On the Optimality of Repetition Coding among Rate-1 DC-offset STBCs for MIMO Optical Wireless Communications

Yerzhan Sapenov, Anas Chaaban, Zouheir Rezki, and Mohamed-Slim Alouini

Abstract

In this paper, an optical wireless multiple-input multiple-output communication system employing intensity-modulation direct-detection is considered. The performance of direct current offset space-time block codes (DC-STBC) is studied in terms of pairwise error probability (PEP). It is shown that among the class of DC-STBCs, the worst case PEP corresponding to the minimum distance between two codewords is minimized by repetition coding (RC), under both electrical and optical individual power constraints. It follows that among all DC-STBCs, RC is optimal in terms of worst-case PEP for static channels and also for varying channels under any turbulence statistics. This result agrees with previously published numerical results showing the superiority of RC in such systems. It also agrees with previously published analytic results on this topic under log-normal turbulence and further extends it to arbitrary turbulence statistics. This shows the redundancy of the time-dimension of the DC-STBC in this system. This result is further extended to sum power constraints with static and turbulent channels, where it is also shown that the time dimension is redundant, and the optimal DC-STBC has a spatial beamforming structure. Numerical results are provided to demonstrate the difference in performance for systems with different numbers of receiving apertures and different throughput.

Index Terms

Intensity-modulation direct-detection, optical wireless communications, repetition coding, space-time block codes, maximum likelihood detection.

Y. Sapenov, A. Chaaban, and M.-S. Alouini are with the Computer, Electrical, and Mathematical Sciences and Engineering Division, King Abdullah University of Science and Technology (KAUST), Thuwal, Saudi Arabia. Email: {yerzhan.sapenov,anas.chaaban,slim.alouini}@kaust.edu.sa.

Z. Rezki is with the Department of Electrical and Computer Engineering, University of Idaho, Moscow, ID 83844-1023, USA. Email: zrezki@uidaho.edu.

I. Introduction

The continuous increase in the demand for higher data rates stresses current radio frequency (RF) wireless communication systems and requires new solutions for coping with this increase. One such solution is optical wireless communications (OWC), in which light is used to convey information between communicating nodes. OWC enjoys many attractive features including free broadband spectrum, enhanced security, and reduced electromagnetic interference, and therefore, it has witnessed increasing research attention from academic and industrial worlds (see [1], [2] and references therein). Furthermore, OWC operating in the visible-light spectrum, known as visible-light communications (VLC) [3], [4], allows simultaneous communication and illumination leading to increased energy efficiency. These features make OWC a promising solution for future wireless communication systems, especially in its simple and practical intensity-modulated direct-detection (IM/DD) form.

Recent studies on IM/DD OWC focus on various performance criteria including achievable rates [5], outage probability [6], and error rates [7]–[10], for instance. Diversity is one of the main performance criteria for reliable wireless communication [11], which can be achieved by exploiting channel variation in space, time, frequency, or combinations thereof. Space and time diversity can be exploited by proper design of space-time block codes (STBC) [12]. It is known that orthogonal STBC (OSTBC) are optimal in RF communications in terms of diversity gain [13]. With this in mind, the following question arises: Does this also hold true in IM/DD OWC?

The answer to this question can be obtained by analyzing the error probability of a given coding scheme for IM/DD OWC. This question has been discussed in [8] for instance, where it was demonstrated numerically that simple spatial repetition coding (RC)[1] interestingly outperforms OSTBC in multiple input single output (MISO) IM/DD OWC systems with an individual (per-aperture) power constraint. Note that RC can be considered as a special case of STBC, where the temporal dimension is ignored and the transmit symbol is repeated spatially. Thus, it is interesting to investigate whether RC remains optimal when the temporal dimension is also exploited in a more general STBC. In this paper, we aim to address this problem analytically, by studying the pairwise error probability (PEP) of the class of rate-1 direct current (DC) offset STBC[2] in an $N_t \times N_r$ multiple input multiple output (MIMO) IM/DD system subject to a power constraint.

[1]Throughout the paper, we refer to spatial RC simply as RC.

[2]Throughout the paper, rate-1 codes are codes, whos transmit rate is 1 symbol per channel use.

We focus on the worst case PEP, corresponding to the minimum distance between any distinct codewords, since (i) it dictates the diversity order, and (ii) it can be used to obtain an upper bound and also an approximation of the actual error probability [14], [15].

The contributions of the paper can be summarized as follows. We prove that the worst case PEP is minimized by RC among all DC-offset STBC (DC-STBC) subject to an individual power constraint. The proof is obtained by deriving an upper bound on the minimum distance of an arbitrary DC-STBC, and then showing that this upper bound is indeed achievable by RC. The proof holds for both electrical and optical power constraints. This result is interesting because RC has lower encoding/decoding complexity than more general STBCs. This proves the redundancy of the temporal dimension in DC-STBC in this case. We also extend this result to systems with sum power constraints, where the time dimension is also redundant, and the optimal scheme is spatial beamforming.

This result holds under a given static channel state for some block length, which is of particular interest in indoors VLC where turbulence is absent. This result also holds under any turbulence distribution for multi block transmission, which is of interest for outdoors free-space optical transmission. Note that this result is consistent with [9], [10], which prove the diversity-optimality of RC, among all *space codes*, for $N_t \times N_r$ MIMO system with log-normal distributed channels and a total power constraints. The main difference with our work here is that we do not restrict our analysis to specific turbulence statistics, and that we also consider the temporal dimension in combination with the spatial one, leading to a more general result. Also this result is consistent with [16], which proves the PEP-optimality of RC at high signal-to-noise ratio (SNR) in Gamma-Gamma distributed channels for a system with 2 transmit and N receive apertures and an individual power constraint. The main difference with our work is that we do not restrict the analysis to specific turbulence statistics, and we consider a general MIMO system with an arbitrary amount of transmit apertures. We also do not focus on diversity only, but rather on the worst-case PEP. Note that the worst-case PEP is the decisive factor which dictates the diversity order of the system, and hence worst-case-PEP optimality implies diversity optimality.

The rest of the paper is organized as follows. In Sec. II we introduce the system model and formulate the problem. The summary of main results is provided in Sec. III. The analysis of the problems with individual constraints and their solutions are discussed in Sec. IV. Extensions to sum power constraints and turbulence scenarios are discussed in Sec. V and VI respectively. Then, in Sec. VII, we provide numerical simulations for different VLC scenarios. Finally, we conclude in Sec. VIII.

Notations: Throughout the paper, we use bold lower-case and upper-case letters to denote vectors and matrices, respectively. We write $\mathbf{x}^T$ and $\|\mathbf{x}\|_p$ to denote the transpose and the ℓ_p-norm of $\mathbf{x}$. We also use $\|\mathbf{X}\|_F$ to denote the Frobenius norm of $\mathbf{X}$. We denote the the $L \times L$ identity matrix, the $M \times N$ all ones matrix, and the $M \times N$ all zeros matrix by $\mathbf{I}_L$, $\mathbf{1}_{M\times N}$, and $\mathbf{0}_{M\times N}$, respectively, and we use $\otimes$ to denote the Kronecker product.

II. System Model and Problem Formulation

A. Channel Model

Consider an IM/DD OWC system consisting of N_t transmit apertures and N_r receive aperture as shown in Fig. 1. The transmission model in time instant t can be represented as

$$\mathbf{y}(t) = \mathbf{H}\mathbf{s}(t) + \mathbf{n}(t), \tag{1}$$

where $\mathbf{s}(t) \in \mathbb{R}_+^{N_t\times 1}$ is the vector of transmit signals, $\mathbf{H}(t) \in \mathbb{R}_+^{N_r\times N_t}$ is the matrix of the channel coefficients, and $\mathbf{n}(t)$ is an N_r-dimensional vector of independent Gaussian noises with zero mean and variance σ^2. Noise is independent through time, and combines thermal noise and background radiation. Here, the nonnegativity of $\mathbf{s}(t)$ and $\mathbf{H}(t)$ follows due to the IM/DD operation [17].

In this work, we consider quasi-static channels, i.e., the channel remains constant for a block of L transmissions and changes independently to another state in the next block. Thus, $\mathbf{H}(mL{+}1) = \cdots = \mathbf{H}((m+1)L)$ for $m \in \mathbb{N}$. This assumption is motivated by the fact that indoors OWC (e.g. VLC) is not affected by turbulence, and that outdoors OWC channels vary very slowly in comparison to the symbol duration, and hence can be assume constant over a block [1]. Note that this quasi-static assumption has been considered also in [8] for numerical comparison of RC and STBC. It is assumed that the channel state information (CSI) is known at the receiver side.

The nonnegativity of $\mathbf{s}(t)$ can be guaranteed by using a DC offset. These signals are constrained by a power constraint. Common power constraints considered in this context are [18]:

1) Individual electrical power constraint $\mathbb{E}[s_i^2(t)] \leq P$, $\forall t$, and $\forall i \in \{1, \ldots, N_t\}$.
2) Individual optical power constraint $\mathbb{E}[s_i(t)] \leq P$ $\forall t$, and $\forall i \in \{1, \ldots, N_t\}$.
3) Sum electrical power constraint $\sum_{i=1}^{N_t} \mathbb{E}[s_i^2(t)] \leq N_t P$ $\forall t$.
4) Sum optical power constraint $\sum_{i=1}^{N_t} \mathbb{E}[s_i(t)] \leq N_t P$ $\forall t$.

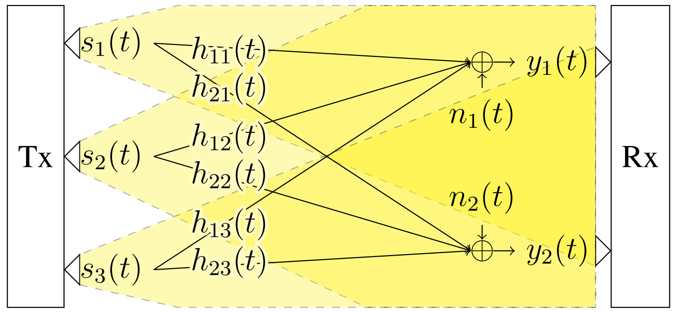


Fig. 1: A MIMO OWC system with three transmit apertures and two detectors at time instant t: $s_j(t) \geq 0$ is the optical intensity of aperture j, $h_{ij}(t) \geq 0$ is the channel gain from aperture j to the detector i, and $n_i(t)$ is Gaussian noise.

We will consider all these constraints in our analysis, starting with the individual constraints, followed by the sum constraints.

Since we are interested in the performance of DC-STBC in this paper, we define it next.

B. DC-offset STBC

Let $\mathbf{x} \in \mathbb{R}^{L \times 1}$ denote a vector of symbols that needs to be sent to the receiver in L transmissions $t \in \{1, \ldots, L\}$ (block $m = 0$), encoded in the transmit signals $\mathbf{s}(1), \ldots, \mathbf{s}(L)$. This is a rate-1 code, where one constellation symbol is sent per transmission on average. If we stack the vectors $\mathbf{s}(1), \ldots, \mathbf{s}(L)$ into one vector $\bar{\mathbf{s}} \in \mathbb{R}^{N_t L \times 1}$ given by $[\mathbf{s}(1)^T, \ldots, \mathbf{s}(L)^T]^T$, then, a DC-offset STBC is can be constructed as follows:

$$\bar{\mathbf{s}} = \mathbf{G}\mathbf{x} + \mathbf{t}, \tag{2}$$

where $\mathbf{G} = [\mathbf{g}_1, \ldots, \mathbf{g}_{N_t L}]^T \in \mathbb{R}^{N_t L \times L}$ is the coding matrix, and $\mathbf{t} = [t_1, \ldots, t_{N_t L}]^T \in \mathbb{R}_+^{N_t L}$ is a DC-offset. Under this construction, the nonnegativity constraint can be stated as follows

$$\mathbf{g}_i^T \mathbf{x} + t_i \geq 0 \text{ for } i \in \{1, \ldots, N_t L\}, \forall \mathbf{x} \in \mathcal{X}_M, \tag{3}$$

where $\mathcal{X}_M$ is the alphabet of $\mathbf{x}$. We fix the constellation of components of $\mathbf{x}$ to be a pulse-amplitude modulation (PAM) constellation with M points, i.e.,

$$\mathcal{P}_M = \{2i - (M-1) | i = 0, \ldots, M-1\}. \tag{4}$$

Thus $\mathcal{X}_M$ is the L-times Cartesian product of $\mathcal{P}_M$, i.e.,

$$\mathcal{X}_M = \mathcal{P}_M^L. \tag{5}$$

Due to this, the minimum DC-offset which satisfies the nonnegativity constraint (3)

$$t_i = (M-1)\|\mathbf{g}_i\|_1. \tag{6}$$

Furthermore, the power constraints can be written as follows:

1) Individual electrical power constraint: $\frac{M^2-1}{3}\|\mathbf{g}_i\|_2^2 + t_i^2 \leq P$, for all $i \in \{1,\ldots,N_tL\}$.
2) Individual optical power constraint: $t_i \leq P$, for all $i \in \{1,\ldots,N_tL\}$.
3) Sum electrical power constraint: $\sum_{i=(k-1)N_t+1}^{kN_t}\left(\frac{M^2-1}{3}\|\mathbf{g}_i\|_2^2 + t_i^2\right) \leq N_tP$, for all $k \in \{1,\ldots,L\}$.
4) Sum optical power constraint: $\sum_{i=(k-1)N_t+1}^{kN_t} t_i \leq N_tP$, for all $k \in \{1,\ldots,L\}$.

The received signal in the L transmissions can be written as

$$\bar{\mathbf{y}} = [\mathbf{y}(1)^T,\ldots,\mathbf{y}(L)^T]^T = \bar{\mathbf{H}}\mathbf{s} + \bar{\mathbf{n}}, \tag{7}$$

where $\bar{\mathbf{n}} = [\mathbf{n}(1)^T,\ldots,\mathbf{n}(L)^T]^T$, and

$$\bar{\mathbf{H}} = \begin{bmatrix} \mathbf{H} & \mathbf{0}_{N_r\times N_t} & \ldots & \mathbf{0}_{N_r\times N_t} \\ \mathbf{0}_{N_r\times N_t} & \mathbf{H} & \ldots & \mathbf{0}_{N_r\times N_t} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0}_{N_r\times N_t} & \mathbf{0}_{N_r\times N_t} & \ldots & \mathbf{H} \end{bmatrix} \tag{8}$$

is the extended-channel matrix for a given block. In what follows, we analyze the system for a given $\mathbf{H}$. The discussion on static channels or channels with turbulence is presented in Sec. VI. Substituting (2) in (7) yields

$$\bar{\mathbf{y}} = \bar{\mathbf{H}}\mathbf{G}\mathbf{x} + \bar{\mathbf{H}}\mathbf{t} + \bar{\mathbf{n}}, \tag{9}$$

as shown in Fig. 2. The signal is decoded by using a maximum likelihood (ML) detector, i.e.

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}\in\mathcal{X}_M} \|\bar{\mathbf{y}} - \bar{\mathbf{H}}(\mathbf{G}\mathbf{x}+\mathbf{t})\|_2. \tag{10}$$

The goal is to design $\mathbf{G}$ so that the system performance is optimized in terms of a criterion of interest defined as follows.

C. The Worst Case PEP Criterion

The main performance criterion in our analysis is the worst case PEP, i.e. the PEP corresponding to the minimum distance between any distinct codewords of the DC-STBC. This criterion is interesting for the following reasons. First, the exact symbol error rate (SER) of the system,

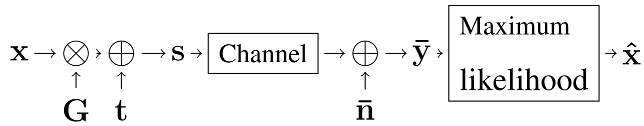


Fig. 2: Signal processing of $N_t \times N_r$ DC-STBC.

denoted $p(\mathbf{G}, \mathbf{H})$, can be approximated in terms of the worst case PEP, denoted $P_{\mathrm{e}}(\mathbf{G}, \mathbf{H})$, by using union bound estimate as follows [15]

$$p(\mathbf{G}, \mathbf{H}) \approx k_{\min} P_{\mathrm{e}}(\mathbf{G}, \mathbf{H}), \tag{11}$$

where k_{min} is the average, with respect to all constellation points, of the number of neighbors at the minimum distance of each constellation point. This approximation is very accurate in the high SNR regime. Second, the worst case PEP can be used to upper bound $p(\mathbf{G}, \mathbf{H})$ as follows [14]

$$p(\mathbf{G}, \mathbf{H}) \leq (L-1) P_{\mathrm{e}}(\mathbf{G}, \mathbf{H}). \tag{12}$$

Thus, by minimizing the worst case PEP we also minimize the approximation of the exact SER and the upper bound. Furthermore, the diversity order achieved by the code is dominated by the diversity order of the worst case PEP [15]. For these reasons, we choose the worst case PEP as our optimization objective. The resulting problem is formulated in the next subsection.

D. Problem Formulation

Since the channel state is known at the receiver and since the vector $\mathbf{t}$ is predefined, $\bar{\mathbf{H}}\mathbf{t}$ can be subtracted at the receiver side leading to

$$\dot{\mathbf{y}} = \mathbf{A}\mathbf{x} + \bar{\mathbf{n}}, \tag{13}$$

where $\mathbf{A} = \bar{\mathbf{H}}\mathbf{G}$. According to the previous discussion, our objective is to minimize the worst case PEP. The PEP between symbols $\mathbf{x}_a$ and $\mathbf{x}_b$ in $\mathcal{X}_M$, is defined as [19]

$$P\{\mathbf{x}_a \rightarrow \mathbf{x}_b | \mathbf{H}\} = Q\left(\frac{\|\mathbf{A}(\mathbf{x}_a - \mathbf{x}_b)\|_2}{2\sigma}\right), \tag{14}$$

where $Q(\cdot)$ is the Q-function. Accordingly, the worst case PEP is given by

$$P_{\mathrm{e}}(\mathbf{G},\mathbf{H}) = \max_{\mathbf{x}_a,\mathbf{x}_b \in \mathcal{X}_M} Q\left(\frac{\|\mathbf{A}(\mathbf{x}_a-\mathbf{x}_b)\|_2}{2\sigma}\right) \tag{15}$$

$$= Q\left(\frac{\min_{\mathbf{x}_a,\mathbf{x}_b \in \mathcal{X}_M} \|\mathbf{A}(\mathbf{x}_a-\mathbf{x}_b)\|_2}{2\sigma}\right), \tag{16}$$

since $Q(x)$ is a decreasing function of x. Thereby, our objective is to minimize $P_{\mathrm{e}}(\mathbf{G},\mathbf{H})$ with respect to $\mathbf{G}$. So, the minimum worst case PEP can be written as

$$P_{\mathrm{e,min}}(\mathbf{H}) = \min_{\mathbf{G}} Q\left(\frac{\min_{\mathbf{x}_a,\mathbf{x}_b \in \mathcal{X}_M} \|\mathbf{A}(\mathbf{x}_a-\mathbf{x}_b)\|_2}{2\sigma}\right) \tag{17}$$

$$= Q\left(\frac{\max_{\mathbf{G}} \min_{\mathbf{x}_a,\mathbf{x}_b \in \mathcal{X}_M} \|\mathbf{A}(\mathbf{x}_a-\mathbf{x}_b)\|_2}{2\sigma}\right), \tag{18}$$

again since $Q(x)$ is a decreasing function of its argument x. Note here that we minimize only with respect to $\mathbf{G}$ since $\mathbf{t}$ is a function of $\mathbf{G}$ as in (6). Therefore, $P_{\mathrm{e,min}}$ can be derived by considering the following problem:

$$\begin{aligned} \max_{\mathbf{G}} \quad & \min_{\mathbf{x}_a,\mathbf{x}_b \in \mathcal{X}_M} \|\mathbf{A}(\mathbf{x}_a-\mathbf{x}_b)\|_2^2 \\ \text{s.t.} \quad & \text{power constraint.} \end{aligned} \tag{19}$$

In the rest of the paper, we treat this problem. We summarize the main results of the paper in the following section.

Remark. *To avoid repetition, whenever the word 'optimal' is used in the paper, it refers to optimality in terms of worst case PEP.*

III. Summary of Main Results

The main results of the paper can be summarized as follows. The optimal rate-1 DC-STBC is in fact a simple *DC-offset space code* in the form of RC, beamforming, or more generally, weighted RC.[3] Thus, the time dimension is redundant. This is true for all 4 possible constraints given in Sec. II-A. This owes to the fact that in IM/DD OWC, the channels are always positive valued, which prevents any 'destructive interference' of symbols at the receiver side. This is contrary

[3]Throughout the paper, we use 'beamforming' to refer to a scheme, where an M-PAM symbol is repeated across the N_t transmit apertures possibly with different weights.

to RF where this effect can occur, and thus, one should capitalize on multiple transmission to harness benefit from multiple channel realizations.

We first introduce the results derived for a single block transmission. We discuss transmission over multiple blocks transmission afterwards.

A. Single Block Transmission

Under individual power constraints, it is optimal to use RC. This is summarized in the following two theorems.

Theorem 1 (Individual Electrical Constraint)**.** *The optimal M-PAM DC-STBC under an individual electrical power constraint in MIMO system is RC corresponding to $\mathbf{G}^{[e]} = \gamma \mathbf{I}_L \otimes \mathbf{1}_{N_t \times 1}$,*[4] *where $\gamma = \sqrt{\frac{3P}{4M^2-6M+2}}$. The corresponding minimum worst-case PEP is*

$$P_{\text{e,min}}^{[\text{e}]}(\mathbf{H}) = Q\left(\frac{\gamma}{\sigma}\|\mathbf{H}\mathbf{1}_{N_t \times 1}\|_2\right). \tag{20}$$

Proof: The proof is given in Sec. IV-A. ∎

Theorem 2 (Individual Optical Constraint)**.** *The optimal M-PAM DC-STBC under an individual optical power constraint in MIMO system is RC corresponding to $\mathbf{G}^{[o]} = \frac{P}{M-1}\mathbf{I}_L \otimes \mathbf{1}_{N_t \times 1}$. The corresponding minimum worst-case PEP is*

$$P_{\text{e,min}}^{[\text{o}]}(\mathbf{H}) = Q\left(\frac{P\|\mathbf{H}\mathbf{1}_{N_t \times 1}\|_2}{(M-1)\sigma}\right). \tag{21}$$

Proof: The proof is given in Sec. IV-B. ∎

Note the RC structure of $\mathbf{G}^{[e]}$ and $\mathbf{G}^{[o]}$. With this structure, every symbol of $\mathbf{x}$ is sent over all transmit apertures with the same amplitude in one and only one channel use. Thus, the DC-STBC structure can be reduced to a simple DC-offset spatial repetition code, which significantly reduces the encoding and decoding complexity.

In the MISO case, the matrix $\mathbf{H}$ becomes a row vector $\mathbf{h}^T$. The optimal solutions in this case can be obtained as corollaries of Theorems 1 and 2. The special case of BPSK can also be obtained as a corollaries of Theorems 1 and 2 by setting $M = 2$.

[4] A column permutation of the optimal $\mathbf{G}^{[e]}$ is also optimal, as it only changes the order of transmission of components of x_i.

On the other hand, under a sum power constraint, the time dimension of the DC-STBC is also redundant, and it is optimal to use beamforming. This result is summarized in the following statements.

Theorem 3 (Sum Electrical Constraint)**.** *The optimal M-PAM DC-STBC under a sum electrical power constraint in MIMO system is beamforming corresponding to* $\mathbf{G}^{[\mathrm{e,s}]}(\mathbf{v}) = \gamma\sqrt{N_t}\mathbf{I}_L \otimes \mathbf{v}$*, for some unit ℓ_2-norm vector $\mathbf{v} \in \mathbb{R}_+^{N_t}$, where γ is as defined in Theorem 1. The corresponding minimum worst-case PEP is*

$$P_{\mathrm{e,min}}^{[\mathrm{e,s}]}(\mathbf{H}, \mathbf{v}) = Q\left(\frac{\gamma\|\mathbf{H}\mathbf{v}\|_2}{\sigma}\sqrt{N_t}\right). \tag{22}$$

Proof: The proof is given in Sec. V-A. ■

Theorem 4 (Sum Optical Constraint)**.** *The optimal M-PAM DC-STBC under a sum optical power constraint in MIMO system is beamforming corresponding to* $\mathbf{G}^{[\mathrm{o,s}]}(\mathbf{v}) = \frac{N_t P}{M-1}\mathbf{I}_L \otimes \mathbf{v}$*, for some unit ℓ_1-norm vector $\mathbf{v} \in \mathbb{R}_+^{N_t}$. The corresponding minimum worst-case PEP is*

$$P_{\mathrm{e,min}}^{[\mathrm{o,s}]}(\mathbf{H}, \mathbf{v}) = Q\left(\frac{N_t P}{(M-1)\sigma}\|\mathbf{H}\mathbf{v}\|_2\right). \tag{23}$$

Proof: The proof is given in Sec. V-B. ■

Theorems 3 and 4 provide the solution for one block. Since the optimal solution leads to disjoint signaling, where one and only one M-PAM symbol is transmitted in every transmission, then there is no need to construct a DC-STBC but rather use DC-offset beamforming. It means that a weighted RC is optimal for both cases with and without CSI at the transmitter (CSIT).

If CSIT is available, then the optimal solution is beamforming as given next. Note that CSIT availability can be justified in two-way systems within a static/quasi-static environments. In this case, channel estimation and feedback can be performed in negligible time, since the symbol duration in OWC is much smaller than the coherence time.

Corollary 1. *If CSIT is available, then the optimal $\mathbf{v}$ in Theorem 3 is $\mathbf{w}$, the eigenvector corresponding to the largest eigenvalue λ of $\mathbf{H}^T\mathbf{H}$. The corresponding minimum worst case PEP becomes* $P_{\mathrm{e,min}}^{[\mathrm{e,s}]}(\mathbf{H}, \mathbf{w}) = Q\left(\frac{\gamma}{\sigma}\sqrt{N_t\lambda}\right)$.

Proof: Follows since $\mathbf{v} = \mathbf{w}$ maximizes $\|\mathbf{H}\mathbf{v}\|_2$. ■

Corollary 2. *If CSIT is available, then the optimal* $\mathbf{v}$ *in Theorem 4 is* $\mathbf{v} = \frac{\mathbf{w}}{\|\mathbf{w}\|_1}$*, where* $\mathbf{w}$ *is the eigenvector corresponding to the largest eigenvalue* λ *of* $\mathbf{H}^T\mathbf{H}$*. The corresponding minimum worst case PEP becomes* $P_{\mathrm{e,min}}^{[\mathrm{o,s}]}(\mathbf{H},\mathbf{w}) = Q\left(\frac{N_t P}{(M-1)\sigma}\sqrt{\lambda}\right)$.

Proof: Follows since $\mathbf{v} = \mathbf{w}$ maximizes $\|\mathbf{H}\mathbf{v}\|_2$. ■

The redundancy of temporal dimension which follows from Theorems 1–4 implies that the vector ML detection of the optimal DC-STBC can be simplified to scalar ML detection. For instance, under individual electrical power constraints, instead of detecting $\hat{\mathbf{x}}$ as in (10), we can detect component-wise as

$$\hat{x}(i) = \arg\max_{x(i)\in\mathcal{P}_M} \|\mathbf{y}(i) - (x(i) + (M-1))\gamma\mathbf{H}\mathbf{1}_{N_t\times 1}\|_2,$$

for $i = 1, \ldots, L$. Similarly for the other power constraints.

These results can be immediately applied for static channels. Transmission over multiple blocks of a quasi-static channel is considered next.

B. Multiple Blocks Transmission

With multiple blocks transmission over a quasi-static channel, the criterion of interest becomes the minimum average worst case PEP defined as

$$\bar{P}_{\mathrm{e,min}} = \min_{\mathbf{G}} \mathbb{E}_{\mathbf{H}}[P_{\mathrm{e}}(\mathbf{G},\mathbf{H})], \tag{24}$$

where $\mathbf{H}$ is a random matrix. Under individual constraints, the optimal solution is RC regardless whether CSIT is available or not, as expressed next.

Theorem 5 (Quasi-static/Individual Constraints)**.** *Under an individual electrical power constraint,* $\bar{P}_{\mathrm{e,min}} = \mathbb{E}_{\mathbf{H}}[P_{\mathrm{e,min}}^{[\mathrm{e}]}(\mathbf{H})]$ *achieved by RC with* $\mathbf{G}^{[\mathrm{e}]}$ *as given in Theorem 1. Similarly, under an individual optical power constraint,* $\bar{P}_{\mathrm{e,min}} = \mathbb{E}_{\mathbf{H}}[P_{\mathrm{e,min}}^{[\mathrm{o}]}(\mathbf{H})]$ *achieved by RC with* $\mathbf{G}^{[\mathrm{o}]}$ *as given in Theorem 2.*

Proof: This is a simple consequence of the optimality of RC in this case for any $\mathbf{H}$. Details are given in Sec. VI-B1. ■

Under a sum power constraint, we have to distinguish between two cases according to CSIT availability. First, if CSIT is available, then the transmitter can use optimal beamforming in each block according to Theorem 3. In this case, the following statement holds.

Theorem 6 (Quasi-static/Sum Constraints/CSIT)**.** *Under a sum electrical power constraint with CSIT,* $\bar{P}_{\rm e,min} = \mathbb{E}_{\mathbf{H}}[P_{\rm e,min}^{\rm [e,s]}(\mathbf{H})]$ *achieved using* $\mathbf{G}^{\rm [e,s]}(\mathbf{v}(\mathbf{H}))$ *as given in Theorem 3, with* $\mathbf{v}(\mathbf{H})$ *as defined in Corollary 1. Similarly, Under a sum optical power constraint with CSIT,* $\bar{P}_{\rm e,min} = \mathbb{E}_{\mathbf{H}}[P_{\rm e,min}^{\rm [o,s]}(\mathbf{H})]$ *achieved using* $\mathbf{G}^{\rm [o,s]}(\mathbf{v}(\mathbf{H}))$ *as given in Theorem 4, with* $\mathbf{v}(\mathbf{H})$ *as defined in Corollary 2.*

Proof: This follows from the optimality of these solutions for any $\mathbf{H}$ under CSIT availability. Details are given in Sec. VI-B2. ■

Second, if CSIT is not available, then beamforming is still optimal as described in the following statement.

Theorem 7 (Quasi-static/Sum Const./no CSIT)**.** *Under a sum electrical power constraint with no CSIT,*

$$\bar{P}_{\rm e,min} = \min_{\mathbf{r}:\|\mathbf{r}\|_2^2=N_t} \mathbb{E}_{\mathbf{H}}\left[Q\left(\frac{\gamma}{\sigma}\|\mathbf{H}\mathbf{r}\|_2\right)\right], \tag{25}$$

achieved using $\mathbf{G} = \gamma\mathbf{I}_L \otimes \mathbf{r}^*$*, and under a sum optical power constraint with no CSIT,*

$$\bar{P}_{\rm e,min} = \min_{\mathbf{r}:\|\mathbf{r}\|_1=N_t} \mathbb{E}_{\mathbf{H}}\left[Q\left(\frac{P}{(M-1)\sigma}\|\mathbf{H}\mathbf{r}\|_2\right)\right], \tag{26}$$

achieved using $\mathbf{G} = \frac{P}{M-1}\mathbf{I}_L \otimes \mathbf{r}^*$*, where* $\mathbf{r}^*$ *is the optimal* $\mathbf{r}$ *for the respective minimization.*

Proof: The proof is given in Sec. VI-B3. ■

Thus, even without CSIT, temporal coding is not necessary in a DC-STBC, and it suffices to code spatially.

The following sections prove these statements, and numerical evaluations are given in Sec. VII.

IV. MIMO Channel with Individual Constraints

In this section we prove Theorems 1 and 2. We start by treating the case with an individual electrical constraint.

A. Electrical Power Constraint (Proof of Theorem 1)

In this case, problem (19) is formulated as follows

$$\Theta^{\rm [e]} \triangleq \max_{\mathbf{G}\in\mathcal{G}_M^{\rm [e]}} \min_{\mathbf{x}_a,\mathbf{x}_b\in\mathcal{X}_M} \|\mathbf{A}(\mathbf{x}_a-\mathbf{x}_b)\|_2^2, \tag{27}$$

where $\mathcal{G}_M^{[\mathrm{e}]}$ is defined as

$$\mathcal{G}_M^{[\mathrm{e}]} = \Big\{ \mathbf{G} \in \mathbb{R}^{N_t L \times L} \mid \text{for } i \in \{1, \dots, N_t L\} \\ \frac{M^2-1}{3} \|\mathbf{g}_i\|_2^2 + (M-1)^2 \|\mathbf{g}_i\|_1^2 \leq P \Big\}. \tag{28}$$

Note that

$$\Psi_{ab} \triangleq \|\mathbf{A}(\mathbf{x}_a - \mathbf{x}_b)\|_2^2 \in S_\Delta, \tag{29}$$

where

$$S_\Delta = \Bigg\{ \Big\| \sum_{i=1}^{L} \delta_i \mathbf{a}_i \Big\|_2^2 \Big| \delta_i = -2(M-1) + 2j, \\ j \in \{0, \dots, 2(M-1)\},\ i \in \{1, \dots, L\} \Bigg\}, \tag{30}$$

and where $\mathbf{a}_i$ is the ith column of matrix $\mathbf{A}$. Now consider the following subset of S_Δ:

$$S'_\Delta = \big\{ 4\|\mathbf{a}_i\|_2^2 \mid i \in \{1, \dots, L\} \big\} \subseteq S_\Delta. \tag{31}$$

Note that

$$\min_{\Psi_{ab} \in S_\Delta} \Psi_{ab} \leq \min_{\Psi_{ab} \in S'_\Delta} \Psi_{ab}, \tag{32}$$

and thus,

$$\min_{\mathbf{x}_a, \mathbf{x}_b \in \mathcal{X}_M} \|\mathbf{A}(\mathbf{x}_a - \mathbf{x}_b)\|_2^2 \leq \min_{\Psi_{ab} \in S'_\Delta} \Psi_{ab}. \tag{33}$$

But the minimum of a set is not larger than its average, i.e.,

$$\min_{\Psi_{ab} \in S'_\Delta} \Psi_{ab} \leq \frac{1}{L} \sum_{i=1}^{L} 4\|\mathbf{a}_i\|_2^2, \tag{34}$$

and hence

$$\min_{\mathbf{x}_a, \mathbf{x}_b \in \mathcal{X}_M} \|\mathbf{A}(\mathbf{x}_a - \mathbf{x}_b)\|_2^2 \leq \frac{4}{L} \|\mathbf{A}\|_F^2. \tag{35}$$

Thus, $\Theta^{[e]}$ in (27) is upper bounded by

$$\Theta^{[\mathrm{e}]} \leq \max_{\mathbf{G} \in \mathcal{G}_M^{[\mathrm{e}]}} \frac{4}{L} \|\mathbf{A}\|_F^2. \tag{36}$$

Now, let us write $\mathbf{G}$ and $\mathbf{A}$ as

$$\mathbf{G} = [\mathbf{F}(1)^T, \dots, \mathbf{F}(L)^T]^T, \tag{37}$$

$$\mathbf{A} = [\mathbf{A}(1)^T, \dots, \mathbf{A}(L)^T]^T, \tag{38}$$

where for $k \in \{1,\ldots,L\}$, $\mathbf{F}(k) \in \mathbb{R}^{N_t \times L}$ consists of the k^{th} N_t rows of $\mathbf{G}$ and $\mathbf{A}(k) = \mathbf{H}\mathbf{F}(k) \in \mathbb{R}^{N_r \times L}$ consists of the k^{th} N_r rows of $\mathbf{A}$. Thus, the upper bound in (36) can be written as

$$\Theta^{[\text{e}]} \leq \max_{\mathbf{G}\in\mathcal{G}_M^{[\text{e}]}} \frac{4}{L} \sum_{k=1}^{L} \|\mathbf{A}(k)\|_F^2 = \sum_{k=1}^{L} \hat{\Theta}_k^{[\text{e}]}, \tag{39}$$

where

$$\hat{\Theta}_k^{[\text{e}]} = \max_{\mathbf{F}(k)\in\mathcal{F}_M^{[\text{e}]}} \frac{4}{L} \|\mathbf{A}(k)\|_F^2, \tag{40}$$

and

$$\mathcal{F}_M^{[\text{e}]} = \Bigg\{ \mathbf{F} \in \mathbb{R}^{N_t \times L} \Big| \text{ for } i \in \{1,\ldots,N_t\} \tag{41}$$
$$\frac{M^2-1}{3} \sum_{j=1}^{L} f_{ij}^2 + (M-1)^2 \left[\sum_{j=1}^{L} |f_{ij}| \right]^2 \leq P \Bigg\},$$

with f_{ij} being the i^{th} component in $\mathbf{f}_j$, the j^{th} column of $\mathbf{F}$. Thus, we have split the problem into L similar sub problems $\hat{\Theta}_k^{[\text{e}]}$. Writing

$$\mathbf{H} = [\mathbf{h}_1, \ldots, \mathbf{h}_{N_r}]^T, \tag{42}$$

we can write $\hat{\Theta}_k^{[\text{e}]}$ as

$$\begin{aligned} \hat{\Theta}_k^{[\text{e}]} &= \max_{\mathbf{F}(k)\in\mathcal{F}_M^{[\text{e}]}} \frac{4}{L} \sum_{r=1}^{N_r} \|\mathbf{h}_r^T \mathbf{F}(k)\|_2^2 \\ &\leq \sum_{r=1}^{N_r} \max_{\mathbf{F}(k)\in\mathcal{F}_M^{[\text{e}]}} \frac{4}{L} \|\mathbf{h}_r^T \mathbf{F}(k)\|_2^2. \end{aligned} \tag{43}$$

In this upper bound, the optimal $\mathbf{F}(k)$ for a given $r \in \{1,\ldots,N_r\}$ might depend on $\mathbf{h}_r$. Consider one such maximization for $r \in \{1,\ldots,N_r\}$:

$$\check{\Theta}_{kr}^{[\text{e}]} \triangleq \max_{\mathbf{F}(k)\in\mathcal{F}_M^{[\text{e}]}} \frac{4}{L} \sum_{i=1}^{L} (\mathbf{h}_r^T \mathbf{f}_i(k))^2. \tag{44}$$

Note here that $\mathbf{h}_r \in \mathbb{R}_+^{N_t}$, i.e., has nonnegative real-valued components. Thus, a necessary condition on the optimal solution $\mathbf{F}(k)$ is that the components of $\mathbf{f}_i(k)$, $i \in \{1,\ldots,L\}$, have the same sign.[5] By symmetry, we can restrict the components of $\mathbf{f}_i(k)$ to positive values. Next, we relax $\mathcal{F}_M^{[\text{e}]}$ by noting that

$$\sum_{j=1}^{L} f_{ij}^2 \leq \left[\sum_{j=1}^{L} |f_{ij}| \right]^2. \tag{45}$$

[5]This is the main difference between this problem and one which arises in the RF context. Since RF channels are complex-valued, this condition is not necessary in the RF case which leads to a different solution.

Hence, $\mathcal{F}_M^{[\text{e}]} \subseteq \tilde{\mathcal{F}}_M^{[\text{e}]}$, where

$$\tilde{\mathcal{F}}_M^{[\text{e}]} = \Big\{ \mathbf{F} \in \mathbb{R}_+^{N_t \times L} \Big| \sum_{j=1}^{L} f_{ij}^2 \le \gamma^2, \forall i \in \{1, \dots, N_t\} \Big\},$$

where $\gamma = \sqrt{\frac{3P}{4M^2 - 6M + 2}}$ as defined in Theorem 1. γ is always bigger than 0, because $M > 1$. Consequently, we have that

$$\check{\Theta}_{kr}^{[\text{e}]} \le \tilde{\Theta}_{kr}^{[\text{e}]} \triangleq \max_{\mathbf{F}(k) \in \tilde{\mathcal{F}}_M^{[\text{e}]}} \frac{4}{L} \sum_{i=1}^{L} (\mathbf{h}_r^T \mathbf{f}_i(k))^2. \tag{46}$$

The maximum $\tilde{\Theta}_{kr}^{[\text{e}]}$ is achieved when only one column of $\mathbf{F}(k)$ has non-zero values. To show this, consider a feasible $\mathbf{F}(k)$ with two nonzero columns $\mathbf{f}_1(k)$ and $\mathbf{f}_2(k)$, and consider another feasible $\mathbf{F}'(k)$ with only one nonzero column $\mathbf{f}_1'(k)$ satisfying

$$f_{1i}'(k) = \sqrt{f_{1i}^2(k) + f_{2i}^2(k)}, \ \forall i \in \{1, \dots, N_t\}, \tag{47}$$

so that the rows of $\mathbf{F}(k)$ and $\mathbf{F}'(k)$ have the same ℓ_2-norm. One can easily verify that

$$(\mathbf{h}_r^T \mathbf{f}_1(k))^2 + (\mathbf{h}_r^T \mathbf{f}_2(k))^2 \le (\mathbf{h}_r^T \mathbf{f}_1'(k))^2. \tag{48}$$

This approach can be extended to show that an $\mathbf{F}'(k)$ with one nonzero column is better than any $\mathbf{F}(k)$ with more than one nonzero column. Thus, the optimal solution of (46) must have only one nonzero column in $\mathbf{F}(k)$. Substituting in (46) yields

$$\begin{aligned} \tilde{\Theta}_{kr}^{[\text{e}]} = & \max_{\mathbf{f}_1(k) \in \mathbb{R}_+^{N_t}} \frac{4}{L} (\mathbf{h}_r^T \mathbf{f}_1(k))^2 \\ & \text{s.t. } f_{i1}^2(k) \le \gamma^2, \ \forall i \in \{1, \dots, N_t\}. \end{aligned} \tag{49}$$

Since the objective is increasing in $f_{i1}(k)$, then it is maximized when the constraints are met with equality. This implies that (49) is maximized by $\mathbf{f}_1(k) = \gamma \mathbf{1}_{N_t \times 1}$ and the maximum value is $\tilde{\Theta}_{kr}^{[\text{e}]} = \frac{4\gamma^2}{L} \|\mathbf{h}_r\|_1^2$.

As a consequence of these steps, we have upper bounded the optimization problem (27) as follows

$$\begin{aligned} \Theta^{[e]} \le \sum_{k=1}^{L} \hat{\Theta}_k^{[\text{e}]} & \le \sum_{k=1}^{L} \sum_{r=1}^{N_r} \check{\Theta}_{kr}^{[\text{e}]} \\ & \le \sum_{k=1}^{L} \sum_{r=1}^{N_r} \tilde{\Theta}_{kr}^{[\text{e}]} = 4\gamma^2 \sum_{r=1}^{N_r} \|\mathbf{h}_r\|_1^2. \end{aligned} \tag{50}$$

This upper bound is achievable by choosing $\mathbf{G}$ as $\mathbf{G}^{[\text{e}]}$ given in Theorem 1, respectively, which proves the desired result.

B. Optical Power Constraint (Proof of Theorem 2)

The problem with the individual optical power constraint is given by

$$\Theta^{[\mathrm{o}]} \triangleq \max_{\mathbf{G}\in\mathcal{G}_M^{[\mathrm{o}]}} \min_{\mathbf{x}_a,\mathbf{x}_b\in\mathcal{X}_M} \|\mathbf{A}(\mathbf{x}_a-\mathbf{x}_b)\|_2^2, \tag{51}$$

where

$$\begin{aligned}&\mathcal{G}_M^{[\mathrm{o}]} \\ &= \Big\{\mathbf{G}\in\mathbb{R}^{N_tL\times L}\Big|(M-1)\|\mathbf{g}_i\|_1 \le P,\ i\in\{1,\dots,N_tL\}\Big\}.\end{aligned} \tag{52}$$

Using similar steps as in Sec. IV-A, we can upper bound $\Theta^{[\mathrm{o}]}$ by

$$\Theta^{[\mathrm{o}]} \le \sum_{k=1}^{L}\hat{\Theta}_k^{[\mathrm{o}]}, \tag{53}$$

where

$$\hat{\Theta}_k^{[\mathrm{o}]} \triangleq \sum_{r=1}^{N_r}\max_{\mathbf{F}(k)\in\mathcal{F}_M^{[\mathrm{o}]}} \frac{4}{L}\|\mathbf{h}_r^T\mathbf{F}(k)\|_2^2, \tag{54}$$

and

$$\begin{aligned}\mathcal{F}_M^{[\mathrm{o}]} = \Big\{\mathbf{F}\in\mathbb{R}^{N_t\times L}\Big|&\text{for } i\in\{1,\dots,N_t\}, \\ &(M-1)\sum_{j=1}^{L}|f_{ij}|\le P\Big\}.\end{aligned} \tag{55}$$

Similar to Sec. IV-A, for any $r\in\{1,\dots,N_r\}$, the optimal $\mathbf{F}(k)$ has only one nonzero column with positive components, leading to

$$\hat{\Theta}_k^{[\mathrm{o}]} = \frac{4P^2}{(M-1)^2L}\sum_{r=1}^{N_r}\|\mathbf{h}_r\|_1^2. \tag{56}$$

Consequently, we obtain

$$\Theta^{[\mathrm{o}]} \le \sum_{k=1}^{L}\hat{\Theta}_k^{[\mathrm{o}]} = \frac{4P^2}{(M-1)^2}\sum_{i=1}^{N_r}\|\mathbf{h}_r\|_1^2. \tag{57}$$

This upper bound is achievable by setting $\mathbf{G}$ to $\mathbf{G}^{[\mathrm{o}]}$ as in Theorem 2, respectively, which proves the desired result.

Next, we consider the system with a sum-power constraint instead of an individual power constraint.

V. MIMO Channel with Sum Constraints

In the next two subsections, we consider the MIMO IM/DD system with sum electrical and optical power constraints, and prove Theorems 3 and 4.

A. Electrical Power Constraint (Proof of Theorem 3)

Problem (19) with a sum electrical power constraint can be written as follows

$$\Theta^{[\text{e,s}]} \triangleq \max_{\mathbf{G}\in\mathcal{G}_M^{[\text{e,s}]}} \min_{\mathbf{x}_a,\mathbf{x}_b\in\mathcal{X}_M} \|\mathbf{A}(\mathbf{x}_a-\mathbf{x}_b)\|_2^2. \tag{58}$$

where

$$\begin{aligned}\mathcal{G}_M^{[\text{e,s}]} = \Big\{\mathbf{G}\in\mathbb{R}^{N_tL\times L}\Big|\text{for } k\in\{1,\ldots,L\},\\ \sum_{i=(k-1)N_t+1}^{kN_t}\left(\frac{M^2-1}{3}\|\mathbf{g}_i\|_2^2+(M-1)^2\|\mathbf{g}_i\|_1^2\right)\leq N_tP\Big\}.\end{aligned} \tag{59}$$

Using similar steps as (29)–(36) we can write

$$\Theta^{[\text{e,s}]} \leq \max_{\mathbf{G}\in\mathcal{G}_M^{[\text{e,s}]}} \frac{4}{L}\|\mathbf{A}\|_F^2. \tag{60}$$

Then, we can split this upper bound into L subproblems as in (39) leading to

$$\Theta^{[\text{e,s}]} \leq \sum_{k=1}^{L}\hat{\Theta}_k^{[\text{e,s}]}, \tag{61}$$

where

$$\hat{\Theta}_k^{[\text{e,s}]} \triangleq \max_{\mathbf{F}(k)\in\mathcal{F}_M^{[\text{e,s}]}} \frac{4}{L}\|\mathbf{A}(k)\|_F^2, \tag{62}$$

and

$$\begin{aligned}\mathcal{F}_M^{[\text{e,s}]} = \Bigg\{\mathbf{F}\in\mathbb{R}^{N_t\times L}\Bigg|\frac{M^2-1}{3}\sum_{i=1}^{N_t}\sum_{j=1}^{L}f_{ij}^2\\ +(M-1)^2\sum_{i=1}^{N_t}\left[\sum_{j=1}^{L}|f_{ij}|\right]^2\leq N_tP\Bigg\}.\end{aligned} \tag{63}$$

We further split this problem in N_r subproblems, dependent only on one row vector of $\mathbf{H}$, i.e.,

$$\hat{\Theta}_k^{[\text{e,s}]} = \max_{\mathbf{F}(k)\in\mathcal{F}_M^{[\text{e,s}]}} \frac{4}{L}\sum_{r=1}^{N_r}\|\mathbf{h}_r^T\mathbf{F}(k)\|_2^2 \leq \sum_{r=1}^{N_r}\check{\Theta}_{kr}^{[\text{e,s}]}, \tag{64}$$

where

$$\check{\Theta}_{kr}^{[\text{e,s}]} \triangleq \max_{\mathbf{F}(k)\in\mathcal{F}_M^{[\text{e,s}]}} \frac{4}{L}\sum_{i=1}^{L}(\mathbf{h}_r^T\mathbf{f}_i(k))^2, \tag{65}$$

where $\mathbf{f}_i(k)$ is the i^{th} column of $\mathbf{F}(k)$.

Since $\mathbf{h}_r \in \mathbb{R}_+^{N_t}$, then a necessary condition for optimality is that the components of $\mathbf{f}_i(k)$ have the same sign. Without loss of generality, we can restrict these components to positive values by symmetry. Next, we upper bound (65) by relaxing the constraints using the fact that

$$\sum_{j=1}^{L} f_{ij}^2 \leq \left[\sum_{j=1}^{L}|f_{ij}|\right]^2, \tag{66}$$

to obtain

$$\check{\Theta}_{kr}^{[\text{e,s}]} \leq \tilde{\Theta}_{kr}^{[\text{e,s}]} \triangleq \max_{\mathbf{F}(k)\in\tilde{\mathcal{F}}_M^{[\text{e,s}]}} \frac{4}{L}\sum_{i=1}^{L}(\mathbf{h}_r^T\mathbf{f}_i(k))^2, \tag{67}$$

where

$$\tilde{\mathcal{F}}_M^{[\text{e,s}]} = \left\{\mathbf{F}\in\mathbb{R}_+^{N_t\times L}\,\middle|\,\sum_{j=1}^{L}\|\mathbf{f}_j\|_2^2 \leq \gamma^2 N_t\right\}, \tag{68}$$

where $\gamma = \sqrt{\frac{3P}{4M^2-6M+2}}$.

The maximum of this relaxed problem is achieved when only one column of $\mathbf{F}(k)$ is nonzero, with positive components. To show this, we can use the same arguments as in (47) – (48). Using this property in (67) yields

$$\tilde{\Theta}_{kr}^{[\text{e,s}]} = \max_{\mathbf{f}_1(k)\in\mathbb{R}_+^{N_t}} \frac{4}{L}(\mathbf{h}_r^T\mathbf{f}_1(k))^2 \tag{69}$$

$$\text{s.t. } \|\mathbf{f}_1(k)\|_2^2 \leq \gamma^2 N_t. \tag{70}$$

Since the objective is increasing in $f_{i1}(k)$, then the maximum is acheived when the constraint is met with equality. Note that this fixes the magnitude of $\mathbf{f}_1(k)$ but not its direction. Let the optimal direction be that of a unit vector $\mathbf{v}\in\mathbf{R}_+^{N_t}$, which is independent of k, because all $\tilde{\Theta}_{kr}^{[\text{e,s}]}$ are equivalent. Writing the optimal $\mathbf{f}_1(k)$ as $\gamma\sqrt{N_t}\mathbf{v}$ leads to

$$\tilde{\Theta}_{kr}^{[\text{e,s}]} = \frac{4\gamma^2 N_t}{L}(\mathbf{h}_r^T\mathbf{v})^2, \tag{71}$$

for some $\mathbf{v}$.

As a conclusion, we have that

$$\Theta^{[\mathrm{e,s}]} \leq \sum_{k=1}^{L} \hat{\Theta}_k^{[\mathrm{e,s}]} \leq \sum_{k=1}^{L}\sum_{r=1}^{N_r} \check{\Theta}_{kr}^{[\mathrm{e,s}]} \tag{72}$$

$$\leq \sum_{k=1}^{L}\sum_{r=1}^{N_r} \tilde{\Theta}_{kr}^{[\mathrm{e,s}]} = 4\gamma^2 N_t \|\mathbf{H}\mathbf{v}\|_2^2. \tag{73}$$

This upper bound is achievable by setting $\mathbf{G}$ to $\mathbf{G}^{[\mathrm{e,s}]}$ given in Theorem 3, which proves the statement of the theorem.

B. Optical Power Constraint (Proof of Theorem 4)

In the case of a sum optical power constraint, the problem becomes

$$\Theta^{[\mathrm{o,s}]} \triangleq \max_{\mathbf{G}\in\mathcal{G}_M^{[\mathrm{o,s}]}} \min_{\mathbf{x}_a,\mathbf{x}_b\in\mathcal{X}_M} \|\mathbf{A}(\mathbf{x}_a - \mathbf{x}_b)\|_2^2, \tag{74}$$

where

$$\mathcal{G}_M^{[\mathrm{o,s}]} = \Bigg\{ \mathbf{G} \in \mathbb{R}^{N_t L \times L} \Bigg| \text{ for } k \in \{1,\ldots,L\} \quad (M-1)\sum_{i=(k-1)N_t+1}^{kN_t} \|\mathbf{g}_i\|_1 \leq N_t P \Bigg\}. \tag{75}$$

After applying similar steps as (29)–(36), $\Theta^{[\mathrm{o,s}]}$ can be upper bounded by

$$\Theta^{[\mathrm{o,s}]} \leq \max_{\mathbf{G}\in\mathcal{G}_M^{[\mathrm{o,s}]}} \frac{4}{L}\|\mathbf{A}\|_F^2. \tag{76}$$

As mentioned in the previous subsection, this maximization problem can also be split into L similar independent subproblems, each one depends on N_t row vectors of the matrix $\mathbf{G}$. Namely,

$$\Theta^{[\mathrm{o,s}]} \leq \sum_{k=1}^{L} \hat{\Theta}_k^{[\mathrm{o,s}]}, \tag{77}$$

where

$$\hat{\Theta}_k^{[\mathrm{o,s}]} \triangleq \sum_{r=1}^{N_r} \max_{\mathbf{F}(k)\in\mathcal{F}_M^{[\mathrm{o,s}]}} \frac{4}{L}\|\mathbf{h}_r^T\mathbf{F}(k)\|_2^2, \tag{78}$$

and

$$\mathcal{F}_M^{[\mathrm{o,s}]} = \big\{\mathbf{F} \in \mathbb{R}^{N_t\times L} \big| (M-1)\sum_{j=1}^{L} \|\mathbf{f}_j\|_1 \leq N_t P\big\},$$

where $\mathbf{f}_j$ is the j^{th} column of $\mathbf{F}$.

Similar to Sec. IV-A, the maximum of this upper bound is achieved when only one column of $\mathbf{F}(k)$ is nonzero. This can be restricted to be positive component-wise, leading to

$$\begin{aligned}\hat{\Theta}_k^{[\mathrm{o,s}]} = & \sum_{r=1}^{N_r} \max_{\mathbf{f}_1(k)} \frac{4}{L} \left(\mathbf{h}_r^T \mathbf{f}_1(k)\right)^2 \\ & \text{s.t.} \quad (M-1)\|\mathbf{f}_1(k)\|_1 \leq N_t P. \end{aligned} \tag{79}$$

The objective is maximized when the constraints are met with equality, because it is increasing in $f_{i1}(k)$. Thus, we can write $\mathbf{f}_1(k)$ as $\frac{N_t P}{(M-1)}\mathbf{v}$ for some vector $\mathbf{v} \in \mathbb{R}_+^{N_t}$ satisfying $\|\mathbf{v}\|_1 = 1$. Hence,

$$\hat{\Theta}_k^{[\mathrm{o,s}]} = \frac{4N_t^2 P^2}{(M-1)^2 L} \sum_{r=1}^{N_r} \left(\mathbf{h}_r^T \mathbf{v}\right)^2 \tag{80}$$

for some unit ℓ_1-norm $\mathbf{v}$.

As a conclusion, we have

$$\Theta^{[\mathrm{o,s}]} \leq \sum_{k=1}^{L} \hat{\Theta}_k^{[\mathrm{o,s}]} = \frac{4N_t^2 P^2}{(M-1)^2} \sum_{r=1}^{N_r} \left(\mathbf{h}_r^T \mathbf{v}\right)^2. \tag{81}$$

This upper bound is achievable by setting $\mathbf{G}$ to $\mathbf{G}^{[\mathrm{o,s}]}$ given in Theorem 4, which concludes the proof.

Next, we consider application of the results to static and turbulent channels.

VI. Static and Turbulence Channels

The results discussed so far focus on a single block transmission. However, in practice, transmission spans multiple blocks. In this scenario cases, the interesting performance criterion, the worst case PEP, needs to be averaged over the blocks. For a large number of blocks, this average can be captured by averaging with respect to the channel state, i.e., the average worst case PEP for a given $\mathbf{G}$ becomes[6]

$$\bar{P}_{\mathrm{e}}(\mathbf{G}) = \mathbb{E}_{\mathbf{H}}[P_{\mathrm{e}}(\mathbf{G}, \mathbf{H})]. \tag{82}$$

The minimum thereof can be written as

$$\bar{P}_{\mathrm{e,min}} = \min_{\mathbf{G}} \bar{P}_{\mathrm{e}}(\mathbf{G}). \tag{83}$$

Two cases arise: static channel and quasi-static channel.

[6]Here, $\mathbf{G}$ can be a function of $\mathbf{H}$ if CSIT is given.

A. Static Channels

Let us first consider static channels. In this case, $\mathbf{H}$ is the same throughout the blocks. This captures indoors VLC scenarios with no mobility for instance. So, we may write

$$\bar{P}_{\mathrm{e,min}} = \min_{\mathbf{G}} \mathbb{E}_{\mathbf{H}}[P_{\mathrm{e}}(\mathbf{G},\mathbf{H})] \tag{84}$$

$$= \min_{\mathbf{G}} P_{\mathrm{e}}(\mathbf{G},\mathbf{H}) \tag{85}$$

$$= P_{\mathrm{e,min}}(\mathbf{H}). \tag{86}$$

Here, Theorems 1–4 can be used depending on the power constraint. Indeed, in this case the assumption of availability of CSIT is well justified because the channel is static, and can be estimated and fedback before the transmission starts.

B. Quasi-static Channels

The channel can be quasi-static due to mobility in an indoor scenario, or due to turbulence in an outdoor one. Here, we distinguish between three cases: (i) Individual power constraints, (ii) sum power constraints with CSIT, and (iii) sum power constraints without CSIT.

1) Individual Constraints: For this case, the derivation of the optimal solution is explained next. We first lower bound $\bar{P}_{\mathrm{e,min}}$ as follows

$$\bar{P}_{\mathrm{e,min}} \geq \mathbb{E}_{\mathbf{H}}[\min_{\mathbf{G}} P_{\mathrm{e}}(\mathbf{G},\mathbf{H})] = \mathbb{E}_{\mathbf{H}}[P_{\mathrm{e,min}}(\mathbf{H})]. \tag{87}$$

This lower bound is achievable using $\mathbf{G}^{[\mathrm{e}]}$ and $\mathbf{G}^{[\mathrm{o}]}$ in Theorems 1 and 2, which are independent of $\mathbf{H}$. That is, *RC is optimal among all DC-STBCs for quasi-static channels with individual power constraints.* Note that this result agrees with the numerical results in [8].

2) Sum Constraint with CSIT: In this case, the lower bound above is also achievable by using the optimal $\mathbf{G}^{[\mathrm{e,s}]}$ and $\mathbf{G}^{[\mathrm{o,s}]}$ in Theorems 3 and 4 in each block transmission. Thus, *beamforming is optimal among all DC-STBCs for quasi-static channels with sum power constraints and CSIT.*

3) Sum Constraint without CSIT: In this case, the lower bound in (87) is not achievable. We will derive an alternative lower bound for this case. Firstly, consider a sum electrical power constraint. The objective in this case is to find DC-STBC, $\mathbf{s} = \mathbf{G}\mathbf{x} + \mathbf{t}$, which is the solution of the following problem

$$\bar{P}_{\mathrm{e,min}} = \min_{\mathbf{G}} \mathbb{E}_{\mathbf{H}}[P_{\mathrm{e}}(\mathbf{G},\mathbf{H})], \tag{88}$$

where $\mathbf{G}$ is independent of $\mathbf{H}$, and satisfies nonnegativity and sum electrical power constraints. These constraints can be combined as

$$\sum_{i=(k-1)N_t+1}^{kN_t} \rho_i^2 \leq N_t, \tag{89}$$

for $k \in \{1, \ldots, L\}$, where

$$\rho_i = \sqrt{\frac{M^2-1}{3P}\|\mathbf{g}_i\|_2^2 + \frac{(M-1)^2}{P}\|\mathbf{g}_i\|_1^2}. \tag{90}$$

(cf. Sec. V-A). Let us denote $[\rho_{(k-1)N_t+1}, \ldots, \rho_{kN_t}]^T$ by $\mathbf{r}_k$. Since $\mathbf{H}$ preserves its value throughout a block, then by symmetry, the optimal solution satisfies $\mathbf{r}_1 = \mathbf{r}_2 = \cdots = \mathbf{r}_L \triangleq \mathbf{r}$. Let us write

$$\mathbf{G} = \bar{\mathbf{R}}\tilde{\mathbf{G}}, \tag{91}$$

where $\bar{\mathbf{R}} = \mathbf{I}_L \otimes \mathbf{R}$, and $\mathbf{R}$ is a diagonal matrix with $\mathbf{r}$ as its diagonal. Then, this DC-STBC can be expressed as $\bar{\mathbf{R}}(\tilde{\mathbf{G}}\mathbf{x} + \tilde{\mathbf{t}})$, where

$$\tilde{\mathbf{G}} = [\tilde{\mathbf{g}}_1, \ldots, \tilde{\mathbf{g}}_{N_tL}]^T, \qquad \tilde{\mathbf{g}}_i = \frac{\mathbf{g}_i}{\rho_i}, \tag{92}$$

$$\tilde{\mathbf{t}} = [\tilde{t}_1, \ldots, \tilde{t}_{N_tL}]^T, \qquad \tilde{t}_i = \frac{(M-1)\|\mathbf{g}_i\|_1}{\rho_i}, \tag{93}$$

in order to satisfy constraint in (89). So, we can write

$$\bar{P}_{\text{e,min}} = \min_{\mathbf{G}} \mathbb{E}_{\mathbf{H}}[P_{\text{e}}(\mathbf{G}, \mathbf{H})] \tag{94}$$

$$= \min_{\mathbf{r},\tilde{\mathbf{G}}} \mathbb{E}_{\mathbf{H}}[P_{\text{e}}(\bar{\mathbf{R}}\tilde{\mathbf{G}}, \mathbf{H})] \tag{95}$$

$$\geq \min_{\mathbf{r}} \mathbb{E}_{\mathbf{H}}[\min_{\tilde{\mathbf{G}}} P_{\text{e}}(\bar{\mathbf{R}}\tilde{\mathbf{G}}, \mathbf{H})]. \tag{96}$$

Using the definition of $P_{\text{e}}(\mathbf{G}, \mathbf{H})$ in (15), we can write this lower bound as

$$\bar{P}_{\text{e,min}} \geq \min_{\mathbf{r}} \mathbb{E}_{\mathbf{H}}[\min_{\tilde{\mathbf{G}}} P_{\text{e}}(\tilde{\mathbf{G}}, \mathbf{HR})]. \tag{97}$$

The inner problem corresponds to a DC-STBC with individual power constraint (cf. Sec. IV-A), with a channel matrix $\mathbf{HR}$. Thus, the optimal $\tilde{\mathbf{G}}$ corresponds to RC, i.e., $\tilde{\mathbf{G}}^{[\text{e}]} = \gamma \mathbf{I}_L \otimes \mathbf{1}_{N_t\times 1}$, and the corresponding minimum worst case PEP for a given $\mathbf{HR}$ is $P_{\text{e,min}}^{[\text{e}]}(\mathbf{HR}) = Q(\frac{\gamma}{\sigma}\|\mathbf{HR}\mathbf{1}_{N_t\times 1}\|_2)$ as in Theorem 1. Thus,

$$\bar{P}_{\text{e,min}} \geq \min_{\mathbf{r}} \mathbb{E}_{\mathbf{H}}\left[Q\left(\frac{\gamma}{\sigma}\|\mathbf{Hr}\|_2\right)\right], \tag{98}$$

with $\|\mathbf{r}\|_2^2 \leq N_t$. This lower bound is achievable by using $\mathbf{G} = \gamma \mathbf{I}_L \otimes \mathbf{r}^*$, where $\mathbf{r}^*$ is the optimal solution of the minimization in (98), which can be found numerically.

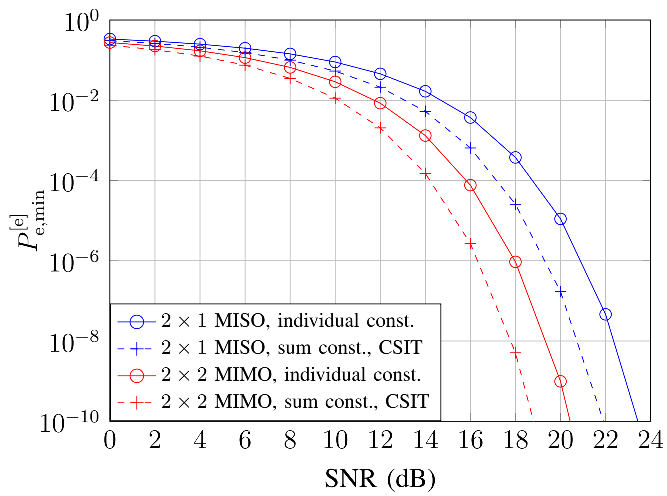


Fig. 3: Comparison of the minimum worst case PEP as a function of SNR (P/σ^2) under electrical power constraints for a 2×1 MISO system with $\mathbf{h}^T = [0.1,\ 0.5]$ and a 2×2 MIMO system with $\mathbf{H} = [\mathbf{h},\ \mathbf{h}]$ (static channels).

A similar derivation can be used for a sum optical power constraint leading to

$$\bar{P}_{\mathrm{e,min}} \geq \min_{\mathbf{r}} \mathbb{E}_{\mathbf{H}}\left[Q\left(\frac{P}{(M-1)\sigma}\|\mathbf{Hr}\|_2\right)\right], \tag{99}$$

where $\|\mathbf{r}\|_1 \leq N_t$, and this lower bound is achievable using $\mathbf{G} = \frac{P}{M-1}\mathbf{I}_L \otimes \mathbf{r}^*$ where $\mathbf{r}^*$ is the optimal $\mathbf{r}$ for the above minimization. This proves that *beamforming is optimal among all DC-STBCs for quasi-static channels with a sum power constraint and without CSIT.*

Finding the optimal $\mathbf{r}$ accounts for finding the optimal power allocation across the apertures. Instead of finding this optimal $\mathbf{r}$, one could use the practical solution of allocating powers equally across apertures. Under a sum electrical power constraint, this leads to $\mathbf{r} = \mathbf{1}_{N_t\times1}$, for which the average worst case PEP becomes $\mathbb{E}_{\mathbf{H}}\left[Q\left(\frac{\gamma}{\sigma}\|\mathbf{H}\mathbf{1}_{N_t\times1}\|_2\right)\right]$.[7] Under a sum optical power constraint, this leads to $\mathbf{r} = \mathbf{1}_{N_t\times1}$ leading to an average worst case PEP of $\mathbb{E}_{\mathbf{H}}\left[Q\left(\frac{P\|\mathbf{H}\mathbf{1}_{N_t\times1}\|_2}{(M-1)\sigma}\right)\right]$.

By comparing obtained result with that in [9], we notice that our result is more general due to the following aspects. First, it considers both spatial and temporal coding and shows the redundancy of the temporal dimension, whereas [9] considers only spatial coding. Second, it holds for any turbulence distribution and not restricted to log-normal as [9].

[7]Note that this is an achievable average worst case PEP corresponding to this choice of $\mathbf{r}$, whereas the optimal $\mathbf{r}$ of (98) or (99) leads to the minimum average worst case PEP.

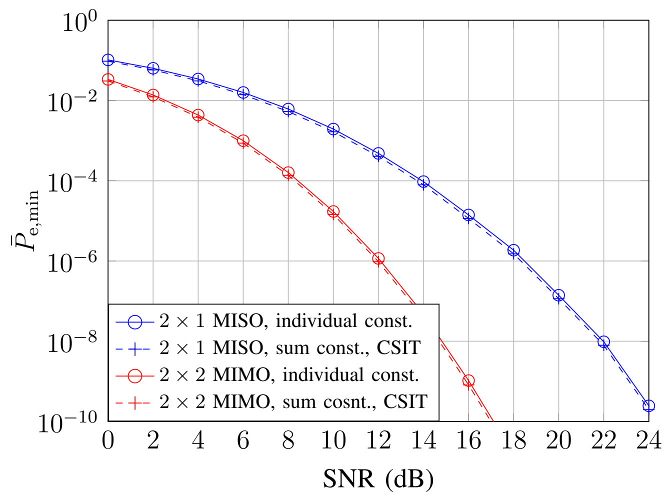


Fig. 4: Comparison of the minimum average worst case PEP as a function of SNR (P/σ^2) under electrical power constraints for 2×1 MISO and 2×2 MIMO systems with log-normal distributed channels with Rytov variance $\sigma_R^2 = 0.25$.

VII. Numerical Analysis

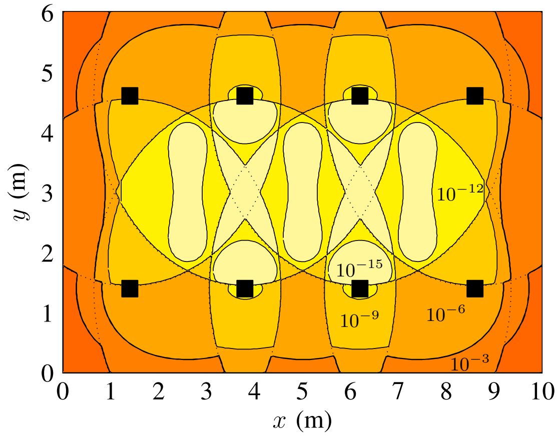
As discussed in Section II-C, the probability of error can be approximated as a linear function of the worst case PEP. Thus, comparing the worst case PEP can reveal similar conclusions as comparing error probability. Thus, we use the worst case PEP to compare the performance of MISO and MIMO systems. Recall that the minimum worst case PEP for the MIMO case with individual electrical power constraints is as follows

$$P_{\text{e,min}}^{[\text{e}]} = Q\left(\frac{\gamma}{\sigma}\|\mathbf{H}\mathbf{1}_{N_t\times 1}\|_2\right). \tag{100}$$

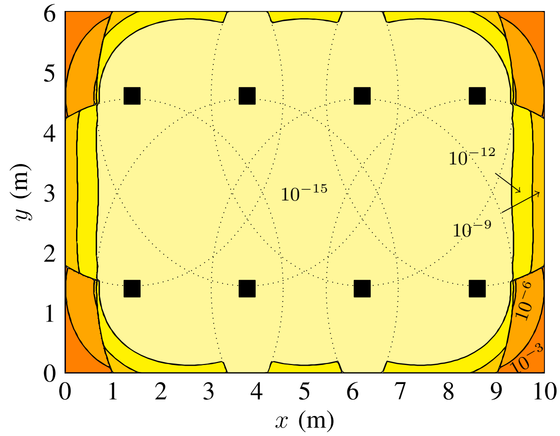
The result for the MISO case can be obtained by replacing $\mathbf{H}$ by $\mathbf{h}^T$. This expression quantifies the performance gain of MIMO over MISO. This is plotted in Fig. 3, which shows $P_{\text{e,min}}^{[\text{e}]}$ for $M = 2$ (BPSK) with $N_t = 2$ and $N_r = 1$ MISO or $N_r = 2$ MIMO. In this comparison, we need a 3 dB increase in the signal-to-noise ratio (SNR) in order to achieve as similar performance in the MISO system as in the MIMO one.

A performance comparison under lognormal distributed channels for 2×1 MISO and 2×2 MIMO systems is given in Fig. 4. Therein, the probability density function of channel h_{ij} is [20], [21]

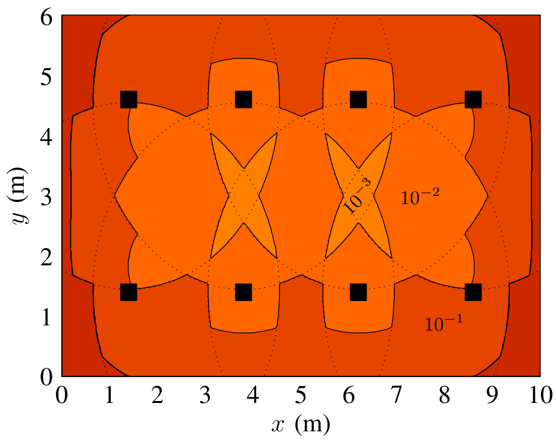
$$f(h_{ij}) = \frac{1}{h_{ij}\sqrt{2\pi\sigma_R^2}} \exp\left(-\frac{\left(\ln(h_{ij}) + \frac{\sigma_R^2}{2}\right)^2}{2\sigma_R^2}\right), \tag{101}$$



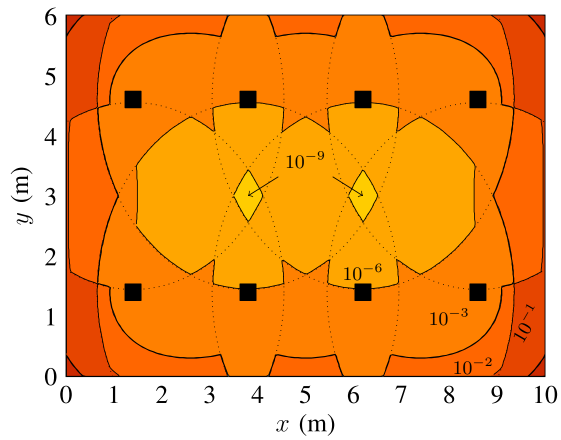
(a) $(M, N_r) = (2, 1)$: BPSK with 1 receive aperture.



(b) $(M, N_r) = (2, 3)$: BPSK with 3 receive aperture.



(c) $(M, N_r) = (4, 1)$: 4-PAM with 1 receive apertures.



(d) $(M, N_r) = (4, 3)$: 4-PAM with 3 receive apertures.

Fig. 5: SER for the VLC system described in Table I. The dotted circles depict the coverage areas of each light fixture (black squares), while the shading depicts different levels of guaranteed SER, i.e., the SER in each area is less than the indicated value.

where σ_R^2 is the Rytov variance, assumed here equal to 0.25. We can observe from Fig. 3 that considering a sum constraint and CSIT availability might improve performance for a given channel in comparison to individual constraints. However, Fig. 4 reveals that this does not have significant influence on average in a quasi-static channel.

Next, we provide numerical comparisons for indoors VLC scenarios with different number of receiving apertures and different constellation sizes. The VLC system parameters are presented in the Table I. The symbol error rate (SER) versus the location of the receiver in this VLC

TABLE I: VLC System Parameters

Name of Parameter	Value
Room dimensions	$10 \times 6 \times 4$ m
Transmitter x coordinates	1.4, 4.6 m
Transmitter y coordinates	1.4, 3.8, 6.2, 8.6 m
Height of the desk	0.85 m
Bandwidth	20 MHz
Noise spectral density	10^{-19} A^2/Hz
Field of view of the receiver	45°
Transmitted power	3 W
Transmitter Lambertian index	1
PD effective area	10^{-4} m^2

scenario is illustrated in Fig. 5 for $(M, N_r) \in \{2, 4\} \times \{1, 3\}$. Note that in BPSK, we have

$$p(\mathbf{G}, \mathbf{H}) = P_{\mathrm{e}}(\mathbf{G}, \mathbf{H}). \tag{102}$$

The SER is equal to the bit-error rate (BER) in this case. On the other hand, in 4-PAM, we have

$$p(\mathbf{G}, \mathbf{H}) \approx 1.5 P_{\mathrm{e}}(\mathbf{G}, \mathbf{H}) \tag{103}$$

according to (11). This SER is approximately equal to the BER at high SNR if we use gray coding.

Suppose that a user in this room uses a service that runs reliably if SER is smaller than 10^{-3}. Then, it is desired to increase the area in the room where this condition is met. From Fig. 5a we see that only the corners of the room have bad performance, since these positions is covered by only one light fixture. [8] Using the MIMO advantage by adding two receive apertures allows the user to enjoy (almost) error free transmission anywhere in the room as illustrated in Fig. 5b. If we consider a MISO system with higher throughput using 4-PAM, the reliable communication area shrinks dramatically as shown in Fig. 5c; and now lies in areas which are covered by 6 transmit apertures. This is not a good situation in practice, since the user might be located anywhere in the room. Increase the throughput by using M-PAM has a tremendous negative effect on the size of the reliable communication area. However, this area can be expanded significantly by adding using three receive apertures instead of one, as shown in Fig. 5d. To sum up, there is

[8] We use 'coverage' of a light fixture to indicate areas wherein a receive is able to see the light fixture given its field of view.

definite advantage of using multiple receiving apertures in ensuring that reliable communication can be achieved anywhere in a room in a VLC scenario. Similar observations hold in the optical power constraint scenario.

VIII. Conclusion

In this paper, we studied MIMO IM/DD OWC employing DC-offset STBCs. The worst case pairwise error probability was considered as the main criterion for performance evaluation. We analyzed the problem of minimizing this criterion under electrical and optical power constraints. We showed that spatial repetition coding is optimal from this perspective, among all DC-offset STBCs, for a system with individual power constraints. This result holds true for a static channels and also for quasi-static channel with arbitrary channel statistics. The importance of this result is that it means that the temporal dimension is redundant DC-offset STBCs in such systems, and hence, it is enough to code spatially. Moreover, this result is important since repetition coding is much simpler in practice than STBCs, thus reducing the system's computational complexity. We also prove a similar result for systems with sum power constraints, where we show that spatial coding in the form of beamforming is optimal. The optimal beamforming depends on the channel (static or quasi-static) and the availability of channel state information at the transmitter.

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