High-Resolution Fracture Characterization Using Elastic Full-Waveform Inversion

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Summary

Current methodologies to characterize fractures at the reservoir scale have serious limitations in spatial resolution. Here, we propose to estimate both the spatial distribution and physical properties of fractures using full waveform inversion (FWI) of multicomponent surface seismic data. An effective orthorhombic medium with five clusters of vertical fractures distributed in a checkboard fashion is used to test the algorithm. To better understand the inversion results, we analyze the FWI radiation patterns of the fracture weaknesses. A shape regularization term is added to the objective function to improve the inversion for the horizontal weakness, which is otherwise poorly constrained. Alternatively, a simplified model of penny-shaped cracks is used to reduce the nonuniqueness in the inverted weaknesses and achieve a faster convergence.
Introduction

The flow and transportation of fluids in the subsurface are controlled by fractures, which are of primary importance for oil and gas exploration and production. The estimation of fracture properties from geophysical measurements is of significant interest for the seismic exploration community. In theory, these properties can be estimated from parameters of the fracture-induced azimuthally anisotropic effective medium. Bakulin et al. (2000a) provide a comprehensive analysis of the anisotropic seismic signatures associated with vertical fractures. They show how to estimate physical properties such as the normal and tangential compliances of the fractures from multicomponent data. The compliances can then be used to infer fracture density and infill, as also discussed by Tsvankin and Grechka (2011). However, existing methods of fracture detection are mostly based on reflection traveltimes and/or amplitudes (AVO analysis). Here we show that waveform information in the recorded data can improve estimates of both the spatial distribution and physical properties of fractures.

In general, full-waveform inversion (FWI) aims to employ entire waveforms of diving and/or reflected waves to reconstruct the subsurface structure and velocity field. Masmoudi et al. (2016) analyzed the feasibility of high-resolution fracture characterization using a waveform-inversion method based on an acoustic approximation for orthorhombic media. To simulate wave propagation in a realistic fractured medium (e.g., fractured shale), we consider here an effective elastic orthorhombic medium formed by parallel vertical fractures embedded in a transversely isotropic background with a vertical symmetry axis (VTI). It is one of the most common physical models for fractured reservoirs (Bakulin et al., 2000b). The effective compliance tensor is obtained from linear-slip theory (Schoenberg, 1980), and the elastic wave equation is solved by a staggered finite-difference method. In the linear-slip theory, fractures are described by three dimensionless parameters, one normal (△N) and two tangential (△V and △H) weaknesses (normalized compliances). These parameters are much smaller than unity, which allows us to linearize the modeling procedure. An iterative local optimization method can be applied to update the model.

Here, we describe the method and test it on an effective orthorhombic medium with five clusters of vertical fractures distributed in a checkboard fashion. One cluster is assumed to be a sweet spot with relatively large tangential weaknesses. Initially, we only invert for the fracture weaknesses with a known background VTI model.

Theory

We have devised a multiparameter elastic FWI algorithm to estimate the normal and tangential fracture weaknesses. The objective function contains a standard data misfit term and a shape regularization term (Gallardo and Meju, 2003):

\[ J(\mathbf{m}) = \| \mathbf{W}_d (\mathbf{d}^{\text{pre}} - \mathbf{d}^{\text{obs}}) \|^2 + \alpha \| \nabla \mathbf{m} \triangle \nabla \times \nabla \mathbf{m} \triangle \mathbf{H} \|^2, \]  

(1)

where \( \mathbf{d}^{\text{pre}} \) and \( \mathbf{d}^{\text{obs}} \) are the vectors of the multicomponent predicted and observed data, \( \mathbf{W}_d \) is a weighting operator (\( \mathbf{W}_d = \sigma_d \mathbf{I} \)), and \( \sigma_d \) is the standard deviation of the predicted data. The shape regularization term imposes a consistent spatial behavior on the inverted vertical (\( \triangle \nabla \)) and horizontal (\( \triangle \mathbf{H} \)) tangential weaknesses. The parameter \( \alpha \) controls the level of similarity between these two parameters. As shown previously (Bakulin et al., 2000b; Masmoudi et al., 2016) and follows from our analysis of the radiation patterns of the weaknesses, \( \triangle \nabla \) is expected to be better recovered than \( \triangle \mathbf{H} \) from the data misfit term. The shape regularization term can help improve the accuracy of the inverted \( \triangle \mathbf{H} \). The elastic wave equation can be written as:

\[
\left( \rho \mathbf{I}_3 \begin{array}{cc} 0 & 0 \\ 0 & \mathbf{c}^{-1} \end{array} \right) \frac{\partial \Psi}{\partial t} - \begin{pmatrix} 0 & E^T \\ E & 0 \end{pmatrix} \Psi - \mathbf{s} = 0,
\]  

(2)

where \( \mathbf{I}_3 \) is a 3×3 identity matrix, \( \mathbf{c} \) is the invertible stiffness matrix, \( \Psi = (\mathbf{v}, \mathbf{a})^T \) is the wavefield vector that includes the particle velocity and stress fields, \( E \) is the differential matrix, and "\( T \)" denotes a transpose.
According to the linear-slip theory (Schoenberg, 1980), the effective compliance tensor of a fractured medium can be obtained by adding the fracture compliances to that of the background:
\[
c = (s_b + s_f)^{-1} = [(I + s_f s_b^{-1}) s_b]^{-1} = c_b [I + s_f c_b]^{-1},
\]
Equation 3 can be expanded in a Taylor series if all elements of \(s_f c_b\) are much smaller than unity, which is the case if the weaknesses are sufficiently small. Then equation 3 can be approximated by retaining only linear terms:
\[
c \approx c_b - c_b s_f c_b = c_b - \delta c.
\]
We assume a set of parallel fractures with the normal in the \(x_1\)-direction embedded in a VTI medium. The linearized fracture compliance matrix and the corresponding effective stiffness tensor are given by (Bakulin et al., 2000b):
\[
s_f^{lin} = \begin{bmatrix}
\frac{\Delta N}{c_{11b}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{\Delta V}{c_{55b}} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{\Delta H}{c_{66b}}
\end{bmatrix}, \quad c_f^{lin} = c_b - \begin{bmatrix}
\frac{\Delta N c_{11b}}{c_{11b}} & \frac{\Delta N c_{12b}}{c_{12b}} & \frac{\Delta N c_{13b}}{c_{13b}} & 0 & 0 & 0 \\
\frac{\Delta N c_{12b}}{c_{11b}} & \frac{\Delta N c_{13b}}{c_{12b}} & \frac{\Delta N c_{13b}}{c_{13b}} & 0 & 0 & 0 \\
\frac{\Delta N c_{13b}}{c_{11b}} & \frac{\Delta N c_{13b}}{c_{12b}} & \frac{\Delta N c_{13b}}{c_{13b}} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{\Delta V c_{55b}}{c_{55b}} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{\Delta V c_{55b}}{c_{66b}} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{\Delta H c_{66b}}{c_{66b}}
\end{bmatrix}.
\]
For more general cases of multiple fracture sets and rotated fracture planes (i.e., not perpendicular to the \(x_1\)-direction), the effective stiffness tensor can be obtained using summation over the additional fracture compliances and Bond rotation.

**Examples**

We consider part of the 3D SEAM VTI model as the background medium. Five clusters of vertical fractures are embedded in the VTI background. One fracture set, which has relatively large tangential compliances (due to the higher crack density), is assumed to be a potential sweet spot. The staggered finite-difference method is implemented to solve equation 2. There are 24 inlines and crosslines evenly covering the surface, each with 24 explosive sources and 48 receivers placed at constant increments. The recorded data are three-component (3C) particle velocities. To illustrate the distributions of fractures, the actual and initial \(C_{11}\)-models are shown in Figure 1. An assumed sweet spot, which has a relatively large crack density, is located in the center of the model (Figure 2). Four more regions of fractures are used to test the resolution of the proposed method. The initial weaknesses \(\Delta N, \Delta V, \text{and } \Delta H\) are set to zero throughout the model.

**Figure 1** Actual (a) and initial (b) stiffness \(C_{11}\). The shaded blocks (green) mark the fractured areas.

Four frequency bands, 0-7 Hz, 0-10 Hz, 0-15 Hz, and 0-21 Hz, are used in a multistage inversion. The inverted weaknesses \(\Delta N, \Delta V, \text{and } \Delta H\) are shown in Figure 3. The assumed sweet spot, which has larger tangential weaknesses, can be reliably identified. However, \(\Delta N\) and \(\Delta H\) are not sufficiently well resolved in this test mainly because these two parameters mostly influence the far-offset data, which were missing in our acquisition. Vertical profiles across the assumed sweet spot and horizontal profiles...
Across two other fracture sets are shown in Figures 4 and 5, respectively. As expected, fractures located near the edge of the model are inverted with a lower resolution compared to those close to the center.

**Figure 3** Inverted $\triangle_N$ (left), $\triangle_V$ (middle), and $\triangle_H$ (right). The sweet spot near the center of the model is successfully resolved.

**Figure 4** Vertical profiles of $\triangle_N$ (left), $\triangle_V$ (middle), and $\triangle_H$ (right) at point $(x = 0.5 \text{ km}, y = 0.5 \text{ km})$. Cyan: actual model; Pink: initial model; Blue: inverted model.

**Figure 5** Horizontal profiles of $\triangle_N$ (left), $\triangle_V$ (middle), and $\triangle_H$ (right) at point $(z = 0.47 \text{ km}, y = 0.7 \text{ km})$. Cyan: actual model; Pink: initial model; Blue: inverted model.

**Figure 6** Azimuthally varying radiation patterns of the incident P-wave and reflected P-wave (top) / SH-wave (bottom) for the fracture weaknesses. The radial direction indicates the opening angle. The P-SV wave (not shown) has a behavior similar to that of P-P wave.

To better understand the inversion results, Figure 6 shows the radiation scattering patterns of the fracture weaknesses (Oh and Alkhalifah, 2016). The parameters $\triangle_N$ and $\triangle_H$ are more sensitive to large opening angles and, therefore, need large offset-to-depth ratios to be resolved. The weakness $\triangle_V$ is more sensitive to medium opening angles, so it can be inverted with a high resolution from conventional-spread...
data. Since only one tangential weakness can be resolved with surface acquisition, next we assume the model of penny-shaped cracks, which have the same vertical and horizontal tangential weaknesses ($\Delta_V = \Delta_H$). As illustrated by the estimated weaknesses in Figure 7, the inversion for penny-shaped cracks provides a more accurate normal weakness (further improvement in $\Delta_N$ can be achieved by including larger offsets) and a well-resolved tangential weakness. Using the simpler crack model provides comparable rate of convergence (Figure 8) in this limited offset example.

**Conclusions**

We proposed a new approach to estimate the spatial distribution and physical properties of fractures by waveform inversion of multicomponent surface data. Due to the high-resolution potential of elastic full waveform inversion involved in the scattering components of the data, the developed algorithm can recover the spatial fracture distribution and identify localized "sweet spots" of intense fracturing. The numerical example also shows that the vertical tangential weakness $\Delta_V$ is better resolved than the other two weaknesses. The radiation patterns indicate that reliable estimation of the normal weakness $\Delta_N$ requires a relatively large offset-to-depth ratio. The weakness $\Delta_H$ cannot be obtained with sufficient accuracy, which is also confirmed by previous studies. A shape-regularization term is added to the objective function to improve the quality of the inverted parameter $\Delta_H$. Alternatively, the simplified model of penny-shaped cracks, often employed in practice, yields good inversion results and comparable convergence for this limited offset data. The proposed method has the same limitations as more conventional FWI algorithms: it requires a good background model and sufficient subsurface illumination.

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**References**


