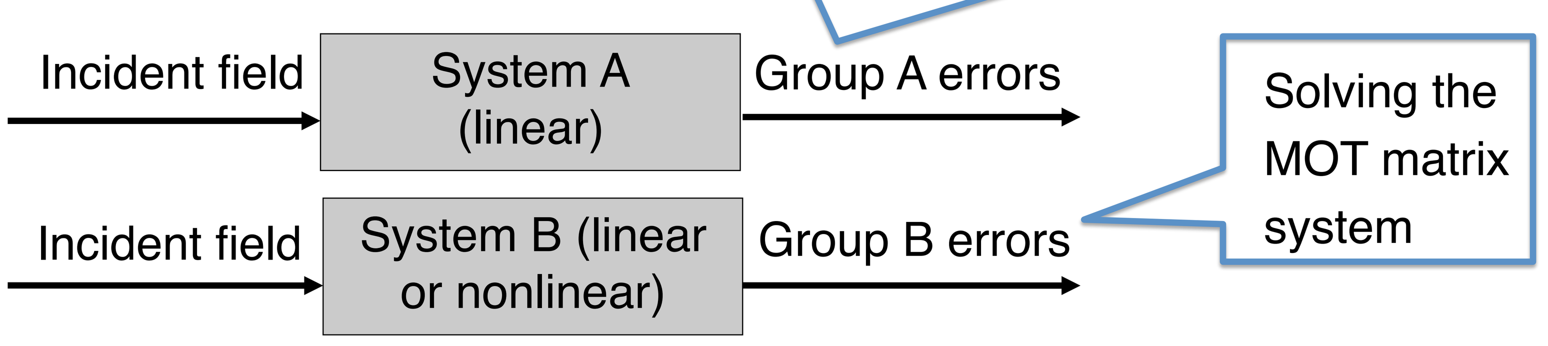
The divergence of RWG functions is a constant $a_1L_1 = a_2L_2 = \cdots = a_{11}L_{11}$, L_n : length of the edge

$$0 \Leftarrow -\frac{1}{4\pi\varepsilon_0}\int_S ds\left[\nabla\cdot\mathbf{S}_m(\mathbf{r})\right]\int_S\frac{ds'}{R}\sum_{n=1}^{11}a_n\left[\nabla'\cdot\mathbf{S}_n(\mathbf{r}')\right]\int_{-\infty}^{t^*-R/c}\sum_l T_l(t')dt'$$

$$0 \Leftarrow -\frac{\mu_0}{4\pi}\int_S ds\mathbf{S}_m(\mathbf{r})\cdot\int_S\frac{ds'}{R}\sum_{n=1}^{11}a_n\mathbf{S}_n(\mathbf{r}')\sum_l\frac{dT_l(t')}{dt'}\bigg|_{t'=t^*-R/c}$$

Temporal derivative of DC is 0

- Space-time discretization
- Sampling the incident field
- Evaluation of the surface integrations

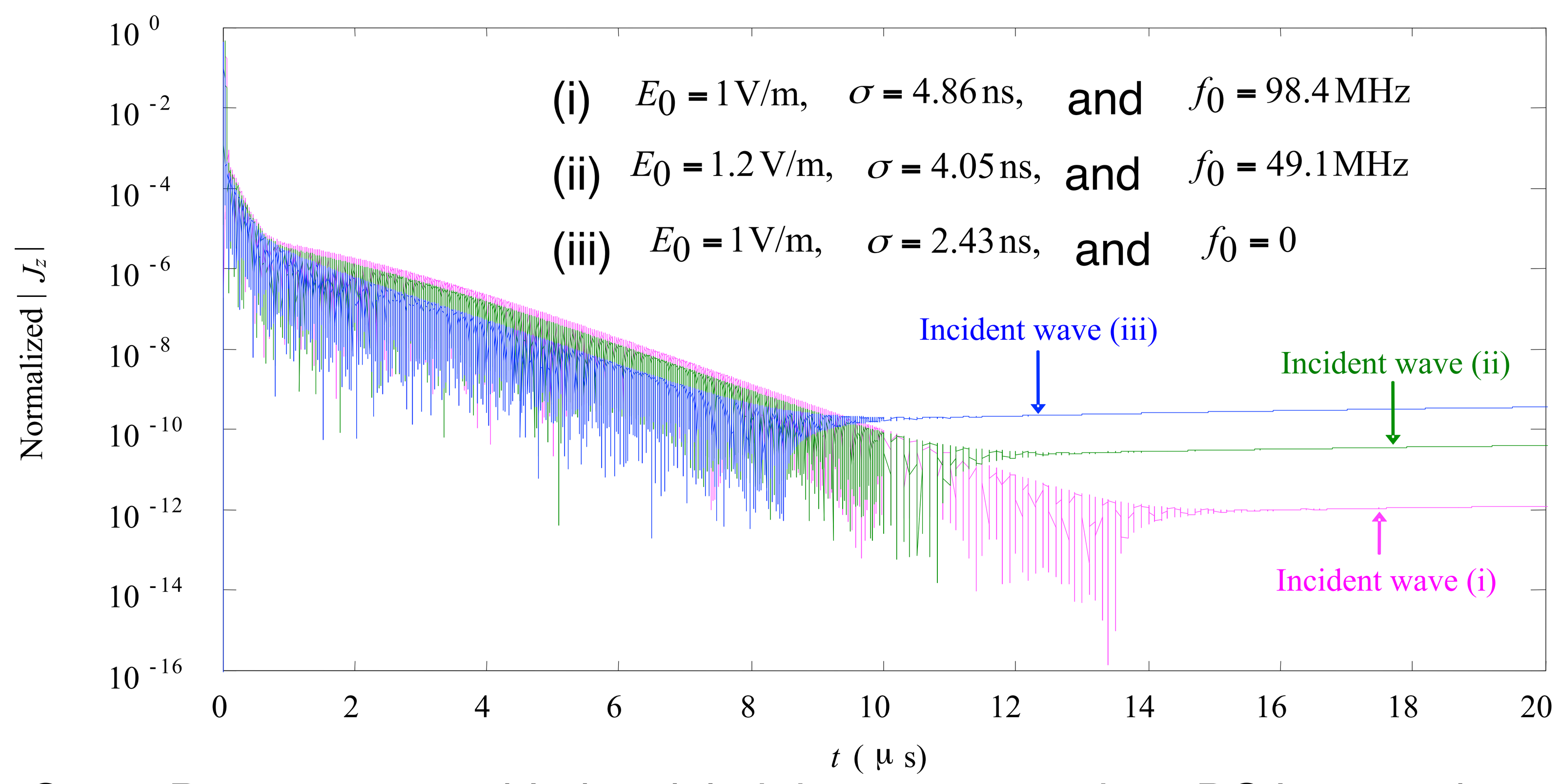


NUMERICAL RESULTS

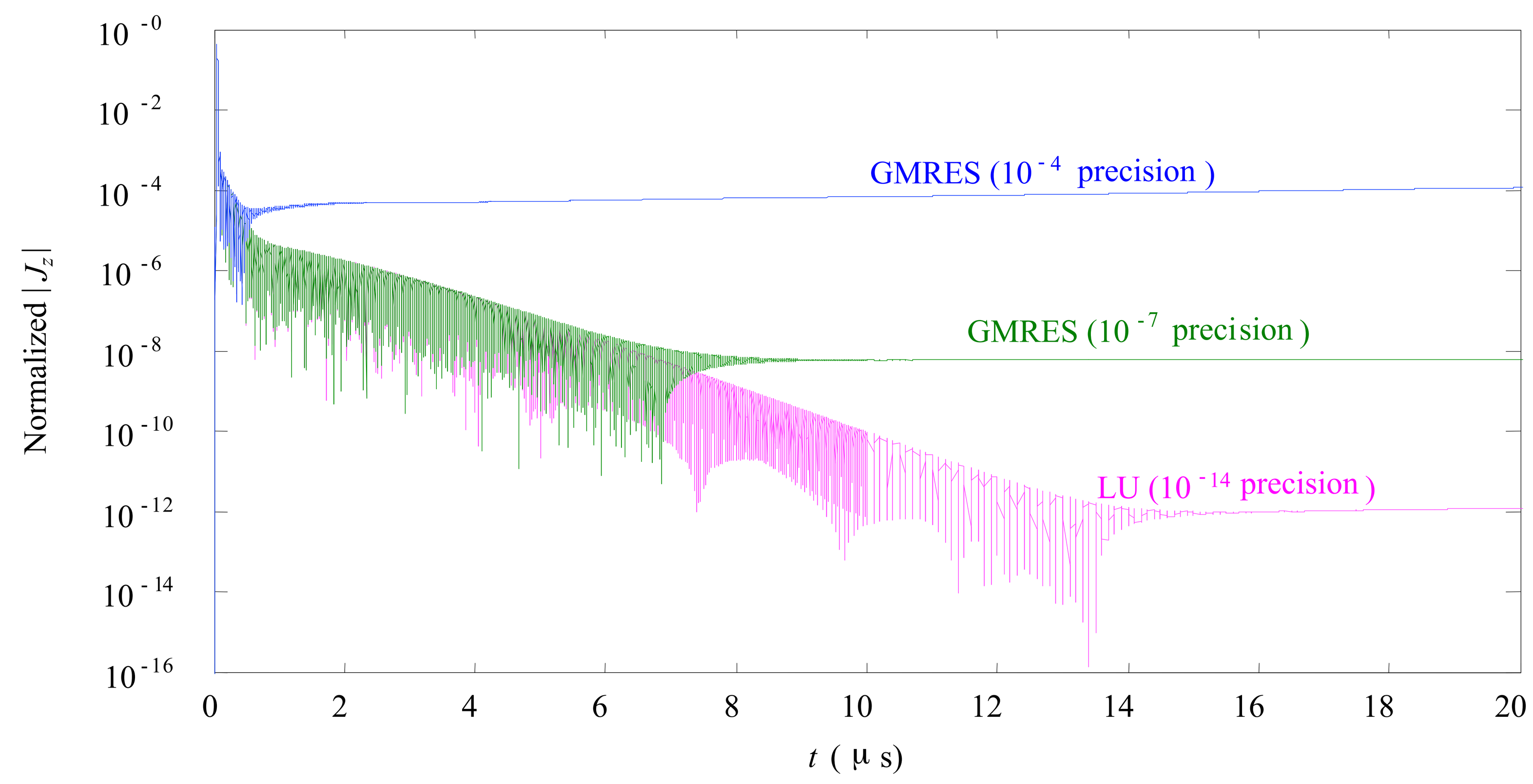
The scatterer is a PEC sphere with radius 1 meter and centered at the spatial origin. 963 RWG functions and linear Lagrange interpolation function are used with time step size 0.51 ns. The component of the current density at (1m, 0, 0) is recorded. The excitation is a plane wave propagating a modulated Gaussian pulse expressed:

$$\mathbf{E}^i(\mathbf{r},t) = \hat{\mathbf{x}}E_0G(t-\mathbf{r}\cdot\hat{\mathbf{z}}/c) \qquad G(t) = e^{-(t-10\sigma)^2/(2\sigma^2)}\cos[2\pi f_0(t-10\sigma)]$$

LU decomposition method is applied to solve the MOT matrix system. These results also demonstrate that Group A errors are linear with respect to the incident field.



Group B errors are enabled and their impact on spurious DC loop modes is demonstrated. Incident field (i) is used in three simulations



CONCLUSIONS

Errors due to space-time discretization carried out using RWG and Lagrange interpolation functions has zero projection onto DC loop modes. Numerical experiments demonstrate that errors due to the approximate solution of the MOT matrix system using an iterative solver have dominant impact on the generation of spurious DC loop modes in the MOT solution.

References:

1. S. M. Rao and D. R. Wilton, "Transient scattering by conducting surfaces of arbitrary shape," IEEE Trans. Antennas Propag., vol. 39, no. 1, pp. 56-61, Jan. 1991.
2. S. M. Rao, D. R. Wilton, and A. W. Glisson, "Electromagnetic scattering by surfaces of arbitrary shape," IEEE Trans. Antennas Propag., vol. 30, no. 3, pp. 409-418, May 1982.
3. Y. Shi, H. Bağcı, and M. Lu, "On the internal resonant modes in marching-on-in-time solution of the time domain electric field integral equation," IEEE Trans. Antennas Propag., vol. 61, no. 8, pp. 4389-4392, Aug. 2013.

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