Geological storage of CO2 is an attempt at controlling future climate changes. Modelling and simulation of underground CO2 storage are however subjected to significant sources of geological uncertainties, which require the use of stochastic approaches. Sources of uncertainties in the CO2 storage problem can be classified as either geological, physical, or operational uncertainties. Ranking the importance of the model parameters based on their influence can provide a better understanding of the system. Computationally efficient methods for uncertainty quantification are therefore needed. Furthermore, due to the computational complexity of such problems, even a single deterministic simulation may require parallel high-performance computing, stochastic simulation techniques based on standard Monte Carlo are currently inefficient for these problems. To overcome the prohibitive computational cost of standard Monte Carlo, we propose a Multi-Level Monte Carlo technique to estimate statistical quantities of interest within some prescribed accuracy constraint. We illustrate and verify our proposed approach by comparing it against Monte Carlo simulation using a common problem for CO2 injection. A study of computational speed-up was performed with Monte Carlo.

### Parameter uncertainty with MLMC

In order to define the mathematical model, we write the mass balance equation for the fluid phase, the capillary pressure satisfies

\[ \frac{\partial (\rho \nabla \cdot \mathbf{Q})}{\partial t} + \nabla \cdot (\rho \mathbf{Q}) = 0, \]

where \( K \) is the absolute permeability, \( g \) is the gravity and \( k_{iw} \) is the relative permeability.

The diffusive flux of a component \( j \) in the \( n \) phase is in a porous media is given by

\[ J_n^j = D_n^{\text{diff}} \nabla p_n^j, \quad J_n^j = 0, \]

where \( D_n^{\text{diff}} \) is the diffusion component \( j \) in the \( n \) phase.

The domain dimension is \( 100 \times 100 \) m. CO2 is injected in the left side of the domain, we consider a non flux at the top and bottom boundary and we suppose a open boundaries on the right part of the domain which some of the CO2 would have to escape.

### Physical Hypothesis and Model

In order to define the mathematical model, we write the mass conservation of each component (here \( \beta = \alpha \) denote water (wetting) and \( \beta = \alpha \) denote liquid (non-wetting) phase). (In the following, the index \( r \) will refer to the phase and the exponent to the component).

\[ \begin{align*}
\phi ( \rho_{\text{m,\text{a}}r} \mathbf{V}_r + \rho_{\text{m,\text{n}}r} \mathbf{V}_r ) & = \text{div} \ ( \rho_{\text{m,\text{a}}r} \mathbf{V}_r + \rho_{\text{m,\text{n}}r} \mathbf{V}_r ) + \left( \frac{\partial \rho_{\text{a}}}{\partial t} + \nabla \cdot (\rho_{\text{a}} \mathbf{Q}) \right), \\
\phi ( \rho_{\text{m,\text{a}}r} \mathbf{V}_r + \rho_{\text{m,\text{n}}r} \mathbf{V}_r ) & = \text{div} \ ( \rho_{\text{m,\text{a}}r} \mathbf{V}_r + \rho_{\text{m,\text{n}}r} \mathbf{V}_r ) + \left( \frac{\partial \rho_{\text{a}}}{\partial t} + \nabla \cdot (\rho_{\text{a}} \mathbf{Q}) \right), \end{align*} \]

Here, we note \( \alpha \) the porosity, \( \rho_{\text{m,\text{a}}} \) the molar density of the \( \alpha \) phase, \( \phi \) the saturation, \( V_r \) the velocity of the \( r \) phase, \( \mathbf{Q} \) a diffusion flux of the \( \beta \) component into the \( \alpha \) phase, \( \chi_r \) the molar fraction of \( \beta \) in the \( \alpha \) phase, \( \chi_r^\alpha \) a source term of the \( \alpha \) phase and \( g \) the gravity. In the case of non-isothermal systems, we further add the energy balance equation.

\[ \frac{\partial \rho_{\text{a}}}{\partial t} + \nabla \cdot (\rho_{\text{a}} \mathbf{Q}) = -\text{div} \ (\lambda_{\text{m,\text{a}}} \nabla T_r) + \sum_{\alpha} \phi \rho_{\text{m,\text{a}}} \Delta h_{\alpha} K_{\alpha \beta \text{w}} (p_{\beta \text{w}} - p_{\beta \text{a}}) \]

where \( \rho_{\text{a}} \) is the mass density of the \( \alpha \) phase, \( T_r \) is the temperature, \( \chi_w \) is the specific internal energy, \( h_{\alpha} \) is the specific heat enthalpy, \( \lambda_{\text{m,\text{a}}} \) is the heat conductivity. The saturation of the \( \alpha \) phases and the capillary pressure satisfies

\[ S_\alpha = S_\alpha(r) = \text{min} \left( p_{\alpha w} - p_{\alpha b}, 1 \right), \]

and the molar fraction (used to describe compositions of phases) satisfy

\[ \chi_\alpha + \chi_w = 1, \chi_\alpha^\alpha + \chi_w = 1. \]

The phase velocities \( V_\alpha \) are given by Darcy’s law

\[ V_\alpha = -\frac{k_{\alpha w}}{\mu_{\alpha w}} (\nabla p_{\alpha w} - \rho_{\alpha w} \mathbf{g}), \]

\[ V_w = -\frac{k_{\alpha w}}{\mu_{\alpha w}} (\nabla p_{\alpha w} - \rho_{\alpha w} \mathbf{g}). \]

### References