

# A Hybrid DGTD-MNA Scheme for Analyzing Complex Electromagnetic Systems



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## Abstract

A hybrid electromagnetics (EM)-circuit simulator for analyzing complex systems consisting of EM devices loaded with nonlinear multi-port lumped circuits is described. The proposed scheme splits the computational domain into two subsystems: EM and circuit subsystems, where field interactions are modeled using Maxwell and Kirchhoff equations, respectively. Maxwell equations are discretized using a discontinuous Galerkin time domain (DGTD) scheme while Kirchhoff equations are discretized using a modified nodal analysis (MNA)-based scheme. The coupling between the EM and circuit subsystems is realized at the lumped ports, where related EM fields and circuit voltages and currents are allowed to “interact” via numerical flux. To account for nonlinear lumped circuit elements, the standard Newton-Raphson method is applied at every time step. Additionally, a local time-stepping scheme is developed to improve the efficiency of the hybrid solver. Numerical examples consisting of EM systems loaded with single and multiport linear/nonlinear circuit networks are presented to demonstrate the accuracy, efficiency, and applicability of the proposed solver.

## Problem Description

With increasing operating frequencies of electronic circuits, and integration of multifunctional capabilities, any simulation tool developed for circuit-system modeling must take into account unintentional emissions and couplings between the distributive and lumped circuit networks. This requirement can be fulfilled by a hybrid approach that solves the Maxwell and Kirchhoff equations simultaneously. Since interactions between electromagnetic fields and active devices have significant influence on the system performance, incorporation of nonlinear lumped element modeling within the simulation tool is also required. Presence of nonlinear interactions renders the system response highly susceptible to small changes in the EM and circuit subsystems. Therefore, utmost accuracy that can only be obtained by coupled simulation, which takes into account all physical interactions between the two subsystems.

## Formulation

### DGTD for Maxwell Equations

Maxwell equations are discretized using DGTD. The resulting system of equations is expressed as:

$$\mathbf{M}_e^{(i)} \mathbf{e}_{n+1}^{(i)} + \delta t \frac{\mathbf{j}_{e,n+1}^{(i)}}{2} = \mathbf{M}_e^{(i)} \mathbf{e}_n^{(i)} + \delta t \left[ \mathbf{S}_e^{(i)} \mathbf{h}_{n+1/2}^{(i)} - \mathbf{j}_{im,n+1/2}^{(i)} - \mathbf{j}_{e,n}^{(i)} / 2 + \mathbf{F}_{ee}^{(ii)} \mathbf{e}_n^{(i)} - \mathbf{F}_{ee}^{(ij)} \mathbf{e}_n^{(j)} - \mathbf{F}_{eh}^{(ii)} \mathbf{h}_{n+1/2}^{(i)} + \mathbf{F}_{eh}^{(ij)} \mathbf{h}_{n+1/2}^{(j)} \right]$$

$$\mathbf{M}_h^{(i)} \mathbf{h}_{n+3/2}^{(i)} = \mathbf{M}_h^{(i)} \mathbf{h}_{n+1/2}^{(i)} - \delta t \left[ \mathbf{S}_h^{(i)} \mathbf{e}_{n+1}^{(i)} - \mathbf{j}_{h,n+1}^{(i)} - \mathbf{F}_{hh}^{(ii)} \mathbf{h}_{n+1/2}^{(i)} + \mathbf{F}_{hh}^{(ij)} \mathbf{h}_{n+1/2}^{(j)} - \mathbf{F}_{he}^{(ii)} \mathbf{e}_{n+1}^{(i)} - \mathbf{F}_{he}^{(ij)} \mathbf{e}_{n+1}^{(j)} \right]$$

### MNA for Kirchhoff's Current Equations

Kirchhoff's current equations are discretized by MNA

$$\begin{bmatrix} [\mathbf{G}] & [\mathbf{B}_1] \\ [\mathbf{B}_2] & [\mathbf{D}] \end{bmatrix} \begin{bmatrix} \mathbf{V}_{n+1}^{\text{CKT}} \\ \mathbf{I}_{n+1}^{\text{CKT}} \end{bmatrix} + \mathbf{I}_{n+1}^{\text{CKT,nl}}(\mathbf{V}_{n+1}^{\text{CKT}}) = \begin{bmatrix} \mathbf{I}_n^{\text{CP}} + \mathbf{I}_{n+1}^{\text{ind}} \\ \mathbf{V}_{n+1}^{\text{Port}} + \mathbf{V}_{n+1}^{\text{ind}} \end{bmatrix}$$

### Coupling of EM and Circuit Subsystems

Coupling between the EM to circuit systems is achieved by a voltage source obtained from the line integral of the electric field along the lumped port.

$$\mathbf{F}_f(\mathbf{x}_{n+1,f}) = \begin{bmatrix} [\mathbf{M}_e^f] & 0 & \delta t [\mathbf{T}_e^f / 2] \\ 0 & [\mathbf{G}^f] & [\mathbf{B}_1^f] \\ [\mathbf{C}^f] & -[\mathbf{B}_2^f] & -[\mathbf{D}^f] \end{bmatrix} \begin{bmatrix} \mathbf{e}_{n+1}^{(f)} \\ \mathbf{V}_{n+1,f}^{\text{CKT}} \\ \mathbf{I}_{n+1,f}^{\text{CKT}} \end{bmatrix} + \begin{bmatrix} 0 \\ \mathbf{I}_{n+1,f}^{\text{CKT,nl}} \\ \mathbf{V}_{n+1,f}^{\text{ind}} \end{bmatrix} = \begin{bmatrix} \mathbf{b}_{\text{EM}}^f \\ \mathbf{I}_{n,f}^{\text{CP}} \\ 0 \end{bmatrix}$$

## Numerical Results

### Parallel Plate Waveguide Loaded by Silicon Diodes.

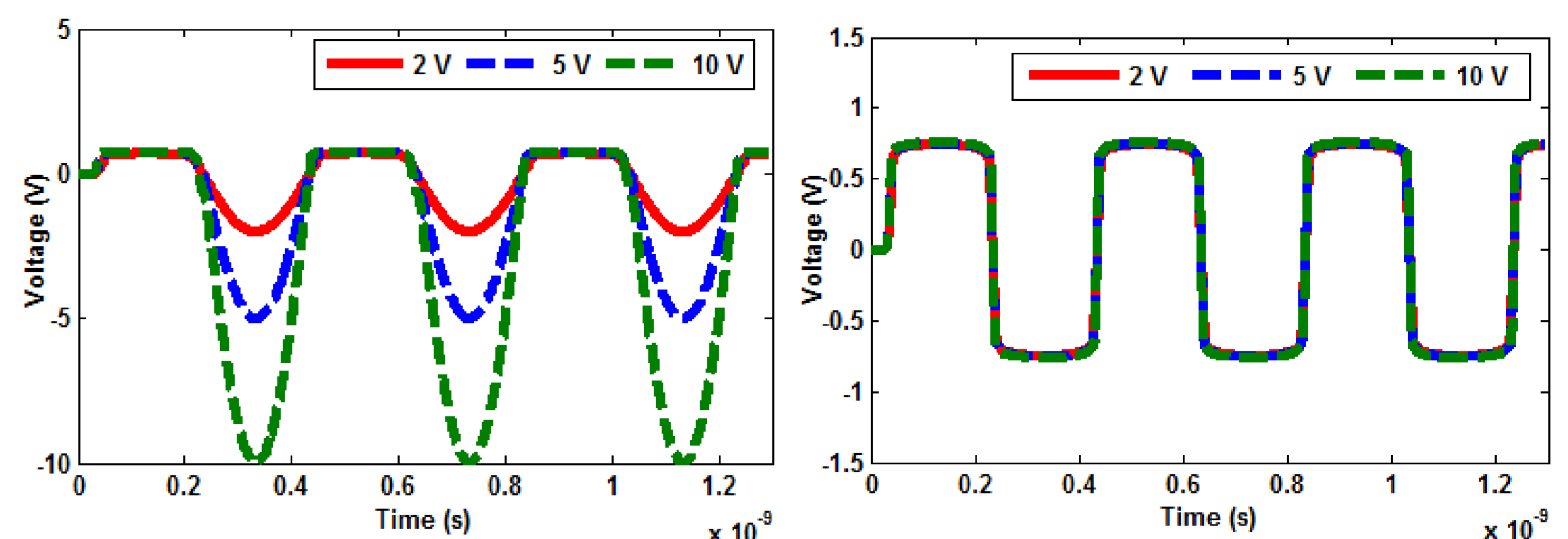


Fig.1 Left: The output voltage of a single diode. Right: The output voltage of an anti-parallel placed diode pair

### Large Signal MESFET Power Amplifier.

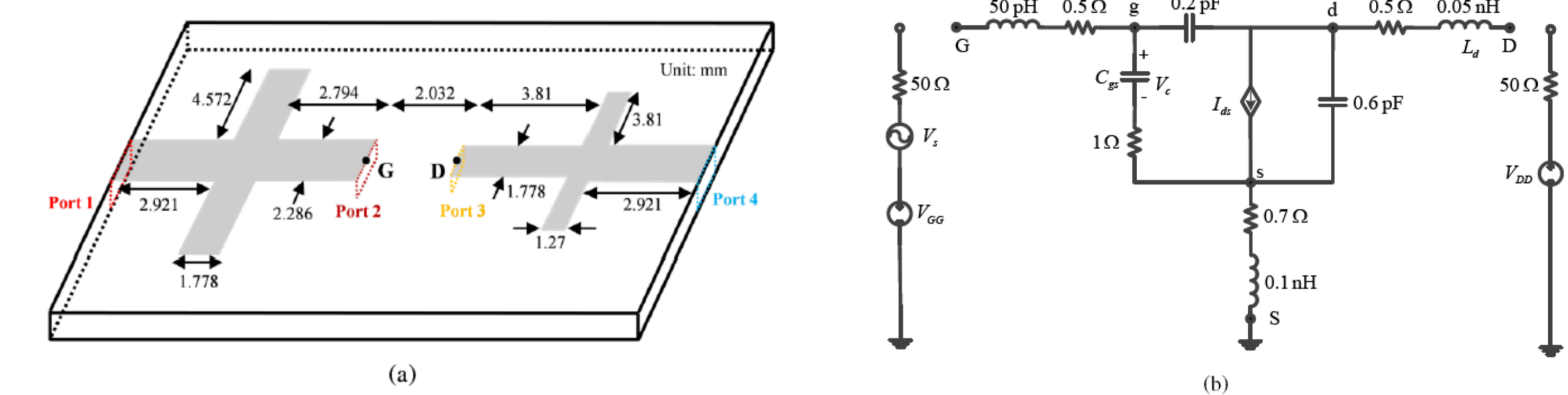


Fig.2 (a) Layout of the circuit configuration. (b) The extrinsic equivalent circuit of the MESFET power amplifier.

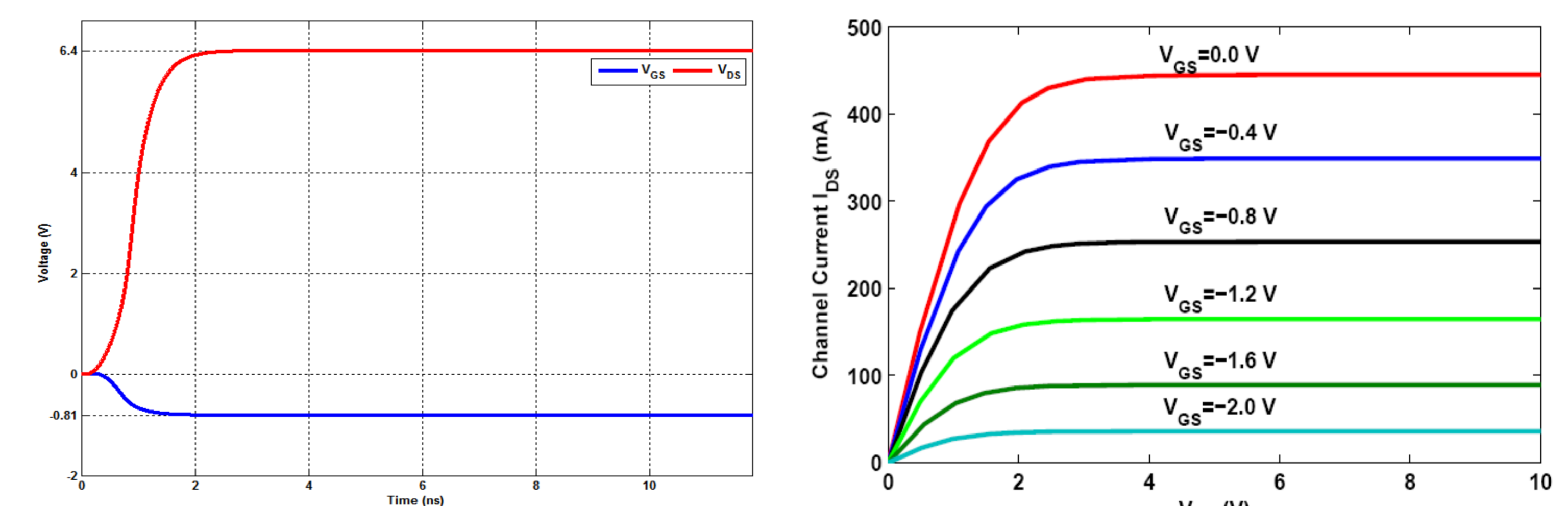


Fig.3 Left: The DC voltage drop at the gate and Drain terminal versus time. Right: Basic DC operating property

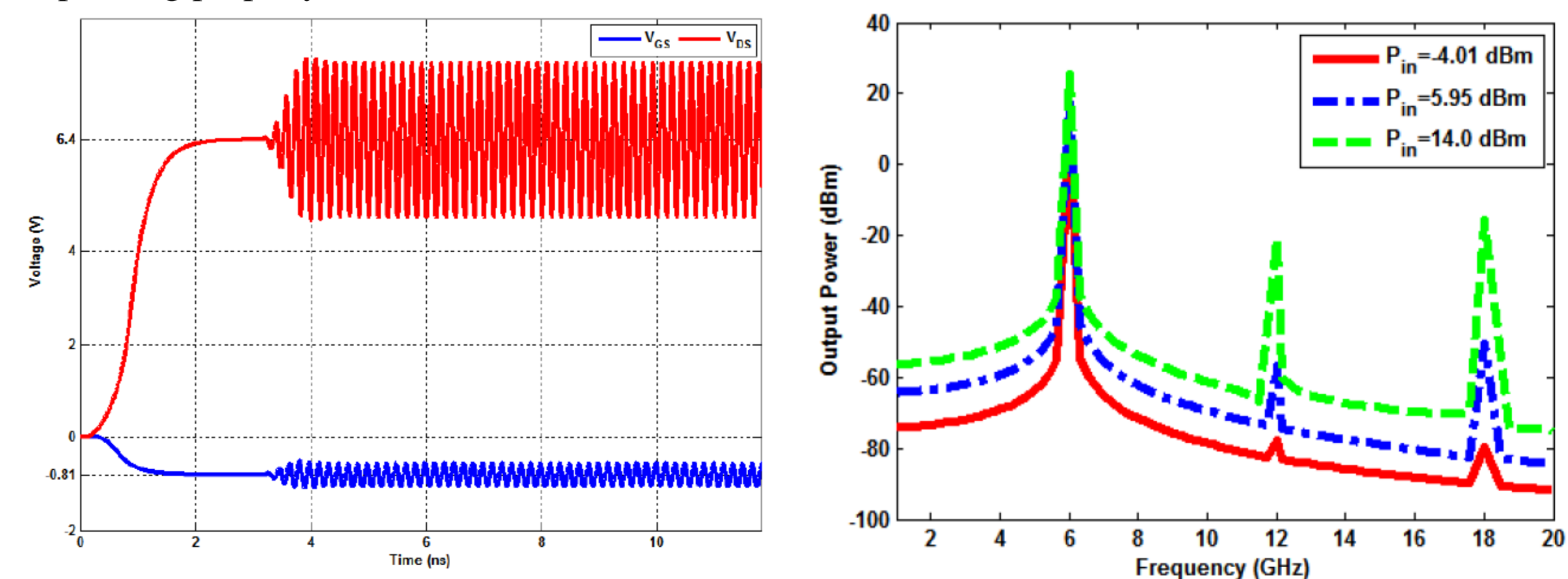


Fig.4 Left: Output voltage at the Drain and Gate terminals when an AC signal is applied at Port I. Right: Output power spectrum when a 6 GHz signal is applied at Port I

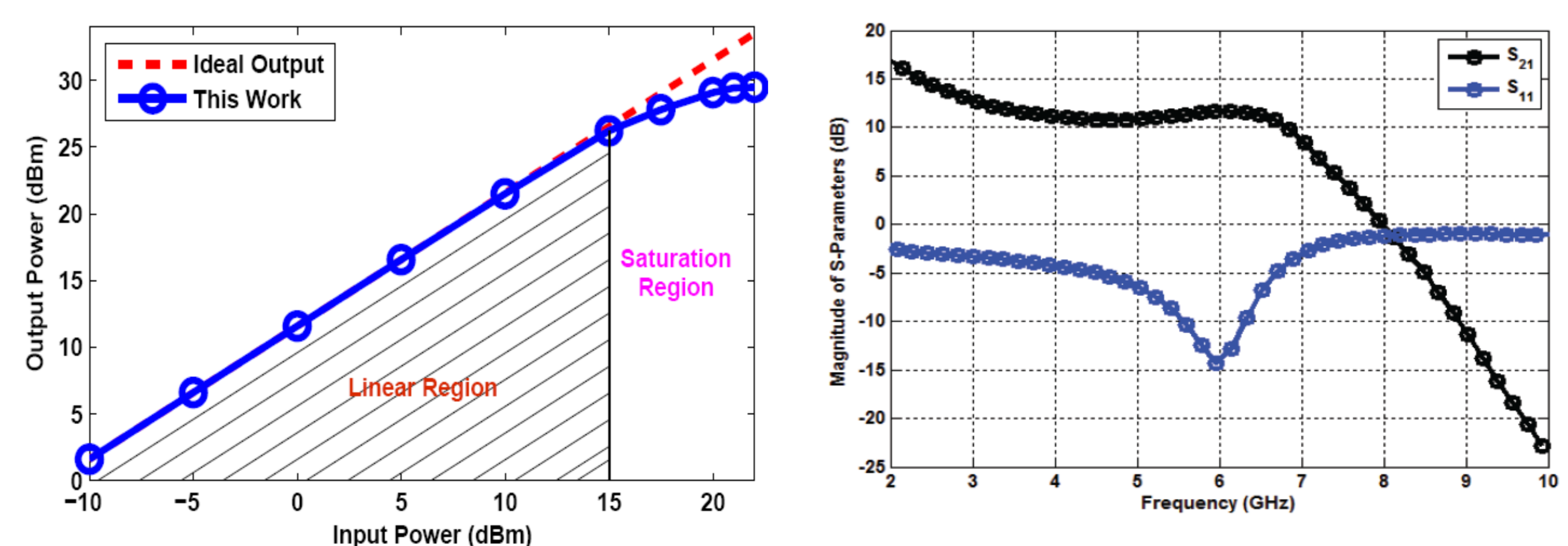


Fig.5 Left: Output power at 6 GHz versus different input power levels. Right: Calculated S-parameters from 2-10 GHz of this MESFET power amplifier.