Time Reversal Migration for Passive Sources Using a Maximum Variance Imaging Condition

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Summary

The conventional time-reversal imaging approach for micro-seismic or passive source location is based on focusing the back-propagated wavefields from each recorded trace in a source image. It suffers from strong background noise and limited acquisition aperture, which may create unexpected artifacts and cause error in the source location. To overcome such a problem, we propose a new imaging condition for microseismic imaging, which is based on comparing the amplitude variance in certain windows, and use it to suppress the artifacts as well as find the right location for passive sources. Instead of simply searching for the maximum energy point in the back-propagated wavefield, we calculate the amplitude variances over a window moving in both space and time axis to create a highly resolved passive event image. The variance operation has negligible cost compared with the forward/backward modeling operations, which reveals that the maximum variance imaging condition is efficient and effective. We test our approach numerically on a simple three-layer model and on a piece of the Marmousi model as well, both of which have shown reasonably good results.
Introduction

Hydraulic fracturing is a commonly used method in oil and gas extraction, especially in reservoirs with dense rock such as shale. Usually, water or other liquids are injected into the reservoir area to create high pressure conditions, which may crack the dense rock in order to make the oil and gas flow more freely. These induced cracks cause seismic events, which can be monitored in with passive sensors in the well or on the Earth surface. By monitoring these events, we could ultimately attain useful information of the deformation process, such as fracture development, reservoirs geomechanical conditions and so on (Maxwell et al., 2010; Kamei et al., 2015). We can usually locate these passive seismic sources, which in turn can help us monitor the fracturing process. Such information helps the engineers to optimize the injection strategy.

There are many ways to find the passive seismic sources. Some methods use both P-wave and S-wave traveltine picks. These ray based methods suffer in low-signal-to-noise ratio data causing errors in picking the arrivals, as well as in complex media, where the high frequency asymptotic approximation does not work. Other methods are based on migration techniques, such as time reversed migration. Such methods utilize not only the traveltine information, but also the whole recorded data. Usually, a good enough velocity model is assumed available, and the recorded traces are back-propagated into the model from their receiver locations as virtual sources. Then a proper imaging condition is applied to determine to the source. Many different imaging conditions exist, such as the maximum energy imaging condition (Artman et al., 2010) and the cross-correlation based imaging condition (Nakata and Beroza, 2016). The energy focus on the source location, with the help of a correct velocity model and the proper wave equation numerical solution. Since the migration based methods inject the recorded data simultaneously, one can improve the poor S/N by using more sensors. Thus, a dense and uniform acquisition system is usually necessary. Some others invert for the sources based on the full waveform inversion theory. Only an approximate starting velocity model is needed since the inversion is also responsible for estimating the velocity model by minimizing the data difference between the synthetic and the recorded ones (Sun et al., 2016). The sources spatial and temporal components can also be inverted by the adjoint state method (Kaderli et al., 2015; Wang, 2016). The FWI based methods provide good results, but the computational cost is still a big challenge for real cases.

In this abstract, we have developed a new imaging condition based on a migration technique for passive source imaging, which we refer to as the maximum variance imaging condition. The key idea is that the amplitude at the source is way bias from the average wavefield amplitude. We could utilize such a property to determine the source location and onset, as well as to suppress the noise elsewhere with a quite cheap cost.

Theory

Acoustic wave propagation in a constant density 2D medium obeys the following wave equation:

\[ \frac{1}{c^2(x,z)} \frac{\partial^2 u(x,z,t)}{\partial t^2} - \nabla^2 u(x,z,t) = f(x,z,t_0) \] (1)

where \( u(x,z,t) \) is the acoustic wavefield in 2D media described location wise by \( x \) laterally, and \( z \) in depth, \( c(x,z) \) is the velocity, and \( f(x_0,z_0,t_0) \) is the source term, which is ignited at \( (x_0,z_0) \) position and \( t_0 \) time. Time reversal can then be used to locate the seismic source. All the imaging conditions mentioned above require the information of source onset time, which is not available in the passive seismic monitoring process. So most of the migration based methods require wavefield scanning for the potential source onset time. In our method, instead of searching the whole back-propagated wavefield point by point, we define the seismic sources as the locations where the amplitude variance is the largest compared to neighboring grid points. In reverse time migration, the adjoint wavefield (back-propagated wavefield) of the surface recorded data is given by the following wave equation:

\[ \frac{1}{c^2(x,z)} \frac{\partial^2 \pi(x,z,t)}{\partial t^2} - \nabla^2 \pi(x,z,t) = d_{obs}(x_r,t) \] (2)
where $\pi(x,z,t)$ is the adjoint wavefield, $\overline{d_{obs}}$ is the time reversed surface recorded data injected at its receiver location as a virtual source. The maximum variance imaging condition can be formulated as:

$$I(x,z) = \text{var}_{(xw,zw)} \left[ IC \left( \sum_{i=1}^{N} \pi_i(x,z,t) \right) \right]$$

(3)

where $IC$ means applying the imaging condition and $\text{var}_{(xw,zw)}$ means calculating the variance at all the source image grid points along with the points around it in a proper window of size $(xw,zw)$. Considering $R_i$ as the back propagated wavefield corresponding to each recorded trace $i$, taking in mind that in practice, all the $N$ traces are injected at the same time to save computational cost, the variance calculation is nothing but a noise suppression process. We first determine the passive source locations and then utilize the variance technique to enhance the resolution of the source image. As shown in Figure 1, the window moves alone $x$ and $z$ axis in the source image, and at each point, the amplitude variance in the window is calculated and is placed at the center. It works because the amplitude difference (variance) in the window centered at the source location is much larger than any other differences of the windows. A more advanced maximum variance imaging condition can be described as:

$$I(x,z) = \text{var}_{(xw,zw, tw)} \left( \sum_{i=1}^{N} R_i(x,z,t) \right)$$

(4)

where the $\text{var}_{(xw,zw, tw)}$ means the variance calculation operation along all potential image points and the points around it in a window sized $(xw,zw, tw)$. Comparing Equation 4 with Equation 3, the difference is that instead of determining the source onset before the variance operation, we now determine the source onset by calculating the amplitude variance along all the spatial and temporal points. The approach utilizes the fact that at the true source location we obtain a large magnitude in the variance as a function of space and time. Through this imaging process, we no longer need to scan the wavefield to find the source location. The source location is evident by the variance map in space. In addition, the cost of variance calculation is negligible compared to the wavefield extrapolation.

**Numerical Examples**

We first test our imaging condition, provided by equation 3, on a simple three horizontal layer model. The velocity model is shown as Figure 2(a), which has 400 * 200 grid points with a 10m spatial interval. We place a single source at location $(x = 2 km, z = 1.2 km)$ and each grid point on the top surface works as a receiver to make sure that the acquisition system is dense enough. We have tested the method with both clean and noisy data. Figure 2(b) shows the observed data without any noise and Figure 2(c) shows the data with strong Gaussian distributed noise added to it. We consider a variance calculation window of 5 * 5 samples, which is mainly corresponds to the dominant wavelength. A Ricker wavelet of 20Hz peak frequency is injected as a source. The true velocity is assumed known. The results of our maximum variance imaging condition with both clear and noisy data are shown in Figure 3(a) and Figure 3(b), respectively. As we can see, the maximum variance imaging condition can image the source location very well with a quite satisfying resolution. On the other hand, the conventional max-energy imaging condition, which are shown in Figure 3(c) and Figure 3(d), cannot focus all the energy to the right location even if the true model is used for back propagation. This is mainly caused by the limited acquisition aperture. If the receivers are placed on all four boundaries of the model, we may obtain more focused source images. In our approach, although the acquisition system and the back-
Figure 2 (a) A three horizontal layer velocity model and the corresponded recorded data (b) without noise and (c) with strong random noise.

propagation are the same, all the unfocused energy are suppressed to nearly zero and only the energy at the source location is left. Then we test our second imaging condition, which is shown in Equation 4, on

Figure 3 Maximum variance imaging condition using (a) noise free data and (b) noisy data. And conventional maximum energy imaging condition with (a) noise free data and (b) noisy data.

a small piece of the Marmousi model (shown in Figure 4(a)). Three sources are triggered at locations of \((x = 1.5km, z = 1.1km), (x = 2km, z = 1.2km), (x = 2.5km, z = 1.3km)\). The model has 400 \(\times\) 200 grid points with a 10m spatial interval. Each grid point on the top surface acts as a receiver. Both clean and noisy data are generated and used. The window size of this experiment is chosen as 5 \(\times\) 5 \(\times\) 200 samples in space and time. The three sources have wavelets that are the same given by a Ricker wavelet with a dominant frequency of 20Hz. Instead of the true velocity model, we now assume only a smoothed model is given, which is shown in Figure 4(b). The results of our approach are shown in Figure 5(a) and Figure 5(b), without and with random noise. We also show the results of the conventional max-energy imaging condition in Figure 5(d) and the cross-correlation based methods in Figure 5(c). There are artifacts in Figure 5(d), which are caused by the noisy data and the tradeoff between different sources. The artifacts practically disappeared in the other three figures. It means the conventional max-energy imaging condition is more sensitive to noise compared with the other two imaging conditions. We

Figure 4 (a) A piece of the Marmousi model and (b) the given smoothed model for back propagating.
can easily draw the conclusion that the imaging results of our maximum variance imaging condition II and the cross-correlation based methods are much better than the conventional max-energy imaging condition. Although the imaging results of the maximum variance and cross-correlation look similar, the computational cost is different. The maximum variance imaging condition requires only one modeling (back-propagate the recorded data), while the cross-correlation based methods require several separated modeling (back inject data as groups), which will certainly take more computing time. Thus, by getting similar results, our approach is much cheaper than the cross-correlation based ones.

Conclusions

We develop an approach to image the source location based on calculating the amplitude difference between grid points, which is the variance of amplitude. It not only suppresses the artifacts caused by background noise or imperfect acquisition system, but it also automatically determines the source ignition time. It is able to deal with the multi-source situation without tradeoff between different sources. We have tested our approach on a simple three layer model and on a piece of the Marmousi model. All the experiments show satisfying results with high resolution, which reveal the effectiveness.

References


