Performance Limits of Energy Harvesting Communications under Imperfect Channel

State Information
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Abstract
In energy harvesting communications, the transmitters have to adapt transmission to availability of energy harvested during the course of communication. The performance of the transmission depends on the channel conditions which vary randomly due to mobility and environmental changes. In this work, we consider the problem of power allocation taking into account the energy availability over time and the degree of channel state information (CSI) available at the transmitter, in order to maximize the throughput. In this work, the CSI at the transmitter is not perfect and may include estimation errors. We solve this problem with respect to the causality and energy storage constraints. We determine the optimal offline policy in the case where the channel is assumed to be perfectly known at the receiver. Different cases of CSI availability are studied for the transmitter. We obtain the power policy when the transmitter has either perfect CSI or no CSI. We also investigate of utmost interest the case of fading channels with imperfect CSI. Furthermore, we analyze the asymptotic average throughput in a system where the average recharge rate goes asymptotically to zero and when it is very high.

1. System model

![Figure 1: An energy harvesting communication system model with CSI feedback.](image)

Figure 1: An energy harvesting communication system model with CSI feedback.

![Figure 2: Energy arrival and fading channel.](image)

Definitions: We define:
- The average throughput (AT): the throughput of the communication system per second, i.e.,
\[ AT = \frac{1}{T} \sum_{t=1}^{T} T_i, \]
- The average recharge rate (ARR): the average energy harvested over the duration T, i.e.,
\[ ARR = \frac{1}{T} \sum_{t=1}^{T} E_r(t), \]

2. Optimal power policies

2.1 Static channel

Corresponding Optimization Problem
\[
\max_{P_i} \sum_{t=1}^{T} \log(1 + \gamma p_i)
\]
subject to \[
\sum_{t=1}^{T} E_r(t) \leq \sum_{t=1}^{T} E_{n}(t) \leq E_{\text{max}}, \quad \forall i = 1, \ldots, N.
\]

Solution: Water-Filling algorithm.
\[ p_i^* = \frac{1}{\sum_{t=1}^{T} \gamma^t}. \]

2.2 Fading channel without CSI

Corresponding Optimization Problem
\[
\max_{p_i} \sum_{t=1}^{T} \log(1 + \sum_{j=1}^{N} p_{ij} \gamma_j)
\]
subject to \[
\sum_{t=1}^{T} E_r(t) \leq \sum_{t=1}^{T} E_{n}(t) \leq E_{\text{max}}, \quad \forall i = 1, \ldots, N.
\]

Solution: The optimal policy when the CSI is unavailable at the transmitter is the same as the power policy in the case when the channel is static during communication.

2.3 Fading channel with perfect CSI

Corresponding Optimization Problem
\[
\max_{p_i} \sum_{t=1}^{T} \log(1 + \gamma p_i)
\]
subject to \[
\sum_{t=1}^{T} E_r(t) \leq \sum_{t=1}^{T} E_{n}(t) \leq E_{\text{max}}, \quad \forall i = 1, \ldots, N.
\]

Solution: Directional Water-Filling algorithm.
\[ p_i^* = \frac{1}{\sum_{t=1}^{T} \gamma^t}. \]

2.4 Fading channel with imperfect CSI

Channel model
\[ h = \sqrt{T} - \alpha h + \sqrt{h}. \]
where \[ \gamma \] is the error variance, $\alpha \in [0, 1]$. Corresponding Optimization Problem
\[
\max_{p_i} \sum_{t=1}^{T} \log(1 + \gamma p_i)
\]
subject to \[
\sum_{t=1}^{T} E_r(t) \leq \sum_{t=1}^{T} E_{n}(t) \leq E_{\text{max}}, \quad \forall i = 1, \ldots, N.
\]

Solution:
\[ p_i^* = \frac{1}{\sum_{t=1}^{T} \gamma^t}. \]

3. Asymptotic behavior of EH systems

3.1 Low average recharge rate regime

Static channel
\[
\lim_{\text{ARR} \to 0} \frac{AT_{\text{Static}}}{\text{ARR}} = \frac{1}{2}
\]
Fading channel without CSI
\[
\lim_{\text{ARR} \to 0} \frac{AT_{\text{Fading}}}{\text{ARR}} = \frac{1}{2}
\]
Fading channel with perfect CSI
\[
\lim_{\text{ARR} \to 0} \frac{AT_{\text{Fading}}}{{\text{ARR}}} = \frac{1}{2}
\]
Fading channel with imperfect CSI
\[
\lim_{\text{ARR} \to 0} \frac{AT_{\text{Fading}}}{{\text{ARR}}} = \frac{1}{2}
\]

3.2 High average recharge rate regime

Static channel
\[
\lim_{\text{ARR} \to \infty} \frac{AT_{\text{Static}}}{\text{ARR}} = \frac{1}{2}
\]
Fading channel without CSI
\[
\lim_{\text{ARR} \to \infty} \frac{AT_{\text{Fading}}}{\text{ARR}} = \frac{1}{2}
\]
Fading channel with perfect CSI
\[
\lim_{\text{ARR} \to \infty} \frac{AT_{\text{Fading}}}{\text{ARR}} = \frac{1}{2}
\]
Fading channel with imperfect CSI
\[
\lim_{\text{ARR} \to \infty} \frac{AT_{\text{Fading}}}{\text{ARR}} = \frac{1}{2}
\]

4. Results

![Figure 3: Performances of the offline policies for various energy arrival rates under exponential fading channel.](image)

![Figure 4: Performances of the offline policies at low Average Recharge Rate.](image)

![Figure 5: Performances of the offline policies at high Average Recharge Rate.](image)

5. Conclusion

We have investigated the optimal offline power policy for a point-to-point data transmission with energy harvesting transmitter where the CSI is known perfectly at the receiver. For the transmitter, we considered three scenarios depending on the availability of the CSI. We determined the optimal power policy in each scenario, that maximizes the throughput with the knowledge of the energy harvesting profile during the communication. On the other hand, we studied the asymptotic behavior of the average throughput when the average recharge rate is either low or high.

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