A Numerical Framework for Linear and Nonlinear Simulations of Richtmyer-Meshkov Instability in Magnetohydrodynamics

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Abstract: Richtmyer-Meshkov (RMI) instability is a fundamental physical process that occurs during high-speed compression, and is critical for understanding the generation of turbulence and shock waves in astrophysics and geophysics. We present a comprehensive numerical framework that is capable of simulating both linear and nonlinear RMIs. This framework employs state-of-the-art numerical methods and can be used to explore a wide range of physical scenarios.

Introduction

- Magnetohydrodynamics (MHD) is a widely used fluid model for plasma dynamics and has been widely used to investigate shock-driven instabilities, such as the Richtmyer-Meshkov instability (RMI), in the context of inertial confinement fusion.
- A numerical framework for linear and nonlinear simulations of ideal MHD flows is developed.
- Linearized initial value problem is solved by an explicit time marching upwind approach using seven wave method[6].
- A second-order adaptive mesh (AMR) MHD code is developed by combining both upwind method[1] and a Cartesian adaptively refined mesh of Berger-Cola type using the Chombo library[2].
- A robust finite volume scheme for two-fluid ideal MHD model is included in our framework[6].

Linear simulations

Governing equations in cylindrical geometry: \( \frac{\partial U}{\partial t} + \nabla \cdot F = S \)

- \( U(r, \theta, t) = (\rho, \rho u, \rho v, B_r, B_\theta) \)
- \( F = (F_r, F_\theta, F_z) \)

Linear Stability Analysis: The ideal MHD equations are linearized by \( \tilde{U} = \tilde{U}(r, \theta, t) + e \tilde{U}(r, \theta, t) \)

Nonlinear simulations

- MHD solver with AMR
- Usplit method
- Chombo library
- Projection method for divergence cleaning

Converging shock-interface intersection: \( x \approx 0 \) \( \beta \approx 1 \)

Effect of different \( \beta \) and \( m = 256 \) for \( A > 0 \)

Effect of different \( \beta \) and \( m = 256 \) for \( A < 0 \)

Evolution of the perturbations

Two-fluid MHD simulations

Non-dimensional two-fluid MHD equations are \( \frac{\partial U}{\partial t} + \nabla \cdot F = S \)

Explicit-implicit method[8]:
1. Compute \( U^1 \) without source terms.
2. Update \( U^{n+1} \) by solving
   \( \frac{\partial U^{n+1}}{\partial t} + \nabla \cdot F = S \)

Generalize Ri-Wo problem

Summary

- We have developed a numerical framework for linear and nonlinear simulations of RMI.
- For linear MHD simulations, a numerical scheme is developed to remove the singularity of the magnetic field.
- For nonlinear MHD simulations, a second-order AMR code and a fourth-order finite volume scheme are developed.

References
2. P. Colella and et al. Chombo software package for AMR applications design document.
8. R. Abgrall, H. Kumar, Robust finite volume schemes for two-fluid plasma equations, J. Sci. Comp. 60, 584 (2014)

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