Multiscale Modeling of Wear Degradation

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Abstract

Cylinder liners of diesel engines used for marine propulsion are naturally subjected to a wear process, and may fail when their wear exceeds a specified limit. Since failures often represent high economical costs, it is utterly important to predict and avoid them.

In this work [4], we model the wear process using a pure jump process. Therefore, the inference goal here is to estimate: the number of possible jumps, its sizes, the coefficients and the shapes of the jump intensities.

We propose a multiscale approach for the inference problem that can be seen as an indirect inference scheme.

We found that using a Gaussian approximation based on moment expansions, it is possible to accurately estimate the jump intensities and the jump amplitudes. We obtained results equivalent to the state of the art but using a simpler and less expensive approach.

The data

The data set consists of wear levels observed on 32 cylinder liners of eight-cylinder SULZER engines and measured by a caliper with a precision of 0.05 mm (see Figure 1). Warranty clauses specify that the liner should be substituted before it accumulates a wear level of 4.0 mm, in order to avoid catastrophic and very expensive failures. Data are shown in Figure 2.

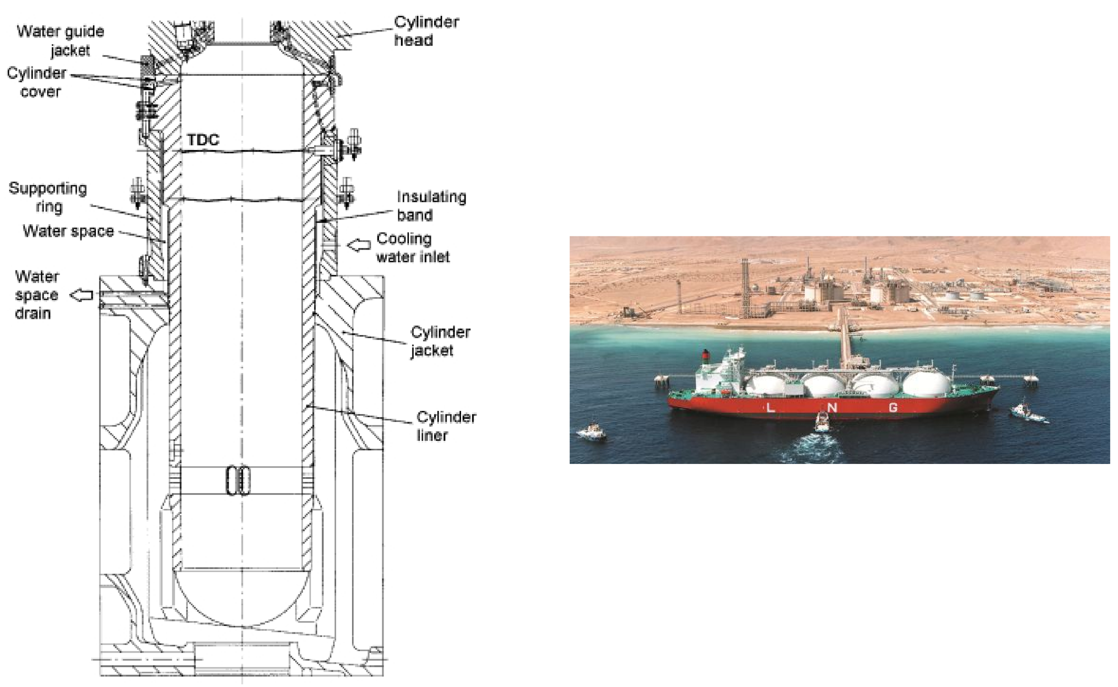

Figure 1: *Left: Cross-section of the cylinder liner of a SULZER RTA 58 engine. Right: Data refer to cylinder liners, equipping natural gas transport ships like the one shown here.*

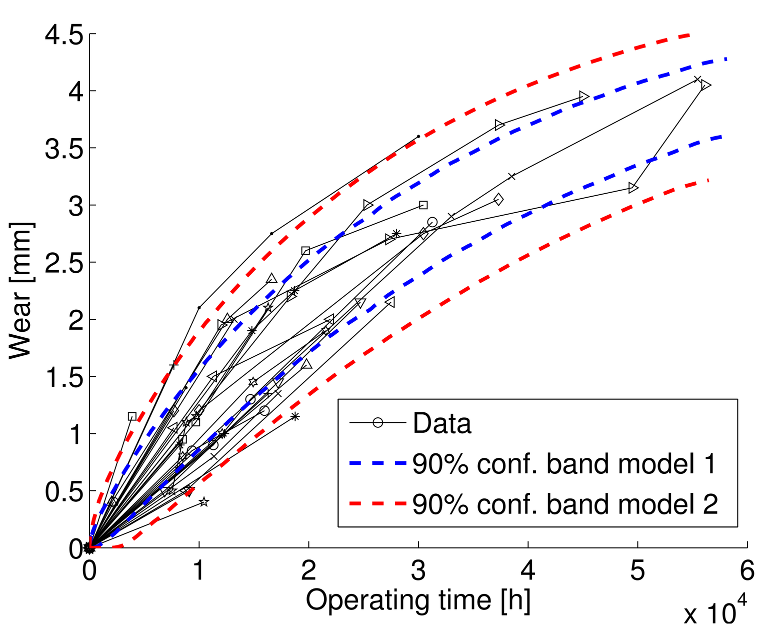

Figure 2: *Data and 90% confidence bands for models 1 and 2. The confidence band for the model 2 is essentially the same as the one obtained by Giorgio et al. in [2] using a stochastic differential equation driven by gamma noise.*

The mathematical model

The Pure Jump Process.

Consider a Markov pure jump process X taking values in a lattice $\mathcal{S}$. The evolution of the state vector $X(t) = (X_1(t), \dots, X_d(t))$ is modeled as a continuous-time Markov chain (see [5]).

Assume that each possible jump in the system occurs according to one of the pairs $\{(a_j(x;\theta), \nu_j)\}_{j=1}^J$, where $a_j : \mathbb{R}^d \times \Theta \to \mathbb{R}_+$ is known as the propensity function associated to the jump ν_j. The propensity functions depend on a parameter $\theta \in \Theta$, where Θ is finite dimensional.

The probability that the system jumps from $x \in \mathcal{S}$ to $x+\nu_j \in \mathcal{S}$ during the small interval $(t, t+\mathrm{d}t)$ is given by

$$\mathrm{P}\left(X(t, t+\mathrm{d}t) = x+\nu_j \mid X(t) = x\right) = a_j(x;\theta)\mathrm{d}t + o\left(\mathrm{d}t\right).$$

Example 0.1 (Simple decay model) *The Simple Decay model is a pure jump process X in the lattice $\mathcal{S} = \Delta\mathbb{N}$, where Δ is a positive real number. The system starts form $x_0 \in \mathcal{S}$ at time $t=0$, and the only reaction allowed is $\nu = -\Delta$. Its associated propensity function is $a(x;c) = c\,x$, where $c > 0$.*

Upscaling and generators.

The generator $\mathcal{L}_X$ of a pure jump Markov process X is a linear operator defined on the set of bounded functions. In our case it is given by (see [3])

$$\mathcal{L}_X(f) = \sum_j a_j(x;\theta)(f(x+\nu_j) - f(x)).$$

Using a first order Taylor expansion of f, we obtain the following approximate generator

$$\mathcal{L}_Z(f) = \sum_j a_j(x;\theta)\partial_x f(x)\nu_j,$$

which corresponds to reaction-rates ODE (mean field)

$$\begin{cases} dZ(t) &= \nu a(Z(t);\theta)dt,\ t \in \mathbb{R}_+ \\ Z(0) &= x_0 \in \mathbb{R}_+. \end{cases}$$

where the j–column of the matrix ν is ν_j, and a is a column vector with components a_j.

Wear process

Let $X(t)$ be the *thickness process* derived from the wear of the cylinder liners up to time t (see [1, 2]), *i.e.*, $X(t) = T_0 - W(t)$, where W is the wear process, and T_0 is the initial thickness. We model $X(t)$ as a sum of two simple decay processes (see example 0.1) with $\Delta = 0.05$ (caliper precision), since one simple decay process is not enough to explain the variance of the data. The two considered intensity-reaction pairs are $(a_1(x;\theta), \nu_1) = (c_1 x, -\Delta)$ and $(a_2(x;\theta), \nu_2) = (c_2 x, -k\Delta)$, where k is a positive integer to be determined, and $\theta = (c_1, c_2)$.

Therefore, the probability of observing a thickness decrease in a small time interval $(t, t+dt)$ is

$$P(X(t+dt) = X(t) - \Delta \mid X(t) = x) = c_1 x\, dt$$
$$P(X(t+dt) = X(t) - k\Delta \mid X(t) = x) = c_2 x\, dt$$

where $X(0) = x_0$ is the initial thickness and c_1, c_2, T_0 and k are the unknown parameters.

Inference for the wear process

Mean field approximation (model 1).

Let us consider a mean field approximation, *i.e.*, the data $\mathbf{y} = \{y_i\}_{i=1}^n$ are modeled according to $y_i = Z(t_i) + \epsilon_i$, where: $Z(t)$ satisfies the mean field ODE

$$\begin{cases} dZ(t) &= c\nu Z(t)dt,\ t \in \mathbb{R}_+ \\ Z(0) &= x_0 \in \mathbb{R}_+. \end{cases}$$

and ϵ_i are i.i.d. realizations of $\mathcal{N}(0, \sigma_E^2)$ for $i = 1, \dots, n$, where $\sigma_E^2 > 0$ is the experimental measurement error. It is natural to take for instance $\sigma_E = \Delta$.

In this case, the likelihood can be written as

$$L(\theta;\mathbf{y}) \propto \prod_{i=1}^n \exp\left\{-\frac{(y_i - Z(t_i;\theta))^2}{2\sigma_E^2}\right\},$$

where $\theta = (c_1, c_2, T_0, k) \in \Theta = \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{Z}_+^d \times \mathbb{N}$

Now, the maximum likelihood estimator (MLE) for θ, is given by the minimizer of minus the log likelihood,

$$\theta^* = \arg\min_{\theta \in \Theta} \sum_{i=1}^n \left\{(y_i - Z(t_i;\theta))^2\right\}.$$

Gaussian approximation based on moment expansion (model 2).

Let us consider a Gaussian approximation up to second order based on moment expansion, *i.e.*the data $\mathbf{y} = \{y_i\}_{i=1}^n$ are modeled according to $y_i = Z(t_i) + \epsilon_i$, where: $Z(t) \sim \mathcal{N}(m(t), \sigma^2(t))$, $m(t)$ and $\sigma^2(t)$ satisfy the following system of ordinary differential equations

$$\begin{cases} dm(t) &= (c_1\nu_1 + c_2\nu_2)m(t)dt, \\ d\sigma^2(t) &= (2(c_1\nu_1 + c_2\nu_2)\sigma^2(t) + (c_1\nu_1^2 + c_2\nu_2^2)m(t))dt, \\ (m(0), \sigma^2(0)) &= (x_0, 0),\ x_0 \in \mathbb{R}_+,\ t \in \mathbb{R}_+, \end{cases}$$

and ϵ_i are i.i.d. realizations of $\mathcal{N}(0, \sigma_E^2)$ for $i = 1, \dots, n$, where $\sigma_E^2 > 0$ is the experimental measurement error. In this work we set $\sigma_E = \Delta$.

In this case, the likelihood can be written as

$$L(\theta;\mathbf{y}) = \prod_{i=1}^n \frac{1}{\sqrt{2\pi(\sigma_E^2 + \sigma^2(t_i;\theta))}} \exp\left\{-\frac{(y_i - m(t_i;\theta))^2}{2(\sigma_E^2 + \sigma^2(t_i;\theta))}\right\}.$$

The MLE for θ is given by the minimizer of minus the log likelihood,

$$\theta^* = \arg\min_{\theta \in \Theta} \sum_{i=1}^n \left\{\frac{(y_i - m(t_i;\theta))^2}{\sigma_E^2 + \sigma^2(t_i;\theta)} + \log(\sigma_E^2 + \sigma^2(t_i;\theta))\right\}.$$

We determine first the minimum conditioned on k and T_0 and then the global optimizer.

Hitting times

The Master Equation.

Let us define τ_B as the first and sole time that the process X is less or equal than B. We have that $F_{\tau_B;\theta}(t) = \mathrm{P}\left(X(t) \le B|\theta\right) = \sum_{x \le B} p(x,t;\theta)$, where $p(x,t;\theta)$ is the probability that $X(t) = x$ given the value of the parameter vector θ. We know that $p(x,t;\theta)$ satisfies the Master Equation [6]:

$$\begin{cases} \frac{dp_x}{dt}(t;\theta) &= \sum_j (p_{x+\nu_j}(t;\theta)a_j(x+\nu_j;\theta) - p_x(t;\theta)a_j(x;\theta)),\ t \in \mathbb{R}_+, x, x+\nu_j \in \mathcal{S} \\ p_x(0;\theta) &= \mathbf{1}_{\{x=x_0\}}, \end{cases} \tag{1}$$

where $\mathbf{1}_{\{x=x_0\}}$ is the indicator function of the set $\{x_0\}$. This is an ODE system which can be efficiently solved by any standard numerical technique.

Conditional residual reliability.

Suppose that we know that the wear process, W, is at level w_0 at time $t_0 \ge 0$. Assume that there exists a critical stopping level, $w_{max} > w_0$, that determines the residual lifetime $\tau_{max} - t_0$. For $t > 0$, the residual lifetime is greater than t, if and only if $W(t_0 + t) < w_{max}$. Therefore, the conditional probability

$$\mathrm{P}\left(\tau_{max} - t_0 > t|W(t_0) = w_0\right) = \mathrm{P}\left(W(t_0+t) < w_{max}|W(t_0) = w_0\right).$$

Taking into account the relation between the wear and the thickness processes, we have that the conditional residual reliability function defined as

$$R(t;t_0,w_0) := \mathrm{P}\left(\tau_{max} - t_0 > t|W(t_0) = w_0\right)$$

can be written as $\mathrm{P}\left(X(t;T_0 - w_0) > T_0 - w_{max}\right)$, where $X(\cdot, x_0)$ is the thickness process starting from x_0.

Figure 3 shows the behavior of the conditional residual reliability function, $R(t;0,w_0)$, for some values of w_0. In this case, we set $w_{max} = 4$. As expected, for a fixed residual lifetime t, we have that $R(t;0,w_0)$ is a decreasing function of w_0.

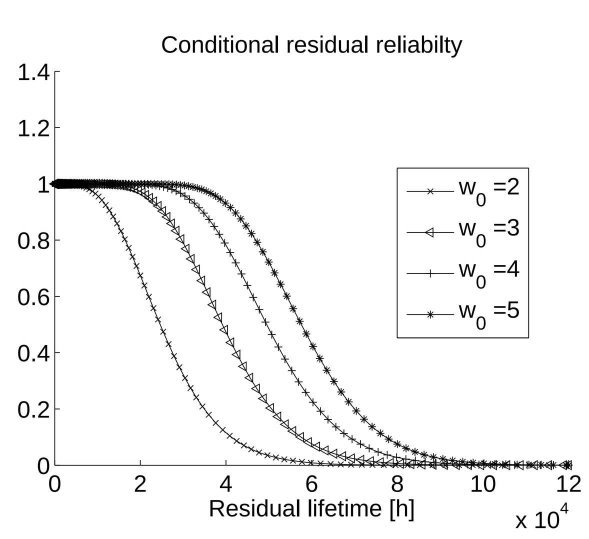

Figure 3: *The conditional residual reliability function, $R(t;0,w_0)$, for some values of w_0.*

Numerical results

The MLE of θ in model 2 is given by $c^* = (0.63 \cdot 10^{-4}, 1.2 \cdot 10^{-4})$, $T_0^* = 5.0$ and $k^* = 4$.

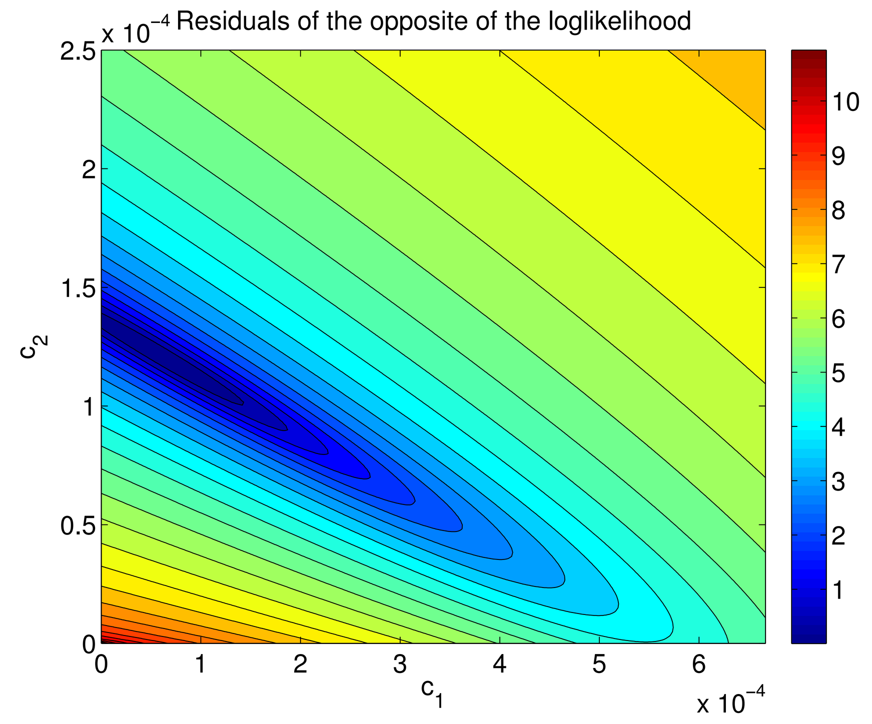

Figure 4: *Square residuals of the likelihood.*

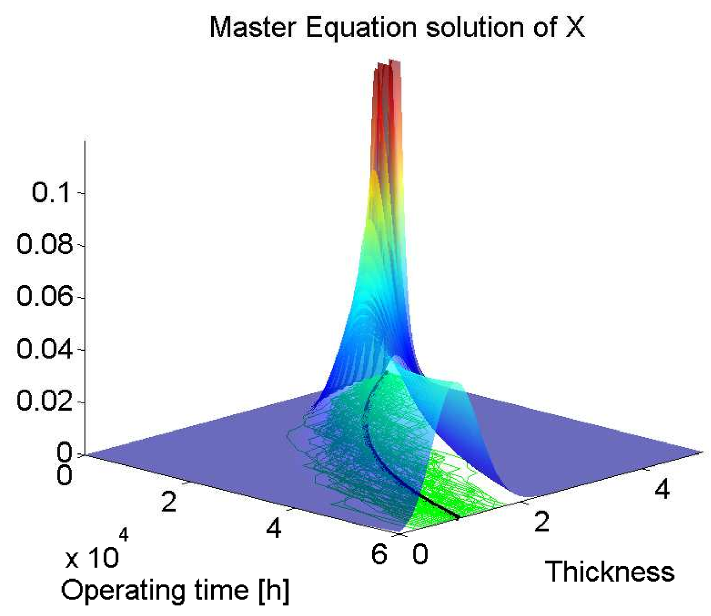

Figure 5: *Solution of the ME* (1) *and 100 exact simulated paths.*

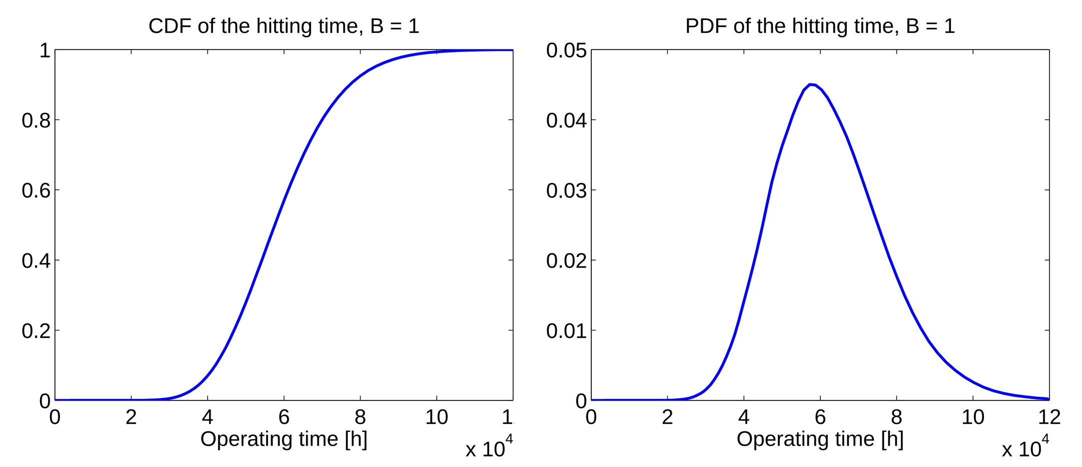

Figure 6: ***Left panel**: CDF of the hitting time for $B = 1$. **Right panel**:PDF of the hitting time to the critical level.*

Conclusions

- This is a novel approach to the problem of modeling the wear process of cylinder liners. We use a continuous time Markov chain in a lattice determined by the caliper precision.
- In contrast to Giorgio et al. [1, 2], we conclude that age-dependent propensity functions are not needed. Also, our noise term is Poissonian, rather than Gamma.
- Main contribution: a multiscale inference approach. The coefficients of the propensity functions were inferred using an upscaled Gaussian approximate model.
- Taking advantage of the one-dimensionality of the state-space and its finiteness, we obtained the probability mass function of the process by solving its Master Equation. From this probability mass function, we computed the cumulative distribution function of the hitting time of the thickness process to the value stipulated in the warranty, conditioned on the propensity coefficient.
- Thanks to the remarkable simplicity of our model, we can easily obtain the distribution of any observable of the process directly from the solution of the Master Equation. It is worth mentioning that we did not use Monte Carlo simulation or any other time-consuming sampling procedure.
- This technique will be extended for Condition Based Maintenance in the context of optimal replacement using techniques from Bayesian Decision Analysis.

Acknowledgements

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