Precoding Design of MIMO Amplify-and-Forward Communication System with an Energy Harvesting Relay and Possibly Imperfect CSI

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Abstract—In this paper, we investigate the simultaneous wireless information and power transfer (SWIPT) in a Multiple-Input Multiple-Output (MIMO) Amplify-and-Forward (AF) relay communication system where the relay is an energy harvesting (EH) node and harvests the energy the signals transmitted from the source. The harvested energy is partially used to forward signals from the source to the destination, and the remaining energy is stored for other usages. The SWIPT in relay-assisted communication is interesting as long as the relay stores energy from the source and the destination receives successfully the data from the source. In this context, we propose to investigate the source and relay precoders that characterize the relationship between the achievable stored energy at the relay and the achievable source-to-destination rate, namely the rate-stored energy (R-E) tradeoff region. First, we consider the ideal scheme where there is the simultaneous operation of the EH and ID receivers at the relay. Then, we consider practical schemes such as the power splitting (PS) and the time switching (TS) that separate the operation of EH and information decoding (ID) receivers over power domain or time domain, respectively. Moreover, we study the case of imperfect channel state information (CSI) at the relay and the destination and characterize its impact on the achievable R-E region. Through the simulation results, we show the effect of the position of the relay and the channel uncertainty on the achievable R-E regions of all the schemes when the used energy at the relay is constant or variable. We also show that, although it provides an outer bound on the achievable rate-energy region in one-hop MIMO systems, the ideal scheme provides only an upper bound on the maximum achievable end-to-end rate and not an outer bound on the R-E region.

Index Terms—Simultaneous Wireless Information and Power Transfer (SWIPT), Multiple-Input Multiple-Output (MIMO), Amplify-and-Forward (AF) relay, rate-stored energy (R-E) tradeoff region, ideal scheme, Power Splitting (PS), Time Switching (TS), achievable end-to-end rate, harvested energy, stored energy.

I. Introduction

Harvesting energy from the environment is a promising technique to prolong the lifetime of the battery powered wireless communication system and make it self-sustaining. Solar, wind, and vibration are known to be the classical sources of harvesting energy. However, the radio frequency

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(RF) signals from ambient transmitters have recently been considered as another possible source of wireless power transfer (WPT). Passive radio-frequency identification (RFID) systems [1] and body sensor networks (BSNs) for medical implants [2] are, for instance, flagship applications that have successfully implemented RF energy harvesting. On the other hand, it is well known that RF signals are used for wireless information transfer (WIT). As a result, simultaneous wireless information and power transfer (SWIPT) has been widely investigated where RF signals are simultaneously used to transmit information and scavenge energy.

The study of SWIPT in one-hop communication systems has captured a lot of research interest. Single-input single-output (SISO) flat-fading channels and frequency selective channels were considered in [3] and [4] where the optimal tradeoff between information rate and energy transfer was investigated. These works assumed that the information decoding (ID) and energy harvesting (EH) could be done with the same circuit technologies, namely co-located receiver. However, practical receiver designs are not yet able to decode information and harvest energy simultaneously. Thus, [5] extended the work done in [3] and [4] to the multiple-input multiple-output (MIMO) broadcasting system and considered two practical schemes where the receiver separates the ID and EH transfer over the power domain (known as the power splitting (PS) scheme) or the time domain (known as the time switching (TS) scheme), namely separated receivers. More specifically, in [5], the tradeoff region between the achievable information rate and the achievable harvested energy, namely the rate-energy tradeoff region, was characterized for the co-located and the separated receivers in a one-hop MIMO communication system and it has been shown that the ideal scheme provides an outer bound on the achievable rate-energy region.

Relay communication systems are known to further improve the channel capacity and the energy consumption [6]–[9]. So, it is interesting to investigate the concept of SWIPT in relay-assisted communications. Among relaying protocols, the amplify-and-forward (AF) and the decode-and-forward (DF) are the most famous ones. On the one hand, the SWIPT technique in DF-based relay systems was studied in [10]–[12] where the relay is an EH node and where the nodes are equipped with either single or multiple antennas. In [10], a two-hop SISO orthogonal frequency division multiplexing (OFDM) DF relay system was investigated where the relay harvests the energy from the source. The PS scheme was

considered at the relay and the resource allocation was studied to maximize the total achievable transmission rate. In [11], a wireless DF cooperative network with multiple pairs of source and destination and an EH single antenna relay using the PS scheme was studied, and the impact of various power allocation strategies on the end-to-end transmission reliability was investigated. In [12], a DF MIMO relay system was considered where the source and the destination have single antennas and the relay is an EH multiple-antenna node. An antenna switching (AS) policy was considered where the strongest antennas are exploited for information decoding while the others are used for energy harvesting (and vice versa). In [13], the throughput maximization problem was considered in MIMO DF relay system with an EH relay and possibly imperfect CSI.

On the other hand, the SWIPT technique in AF-based relay systems was also studied along with an EH node and with single/multiple antennas at each node in [14]–[20]. In [14], a two-hop SISO AF relay system was considered where the relay harvests the energy from the source and uses it to forward the signal to the destination. This work analyzed the outage probability and the ergodic capacity for delay-limited and delay-tolerant transmission modes. However, the channel state information (CSI) is assumed to be known only at the destination. In [15], the authors extended the work done in [14] to a continuous and discrete adaptive TS protocol where both AF and DF relay networks were considered. Analytic expressions of the achievable throughput for both cases were derived. In [16], a two-hop multi-antenna AF relay system was investigated in the presence of a separate multi-antenna EH receiver where the source and the relay nodes employ orthogonal space-time block codes (STBC) for data transmission. When full instantaneous CSI is available, the optimal source and relay precoders were jointly optimized to achieve the rate-energy tradeoffs between the harvested power at the EH receiver and the information rate at the destination node. When only second order statistics of the CSI is available, the tradeoff between the outage probability and energy was characterized. In [16], the harvested power from the signal transmitted by the source and the relay was gathered in an independent node that coexists with the two-hop MIMO relay system. In [17], a two-hop multi-antenna AF EH relay system was considered where the source and destination have single antennas, and co-channel interference (CCI) may occur at the EH relay. The expressions of the outage probability and ergodic capacity were investigated with/without CCI. In [18], a harvest-then-cooperate protocol was proposed in a cooperative network consisting of a hybrid access point (AP), a source and an AF-based relay where the source and relay harvest energy from the AP in the downlink and cooperate in the uplink for the source information transmission. The approximate expression of the average throughput was derived for Rayleigh fading channels assuming delay-limited transmission mode and the results were extended to the multi-relay scenario. In [19], a two-hop full-duplex relay system was considered where the relay was an EH node equipped with two antennas, and the TS technique was adopted. Both AF and DF relaying protocols were studied, and analytical characterization of the

achievable throughput of three different communication modes was analyzed. In [20], the beamforming design of the SWIPT in a two-way AF relay system with imperfect CSI where two sources communicate via K relays and all the nodes are equipped with single antennas.

In this paper, we propose to study the source/relay precoders design for the SWIPT in MIMO AF relay system with an EH relay. The relay harvests the energy from the source's signals, uses partially the harvested energy to forward the source's signals to the destination and stores the remaining energy for other usages. In our work, we optimize the source and relay precoders to characterize the R-E tradeoff region between the achievable end-to-end rate and the achievable stored energy at the relay. The main contributions of this work are listed as follows:

- The investigation of the joint source/relay precoders design in MIMO AF relays systems with an EH multiple antenna relay and with possibly imperfect CSI.
- The study of the partial use of harvested energy at the relay and the storage of the remaining energy, by contrast to most works where the relay fully uses the harvested energy to assist the transmission from the source to the destination.
- The characterization of the tradeoff region between the achievable end-to-end rate and the stored energy at the relay in order to characterize the mutual benefit at the relay (in terms of stored energy) and the destination (in terms of data).
- The study of the ideal scheme which usually provides an outer bound on the rate-energy region in one-hop systems and the illustration that its R-E region is no more an outer bound on the R-E regions of the other schemes in two-hop relay systems.
- The study of PS and TS practical schemes in MIMO AF relay systems and the characterization of their R-E regions.
- The study of imperfect CSI at the AF-based relay and the destination and its impact on the achievable R-E tradeoff region of the ideal scheme.

II. System Model

We consider a MIMO AF relay communication system where the source, the relay, and the destination are equipped with N_s, N_r, and N_d antennas, respectively. The channel between the source and the relay and the channel between the relay and the destination are denoted $\boldsymbol{H} \in \mathbb{C}^{N_r \times N_s}$ and $\boldsymbol{G} \in \mathbb{C}^{N_d \times N_r}$, respectively, and are assumed to be quasi-static block-fading channels. The slot duration is considered to be sufficiently small compared to the coherence time of the channel. It is assumed to be normalized to 1, without loss of generality. Let us denote the singular value decomposition (SVD) of $\boldsymbol{H}$ and $\boldsymbol{G}$ as

$$\boldsymbol{H} = \boldsymbol{U}_H \boldsymbol{D}_H^{1/2} \left(\boldsymbol{V}_H\right)^H, \qquad (1)$$

$$\boldsymbol{G} = \boldsymbol{U}_G \boldsymbol{D}_G^{1/2} \left(\boldsymbol{V}_G\right)^H, \qquad (2)$$

respectively, where $\boldsymbol{U}_H$, $\boldsymbol{V}_H$, $\boldsymbol{U}_G$, and $\boldsymbol{V}_G$ are unitary matrices with dimensions $N_r \times r_1$, $r_1 \times N_s$, $N_d \times r_2$, and $r_2 \times N_r$,

respectively, $\boldsymbol{D}_H$, and $\boldsymbol{D}_G$ are the diagonal matrices containing the eigenvalues of $\boldsymbol{H}\boldsymbol{H}^H$ and $\boldsymbol{G}\boldsymbol{G}^H$ arranged in a decreasing order, respectively, and r_1 and r_2 are the ranks of $\boldsymbol{H}$ and $\boldsymbol{G}$, respectively. Unless specified, we assume that the CSI is perfectly known at the three nodes.

We assume that the source cannot directly transmit to the destination because the direct link between the source and destination suffers from severe path loss and shadowing. The source and destination are battery-powered, while the relay is an EH node equipped with EH and ID receivers and harvests the energy from the received signals from the source while assisting the transmission to the destination. We assume that the EH and ID receivers are operating at the same frequency.

The ID receiver at the relay operates in a half-duplex mode, i.e. the signal from the source to the relay and the signal from the relay to the destination are transmitted over two separate time instants. Note that, unless specified, the energy and information transmissions from the source to the relay occur simultaneously during the first half of the time slot and the information transmission from the relay to the destination occurs during the second half of the time slot.

A. Source-to-Relay Transmission

First, the source transmits the $N_s \times 1$ precoded vector $\boldsymbol{x}_s$ to the relay through the $N_r \times N_s$ block-fading channel $\boldsymbol{H}$ which is corrupted by the additive white Gaussian noise (AWGN) vector $\boldsymbol{n}_r$. The $N_r \times 1$ received vector at the relay $\boldsymbol{y}_r$ is given by

$$\boldsymbol{y}_r = \boldsymbol{H}\boldsymbol{x}_s + \boldsymbol{n}_r, \tag{3}$$

where $\boldsymbol{x}_s$ is the transmitted signal from the source whose covariance matrix is $\boldsymbol{S} = \mathbb{E}\left[\boldsymbol{x}_s\boldsymbol{x}_s^H\right]$, and $\boldsymbol{n}_r$ is the $N_r \times 1$ AWGN vector whose entries are independent identically distributed (i.i.d.) and drawn from the Gaussian distribution with zero mean and variance equal to σ_r^2. Furthermore, we assume that the source has an average power constraint, i.e. $\mathbb{E}\left[\|\boldsymbol{x}_s\|^2\right] = tr(\boldsymbol{S}) \le P_s$, where $\mathbb{E}[\cdot]$ and $tr(\cdot)$ denote the mean and the trace operators, respectively.

Then, the ID receiver at the relay amplifies the received signal $\boldsymbol{y}_r$ by the $N_r \times N_r$ relay precoder matrix $\boldsymbol{F}$. The baseband transmitted signal from the relay, denoted as $\boldsymbol{x}_r$, is given by

$$\boldsymbol{x}_r = \boldsymbol{F}\boldsymbol{y}_r = \boldsymbol{F}\boldsymbol{H}\boldsymbol{x}_s + \boldsymbol{F}\boldsymbol{n}_r, \tag{4}$$

whose covariance matrix is given by

$$\mathbb{E}\left[\boldsymbol{x}_r\boldsymbol{x}_r^H\right] = tr\left(\boldsymbol{F}\left(\boldsymbol{H}\boldsymbol{S}\boldsymbol{H}^H + \sigma_r^2 I_{N_r}\right)\boldsymbol{F}^H\right), \tag{5}$$

which should be constrained by the average transmit power at the relay $P_r(\boldsymbol{S})$, defined in what follows. As said previously, the relay is not battery powered and its power comes from the energy harvesting process explained in what follows.

B. Energy Harvesting at the Relay

The EH receiver at the relay converts the RF received signal $\boldsymbol{y}_r$ to a direct current (DC) by the rectifier without the need for the conversion from RF band to baseband [21]. Thanks to the law of energy conservation, the harvested energy at the relay (unit of energy), denoted as Q_r, is given by

$$Q_r(\boldsymbol{S}) = \frac{1}{2}\zeta\,\mathbb{E}\left[\|\boldsymbol{H}\boldsymbol{x}_s\|^2\right] = \frac{1}{2}\zeta\,tr\left(\boldsymbol{H}\boldsymbol{S}\boldsymbol{H}^H\right), \tag{6}$$

where $\zeta \in [0, 1]$ is the conversion efficiency. The relay uses a part of the harvested energy to forward the signal from the source to the destination. The used energy at the relay is denoted as $E_u(\boldsymbol{S})$ and is either constant or variable.

$$E_u(\boldsymbol{S}) = \begin{cases} \bar{E}_u, & \text{if the used energy is constant,} \\ \nu\,Q_r(\boldsymbol{S}), & \text{otherwise,} \end{cases} \tag{7}$$

where ν is the ratio between the used energy and the harvested energy and $0 \le \nu \le 1$. Certainly, $E_u(\boldsymbol{S})$ should verify: $E_u(\boldsymbol{S}) \le Q_r(\boldsymbol{S})$, for both cases. The transmit power at the relay $P_r(\boldsymbol{S})$ is given by

$$P_r(\boldsymbol{S}) = 2E_u(\boldsymbol{S}), \tag{8}$$

which is also either constant or variable, depending on $E_u(\boldsymbol{S})$. To alleviate any confusion, recall that the power is the ratio between the energy and the usage duration. We assume that the relay has an infinite capacity of storage. Hence, the remaining energy at the relay after transmission to the destination is stored and given by

$$E_s(\boldsymbol{S}) = \begin{cases} Q_r(\boldsymbol{S}) - \bar{E}_u, & \text{if the used energy is constant,} \\ (1-\nu)\,Q_r(\boldsymbol{S}), & \text{otherwise,} \end{cases} \tag{9}$$

with $E_s(\boldsymbol{S}) \ge 0$, for both cases. Note that if $E_u(\boldsymbol{S}) = \bar{E}_u$, maximizing $E_s(\boldsymbol{S})$ is equivalent to maximizing $Q_r(\boldsymbol{S})$.

C. Relay-to-Destination Transmission

Afterwards, the $N_d \times 1$ received vector at the destination $\boldsymbol{y}_d$ is expressed as

$$\boldsymbol{y}_d = \boldsymbol{G}\boldsymbol{x}_r + \boldsymbol{n}_d = \boldsymbol{G}\boldsymbol{F}\boldsymbol{H}\boldsymbol{x}_s + \boldsymbol{G}\boldsymbol{F}\boldsymbol{n}_r + \boldsymbol{n}_d, \tag{10}$$

where $\boldsymbol{n}_d$ is the $N_d \times 1$ AWGN vector whose entries are i.i.d. and drawn from the Gaussian distribution with zero mean and variance equal to σ_d^2. The source-destination achievable rate of the MIMO AF relay system, in bits/s/Hz, is known to be given by [22]

$$R(\boldsymbol{S},\boldsymbol{F}) = \frac{1}{2}\log_2\left|\boldsymbol{I} + \boldsymbol{G}\boldsymbol{F}\boldsymbol{H}\boldsymbol{S}\boldsymbol{H}^H\boldsymbol{F}^H\boldsymbol{G}^H\left(\sigma_r^2\boldsymbol{G}\boldsymbol{F}\boldsymbol{F}^H\boldsymbol{G}^H + \sigma_d^2\boldsymbol{I}_{N_d}\right)^{-1}\right|. \tag{11}$$

The relay assists the transmission to the destination as long as the stored energy $E_s(\boldsymbol{S})$ responds to its energy storage requirements. The destination allows the relay to take profit from its information data as long as its transmission rate is maximized. Hence, the SWIPT in relaying is interesting as long as the relay is storing energy and the destination receives successfully the transmitted data from the source. But, the more the relay gains energy, the less the transmit power at the relay P_r is and the less the achievable rate R is, and vice versa. Consequently, we need to investigate the tradeoff region between the maximum achievable rate (in bits/s/Hz) for information transfer and the maximum stored energy (in units of energy) for energy transfer at the relay, namely the R-E tradeoff region. First, we consider the achievable R-E region of the ideal scheme where the EH and ID receivers at the relay operate simultaneously. Then, we study the achievable R-E regions for the practical PS and TS schemes which separate the operation of the EH and ID receivers at the relay over the power and the time domain, respectively.

III. Ideal Scheme: An Outer Bound or Not

In this section, we assume that the EH and ID receivers at the relay can operate simultaneously and have access to the whole received signal and its energy. This ideal case serves as an outer bound on the rate-energy tradeoff region in a one-hop MIMO system. Here, we propose to check if it is still an outer bound on the R-E region in the two-hop MIMO systems. For a given transmit power constraint at the source P_s, the R-E tradeoff region of the ideal scheme is given by

$$C_{R-E}(P_s) = \left\{ (R, E_s) ; R \leq \frac{1}{2} \log_2 \left| I + GFHSH^H F^H G^H \right. \right.$$

$$\left. \times \left(\sigma_r^2 GFF^H G^H + \sigma_d^2 I_{N_d} \right)^{-1} \right|, E_s \leq \frac{1}{2} \zeta \ tr\left(HSH^H\right) - E_u(S),$$

$$\left. tr\left(F\left(HSH^H + \sigma_r^2 I_{N_r}\right)F^H\right) \leq 2E_u(S), tr(S) \leq P_s, S \geq 0 \right\}.$$
(12)

Let $(E_{s,max}, R_{EH})$ and $(E_{s,ID}, R_{max})$ be the boundary points of the R-E tradeoff region corresponding to the maximum stored energy at the relay and the maximum achievable end-to-end rate, respectively. The remaining boundary of the R-E region over $R_{EH} < R < R_{max}$ is defined for $E_{s,ID} < E_s < E_{s,max}$. Consequently, the optimal S and F are solutions to the following optimization problem:

$$(P) : \max_{S,F} \quad R(S,F),$$
(13a)

$$\text{s.t.} \quad \frac{1}{2} \zeta \ tr\left(HSH^H\right) - E_u(S) \geq E_s,$$
(13b)

$$tr\left(F\left(HSH^H + \sigma_r^2 I_{N_r}\right)F^H\right) \leq 2E_u(S),$$
(13c)

$$tr(S) \leq P_s, S \geq 0,$$
(13d)

where (13b) is the energy storing constraint at the relay, (13c) is the average power constraint at the relay, and (13d) is the average power constraint at the source.

Obviously, the problem (P) is not a convex optimization problem [23] since the objective function is not concave with respect to F, and the constraints are not over a convex set. Global optimal solutions of S and F are not guaranteed. In this work, we propose to modify the problem (P) in a way to obtain a convex optimization problem. More details can be found in Appendix A. First, we investigate the optimal structure of the source covariance matrix S and the relay amplification matrix F. Assuming the fact that $rank(S) = rank(F) = N_b \leq \min(r_1, r_2)$, the optimal S and F have the following structure

$$S = V_{H,1} D_S (V_{H,1})^H,$$
(14)

$$F = V_{G,1} D_F (U_{H,1})^H,$$
(15)

where D_S and D_F are $N_b \times N_b$ diagonal matrices, and $V_{H,1}$, $V_{G,1}$, and $U_{H,1}$ contain the first N_b columns of V_H, V_G, and U_H, respectively. The proofs of (14) and (15) are given in the Appendix B. The optimal structure of the source and relay amplification matrices jointly diagonalize the source-relay-destination channel that becomes equivalent to a set of parallel SISO channels. This result is alike to the MIMO AF relay system with a non-energy harvesting relay. Given (14) and (15), the equivalent optimization problem uses the moderate-to-high signal-to-noise ratio (SNR) approximation to modify it to an equivalent convex optimization problem

where the global convergence is guaranteed. We can use either the Karush-Kuhn-Tucker (KKT) conditions or the semidefinite programming (SDP) formulation to solve the obtained convex problem. The procedure is explained in more details in Appendix A. Now, let us consider the two operation modes at the relay when the used energy is either constant or variable and explain how to solve (P) for each operation mode.

A. Fixed Used Energy at the Relay

If the used energy is constant, i.e. $E_u(S) = \bar{E}_u$, maximizing the stored energy $E_s(S)$ becomes equivalent to maximizing the harvested energy at the relay $Q_r(S)$. Let us denote by $Q_r = E_s + \bar{E}_u$, where E_s is defined in (13b). After some mathematical manipulations as in Appendix A, we show that Problem (P) is equivalent to the convex optimization problems (P1) or (P2), with $Q_r = E_s + \bar{E}_u$ and $P_r(S) = 2\bar{E}_u$. Here, the two extreme points $E_{s,max}$ and $E_{s,ID}$ are given by

$$E_{s,max} = Q_{r,max} - \bar{E}_u,$$
(16)

$$E_{s,ID} = Q_{r,ID} - \bar{E}_u,$$
(17)

where $Q_{r,max} = \frac{1}{2} \zeta \lambda_{H,1} P_s$ is the maximum harvested energy at the relay defined in (80) in Appendix C, $v_{H,1}$ is the eigenvector of $H^H H$ which corresponds to the maximum eigenvalue $\lambda_{H,1}$ of $H^H H$, and $Q_{r,ID}$ is the harvested energy corresponding to the maximum achievable end-to-end rate defined in Appendix D. The two rates R_{EH} and R_{max} are given by (84) and (86), respectively, for $P_r(S_{EH}) = P_r(S_{ID}) = 2\bar{E}_u$.

B. Variable Used Energy at the Relay

If the used energy is variable, we need to optimize the ratio v, besides to S and F. Recall that the maximum of a multivariable objective function can be computed as $\max_{x,y} f(x,y) = \max_x \max_y f(x,y)$ [23]. Thus, we first fix v and we solve (P) for a given $0 \leq v \leq 1$. The optimal v can be then obtained by any one-dimensional search method like the bisection method or the grid search. For a given $0 \leq v \leq 1$, we consider

$$\max_{S,F} \quad R(S,F),$$
(18a)

$$\text{s.t.} \quad \frac{1-v}{2} \zeta \ tr\left(HSH^H\right) \geq E_s,$$
(18b)

$$tr\left(F\left(HSH^H + \sigma_r^2 I_{N_r}\right)F^H\right) \leq v \zeta \ tr\left(HSH^H\right),$$
(18c)

$$tr(S) \leq P_s, S \geq 0,$$
(18d)

Note that when $v = 0$, the used energy and the transmit power at the relay are null, hence the end-to-end rate is zero, while the stored energy corresponds to its maximum value equal to $Q_{r,max}$ defined in (80) in Appendix C. When $v = 1$, the stored energy is null, while the end-to-end rate corresponds to its maximum value R_{max} in (86) in Appendix D with $P_r(S_{ID}) = 2Q_{r,ID}$. $Q_{r,ID}$ is the harvested energy corresponding to the maximum achievable end-to-end rate defined in Appendix D. For a given $0 < v < 1$, we perform some mathematical manipulations as in Appendix A and we show that (P) is equivalent to the convex optimization problems (P1) or (P2), with $Q_r = \frac{E_s}{1-v}$ and $P_r(S) = v \zeta \ tr\left(HSH^H\right)$. Here, the

two boundary points $(R_{EH}, E_{s,max})$ and $(R_{max}, E_{s,ID})$ correspond to $\nu = 0$ and $\nu = 1$, respectively, and are given by

$$E_{s,max} = Q_{r,max}, \tag{19}$$

$$R_{EH} = 0, \tag{20}$$

$$E_{s,ID} = 0, \tag{21}$$

and the rate R_{max} is given by (86) with $\nu = 1$ and $P_r(S_{ID}) = 2Q_{r,ID}$.

In this section, we considered the ideal scenario where the EH and ID receivers at the relay can simultaneously decode the information and harvest the energy. However, practical energy harvesting circuits are not yet able to decode the information simultaneously. So, we propose to study more practical schemes that can be immediately implemented in the practice which separate the ID and EH transfer over the power transfer, known as the power splitting, or the time domain, known as the time switching [5].

IV. Power Splitting (PS) Scheme

For the power splitting scheme, the received signal at each antenna at the relay is split into two separate signal streams, one for the ID receiver at the relay with power ratio $(1-\rho_i)$ and the other for the EH receiver at the relay with power ratio ρ_i, with $0 \le \rho_i \le 1$, for $i = 1, \dots, N_r$. The power ratio ρ_i at each antenna is not necessarily the same in the general case. For the sake of simplicity, we consider the case when we have uniform power splitting among the receiving antennas. Let $\rho = \rho_i, \forall i$, be the proportion of the power split at all the receiving EH antennas at the relay. Let S^{PS} and F^{PS} be the corresponding source covariance matrix and relay amplification matrix. The harvested energy, used energy, stored energy, transmit power at the relay, and overall rate are given by

$$Q_r^{PS}\left(\rho, S^{PS}\right) = \rho Q_r\left(S^{PS}\right), \tag{22}$$

$$E_u^{PS}\left(\rho, S^{PS}\right) = \begin{cases} \bar{E}_u, & \text{if the used energy is constant,} \\ \rho\nu Q_r(S), & \text{otherwise,} \end{cases} \tag{23}$$

$$E_s^{PS}\left(\rho, S^{PS}\right) = Q_r^{PS}\left(\rho, S^{PS}\right) - E_u^{PS}\left(\rho, S^{PS}\right), \tag{24}$$

$$P_r^{PS}\left(\rho, S^{PS}\right) = 2E_u^{PS}\left(\rho, S^{PS}\right), \tag{25}$$

$$R^{PS}\left(\rho, S^{PS}, F^{PS}\right) = \frac{1}{2}\log_2\left| I + (1-\rho)GF^{PS}HS^{PS}H^H\left(F^{PS}\right)^H \right.$$
$$\left. \times G^H\left(\sigma_r^2 GF^{PS}\left(F^{PS}\right)^H G^H + \sigma_d^2 I_{N_d}\right)^{-1} \right|. \tag{26}$$

In this case, the achievable R-E region of the PS scheme is given by

$$C_{R-E}^{PS}(P_s) = \bigcup_{0 \le \rho \le 1}\left\{ \left(R^{PS}, E_s^{PS}\right); R^{PS} \le R^{PS}\left(\rho, S^{PS}, F^{PS}\right), \right.$$
$$E_s^{PS} \le \frac{1}{2}\rho\ \zeta\ tr\left(HS^{PS}H^H\right) - E_u^{PS}\left(\rho, S^{PS}\right),$$
$$tr\left(F^{PS}\left(HS^{PS}H^H + \sigma_r^2 I_{N_r}\right)\left(F^{PS}\right)^H\right) \le 2E_u^{PS}\left(\rho, S^{PS}\right),$$
$$\left. tr\left(S^{PS}\right) \le P_s, S^{PS} \ge 0 \right\}, \tag{27}$$

In order to find the boundary of this R-E region, we first fix $\rho \in [0, 1]$ and we find the boundary of the R-E region for a given ρ. Then, the boundary of the R-E region is the convex envelope of the obtained R-E regions with fixed ρ.

For a given $0 \le \rho \le 1$, the optimal S^{PS} and F^{PS} are solutions to the following optimization problem expressed as

$$\max_{S^{PS}, F^{PS}} \quad R^{PS}\left(\rho, S^{PS}, F^{PS}\right), \tag{28a}$$

$$\text{s.t.} \quad \frac{1}{2}\rho\ \zeta\ tr\left(HS^{PS}H^H\right) - E_u^{PS}\left(\rho, S^{PS}\right) \ge E_s^{PS}, \tag{28b}$$

$$tr\left(F^{PS}\left(HS^{PS}H^H + \sigma_r^2 I_{N_r}\right)\left(F^{PS}\right)^H\right) \le 2E_u^{PS}\left(\rho, S^{PS}\right), \tag{28c}$$

$$tr\left(S^{PS}\right) \le P_s, S^{PS} \ge 0, \tag{28d}$$

where $0 \le E_s^{PS} \le E_{s,max}^{PS}(\rho)$, with $E_{s,max}^{PS}(\rho) = E_s^{PS}\left(\rho, S_{EH}\right)$ and S_{EH} is the source covariance matrix corresponding to the maximum harvested energy at the relay $Q_{r,max}$ in Appendix C. In order to satisfy (28b), we should have $\rho \ge \tau_{low}$, where

$$\tau_{low} = \begin{cases} \frac{\bar{E}_u}{Q_{r,ID}}, & \text{if the used energy is constant,} \\ 0, & \text{otherwise,} \end{cases} \tag{29}$$

where $Q_{r,ID}$ is defined in Appendix D. For a given $\rho \in [\tau_{low}, 1)$, we denote by $\hat{\zeta} = \zeta\frac{\rho}{1-\rho}$ and $\hat{H} = \sqrt{1-\rho}H$. Hence, the optimal S_{PS} and F_{PS} are obtained in a similar way as shown in the previous section III.

Remark 1: Note that:

- When $\rho = 1$, the achievable end-to-end rate is zero, while the stored energy at the relay achieves its maximum value $E_{s,max}$ defined in (16) and (19) depending on the operation mode at the relay. Hence, the maximum achievable stored energy of the PS scheme and ideal scheme are the same.
- When $\rho = \tau_{low}$ and the used energy is constant, the stored energy at the relay is greater or equal to zero and the achievable end-to-end rate is nonzero corresponding to a transmit power at the relay $2\bar{E}_u$. So, the maximum achievable end-to-end rate of the PS scheme happens when the stored energy is zero and is less or equal to the one of the ideal scheme. Note that they are equal if $Q_{r,ID}$ in (17) is equal to $\bar{E}_u$.
- When $\rho = \tau_{low}$ and the used energy is not constant, the used and stored energies at the relay and the end-to-end rate are all zero. Thus, the maximum achievable end-to-end rate of the PS scheme happens for $\rho \in (\tau_{low}, 1)$.

Consequently, we can see that $C_{R-E}^{PS}(P_s) \subset C_{R-E}(P_s)$ defined in (12).

V. Time Switching (TS) Scheme

For the time switching scheme, the transmission block from the source to the relay is divided into two orthogonal time slots, one for energy harvesting, and the other for information transmission from the source to the relay. In the sequel, we examine three variants of this scheme depending on the transmit power of the source covariance matrices during the ID and EH modes. Let $i \in \{1, 2, 3\}$ be the index referring to the three variants TS_i of the TS scheme. Let $S_1^{TS_i}$ and $S_2^{TS_i}$ be the covariance matrices at the source during the ID and EH time slots, respectively, and F^{TS_i} be the relay amplification matrix.

Let α_i denotes the time ratio allocated to the EH mode, with $0 \leq \alpha_i \leq 1$. The three variants of the TS scheme are defined as follows:

- TS_1: Fixed power constraint:
$$tr\left(S_j^{TS_1}\right) \leq P_s, j = 1, 2. \quad (30)$$

- TS_2: Flexible power constraint:
$$\frac{1 - \alpha_2}{2} tr\left(S_1^{TS_2}\right) + \alpha_2 tr\left(S_2^{TS_2}\right) \leq \left(\frac{1 - \alpha_2}{2} + \alpha_2\right) P_s. \quad (31)$$

- TS_3: Peak power constraint: (31) with α_3 instead of α_2, and
$$tr\left(S_j^{TS_3}\right) \leq P_{peak}, j = 1, 2, \quad (32)$$

with $P_{peak} \geq P_s$. For each variant, the harvested energy, used energy, stored energy, transmit power at the relay, and overall rate are given by
$$Q_r^{TS_i}\left(\alpha_i, S_2^{TS_i}\right) = 2\alpha_i Q_r\left(S_2^{TS_i}\right), \quad (33)$$

$$E_u^{TS_i}\left(\alpha_i, S_2^{TS_i}\right) = \begin{cases} \bar{E}_u, & \text{if the used energy is constant,} \\ 2\alpha_i \, v \, Q_r\left(S_2^{TS_i}\right), & \text{otherwise,} \end{cases} \quad (34)$$

$$E_s^{TS_i}\left(\alpha_i, S_2^{TS_i}\right) = Q_r^{TS_i}\left(\alpha_i, S_2^{TS_i}\right) - E_u^{TS_i}\left(\alpha_i, S_2^{TS_i}\right), \quad (35)$$

$$P_r^{TS_i}\left(\alpha_i, S_2^{TS_i}\right) = \frac{2 \, E_u^{TS_i}\left(\alpha_i, S_2^{TS_i}\right)}{1 - \alpha_i}, \quad (36)$$

$$R^{TS_i}\left(\alpha_i, S_1^{TS_i}, F^{TS_i}\right) = (1 - \alpha_i)R\left(S_1^{TS_i}, F^{TS_i}\right). \quad (37)$$

Note that, for the three variants of the TS scheme, the achievable harvested energy at the relay can be greater than the one achieved by the ideal and PS schemes if $\alpha_i \geq \frac{1}{2}$, with $i \in \{1, 2, 3\}$.

A. TS_1: TS with Fixed Power Constraint

First, we assume that the transmitted signals to the EH and ID receivers at the relay have the same power constraint P_s as in (30). In this case, the achievable R-E region of TS_1 is given by
$$C_{R-E}^{TS_1}(P_s) = \bigcup_{0 \leq \alpha_1 \leq 1} \left\{ \left(R^{TS_1}, E_s^{TS_1}\right); R^{TS_1} \leq (1 - \alpha_1)R\left(S_1^{TS_1}, F^{TS_1}\right), \right.$$

$$E_s^{TS_1} \leq \alpha_1 \, \zeta \, tr\left(HS_2^{TS_1}H^H\right) - E_u^{TS}\left(\alpha_1, S_2^{TS_1}\right),$$

$$(1 - \alpha_1) tr\left(F^{TS_1}\left(HS_1^{TS_1}H^H + \sigma_r^2 I_{N_r}\right)\left(F^{TS_1}\right)^H\right) \leq 2 E_u^{TS}\left(\alpha_1, S_2^{TS_1}\right),$$

$$\left. tr\left(S_j^{TS_1}\right) \leq P_s, S_j^{TS_1} \geq 0, j = 1, 2 \right\}, \quad (38)$$

Hence, the optimal α_1, $S_1^{TS_1}$, $S_2^{TS_1}$, and F^{TS_1} are solutions to the following optimization problem
$$\max_{\alpha_1, S_1^{TS_1}, S_2^{TS_1}, F^{TS_1}} (1 - \alpha_1)R\left(S_1^{TS_1}, F^{TS_1}\right), \quad (39a)$$

$$\text{s.t.} \quad \alpha_1 \zeta \, tr\left(HS_2^{TS_1}H^H\right) - E_u^{TS}\left(\alpha_1, S_2^{TS_1}\right) \geq E_s^{TS_1}, \quad (39b)$$

$$(1 - \alpha_1) tr\left(F^{TS_1}\left(HS_1^{TS_1}H^H + \sigma_r^2 I_{N_r}\right)\left(F^{TS_1}\right)^H\right)$$
$$\leq 2 E_u^{TS}\left(\alpha_1, S_2^{TS_1}\right), \quad (39c)$$

$$tr\left(S_j^{TS_1}\right) \leq P_s, S_j^{TS_1} \geq 0, j = 1, 2, \quad (39d)$$

$$0 \leq \alpha_1 \leq 1, \quad (39e)$$

where $0 \leq E_s^{TS_1} \leq E_{s,max}^{TS_1}(\alpha_1)$, with $E_{s,max}^{TS_1}(\alpha_1) = E_s^{TS_1}(\alpha_1, S_{EH})$. Similarly, in order to satisfy (39b), we should have $\alpha_1 \geq \frac{\tau_{low}}{2}$.

For a given $\frac{\tau_{low}}{2} \leq \alpha_1 \leq 1$, the optimal $S_2^{TS_1}$ corresponds to the maximum achievable harvested energy since there is no data transmission during the EH time slot. Hence, $S_2^{TS_1}$ is given by S_{EH} in (79). The corresponding maximum harvested energy is given by
$$Q_{r,max}^{TS_1}(\alpha_1) = 2\alpha_1 Q_{r,max}, \quad (40)$$

and the corresponding used energy, stored energy and transmit power at the relay are given by $E_{u,max}^{TS_1}(\alpha_1) = E_u^{TS}(\alpha_1, S_{EH})$, $E_{s,max}^{TS_1}(\alpha_1) = E_s^{TS}(\alpha_1, S_{EH})$ and $P_{r,max}^{TS_1}(\alpha_1) = P_r^{TS}(\alpha_1, S_{EH})$, respectively. Subsequently, for a given $\frac{\tau_{low}}{2} \leq \alpha_1 < 1$, the optimal $S_1^{TS_1}$, and F^{TS_1} are solutions to the following optimization problem
$$\max_{S_1^{TS_1}, F^{TS_1}} R\left(S_1^{TS_1}, F^{TS_1}\right), \quad (41a)$$

$$\text{s.t.} \quad tr\left(F^{TS_1}\left(HS_1^{TS_1}H^H + \sigma_r^2 I_{N_r}\right)\left(F^{TS_1}\right)^H\right) \leq \frac{2 \, E_{u,max}^{TS_1}(\alpha_1)}{1 - \alpha_1}, \quad (41b)$$

$$tr\left(S_1^{TS_1}\right) \leq P_s, S_1^{TS_1} \geq 0. \quad (41c)$$

For a given $\frac{\tau_{low}}{2} \leq \alpha_1 < 1$, the optimal $S_1^{TS_1}$ and F^{TS_1} can be obtained in a similar way as in Appendix A with $Q_r = 0$ and $P_r(S) = \frac{2}{1-\alpha_1} E_{u,max}^{TS_1}(\alpha_1)$.

Remark 2: Note that:

- If $\alpha_1 = 1$, the achievable end-to-end rate is zero and the harvested energy at the relay is twice the maximum value $Q_{r,max}$. Hence, the stored energy is greater than $E_{s,max}$ defined in (16) and (19) depending on the operation mode at the relay. Hence, the maximum stored energy of the TS scheme is greater than the one achieved by the ideal and PS schemes.

- If $\alpha_1 = \frac{\tau_{low}}{2}$ and the used energy is constant, the stored energy at the relay is greater or equal to zero and the achievable end-to-end rate is nonzero corresponding to a transmit power at the relay $2\bar{E}_u$. So, the maximum achievable end-to-end rate happens when the stored energy is zero and is less or equal to the one of the ideal scheme. Similarly, they are equal if $Q_{r,ID} = \bar{E}_u$ in (17).

- If $\alpha_1 = \frac{\tau_{low}}{2}$ and the used energy is not constant, the used and stored energies at the relay and the end-to-end rate are all zero. Thus, the maximum achievable end-to-end rate happens for $\alpha_1 \in \left(\frac{\tau_{low}}{2}, 1\right)$.

At this point, we can see that the ideal scheme doesn't provide an outer bound on the achievable R-E region of the TS scheme with fixed power constraint. However, the maximum achievable end-to-end rate of the ideal scheme is greater or equal to the one of the TS_1 scheme.

B. TS_2: TS with Flexible Power Constraint:

Next, we assume that the transmitted signals to the EH and ID receivers at the relay have different power constraints given that their average power is less or equal to P_s, as in (31). In

this case, the achievable R-E region of TS_2 is given by

$$C_{R-E}^{TS_2}(P_s) = \bigcup_{0 \leq \alpha_2 \leq 1} \left\{ \left(R^{TS_2}, E_s^{TS_2} \right); R^{TS_2} \leq (1 - \alpha_2) R \left(S_1^{TS_2}, F^{TS_2} \right), \right.$$

$$E_s^{TS_2} \leq \alpha_2 \, \zeta \, tr \left(HS_2^{TS_2} H^H \right) - E_u^{TS} \left(\alpha_2, S_2^{TS_2} \right),$$

$$(1 - \alpha_2) \, tr \left(F^{TS_2} \left(HS_1^{TS_2} H^H + \sigma_r^2 I_{N_r} \right) \left(F^{TS_2} \right)^H \right) \leq 2 \, E_u^{TS} \left(\alpha_2, S_2^{TS_2} \right),$$

$$\left. \frac{1 - \alpha_2}{2} \, tr \left(S_1^{TS_2} \right) + \alpha_2 \, tr \left(S_2^{TS_2} \right) \leq \left(\frac{1 + \alpha_2}{2} \right) P_s, S_j^{TS_2} \geq 0, j = 1, 2 \right\}.$$
(42)

Similarly to the previous part, during the EH time slot, the optimal $S_2^{TS_2}$ corresponds to the maximum achievable harvested energy since there is no data transmission. Hence, $S_2^{TS_2}$ should be given by $S_{EH}^{TS_2} = p \, v_{H,1} \, v_{H,1}^H$ where p is a positive constant satisfying $p = tr \left(S_2^{TS_2} \right)$ and $\frac{P_s}{2} \tau_{low} \leq \alpha_2 p \leq \frac{1+\alpha_2}{2} P_s$. Let us denote the corresponding harvested energy at the relay by

$$q = Q_r^{TS} \left(\alpha_2, S_{EH}^{TS_2} \right) = 2\alpha_2 \frac{p}{P_s} Q_{r,max} \leq (1 + \alpha_2) Q_{r,max}, \quad (43)$$

where $q_{low} \leq q \leq (1 + \alpha_2) Q_{r,max}$ and $q_{low} = \bar{E}_u$ if the used energy is constant, otherwise $q_{low} = 0$. The corresponding used energy, stored energy and transmit power at the relay are given by $E_{u,max}^{TS_2}(\alpha_2, q) = E_u^{TS} \left(\alpha_2, S_{EH}^{TS_2} \right)$, $E_{s,max}^{TS_2}(\alpha_2, q) = E_s^{TS} \left(\alpha_2, S_{EH}^{TS_2} \right)$ and $P_{r,max}^{TS_2}(\alpha_2, q) = P_r^{TS} \left(\alpha_2, S_{EH}^{TS_2} \right)$, respectively. Consequently, the optimal α_2, q, $S_2^{TS_2}$ and F^{TS_2} are solutions to

$$\max_{\alpha_2, q, S_1^{TS_2}, F^{TS_2}} \quad (1 - \alpha_2) R \left(S_1^{TS_2}, F^{TS_2} \right), \quad (44a)$$

$$\text{s.t.} \quad (1 - \alpha_2) \, tr \left(S_1^{TS_2} \right) \leq P_s \left(1 + \alpha_2 - \frac{q}{Q_{r,max}} \right), \quad (44b)$$

$$(1 - \alpha_2) \, tr \left(F^{TS_2} \left(HS_1^{TS_2} H^H + \sigma_r^2 I_{N_r} \right) \left(F^{TS_2} \right)^H \right)$$
$$\leq 2 \, E_{u,max}^{TS_2}(\alpha_2, q), \quad (44c)$$

$$q_{low} \leq q \leq (1 + \alpha_2) \, Q_{r,max}, \quad (44d)$$

$$S_1^{TS_2} \geq 0, 0 \leq \alpha_2 \leq 1. \quad (44e)$$

At this point, we can see that the optimal α_2 that maximizes the achievable rate is when $\alpha_2 \rightarrow 0$. Consequently, the problem simplifies to

$$\max_{q, S_1^{TS_2}, F^{TS_2}} \quad R \left(S_1^{TS_2}, F^{TS_2} \right), \quad (45a)$$

$$\text{s.t.} \quad tr \left(F^{TS_2} \left(HS_1^{TS_2} H^H + \sigma_r^2 I_{N_r} \right) \left(F^{TS_2} \right)^H \right) \leq 2 E_{u,max}^{TS_2}(0, q),$$
$$(45b)$$

$$tr \left(S_1^{TS_2} \right) \leq P_s \left(1 - \frac{q}{Q_{r,max}} \right), \quad (45c)$$

$$q_{low} < q < Q_{r,max}, S_1^{TS_2} \geq 0. \quad (45d)$$

For a given $q_{low} < q < Q_{r,max}$, we denote by $P_s^{TS_2}(q) = P_s \left(1 - \frac{q}{Q_{r,max}} \right)$. Then, we can easily see that the optimal $S_1^{TS_2}$ and F^{TS_2} can be obtained in a similar way as in Appendix A with $P_s = P_s^{TS_2}(q)$, $Q_r = 0$ and $P_r(S) = 2E_{u,max}^{TS_2}(0, q)$.

Remark 3: Note that:

- The maximum achievable harvested energy happens when $q = Q_{r,max}$ which equals to the one achieved by the ideal and PS scheme. The corresponding end-to-end rate is zero and the stored energy at the relay is equal to its maximum

value $E_{s,max}$ defined in (16) and (19) depending on the operation mode at the relay.

- If $q = q_{low}$ and the used energy is constant, the stored energy is zero and the maximum achievable end-to-end rate is less or equal to the one achieved by the ideal scheme. If $\bar{E}_u << Q_{r,max}$, then they are approximatively equal.

- If $q = q_{low}$ and the used energy is not constant, the used and stored energies at the relay and the end-to-end rate are all zero. Thus, the maximum achievable end-to-end rate happens for $q \in (0, Q_{r,max})$.

C. TS_3: TS with Peak Power Constraint:

The TS scheme with flexible power constraint assumes that the transmitter and the receiver can operate under an infinite power ($p \rightarrow \infty$) when $\alpha_2 \rightarrow 0$, so that q is a constant. Hence, the latter can not be implemented in practical circuits. To solve this impracticality, we add a peak transmit power constraint at the source as in (32), with $P_{peak} \geq P_s$. In this case, the achievable R-E region of TS_3 is given by

$$C_{R-E}^{TS_3} \left(P_s, P_{peak} \right) = \bigcup_{0 \leq \alpha_3 \leq 1} \left\{ \left(R^{TS_3}, E_s^{TS_3} \right); \right.$$

$$R^{TS_3} \leq (1 - \alpha_3) R \left(S_1^{TS_3}, F^{TS_3} \right),$$

$$E_s^{TS_3} \leq \alpha_3 \, \zeta \, tr \left(HS_2^{TS_3} H^H \right) - E_u^{TS} \left(\alpha_3, S_2^{TS_3} \right),$$

$$(1 - \alpha_3) \, tr \left(F^{TS_3} \left(HS_1^{TS_3} H^H + \sigma_r^2 I_{N_r} \right) \left(F^{TS_3} \right)^H \right) \leq 2 \, E_u^{TS} \left(\alpha_3, S_2^{TS_3} \right),$$

$$\frac{1 - \alpha_3}{2} \, tr \left(S_1^{TS_3} \right) + \alpha_3 \, tr \left(S_2^{TS_3} \right) \leq \left(\frac{1 + \alpha_3}{2} \right) P_s,$$

$$\left. tr \left(S_j^{TS_3} \right) \leq P_{peak}, S_j^{TS_3} \geq 0, j = 1, 2 \right\}. \quad (46)$$

Note that $C_{R-E}^{TS_3} \left(P_s, P_{peak} \right) \underset{P_{Peak} \rightarrow \infty}{\rightarrow} C_{R-E}^{TS_2} (P_s)$. Similarly to the previous parts, $S_2^{TS_3}$ should be given by $S_{EH}^{TS_3} = P_{peak} \, v_{H,1} \, v_{H,1}^H$. The corresponding harvested energy at the relay is given by

$$Q_{r,max}^{TS_3}(\alpha_3) = Q_r^{TS} \left(\alpha_3, S_{EH}^{TS_3} \right) = 2\alpha_3 \frac{P_{peak}}{P_s} Q_{r,max}, \quad (47)$$

where $Q_{r,max}^{TS_3}(\alpha_3) \geq q_{low}$. The corresponding used energy, stored energy and transmit power at the relay are given by $E_{u,max}^{TS_3}(\alpha_3) = E_u^{TS} \left(\alpha_3, S_{EH}^{TS_3} \right)$, $E_{s,max}^{TS_3}(\alpha_3) = E_s^{TS} \left(\alpha_3, S_{EH}^{TS_3} \right)$ and $P_{r,max}^{TS_3}(\alpha_3) = P_r^{TS} \left(\alpha_3, S_{EH}^{TS_3} \right)$, respectively. Let us denote by $P_s^{TS_3}(\alpha_3) = \frac{(1+\alpha_3)P_s - 2\alpha_3 P_{peak}}{1 - \alpha_3}$, where α_3 is upper bounded by $\frac{P_s}{2P_{peak} - P_s}$, so that $P_s^{TS_3}(\alpha_3) \geq 0$. Also, α_3 is lower bounded by $\frac{P_s}{2P_{peak}} \tau_{low}$, so that $Q_{r,max}^{TS_3}(\alpha_3) \geq q_{low}$. Consequently, the optimal α_3, $S_2^{TS_3}$ and F^{TS_3} are solutions to

$$\max_{\alpha_3, S_1^{TS_3}, F^{TS_3}} \quad (1 - \alpha_3) R \left(S_1^{TS_3}, F^{TS_3} \right), \quad (48a)$$

$$\text{s.t.} \quad tr \left(F^{TS_3} \left(HS_1^{TS_3} H^H + \sigma_r^2 I_{N_r} \right) \left(F^{TS_3} \right)^H \right) \leq \frac{2 \, E_{u,max}^{TS_3}(\alpha_3)}{1 - \alpha_3}, \quad (48b)$$

$$tr \left(S_1^{TS_3} \right) \leq P_s^{TS_3}(\alpha_3), S_1^{TS_3} \geq 0, \quad (48c)$$

$$\frac{P_s}{2P_{peak}} \tau_{low} \leq \alpha_3 \leq \frac{P_s}{2P_{peak} - P_s}. \quad (48d)$$

Subsequently, for a given $\frac{P_s}{2P_{peak}}\tau_{low} \leq \alpha_3 < \frac{P_s}{2P_{peak}-P_s}$, the optimal $S_1^{TS_3}$ and F^{TS_3} can be obtained in a similar way as in Appendix A with $P_s = P_s^{TS_3}(\alpha_3)$, $Q_r = 0$ and $P_r(S) = 2E_{u,max}^{TS_3}(\alpha_3)$.

Remark 4: Note that:

- $tr\left(S_1^{TS_3}\right) \leq P_{peak}$ is implicitly satisfied in (48c) since $P_s^{TS_3}(\alpha_3) \leq P_{peak}$.
- If $\alpha_3 = \frac{P_s}{2P_{peak}-P_s}$, the harvested energy is $\frac{2P_{Peak}}{2P_{peak}-P_s}$ times $Q_{r,max}$. Hence, the maximum achievable harvested and stored energies can be greater than the ones achieved by the ideal and PS schemes. The corresponding end-to-end rate is zero.
- If $\alpha_3 = \frac{P_s}{2P_{peak}}\tau_{low}$, we have the same comments as when $\alpha_1 = \frac{\tau_{low}}{2}$ and $\rho = \tau_{low}$.

VI. Imperfect Channel State Information

In the previous sections, we have assumed that all the source, relay and destination have perfect CSI. This assumption is hard to obtain in practice. Hence, in this section, we assume that we have imperfect knowledge of the channel H at the relay and imperfect knowledge of the channel G at the destination. Taking into account the feedback/forward delay errors and channel estimation errors, the channels H and G can be written in general as [24]

$$H = \hat{H} + E_1 + U_1, \qquad (49)$$

$$G = \hat{G} + E_2 + U_2, \qquad (50)$$

where $\hat{H}$ and $\hat{G}$ are the estimated channels of H and G, E_1 and E_2 are the error matrices associated to the feedback/forward delay errors, and U_1 and U_2 are the error matrices associated to the channel estimation errors. We model E_i and U_i, $i = 1, 2$, as Gaussian random matrices with zero mean and variances equal to $\sigma_{E_i}^2$ and $\sigma_{U_i}^2$, respectively. We assume that the error matrices E_1 and U_1 and the error matrices E_2 and U_2 are uncorrelated with H and G, respectively. We assume that E_1 is known at the relay and E_2 is known at the destination. Note also that we assume that $\sigma_r^2 = \sigma_d^2 = \sigma^2$ to facilitate the mathematical derivations without loss of generality.

Note that the expression of the achievable rate of MIMO AF relay systems with imperfect CSI is not an immediate extension of the one with perfect CSI. The maximization problem of the achievable rate with imperfect CSI was studied in [24], [25] only in function of the relay precoder matrix where the source is assumed to have a uniform power allocation. In order to study the joint source and relay precoders design of MIMO AF relay systems with imperfect CSI, we have derived the expression of the approximate achievable rate thanks to the work previously done in [24] and we showed that it can be written as

$$R(S, F) = \frac{1}{2}\log_2 \left| I + \frac{1}{\psi}\hat{H}S\hat{H}^H \right.$$
$$\left. \times \left[I - \left(I + \kappa_1 F^H \left(\hat{G}^H \hat{G} + \sigma_{E_2}^2 r_2 I \right) F \right)^{-1} \right] \right|, \qquad (51)$$

where $\psi = \left(\sigma_{E_1}^2 + \sigma_{U_1}^2\right)P_s + \sigma^2$, $\kappa_1 = \frac{\psi}{\sigma_{U_2}^2 \bar{P}_r + \sigma^2}$ and $\bar{P}_r$ is the average transmit power at the relay. Due to space limitations, we consider only the case where $E_u(S)$ is fixed

and, hence, $\bar{P}_r = 2\bar{E}_u$. Note that (11) can be rewritten as $\frac{1}{2}\log_2 \left| I + \frac{1}{\sigma^2}HSH^H \left[I - \left(I + F^H G^H GF \right)^{-1} \right] \right|$. The transmit power constraint at the relay is written as

$$tr\left(F\left(\frac{1}{\psi}\hat{H}S\hat{H}^H + I \right)F^H \right) \leq \frac{\bar{P}_r}{\psi} \qquad (52)$$

The harvested energy at the relay can be expressed as

$$Q_r(S) = \psi \left[\frac{1}{2}\zeta\, tr\left(\frac{1}{\psi}\hat{H}S\hat{H}^H \right) + \frac{1}{2}\zeta r_1 \kappa_2 \right], \qquad (53)$$

where $\kappa_2 = \frac{\psi - \sigma^2}{\psi}$. If we assume that the channel estimation at the relay consumes an amount of energy $\bar{E}_c$, the stored energy at the relay is given by

$$E_s(S) = \psi \left[\frac{1}{2}\zeta\, tr\left(\frac{1}{\psi}\hat{H}S\hat{H}^H \right) + \frac{1}{2}\zeta r_1 \kappa_2 \right] - \bar{E}_u - \bar{E}_c, \qquad (54)$$

If we denote by $\tilde{E}_u = -\frac{1}{2}\zeta r_1 \kappa_2 + \frac{\bar{E}_u + \bar{E}_c}{\psi}$, $\tilde{P}_r = \frac{P_r}{\psi}$, $\tilde{H} = \frac{1}{\sqrt{\psi}}\hat{H}$, and $\tilde{G}^H \tilde{G} = \kappa_1\left(\hat{G}^H \hat{G} + \sigma_{E_2}^2 r_2 I_{N_r} \right)$, the approximate achievable rate with imperfect CSI is equivalent to the one with perfect CSI. In addition, the stored energy at the relay with imperfect CSI is equivalent to ψ times the one with perfect CSI. Consequently, the precoder design of S and F can be obtained as in Section III as

$$S = V_{\tilde{H},1}D_S\left(V_{\tilde{H},1} \right)^H, \qquad (55)$$

$$F = V_{\tilde{G},1}D_F\left(U_{\tilde{H},1} \right)^H, \qquad (56)$$

where D_S and D_F are $N_b \times N_b$ diagonal matrices, $V_{\tilde{H},1}$ and $U_{\tilde{H},1}$ contain the first N_b columns of the unitary matrices $V_{\tilde{H}}$ and $U_{\tilde{H}}$ from the SVD of $\tilde{H}$, and $V_{\tilde{G},1}$ contains the first N_b columns of the unitary matrix $V_{\tilde{G}}$ from the eigenvalue decomposition of $\tilde{G}^H \tilde{G}$. Given (55) and (56), the equivalent optimization problem can be solved as shown in Appendix A. Note that the corresponding stored energy at the relay with imperfect CSI is ψ times the one obtained using the procedure in the previous sections.

VII. Numerical Results

In this section, we present some selected numerical simulations of the boundary of the achievable R-E regions. All the simulations are obtained by averaging over 200 independent realizations of H and G. We denote by d_{sr}, d_{rd}, and d_{sd} the distance between the source and the relay, the relay and the destination, and the source and the destination, respectively. We assume that the distance between the source and the relay is of the order of meters since so far the performance of SWIPT was proven to be more effective over short distances, based on practical studies done in [26]. The number of transmit antennas at the source, the relay, and the destination are all equal to 2. The conversion efficiency at the relay is equal to $\zeta = 0.6$. The transmit power at the source is equal to 10 dB. The predefined used energy is equal to $\bar{E}_u = 0.05$. We assume that the noise variances are equal to $\sigma_r^2 = \sigma_d^2 = 1$.

A. Position of the Relay

First, we investigate the achievable R-E region between the end-to-end rate and the stored energy at the relay for different positions of the relay between the source and the

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destination with perfect CSI at all the nodes. We assume that the channels $\boldsymbol{H}$ and $\boldsymbol{G}$ are Rayleigh fading channels with path loss. The Rayleigh fading is assumed to have a normalized variance. The path loss exponent is taken equal to $m = 2.7$ which corresponds to an urban cellular network environment.

In Figs. 1, 2, and 3, we have plotted the achievable R-E regions of the ideal, PS and TS schemes with fixed and variable used energy at the relay where the distance between the source and relay is 0.1, 0.5, and 0.9 times the distance between the source and destination, respectively. In all figures, we can see that the ideal scheme achieves the highest maximum end-to-end rate, but it doesn't provide an outer bound on the achievable R-E of the other schemes. Moreover, we can see that as d_{sr} increases, the achievable stored energy at the relay decreases in all figures, since the harvested energy decreases when the relay is far from the source. In addition, when the used energy is fixed, the maximum achievable end-to-end rate increases as d_{sr} increases. However, when the used energy is variable, the maximum achievable end-to-end rate increases when the relay is closer to the source. We can see also that when the used energy is fixed, the maximum achievable end-to-end rate is the same for all the schemes but it is always lower than the one achieved when the used energy is variable, as d_{sr} varies. But, when $d_{sr}/d_{sd} = 0.9$, both rates are close. So, in this case, there is no interest from optimizing the used energy at the relay.

The TS scheme with fixed power constraint provides the maximum stored energy at the relay compared to the other schemes, while it is almost always the weakest scheme in terms of the maximum end-to-end rate. The TS scheme with flexible power constraint has the closest maximum achievable end-to-end rate to the one achieved by the ideal scheme, but its stored energy at the relay is lower than the other variants of the TS scheme. The performance of the PS scheme in comparison with the three variants of the TS scheme is variable depending on the position of the relay between the source and the destination. At this point, we can conclude that the performance comparison of all the schemes in terms of the achievable end-to-end rate is variable depending on the storage requirements at the relay, the operation mode at the relay and the relay position between the source and destination. The choice of one scheme on the others depends on all these parameters. In terms of the stored energy at the relay, the TS scheme with fixed power constraint outperforms the TS scheme with peak power constraint which outperforms all the other schemes.

In Fig. 4, we have plotted the stored energy at the relay and the end-to-end rate versus the values of ρ and α_i, $i = 1, 3$. The values of ρ and α_i in Fig. 4 are the ones that led to the achievable R-E region in Fig. 2 where $d_{sr} = d_{rd} = 0.5$. We can see that, when the used energy is fixed, the maximum achievable end-to-end rates correspond to the minimum value of ρ and α_i. Note that here $\tau_{low} \approx 0$, since $\bar{E}_u \ll Q_{r,max}$. When the used energy is constant, the maximum achievable end-to-end rate is the same for all the schemes and happens when $\rho = \alpha_i = 0$. In addition, the maximum stored energy achieved by the TS scheme with fixed power constraint is greater than the one achieved by the TS scheme with peak
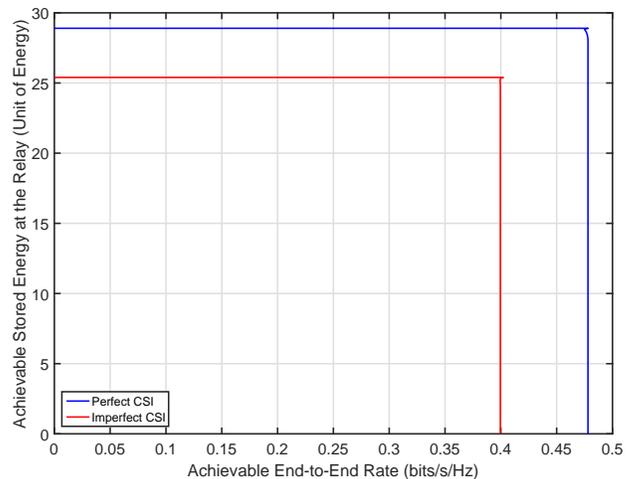

Figure 6. Achievable R-E region for $N_s = N_r = N_d = 4$, $P_s = 10$ dB, $\bar{E}_u = 5 10^{-2}$, $\sigma_r^2 = \sigma_d^2 = 1$, $\zeta = 0.6$, and $\sigma_{E_i}^2 = \sigma_{U_i}^2 = -10$ dB.

power constraint which is also greater than the one achieved by the PS scheme. However, when the used energy is variable, the maximum achievable end-to-end rate of the TS scheme with peak power constraint is higher than the one achieved by the PS scheme which is higher than the one achieved by the TS scheme with fixed power constraint. We can see also that the optimum values of α_1 and α_3 that correspond to the maximum end-to-end rate are such that $\alpha_3 \leq \alpha_1$.

In Fig. 5, we have also plotted the stored energy at the relay and the end-to-end rate versus the value of ν where the used energy is variable. Also, the values of ν in Fig. 5 are the ones that led to the achievable R-E region in Fig. 2 where $D_{sr} = d_{rd} = 0.5$. We can see that the maximum achievable harvested energy corresponds to $\nu = 0$ and the maximum achievable end-to-end rate corresponds to $\nu = 1$. Moreover, the stored energy at the relay decreases and the end-to-end rate increases, when ν increases.

B. Imperfect CSI

In addition, we investigate the impact of imperfect CSI at the relay and the destination on the achievable R-E region of the ideal scheme with fixed used energy at the relay. The channels $\boldsymbol{H}$ and $\boldsymbol{G}$ are Rayleigh fading channels and they are assumed to be full ranked. The full rank assumption is taken only for simplicity. We assume that $\sigma_{E_i}^2 = \sigma_{U_i}^2 = -10$ dB. In Fig. 6, we investigate the achievable R-E region between the end-to-end rate and the stored energy at the relay for the ideal scheme with perfect and imperfect CSI at the relay and destination. We can see the impact of channel uncertainty on both the achievable end-to-end rate and the achievable stored energy at the relay. Both of them decreases when channel uncertainty increases.

VIII. CONCLUSION

In this paper, we considered the SWIPT in a MIMO AF relay communication system with an EH relay and we
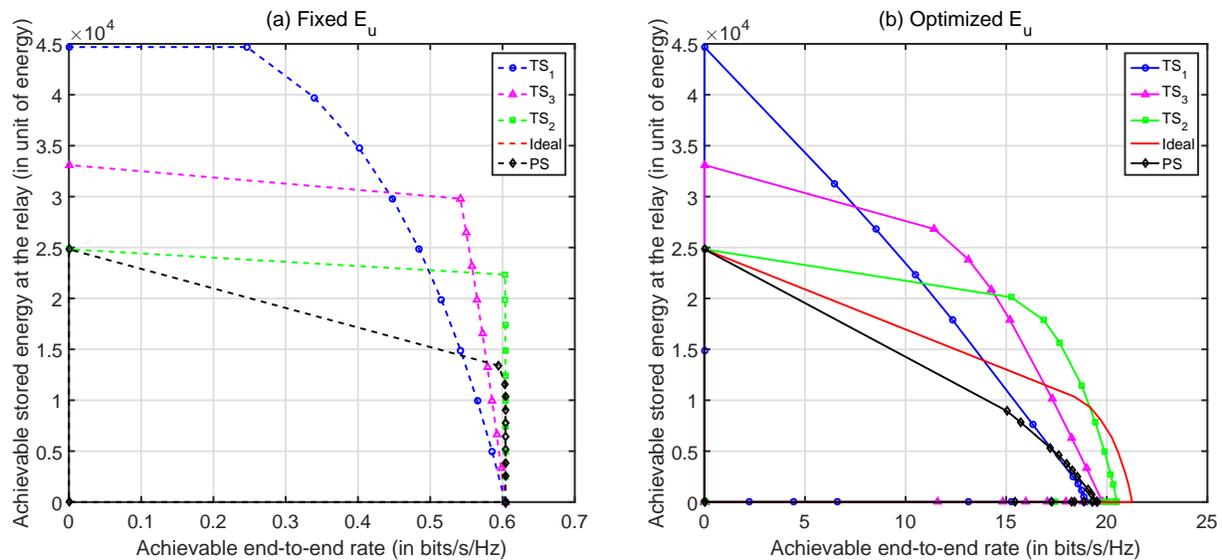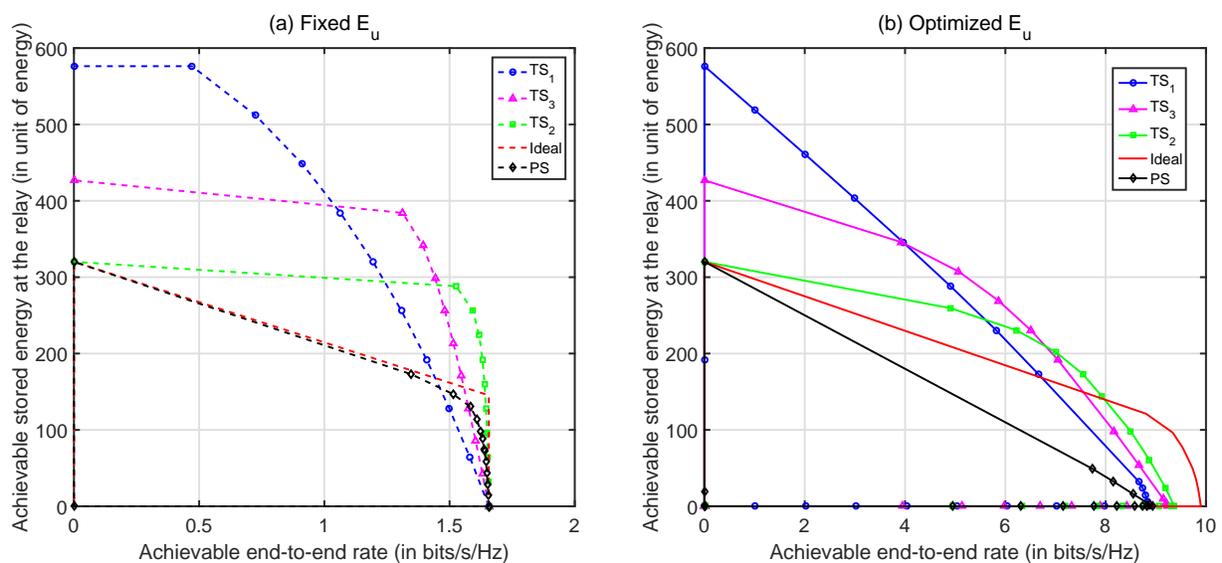
Figure 1. The achievable R-E region of the ideal, PS and TS schemes with fixed/variable E_u for $N_s = N_r = N_d = 4$, $P_s = 10$ dB, $\bar{E}_u = 510^{-2}$, $\sigma_r^2 = \sigma_d^2 = 1$, $\zeta = 0.6$, and $d_{sr} = d_{rd} = 0.1$.

Figure 2. The achievable R-E region of the ideal, PS and TS schemes with fixed/variable E_u for $N_s = N_r = N_d = 4$, $P_s = 10$ dB, $\bar{E}_u = 510^{-2}$, $\sigma_r^2 = \sigma_d^2 = 1$, $\zeta = 0.6$, and $d_{sr} = d_{rd} = 0.5$.

investigated the achievable R-E region between the end-to-end rate and the stored energy at the relay. The used energy at the relay is either constant or variable. First, we considered the ideal scheme where the EH and ID operation at the relay are simultaneously done and we showed that it doesn't provide an outer bound on the achievable R-E region in the two-hop case and only characterizes the upper bound on the maximum achievable end-to-end rate. Then, we have considered the practical PS and TS schemes and investigated their achievable R-E regions. Moreover, we have studied the imperfect CSI at the relay and destination and characterized its impact on the achievable R-E region.

Appendix A
Solution to Problem (P')

Let us consider Problem (P'):

$$(P') : \max_{S,F} \quad R(S,F), \qquad (57a)$$

$$\text{s.t.} \quad tr(S) \leq P_s, \qquad (57b)$$

$$\frac{1}{2} \zeta \, tr\left(HSH^H\right) \geq Q_r, \qquad (57c)$$

$$tr\left(F\left(HSH^H + \sigma_r^2 I_{N_r}\right)F^H\right) \leq P_r(S), \qquad (57d)$$

$$S \succeq 0, \qquad (57e)$$

where $P_r(S)$ is defined in (8). Given the fact that the objective function is Schur-concave (equivalently, $(-R(S,F))$ is Schur-
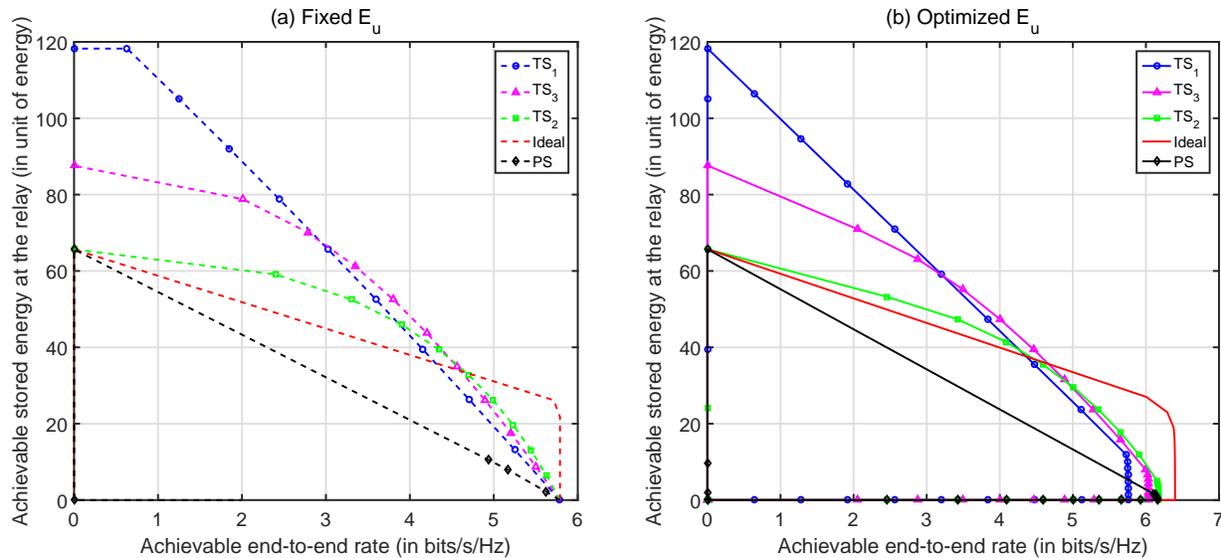
Figure 3. The achievable R-E region of the ideal, PS and TS schemes with fixed/variable E_u for $N_s = N_r = N_d = 4$, $P_s = 10$ dB, $\bar{E}_u = 510^{-2}$, $\sigma_r^2 = \sigma_d^2 = 1$, $\zeta = 0.6$, and $d_{sr} = d_{rd} = 0.9$.
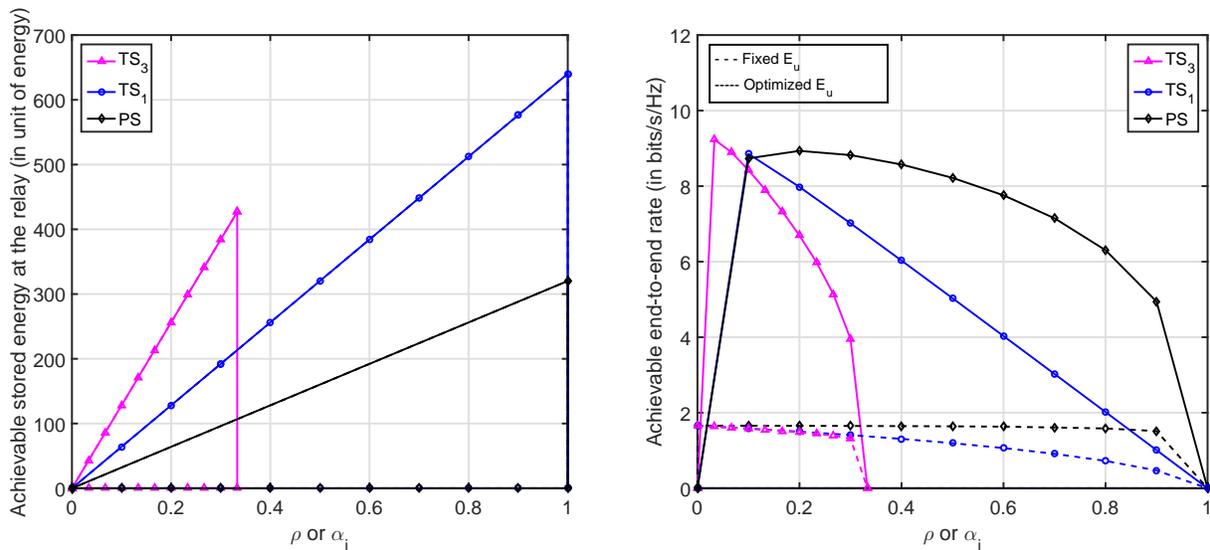

Figure 4. The stored energy at the relay and the end-to-end rate versus the values of ρ and α_i, $i = 1, 3$ for $N_s = N_r = N_d = 4$, $P_s = 10$ dB, $\bar{E}_u = 510^{-2}$, $\sigma_r^2 = \sigma_d^2 = 1$, $\zeta = 0.6$, and $d_{sr} = d_{rd} = 0.5$.

convex [7]), we are able to investigate the structure of the optimal S and F of (P). Assuming the fact that $rank(S) = rank(F) = N_b \leq \min(r_1, r_2)$, the optimal source covariance matrix S and the optimal relay amplification matrix F have the following structure

$$S = V_{H,1} D_S (V_{H,1})^H, \tag{58}$$

$$F = V_{G,1} D_F (U_{H,1})^H, \tag{59}$$

where D_S and D_F are $N_b \times N_b$ diagonal matrices, and $V_{H,1}$, $V_{G,1}$, and $U_{H,1}$ contain the first N_b columns of V_H, V_G, and U_H, respectively. The details of the proof of (58) and (59) are given in the Appendix B. The optimal structure of the source and relay amplification matrices jointly diagonalize the source-

relay-destination channel that becomes equivalent to a set of parallel SISO channels. This result is alike to the MIMO AF relay system with a non-energy harvesting relay. Given (58) and (59), all we need now to optimize is D_S and D_F.

In what follows, we denote by $\hat{\zeta} = \sigma_r^2 \zeta$, $a_i = \frac{1}{\sigma_r^2} D_H(i,i)$, $b_i = \frac{1}{\sigma_d^2} D_G(i,i)$, $x_i = D_S(i,i)$, $y_i = \sigma_r^2 f_i(a_i x_i + 1)$, $f_i = D_F^2(i,i)$, and $\tau_i = \frac{a_i b_i x_i y_i}{b_i y_i + a_i x_i}$, for $i = 1, \ldots, N_b$. We denote also by D_v the diagonal matrix that contain the elements of the vector v. Given (58) and (59), the optimal D_S and D_F verify the
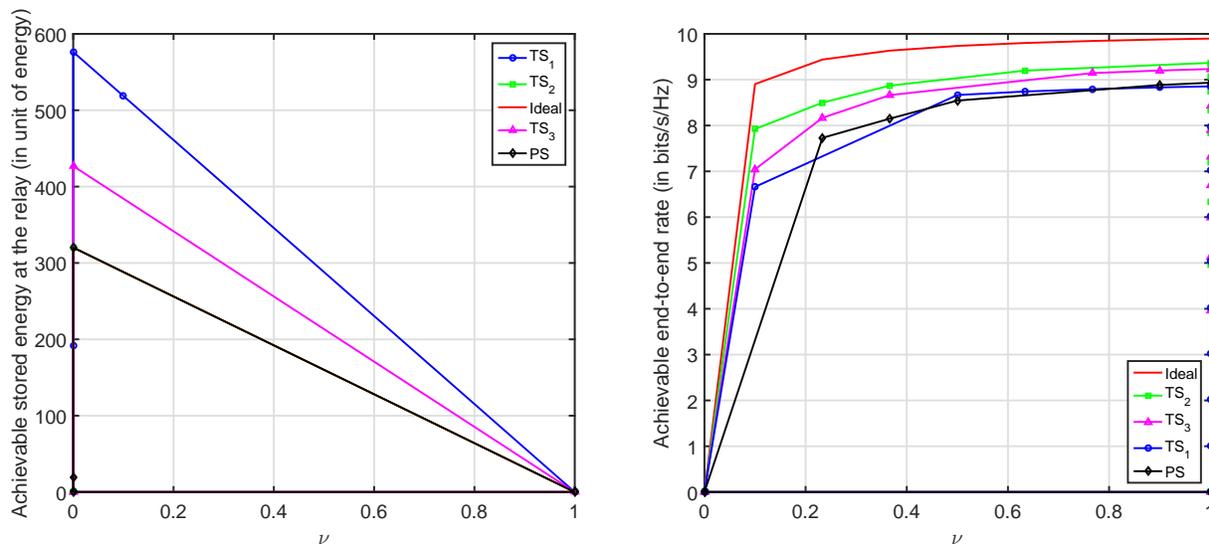
Figure 5. The stored energy at the relay and the end-to-end rate versus the value of ν for $N_s = N_r = N_d = 4$, $P_s = 10$ dB, $\sigma_r^2 = \sigma_d^2 = 1$, $\zeta = 0.6$, and $d_{sr} = d_{rd} = 0.5$.

following optimization problem

$$\max_{x_i, y_i} \quad \frac{1}{2} \sum_{i=1}^{N_b} \log_2 \left(1 + \frac{a_i b_i x_i y_i}{b_i y_i + a_i x_i + 1} \right), \tag{60a}$$

$$\text{s.t.} \quad \sum_{i=1}^{N_b} x_i \leq P_s, \tag{60b}$$

$$\frac{1}{2} \hat{\zeta} \sum_{i=1}^{N_b} a_i x_i \geq Q_r, \tag{60c}$$

$$\sum_i^{N_b} y_i \leq p_r(\boldsymbol{x}), \tag{60d}$$

$$x_i \geq 0, y_i \geq 0, i = 1, ..., N_b, \tag{60e}$$

where

$$p_r(\boldsymbol{x}) = \begin{cases} 2\bar{E}_u, & \text{if the used energy is constant,} \\ \nu \hat{\zeta} \sum_{i=1}^{N_b} a_i x_i, & \text{otherwise.} \end{cases} \tag{61}$$

The objective function of this problem is non-concave, and global optimal solutions are intractable. The local optimal solutions can be obtained iteratively by solving the KKT conditions. However, the iterative algorithm does not guarantee a global convergence and is very sensitive to the initial starting point. Thus, we will present a more efficient and robust method to solve this problem that guarantees global convergence.

As stated previously, the objective function of (60) is not a concave function, even though it is the sum of quasiconcave functions [27]. However, for moderate-to-high SNR values of the links between the source and the relay and between the relay and the destination which occurs when $a_i x_i + b_i y_i \gg 1$, the objective function can be shown to be equivalent to the sum of concave functions and is hence concave [28]. In this case, the problem (60) becomes equivalent to the following convex optimization problem given by

$$\max_{x_i, y_i} \quad \frac{1}{2} \sum_{i=1}^{N_b} \log_2 \left(1 + \frac{a_i b_i x_i y_i}{b_i y_i + a_i x_i} \right), \tag{62a}$$

$$\text{s.t.} \quad (60b) - (60e). \tag{62b}$$

This problem can be solved either using the KKT conditions or can be modified to be equivalent to the SDP formulation. We propose both solutions for completeness.

A. KKT Conditions

Let us consider the Lagragian for the problem (62):

$$L(\boldsymbol{x}, \boldsymbol{y}) = -\frac{1}{2} \sum_{i=1}^{N_b} \log_2 \left(1 + \frac{a_i x_i b_i y_i}{b_i y_i + a_i x_i} \right) + \lambda_1 \left(\sum_{i=1}^{N_b} x_i - P_s \right)$$
$$- \lambda_2 \left(\frac{1}{2} \hat{\zeta} \sum_{i=1}^{N_b} a_i x_i - Q_r \right) + \lambda_3 \left(\sum_{i=1}^{N_b} y_i - p_r(\boldsymbol{x}) \right), \tag{63}$$

where $\lambda_1 \geq 0$, $\lambda_2 \geq 0$ and $\lambda_3 \geq 0$ are the Lagrange multipliers associated to the constraints (60b), (60c) and (60d), respectively. For the convex optimization problem (62), the KKT conditions are necessary and sufficient for the global convergence [6]. Solving the KKT conditions, we show that

$$x_i = \frac{b_i y_i}{2a_i(b_i y_i + 1)} \left[-b_i y_i - 2 + \sqrt{b_i^2 y_i^2 + \frac{4a_i(b_i y_i + 1)}{\lambda_{4,i}}} \right]^+, \tag{64}$$

$$y_i = \frac{a_i x_i}{2b_i(a_i x_i + 1)} \left[-a_i x_i - 2 + \sqrt{a_i^2 x_i^2 + \frac{4b_i(a_i x_i + 1)}{\lambda_3}} \right]^+, \tag{65}$$

where $(z)^+ = \max(0, z)$, and $\lambda_{4,i} \geq 0$ is given by

$$\lambda_{4,i} = \begin{cases} \lambda_1 - \frac{1}{2} \lambda_2 \hat{\zeta} \lambda_{H,1}, & \text{if the used energy is constant,} \\ \lambda_1 - \left(\frac{1}{2} \lambda_2 + \nu \lambda_3 \right) \hat{\zeta} \lambda_{H,1}, & \text{otherwise.} \end{cases} \tag{66}$$

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B. SDP Formulation

Now, let us introduce a slack variable τ_i in the problem (62), for $i = 1, \ldots, N_b$, as follows

$$\max_{\tau_i, x_i, y_i} \quad \frac{1}{2} \sum_{i=1}^{N_b} \log_2 \left(1 + \tau_i \right), \tag{67a}$$

$$\text{s.t. } (60b) - (60e), \tag{67b}$$

$$\tau_i \leq \frac{a_i b_i x_i y_i}{b_i y_i + a_i x_i}. \tag{67c}$$

Given the fact that $\frac{a_i b_i x_i y_i}{b_i y_i + a_i x_i}$ can be written as

$$\frac{a_i b_i x_i y_i}{b_i y_i + a_i x_i} = \frac{1}{2} \Big(a_i x_i + b_i y_i - a_i x_i \left(a_i x_i + b_i y_i \right)^{-1} a_i x_i$$

$$- b_i y_i \left(a_i x_i + b_i y_i \right)^{-1} b_i y_i \Big), \tag{68}$$

and using the Schur-complement theorem [23], we can show that the problem (67) is equivalent to the SDP formulation given by

$$\text{(P1)} : \max_{\tau_i, x_i, y_i} \quad \frac{1}{2} \sum_{i=1}^{N_b} \log_2 \left(1 + \tau_i \right), \tag{69a}$$

$$\text{s.t. } (60b) - (60e), \tag{69b}$$

$$\begin{bmatrix} a_i x_i + b_i y_i & 0 & b_i y_i \\ 0 & a_i x_i + b_i y_i & a_i x_i \\ b_i y_i & a_i x_i & a_i x_i + b_i y_i - 2\tau_i \end{bmatrix} \geq 0, \tag{69c}$$

or equivalently, in matrix notation, as

$$\text{(P2)} : \max_{D_\tau, D_x, D_y} \quad \frac{1}{2} \, tr \left(\log_2 \left(1 + D_\tau \right) \right), \tag{70a}$$

$$\text{s.t. } \begin{bmatrix} \left(D_a D_x + D_b D_y \right)_{ii} & 0 & \left(D_b D_y \right)_{ii} \\ 0 & \left(D_a D_x + D_b D_y \right)_{ii} & \left(D_a D_x \right)_{ii} \\ \left(D_b D_y \right)_{ii} & \left(D_a D_x \right)_{ii} & \left(D_a D_x + D_b D_y - 2D_\tau \right)_{ii} \end{bmatrix} \geq 0, \tag{70b}$$

$$\frac{1}{2} \, \hat{\zeta} \, tr \left(D_a D_x \right) \geq Q_r, \tag{70c}$$

$$tr \left(D_x \right) \leq P_s, tr \left(D_y \right) \leq p_r(x), D_x \geq 0, D_y \geq 0, D_\tau \geq 0, \tag{70d}$$

which can be easily solved by the CVX software [29] in Matlab.

Appendix B
Optimal Structure of S and F

In this section, we propose to investigate the optimal structure of S and F solutions to (P'). Let $N_b = rank \left(S \right) = rank \left(F \right)$, and let B a $N_s \times N_b$ matrix defined such as $S = BB^H$. The achievable rate in (11) is fully characterized by the diagonal elements of the minimum mean square error (MMSE) [7]. In fact, we have [30]

$$R \left(S, F \right) = -\log_2 |\text{MMSE}| = -\log_2 \left| \left(I_{N_s} + \overline{H}^H C^{-1} \overline{H} \right)^{-1} \right|, \tag{71}$$

where $\overline{H} = GFHS^{1/2}$, and $C = \sigma_r^2 GFF^H G^H + \sigma_d^2 I_{N_d}$. As such, minimizing the MMSE is equivalent to maximizing the

achievable rate. Let us denote

$$A = \hat{H}\hat{H}^H = \frac{1}{\sigma_r^2} HSH^H = U_A D_A U_A^H \tag{72}$$

$$X = \frac{\sigma_r}{\sigma_d} GF \left(I_{N_r} + A \right)^{1/2} = U_X D_X V_X^H, \tag{73}$$

where U_A, U_X, and V_X are unitary matrices with dimensions $N_r \times N_b$, $N_d \times N_b$, $N_r \times N_b$, respectively, and D_A and D_X are $N_b \times N_b$ diagonal matrices where the diagonal elements are the singular values sorted in an increasing order of A and X, respectively. In [7], it has been shown that MMSE in (71) can be written as

$$\text{MMSE} = I_{N_s} - V_A D_1 Q_2^H D_2 Q_2 D_1 V_A^H, \tag{74}$$

where V_A is the unitary matrix with dimension with dimension $N_b \times N_b$, $D_1 = \left(I_{N_b} + D_A^{-1} \right)^{-1/2}$, $D_2 = \left(I_{N_b} + D_X^{-2} \right)^{-1}$, and $Q_2 = V_X^H U_A$. Using Lemma 1 and 2 in [7], we have that the column vector containing the diagonal elements of $V_A D_1 Q_2^H D_2 Q_2 D_1 V_A^H$ is weakly submajorized [31] with $V_A = \Phi_{N_b}$ and $Q_2 = \Phi_{N_b}$, where Φ_{N_b} is a $N_b \times N_b$ matrix where the diagonal elements have unit norm and the non-diagonal elements are zero. Given the fact that the achievable rate is Schur-concave and increasing with respect to the column vector containing the diagonal elements of $V_A D_1 Q_2^H D_2 Q_2 D_1 V_A^H$ and using Lemma 3 in [7], the achievable rate is maximized when $V_A = \Phi_{N_b}$ and $Q_2 = \Phi_{N_b}$. In particular, we choose that $V_X = U_A$, without loss of generality. Now let us investigate the optimal structure of S and F.

First, we assume that the optimal S is given and we look for the structure of the optimal F. Recall that it is well known that the maximum of a multivariable objective function can be computed as $\max_{x,y} f(x, y) = \max_x \max_y f(x, y)$ [23]. The problem (P') with respect to F, for a given S, is given by

$$\max_F \quad R \left(S, F \right) \tag{75a}$$

$$\text{s.t. } tr \left(F \left(HSH^H + \sigma_r^2 I_{N_r} \right) F^H \right) \leq \bar{P}_r. \tag{75b}$$

where $\bar{P}_r = P_r \left(S \right)$ which is a constant with respect to F. Hence, the problem becomes exactly like the conventional relay-only design formulation which was previously studied [6]–[9] and it has been shown that if the objective function is Schur-concave [31], the structure of the optimal relay amplification matrix is given by

$$F = V_{G,1} D_F \left(U_{H,1} \right)^H, \tag{76}$$

where D_F is $N_b \times N_b$ diagonal matrix, and $V_{G,1}$, and $U_{H,1}$ contain the first N_b columns of V_G, and U_H, respectively. The structure of F in (76) corresponds to $U_X = U_G^H$, $V_X = U_A = U_H$, and $\Phi_{N_b} = I_{N_b}$.

Now, let us investigate the structure of the optimal S. Recall that the problem (P') is different from the optimization problem in [7] since the constraint (57c) reveals from the energy harvesting constraint and the right hand side of the constraint (57d) depends on S. However, the MMSE in (74) (or equivalently the objective function) and the constraints (57c) and (57d) depend only on the diagonal matrix D_A, independently of the structure of S. The structure of S is only significant in (57b). This implies that the structure of the optimal S is exactly the same as the one when we have only

an average power constraint at the source [7], which equals to

$$S = V_{H,1} D_S \left(V_{H,1}\right)^H, \tag{77}$$

where D_S is $N_b \times N_b$ diagonal matrix, and $V_{H,1}$ contains the first N_b columns of V_H. This concludes our proof.

APPENDIX C
MAXIMUM ACHIEVABLE HARVESTED ENERGY AT THE RELAY

Here, we propose to investigate the maximum achievable harvested energy at the EH receiver of the relay. In this case, the objective function to be maximized is the harvested energy at the relay. Thus, Problem (P) simplifies to

$$\max_S \quad Q_r(S) = \frac{1}{2} \zeta \ tr\left(HSH^H\right) \tag{78a}$$

$$\text{s.t.} \quad tr(S) \leq P_s \tag{78b}$$

$$S \geq 0. \tag{78c}$$

This optimization problem (78) is only with respect to S and independent of F, and hence it is equivalent to the one-hop MIMO systems that were previously studied in [5]. In [5], it has been shown that the optimal source covariance matrix S is given by

$$S_{EH} = P_s \ v_{H,1} \ v_{H,1}^H, \tag{79}$$

where $v_{H,1}$ is the eigenvector of $H^H H$ which corresponds to the maximum eigenvalue $\lambda_{H,1}$ of $H^H H$. The maximum harvested energy at the relay, given S_{EH}, is expressed as

$$Q_{r,max} = Q_r(S_{EH}) = \frac{1}{2} \zeta \lambda_{H,1} P_s. \tag{80}$$

From (79), we can see that the optimal S_{EH} is ranked one, and the maximum harvested energy is obtained by energy beamforming along the strongest eigenmode of $H^H H$. The corresponding used energy and stored energy at the relay are given by

$$E_u(S_{EH}) = \begin{cases} \bar{E}_u, & \text{if the used energy is constant,} \\ \nu Q_{r,max}, & \text{otherwise,} \end{cases} \tag{81}$$

$$\begin{aligned} E_s(S_{EH}) &= Q_{r,max} - E_u(S_{EH}) \\ &= \begin{cases} Q_{r,max} - \bar{E}_u, & \text{if the used energy is constant,} \\ (1-\nu) Q_{r,max}, & \text{otherwise,} \end{cases} \end{aligned} \tag{82}$$

respectively. Consequently, the optimal F_{EH} should be ranked one and is given by

$$F_{EH} = \sqrt{\frac{P_r(S_{EH})}{\sigma_r^2 \left(\lambda_{H,1} P_s + 1\right)}} \ v_{G,1} \ u_{H,1}^H, \tag{83}$$

where $v_{G,1}$, and $u_{H,1}$ are the first column and the first row of V_G, and U_H. The achievable rate corresponding to the maximum harvested energy at the relay R_{EH} is given by

$$R_{EH} = R(S_{EH}, F_{EH}) = \frac{1}{2} \log_2 \left(1 + \frac{\lambda_{H,1} b_1 P_s P_r(S_{EH})}{\lambda_{H,1} P_s + b_1 P_r(S_{EH}) + 1}\right). \tag{84}$$

APPENDIX D
MAXIMUM ACHIEVABLE END-TO-END RATE

Now, we consider a particular case of (P) when E_s is zero, and the constraint (13b) is always satisfied. In this case, the goal becomes to maximize the achievable end-to-end rate

regardless of the harvested energy at the relay. As a result, Problem (P) simplifies to

$$\max_{S,F} \quad R(S,F), \tag{85a}$$

$$\text{s.t.} \quad \frac{1}{2} \zeta \ tr\left(HSH^H\right) - E_u(S) \geq 0, \tag{85b}$$

$$tr\left(F\left(HSH^H + \sigma_r^2 I_{N_r}\right)F^H\right) \leq P_r(S), \tag{85c}$$

$$tr(S) \leq P_s, S \geq 0, \tag{85d}$$

which can be solved in a similar way as in Appendix A where $Q_r = \bar{E}_u$, if the used energy is constant, or $Q_r = 0$, otherwise. Let us denote its solutions by S_{ID} and F_{ID}. Hence, the corresponding achievable rate is denoted as

$$R_{max} = R(S_{ID}, F_{ID}), \tag{86}$$

and the corresponding achievable harvested energy, used energy and stored energy are given by

$$Q_{r,ID} = \frac{1}{2} \zeta \ tr\left(HS_{ID}H^H\right), \tag{87}$$

$$E_u(S_{ID}) = \begin{cases} \bar{E}_u, & \text{if the used energy is constant,} \\ \nu Q_{r,ID}, & \text{otherwise,} \end{cases} \tag{88}$$

$$E_s(S_{ID}) = \begin{cases} Q_{r,ID} - \bar{E}_u, & \text{if the used energy is constant,} \\ (1-\nu) Q_{r,ID}, & \text{otherwise,} \end{cases} \tag{89}$$

respectively. Note that if (85b) is active and the used energy is constant, then $Q_{r,ID} = \bar{E}_u$ and the stored energy is zero.

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