Modeling of MAI in UWB System Using MGGD

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Abstract

Multivariate generalized Gaussian density (MGGD) is used to approximate the multiple access interference (MAI) and additive white Gaussian noise in pulse-based ultrawide bandwidth (UWB) system. The MGGD probability density function (pdf) is shown to be a better approximation of a UWB system as compared to Gaussian, Laplacian and Gaussian-Laplacian mixture (GLM). The similarity between the simulated and the approximated pdf is measured with the help of modified Kullback-Leibler distance (KLD). It is also shown that MGGD has the smallest KLD as compared to Gaussian, Laplacian and GLM densities. Finally, a receiver based on the principles of minimum bit error rate is designed for the MGGD pdf.

Problem Statement

- Accurately modeling the exact probability distributed function (pdf) of MAI can result in an improved bit error rate (BER) performance.
- Modeling the total noise (MAI and AWGN noise) with a single variable is not accurate as we may be overestimating or underestimating the mean, variance and kurtosis of the density.
- Establishing the reliability of characterizing the MAI and noise is not straight forward.

Time-Hopped UWB Systems

UWB Signal Format

A UWB signal for the time-hopping binary phase shift keying (TH-BPSK) symbol is described as

$$s^{(k)}(t)=\sqrt{\frac{E_s}{N_s}}\sum_{j=0}^{N_s-1}d^{(k)}\psi(t-jT_f-c_j^{(k)}T_c), \quad (1)$$

where $\psi(t)$ is the transmitted UWB pulse, which satisfies $\int_{-\infty}^{\infty}\psi^2(t)dt=1$. E_s is the symbol energy, N_s is the length of the repetition code, T_f is the frame duration, $d^{(k)}$ and $c_j^{(k)}$ represents the binary data bit transmitted and the TH sequence by the kth user. T_c is the chip duration, and satisfies $N_hT_c\le T_f$ and N_h stands for number of hops.

UWB Channel Model

The UWB channel model is represented as

$$h(t)=\sum_{v=0}^{V-1}\sum_{u=0}^{U-1}h_{u,v}\delta(t-T_v-T_{u,v})=\sum_{l=0}^{L-1}h_l\delta(t-T_l), \quad (2)$$

where $L=UV$ denotes the number of possible resolvable multipath, V denotes the number of clusters and U represents the number of resolvable rays in each cluster.

Receiver Structure

With K number of active TH-BPSK UWB users, the received signal after being transmitted over independent UWB channels is expressed as

$$r(t)=\sqrt{\frac{E_s}{N_s}}\sum_{k=0}^{K-1}\sum_{l=0}^{L-1}\sum_{j=0}^{N_s-1}h_l^{(k)}d^{(k)}\psi_{rec}\left(t-jT_f-c_j^{(k)}T_c-T_l^{(k)}\right)+n(t), \quad (3)$$

where $n(t)$ represents AWGN process, which has zero-mean and a single-sided power spectral density of $N_0/2$ per dimension, $T_l^{(k)}$ takes into account the lack of synchronisation among the interferer's signals as well as the transmission delay. In this case, $T_l^{(k)}$ is modeled as independent and uniformly distributed between $[-T_f/2,T_f/2)$. In (3), $\psi_{rec}(t)$ represents the received time-domain pulse, which is usually the second derivative of the transmitted pulse. The output of the RAKE receiver after N_s pulses and L multipath is written as

$$\boldsymbol{y}=\boldsymbol{s}+\boldsymbol{i}+\boldsymbol{n}, \quad (4)$$

where

$$\boldsymbol{s}=\left[s_{(0,0)},s_{(0,1)},\cdots,s_{(N_s-1,L-1)}\right]^T, \quad (5)$$

$$\boldsymbol{i}=\left[i_{(0,0)},i_{(0,1)},\cdots,i_{(N_s-1,L-1)}\right]^T, \quad (6)$$

$$\boldsymbol{n}=\left[n_{(0,0)},n_{(0,1)},\cdots,n_{(N_s-1,L-1)}\right]^T, \quad (7)$$

respectively. $\boldsymbol{s}$ is the desired signal, $\boldsymbol{i}$ is the multiuser interference, and $\boldsymbol{n}$ is the AWGN. Further analysis reveal that the desired signal is equivalent to

$$s_{(j,l)}=\sqrt{\frac{E_s}{N_s}}h_l^{(0)}d^{(0)}. \quad (8)$$

While, the multiuser interference is given as

$$i_{(j,l)}=\sum_{k=1}^{K-1}\sqrt{\frac{E_s}{N_s}}h_l^{(k)}d^{(k)}R(c_k+\alpha_k)=\sum_{k=1}^{K-1}\sqrt{\frac{E_s}{N_s}}i_{(j,l)}^{(k)}, \quad (9)$$

where $R(\cdot)$ is the autocorrelation function of the UWB pulse defined as

$$R(x)=\int_{-\infty}^{\infty}\psi_{rec}(t-x)\psi_{rec}^*(t)dt \quad (10)$$

and the time shift between the users and TH code is defined as $\alpha_k=T_l^{(k)}-T_l^{(0)}$ and $c_k=(c_j^{(k)}-c_j^{(0)})T_c$, respectively. $n_{j,l}$, in $\boldsymbol{n}$ is expressed as

$$n_{(j,l)}=\int_{jT_f+T_l^{(0)}}^{(j+1)T_f+T_l^{(0)}}n(t)\psi_{rec}^*\left(t-jT_f-c_j^{(0)}T_c-T_l^{(0)}\right)dt \quad (11)$$

which can be shown as Gaussian distributed with mean zero and a variance of $\sigma_n^2=N_0/2$ per dimension.

Total Noise Modeling

The mean, variance and shape parameter are required to characterize the pdf. Therefore, the characteristic function (CF) of MAI is determined, followed by its moments. With known moments, the mean, variance and the kurtosis can be easily calculated. Kurtosis will assist us in determining the shape parameter.

Characteristic Function (CF) of MAI

The CF for known jth frame, lth channel tap of kth user, conditioned on $d^{(k)}$, α_k and c_k is given as

$$\Phi_{i_{(j,l)}^{(k)}|d^{(k)},\alpha_k,c_k}(\omega)=E\left[\exp\left(j\omega h_l^{(k)}d^{(k)}R(c_k+\alpha_k)\right)\right]. \quad (12)$$

After some manipulation and removing the dependencies, the CF of $i_{(j,l)}^{(k)}$ is written as

$$\Phi_{i_{(j,l)}^{(k)}}(\omega)=\int_{\alpha_k}\Phi_{i_{(j,l)}|\alpha_k}(\omega)f_{\alpha_k}(\alpha_k)d\alpha_k=\frac{1}{N_hT_f}\sum_{n=0}^{N_h-1}\int_{-\frac{T_f}{2}}^{\frac{T_f}{2}}\cos\left(\omega h_l^{(k)}R(nT_c+\alpha_k)\right)d\alpha_k \quad (13)$$

Moments of MAI

As the interference is modeled as MGGD, we are only interested in the first, second, and fourth moment. As all the odd moments are zeroes, the mean of the interference is zero. The second moment is given as

$$E\left(\left(i_{(j,l)}^{(k)}\right)\right)^2=\frac{\left(h_l^{(k)}\right)^2}{N_hT_f}\sum_{n=0}^{N_h-1}\int_{-\frac{T_f}{2}}^{\frac{T_f}{2}}R^2(nT_c+\alpha_k)d\alpha_k=\left(h_l^{(k)}\right)^2\sigma_{MAI}^2, \quad (14)$$

where

$$\sigma_{MAI}^2=\frac{1}{T_f}\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty}\psi_{rec}(t-x)\psi_{rec}^*(t)dt\right]^2dx=\frac{1}{T_f}\int_{-\infty}^{\infty}R^2(x)dx. \quad (15)$$

Fourth moment of jth frame, lth multipath and kth user is

$$E\left(\left(i_{(j,l)}^{(k)}\right)\right)^4=\frac{\left(h_l^{(k)}\right)^4}{N_hT_f}\sum_{n=0}^{N_h-1}\int_{-\frac{T_f}{2}-nT_c}^{\frac{T_f}{2}-nT_c}R^4(\alpha_k)d\alpha_k. \quad (16)$$

As interferers are independent, the mean of jth frame and lth multipath will be zero, while the second moment will be determined as

$$E\left(i_{(j,l)}\right)^2=\frac{E_s}{N_s}\sigma_{MAI}^2\sum_{k=1}^{K-1}\left(h_l^{(k)}\right)^2. \quad (17)$$

The fourth moment of the jth frame and lth multipath is given as

$$E\left(i_{(j,l)}\right)^4=E\left(\sum_{k=1}^{K-1}\sqrt{\frac{E_s}{N_s}}\left(i_{(j,l)}^{(k)}\right)\right)^4+6\left(\frac{E_s}{N_s}\right)^2E\left(\sum_{k=1}^{K-1}\sum_{q>k}^{K-1}\left(i_{(j,l)}^{(k)}\right)^2\left(i_{(j,l)}^{(q)}\right)^2\right)$$
$$=\left(\frac{E_s}{N_s}\right)^2\sum_{k=1}^{K-1}E\left(\left(i_{(j,l)}^{(k)}\right)\right)^4+6\left(\frac{E_s}{N_s}\right)^2\sigma_{MAI}^4\sum_{k=1}^{K-1}\sum_{q>k}^{K-1}\left(h_l^{(k)}\right)^2\left(h_l^{(q)}\right)^2 \quad (18)$$

Moments of Total Noise

The total noise for the jth frame and lth multipath, is

$$g_{(j,l)}=i_{(j,l)}+n_{(j,l)}. \quad (19)$$

The mean of total noise is zero, while the variance is given as

$$E\left(g_{(j,l)}\right)^2=E\left(i_{(j,l)}\right)^2+E\left(n_{(j,l)}\right)^2=\frac{E_s}{N_s}\sigma_{MAI}^2\sum_{k=1}^{K-1}\left(h_l^{(k)}\right)^2+\sigma_n^2. \quad (20)$$

The fourth moment is calculated as

$$E\left(g_{(j,l)}\right)^4=E\left(i_{(j,l)}+n_{(j,l)}\right)^4 \quad (21)$$

where $E\left(i_{(j,l)}\right)^2$ and $E\left(i_{(j,l)}\right)^4$ are calculated in (18) and (19), respectively. The second and fourth moments of AWGN are $E\left(n_{(j,l)}\right)^2=\sigma_n^2$ and $E\left(n_{(j,l)}\right)^4=3\sigma_n^4$.

PDF of Total Noise

The pdf of total noise $\boldsymbol{g}=\boldsymbol{i}+\boldsymbol{n}$ is modeled as

$$p(\boldsymbol{g},\Sigma,\beta_T)=\frac{\Gamma\left(\frac{\bar{p}}{2}\right)}{\pi^{\frac{\bar{p}}{2}}\Gamma\left(\frac{\bar{p}}{2\beta_T}\right)2^{\frac{\bar{p}}{2\beta_T}}}\frac{\beta_T}{|\boldsymbol{\Sigma}|^{\frac{1}{2}}}\exp\left(-\frac{\left(\boldsymbol{g}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{g}\right)^{\beta_T}}{2}\right), \quad (22)$$

where $\bar{p}=N_sL$ is the dimensionality of the probability space, β_T is calculated as

$$\frac{\Gamma(1/\beta_T)\Gamma(5/\beta_T)}{\Gamma^2(3/\beta_T)}=\frac{1}{N_sL}\sum_{j=0}^{N_s-1}\sum_{l=0}^{L-1}\left(\frac{E\left(g_{(j,l)}\right)^4}{E^2\left(g_{(j,l)}\right)^2}-3\right), \quad (23)$$

where Σ is defined as

$$\boldsymbol{\Sigma}=\frac{\bar{p}\Gamma\left(\frac{\bar{p}}{2\beta_T}\right)}{2^{\frac{1}{\beta_T}}\Gamma\left(\frac{\bar{p}+2}{2\beta_T}\right)}\boldsymbol{G}, \quad (24)$$

and

$$\boldsymbol{G}=\mathrm{diag}\left[E\left(g_{(0,0)}\right)^2,\cdots,E\left(g_{(N_s-1,L-1)}\right)^2\right]. \quad (25)$$

Receiver Design for UWB Signals

The received signal multiplied by a weighting vector $\boldsymbol{w}$, will give

$$z=\boldsymbol{w}^T\boldsymbol{y}=\boldsymbol{w}^T\boldsymbol{s}+\boldsymbol{w}^T\boldsymbol{g}. \quad (26)$$

As $\boldsymbol{g}$ is the MGGD, therefore, $\boldsymbol{w}^T\boldsymbol{g}$ is also a GG density, where its mean will be $\boldsymbol{w}^T\boldsymbol{s}$ and variance will be $\boldsymbol{w}^T\boldsymbol{G}\boldsymbol{w}$ and the fourth moment is calculated as

$$E\left(\boldsymbol{w}^T\boldsymbol{g}\right)^4=\sum_{j=0}^{N_s-1}\sum_{l=0}^{L-1}E\left(w_{(j,l)}g_{(j,l)}\right)^4+3\sum_{j=0}^{N_s-1}\sum_{l=0}^{L-1}\sum_{k=0}^{N_s-1}\sum_{m=0}^{L-1}E\left(w_{(j,l)}g_{(j,l)}\right)^2$$
$$\times E\left(w_{(k,m)}g_{(k,m)}\right)^2-3\sum_{j=0}^{N_s-1}\sum_{l=0}^{L-1}\left(E\left(w_{(j,l)}g_{(j,l)}\right)^2\right)^2. \quad (27)$$

The excess kurtosis, κ of $\boldsymbol{w}^T\boldsymbol{g}$ is given as

$$\kappa\left(\boldsymbol{w}^T\boldsymbol{g}\right)=\frac{\Gamma(1/\beta)\Gamma(5/\beta)}{\Gamma^2(3/\beta)}=\frac{E\left(\boldsymbol{w}^T\boldsymbol{g}\right)^4}{E^2\left(\boldsymbol{w}^T\boldsymbol{g}\right)^2}-3, \quad (28)$$

where (28) can be solved numerically to get the optimal β the shape parameter of the generalized gaussian density. Finally, the pdf of z can be represented as

$$p\left(z|d^{(0)}\right)=\frac{\beta}{2\alpha\Gamma\left(\frac{1}{\beta}\right)}\exp\left(-\left(\frac{\left|z-\boldsymbol{w}^T\boldsymbol{s}\right|}{\alpha}\right)^{\beta}\right), \quad (29)$$

where

$$\alpha^2=\frac{\Gamma\left(\frac{1}{\beta}\right)}{\Gamma\left(\frac{3}{\beta}\right)}\boldsymbol{w}^T\boldsymbol{G}\boldsymbol{w}. \quad (30)$$

For equal likely data bits, TH-BPSK UWB system the P_e is calculated as

$$P_e(\boldsymbol{w})=\int_{-\infty}^{0}p(z|d^{(0)}=+1)dz=\frac{1}{2}-\frac{\mathrm{sgn}(\boldsymbol{w}^T\boldsymbol{s})}{2\Gamma\left(\frac{1}{\beta}\right)}\gamma\left(\frac{1}{\beta},\left(\frac{|\boldsymbol{w}^T\boldsymbol{s}|}{\alpha}\right)^{\beta}\right), \quad (31)$$

where, $\mathrm{sgn}(\cdot)$ is the sign function and $\gamma(\cdot)$ is the lower incomplete gamma function.It can be observed from (31) that the P_e is a function of $\boldsymbol{w}$ vector, therefore, in order to minimize the P_e we need to design optimal weights. Now steepest-descent algorithm is applied to find the minima of the function.

$$\boldsymbol{w}(i+1)=\boldsymbol{w}(i)-\mu\nabla P_e(\boldsymbol{w}(i)), \quad (32)$$

where i represents the ith iteration, μ is the step-size and $\nabla P_e(\boldsymbol{w})$ is the gradient with respect to $\boldsymbol{w}$.

Simulation Results and Discussion

A modified KLD is applied

$$K(p_1,p_2)=\int_{-\infty}^{\infty}p_1(x)\ln\frac{p_1(x)}{\frac{1}{2}p_1(x)+\frac{1}{2}p_2(x)}dx. \quad (33)$$

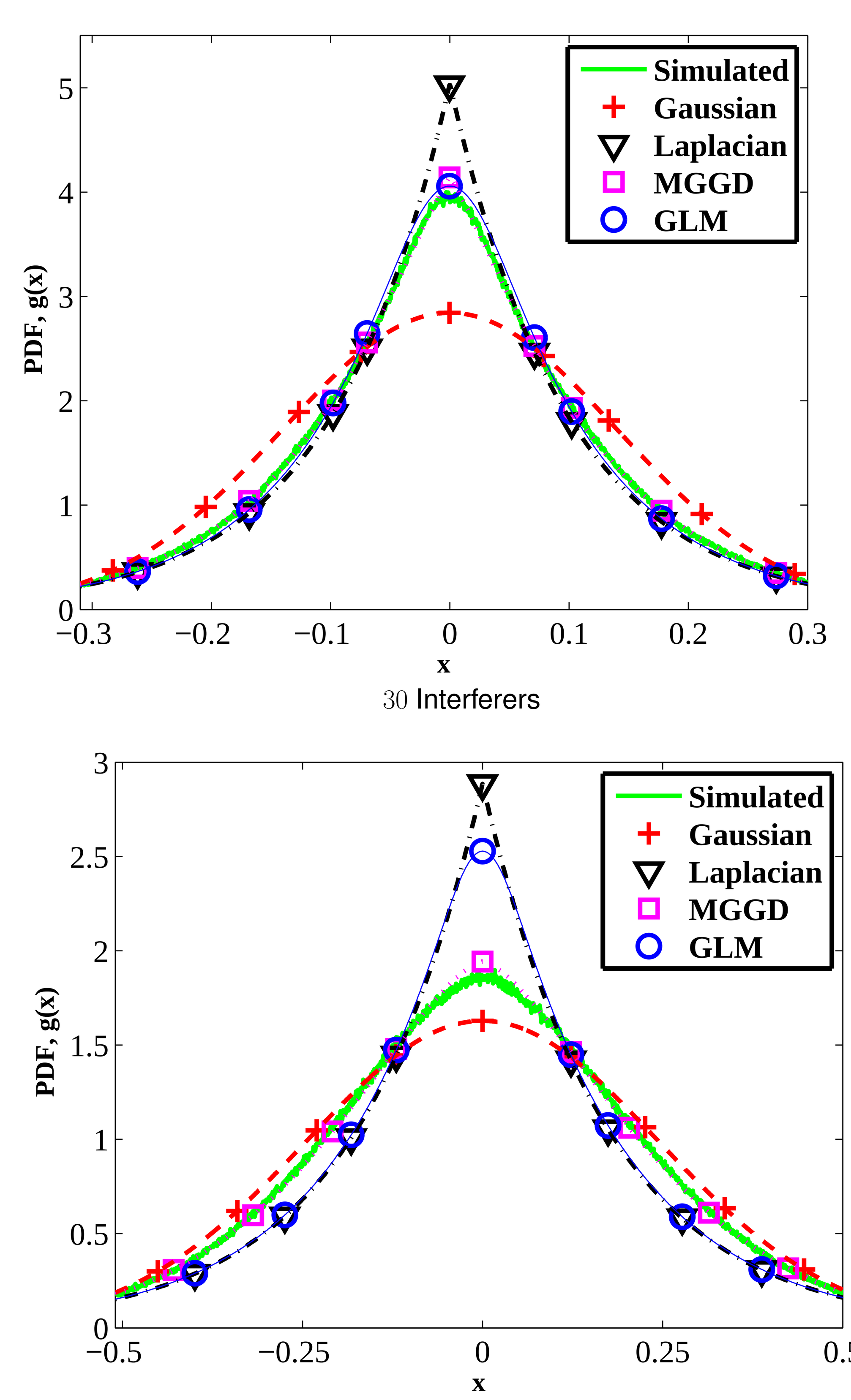

Figure 2: *Comparison of the pdf of the MAI component i plus noise n, with Gaussian, Laplacian, Gaussian-Laplacian mixture and Generalized Gaussian approximations with different number of interferers. The variance of noise is 30-dB.*

Table 1: *Jenson-Shannon Divergence between the simulated PDF and the approximated PDFs.*

Interferers	3	7	10	15	25	30	35	45	50
Gaussian	76.24	30.14	15.52	7.99	3.22	2.37	1.87	1.59	1.14
Laplacian	9.07	0.88	1.57	4.79	8.755	9.64	10.22	10.79	10.77
GLM	23.96	3.24	1.27	3.18	7.10	8.17	8.91	9.75	9.82
MGGD	4.49	0.855	0.468	0.144	0.14	0.15	0.16	0.153	0.148
beta	0.426	0.501	0.59	0.652	0.727	0.75	0.767	0.794	0.802

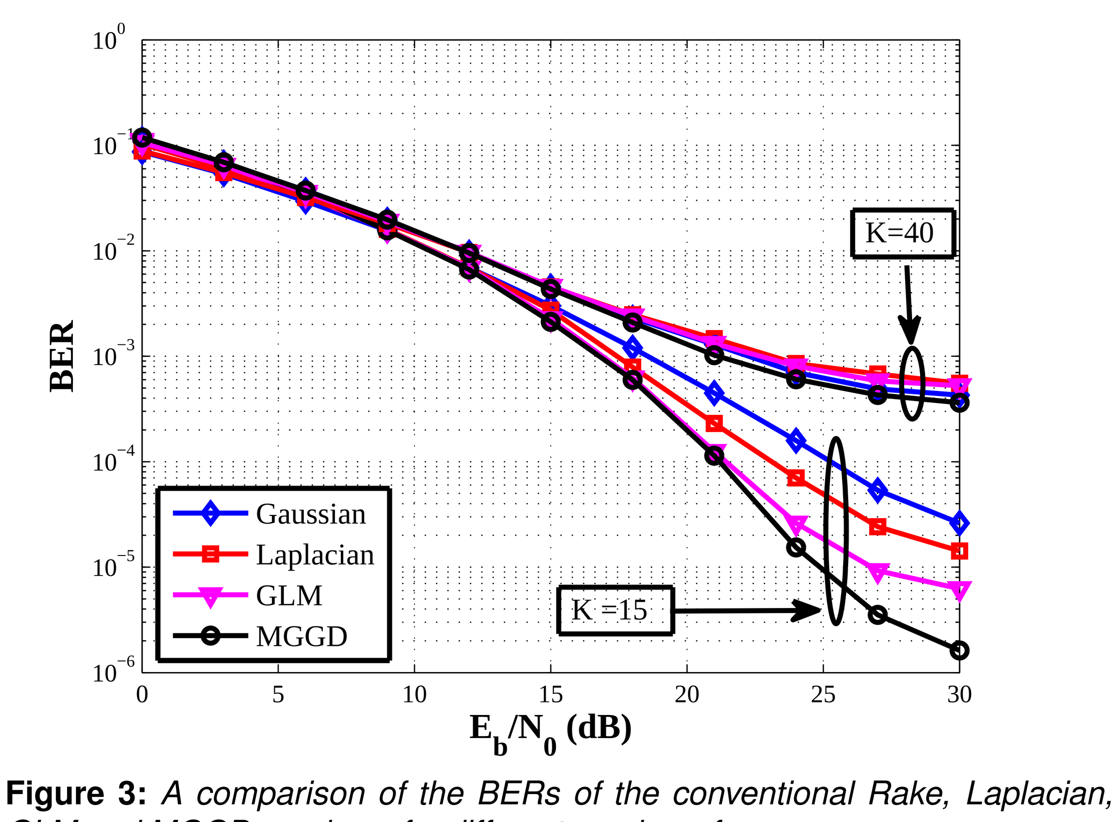

Figure 3: *A comparison of the BERs of the conventional Rake, Laplacian, GLM and MGGD receivers for different number of users.*

Conclusions

- The performance of pulse-based UWB system was investigated when the total noise which is the summation of the MAI and AWGN is modeled as MGGD.
- The modified KLD of MGGD is much smaller than that of the Gaussian, Laplacian and GLM detectors.
- Linear detector for the MGGD pdf is proposed which minimizes the BER of the system.
- It can be observed that this detector outperforms the Gaussian, Laplacian and GLM detector in terms of BER.

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