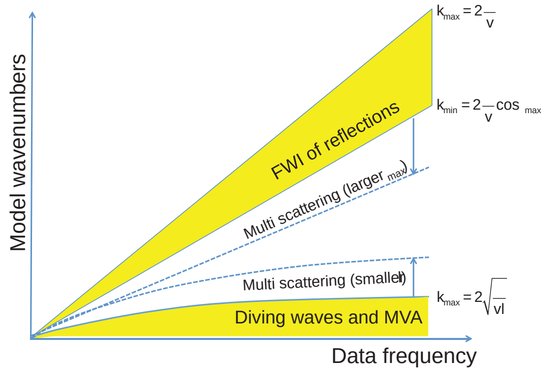

Figure 1 *A diagram showing the range of wavenumbers extracted from the data as a function of dat frequencies for a number of approaches, like reflection FWI, diving wave FWI, and MVA. The variables* $_{max}$ *and l, correspond to the maximum scattering angle for reflections and the wavepath length of transmissions, respectively, while is the angular frequency and v is the velocty at a point in the model.*

The case for sparse frequencies

This was reasonably demonstrated by Sirgue and Pratt (2004); with multi dimensional acquisition only a discrete number of frequencies are needed to obtain gradients with a model wavenumber spectrum necessary to develop a plausible Earth model. Having a fine sampling of frequencies inherently embedded in our conventionally acquired data does not increase our ability to access more information. Its role is to enhance the signal-to-noise ratio of our resolving power. Thus, in frequency domain implementations, we tend to group frequencies together to achieve the effect of enhancing the signal-to-noise ratio for better gradients. This concept is helped by the fact that our typical model of the Earth consists of (or could be represented by) long wavelength changes and very short wavelength interfaces. A frequency that produces wavelengths that are in between these two extremes (the background and the interface) can be used to resolve both the long wavelength information (from the geometrical ray embedded features of the wavefield) and the short wavelength information from the reflections in the data. Actually, the Helmholtz wave equation directly demonstrates that we can explicitly determine the velocity from the knowledge of the wavefield, $u(\mathbf{x}, \ )$, at any frequency, off course, at the resolution of that frequency. The upper bound in the model wavenumber for any frequency is given by Huygen's resolution limit ($k_{max} = 2\frac{}{v(\mathbf{x})}$). The lower bound, actually, extends to zero wavenumber granted we have transmissions (direct, diving, or reflection wave paths). In the absence of classical transmission energy such extensions may be provided by multi scattering. Focusing on reflectivity, we may also access low wavenumbers from Born scattering based inversion of wavepaths (RWI).

Multi scattering

The aforementioned and familiar handicap in FWI in the single scattering regime is in accessing the low and mainly intermediate model wavenumber information embedded in the data. Some low wavenumbers may be provided by diving waves to some limited depth. So we end up utilizing geometrical based methods like tomography and MVA to obtain the low wavenumber information, and hopefully some of the middle wavenumber components. Even with strictly zero-offset data of a single frequency, single scattered energy generally provides velocity model wavelengths confined by the frequency resolution limit ($k_{\text{max}} = 2\frac{}{v}$). On the other hand, in multi scattering, and depending on the location of the scattering, we may obtain longer wavelength updates. Figure 2a shows a piece of the SEG Sigsbee model that contains a canyon within the Salt body with potentially strong scattering off of the salt. The plotted ray paths of doubly scattered energy show the range of scattering angles possible even for a presumed zero-offset acquisition. Based on equation 1, the range of model wavenumbers resulting from the additional scattering angles can push the scattering resolution limit to fill the gap from above (Figure 1). Actually, the inverted perturbation image from triple scattering (Figure 3b), corresponding to conventional acquisition 6 Hz frequency data, is better focussed than the single-scattering case (Figure 3a). This is a direct result of the ability of triple scattering, through the additional scattering angles, to access more wavenumbers (especially low ones) that makes the image more compact. I will show that explicitly later.

Also, through multi scattering, we obtain higher resolution in the wavepath (banana shaped) updates (Figure 2b). As mentioned earlier, the resolution is inversely proportional to the length of the wavepath. Multi scattering offers such high resolution short wave path updates to the inversion process, and thus, helps us fill the model wavenumber gap from below (Figure 1). Even though multi scattering energy may have low amplitude in the data, the update, if properly scaled, will not suffer from such weak energy. Multi scattering, thus, enhances our ability to access more of the model wavenumbers hidden in the data at both ends (high and low).

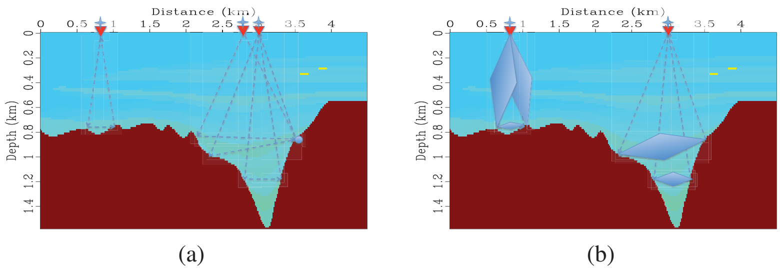

Figure 2 *A diagram of the potential wavepaths for zero-offset acquisition of doubly scattered energy in a piece of the SEG Sigsbee model, for updates focussed on the scattering, perturbation, part (a), and for the wavepath part (b). The dot along the Salt surface in a) points out where multi scattering angles are highlighted.*

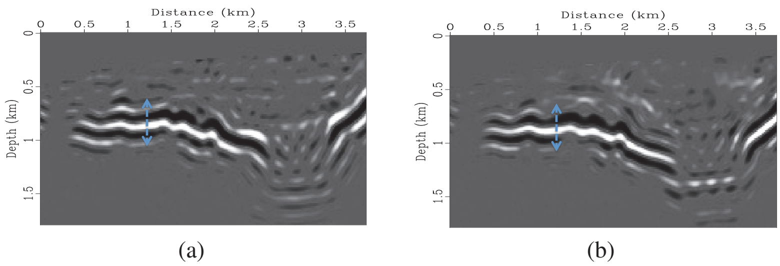

Figure 3 *The optimized images from a single frequency data corresponding to a) single scattering, and b) triple scattering.*

Analysis of the mutiscattering gradient resolution

Scattering angle filtering allows us to extract energy from the gradient corresponding to certain scattering angles, translating to particular resolution (wavenumber) components in the model (Alkhalifah, 2015). We will use the scattering angle filter to verify whether the multi scattered image (Figure 3b) provides additional wavenumber components over the single-scattering image (Figure 3a). If we isolate energy in the single-scattering optimized gradient (Figure 3a) corresponding to scattering angles higher than 160 and 178 degrees, we obtain the filtered versions shown in Figures 4a and 4b, respectively. While the 160 degree gradient shows features consistent with the general trend of the true model (Figure 2a) at a longer wavelength, the 178 degree gradient (at even a longer wavelength) looks erroneous. This reflects the fact that the single-scattering gradient does not include such wavelengths (i.e. low wavenumbers). On the other hand, the triple scattered optimized gradient (Figure 3b), given by Figures 4c and 4d, for 160 and 178 degrees scattering angle filter, respectively, has the general trend of both consistent with the true model.

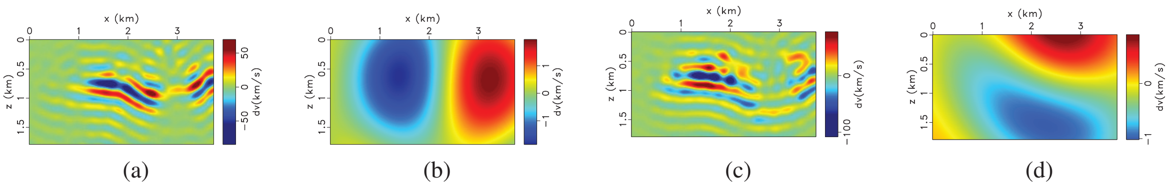

Figure 4 *The optimized perturbation gradient in Figure 3a for single scattering filtered to allow only scattering angles above a) 160, and b) 178 degrees to survive. The same filter applied to the triple scattering gradient (Figure 3b) for c) 160, and d) 178 degrees.*

Acquiring multi scattered energy

The biggest challenge in utilizing multi scattered energy is in our ability to record them. In many cases, doubly scattered waves tend to be an order magnitude lower than its single scattering counterpart. Even if we manage to fully focus such energy to its scattering positions, our limited recording surface guarantees that a big portion of the double (or multi scattered) energy will never be recorded or recovered. One obvious option to overcome this limitation is to inject more energy inside the Earth that allows us to recover more multi-bounce waves with conventional aperture acquisition. Obviously, this maybe provided by high-energy sources and having a lot of them. This leads me to promote the possibility of a seismic acquisition that focuses on increasing the amount of energy injected into the Earth for a limited set of frequencies. If a recorded event is buried in noise, it does not mean it was not recorded. It, however, would be harder to recover it from the noise, unless, enough of these recorded scattered energy can be focused back to their scattering point. If that happens, we can also map the wavepaths, and thus, achieve wavepath updates. In this case, we can optimize the update to fit the data for multi scattering, and that is equivalent to imposing a full Hessian on the update (Alkhalifah and Wu, 2016). Such an acquisition can be accomplished by running the vibrator at a single frequency for the duration of the sweep. Having many of those vibrators running simultaneously, at different frequencies (probably 5 or 7), the energy for the individual sources can be easily deblended if the sweep is long enough. This is a simple property of the Fourier decomposition. A correlation of the sweep signal of every individual frequency with the recorded data, followed by a Fourier transform of the single frequency isolates the energy corresponding to that frequency (Alkhalifah, 2014). The point here is that some frequencies are somewhat redundant (and possibly useless). If our assumption of the Earth is given by smooth variations bordered by abrupt changes, the few sparse frequencies that fall in between these two variations provides most of the information needed to build a velocity model.

Conclusion

Building a high resolution model of the Earth, represented by a broad spectrum of model wavenumbers needed to achieve such resolution, requires a discrete number of acquired data frequencies, reasonable offsets (not necessarily fine or regular), and the inclusion of multi-scattering in the update. This fact leads me to promote an acquisition and inversion strategy that I will explain in more detail in the presentation and show additional results.

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