Asymmetric Hardware Distortions in Receive Diversity Systems: Outage Performance Analysis

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***Abstract*—This paper studies the impact of asymmetric hardware distortion (HWD) on the performance of receive diversity systems using linear and switched combining receivers. The asymmetric attribute of the proposed model motivates the employment of improper Gaussian signaling (IGS) scheme rather than the traditional proper Gaussian signaling (PGS) scheme. The achievable rate performance is analyzed for the ideal and non-ideal hardware scenarios using PGS and IGS transmission schemes for different combining receivers. In addition, the IGS statistical characteristics are optimized to maximize the achievable rate performance. Moreover, the outage probability performance of the receive diversity systems is analyzed yielding closed form expressions for both PGS and IGS based transmission schemes. HWD systems that employ IGS is proven to efficiently combat the self interference caused by the HWD. Furthermore, the obtained analytic expressions are validated through Monte-Carlo simulations. Eventually, non-ideal hardware transceivers degradation and IGS scheme acquired compensation are quantified through suitable numerical results.**

Keywords*—Asymmetric hardware distortions, hardware impairments, improper Gaussian signaling, SIMO systems, achievable rate, outage probability analysis.

I. Introduction

Exponentially growing demand for mobile data traffic with improved quality-of-service is one of the main features of the next generation wireless communications. Immense research has been carried out to develop new techniques/configurations in order to support the expected rapidly growing demand for high data rates [1]. Spatially diverse systems have emerged as a promising candidate to enhance system performance such as energy and spectrum efficiency [2]. However, multiple transceivers accumulate hardware imperfections from various blocks in a communication system. Therefore, the performance of the proposed systems can be highly affected by the non-ideal operation of radio frequency transceivers. Hardware impairments impose a huge challenge on next-generation network planning, deployment, measurement and testing, owing to the degraded system performance due to severe signal contamination [3]. In addition, they can limit the deployment and operation of massive amount of relatively high frequency circuitry [4]. Therefore, hardware-impaired radio frequency transeivers form a key design challenge for developing new techniques/configurations in the next generation wireless communication [5].

Hardware impairments arise in different radio frequency and baseband stages such as analog-to-digital converter imperfections, high power amplifier and low noise amplifier nonlinearity, phase noise and inphase and quadrature (I/Q) imbalance in the radio frequency front-end [5], [6]. The aforementioned imperfections impose a non-linear effect on the signal, in addition to the mismatch between the (I/Q) branches of the signal. Different compenstation schemes and algorithms are developed to dampen the tranceiver imperfections considering different models configurations [7]–[9]. In [7], Boulogeorgos *et al.* proposed a self-interference coordination scheme to mitigate the inphase and quadrature imbalance effect and improve the diversity order but at a cost of reduced transmission rate. Additonaly, Awadin *et al.* adopted an opportunistic relaying approach to deal with the imbalance problem and other interferences in [8]. Moreover, Xia *et al.* considered massive multiple-input multiple-output antenna array system and proposed a transceiver scheme to mitigate radio frequency transmit and receive chain imperfections, by exploiting the participating antenna arrays and the statistical knowledge of the channels in [9].

Analyzing the impact of hardware imperfections on the system performance and evaluating different compensation schemes requires an accurate statistical model of these imperfections. Of various hardware impairments, this work focuses on the modeling, impact and mitigation of the distortion noise from various components in RF front-end. Throughout our study, we consider the accumulated distortion noise caused by the phase noise and non-linear components called as hardware distortion (HWD). HWD is modeled as an additive impairment term at the transmitter and receiver [5], [9]–[12]. Symmetric Gaussian statistical models are used to study the impact of nonlinear distortion noise, i.e, HWD, on the outage probability, channel estimation, energy efficiency, multiplexing gain and capacity limits for different systems such as dual-hop relaying, large-scale multiple-input-single-output and massive multiple-input-multiple-output systems [9]–[12]. Recently, asymmetric or improper Gaussian signaling (IGS) is adopted to provide a general statistical model for the HWD in [13], besides inphase and quadrature imbalance. The IGS characteristics of the source and the relay are optimized in order to maximize the end-to-end rate for full duplex relaying system [13]. Moreover, we used IGS in multiple-input multiple-ouput antenna systems, to analyze the achievable rate performance under HWD and inphase and quadrature imbalance [14].

IGS is a generalized complex Gaussian scehme that is

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appropriately described by the variance as well as the pseudo-covariance in contrast to its counterpart traditional proper Gaussian signaling (PGS) sceheme, which is only described by its variance [15]. Hence, IGS offers an additional degree of design freedom pertinent to its circularly asymmetric characteristics. We should note that IGS can be practically implemented using widely linear precoders, which efficiently maps symmetric information-bearing signals to asymmetric signals at each transmitter [15], [16]. IGS scheme has already been proven to evidently improve system performance in various system configurations such as cognitive radio systems [17]–[20], full-duplex relaying [21] and alternating relaying [22].

In this work, we consider receive diversity system under asymmetric HWD at the transmitter and the multiple receive streams. Generalized asymmetric HWD model is established, pertaining to the absence of highly sensitive, matched and ideal local oscillators and phase shifters. They induce unequal power distribution in the in-phase and quadrature branch, which does not exhibit circularly symmetric characteristics [15], [23]. Then, we investigate the system performance in terms of achievable rate and outage probability analysis under asymmetric HWD. Furthermore, we propose IGS scheme adoption to efficiently mitigate the HWD effects and achieve the appropriate performance for next generation wireless communication system. The main contributions of this paper are summarized as follows:

- We analyze the achievable rate performance for single-input multiple-output (SIMO) systems under asymmetric HWD assuming two combining receivers, linear combining (LC) and selection combining (SC). The achievable rate analysis considers the two signaling schemes, PGS and IGS.
- We optimize the pseudo-variance of the IGS scheme to maximize the achievable rate for both LC and SC combiners.
- We derive closed-form expressions for the system outage probability for SIMO-LC and SC systems under the assumptions of ideal and non-ideal hardware with PGS and IGS transmission schemes.
- We validate the outage probability expressions through Monte-Carlo simulations. Furthermore, we numerically quantify the gain reaped by employing IGS scheme over the conventional PGS scheme, in terms of achievable rate and system outage probability performance.

The rest of the paper is organized as follows. Section II studies the statistical model for SIMO system under asymmetric transceiver distortions. Section III focuses on the information-theoretic achievable rate expressions for the adopted LC and SC systems. It also deals with the optimization framework to fine-tune statistical IGS parameters to achieve optimum system performance. Section IV analyzes the outage probability of the adopted systems under ideal and non-ideal hardware assumption with PGS and IGS transmission. Section V presents a performance comparison between PGS and IGS based schemes using numerical results. Finally, Section VI concludes the paper.

Notations: In this paper, scalars are denoted by lower-case italic letters, while vectors and matrices are denoted by boldfaced lower-and upper-case letters, respectively. For a complex scalar x, the conjugate and absolute value of x are represented by x^* and $|x|$, respectively. On the other hand, for a given vector $\mathbf{x}$, the complex-conjugate, transpose and conjugate-transpose of $\mathbf{x}$ are represented by $\mathbf{x}^*$, $\mathbf{x}^{\mathrm{T}}$ and $\mathbf{x}^{\mathrm{H}}$, respectively. The complex augmented random vector $\underline{\mathbf{x}}$ is defined as $\underline{\mathbf{x}} = \begin{bmatrix} \mathbf{x}^{\mathrm{T}} & \mathbf{x}^{\mathrm{H}} \end{bmatrix}^{\mathrm{T}}$. The identity matrix with N dimension is presented by $\mathbf{I}_N$. For a square matrix $\mathbf{A}$, the complex-conjugate, transpose, conjugate-transpose and inverse are represented by $\mathbf{A}^*$, $\mathbf{A}^{\mathrm{T}}$, $\mathbf{A}^{\mathrm{H}}$, and $\mathbf{A}^{-1}$, respectively. As for the trace and determinant of $\mathbf{A}$, $\mathrm{Tr}\,(\mathbf{A})$ and $|\mathbf{A}|$ are used, respectively. The expected value operator is given by $\mathbb{E}\,[.]$ and the probability of occurrence of an event Ω is expressed as $\Pr\{\Omega\}$.

II. System Description

Consider a receive diversity system where a source with a single radio frequency front-end and a receiver with multiple radio frequency front-ends for signal reception through independent and identically distributed (iid) wireless Rayleigh fading channels. Both the transmitter and the receiver are subject to HWD. In this section, we first introduce some statistical signal characteristics and then we define the mathematical model of the asymmetric HWD.

A. Statistical Signal Model

To characterize the difference between PGS and IGS, we consider a zero-mean complex Gaussian random variable x and introduce the following definitions:

Definition 1. *The variance and the pseudo-variance of x are defined, respectively, as $\sigma_x^2 = \mathbb{E}[|x|^2]$ and $\tilde{\sigma}_x^2 = \mathbb{E}[x^2]$ [24].*

Definition 2. *A complex random variable is called proper if its pseudo-variance is equal to zero, otherwise it is called improper [24].*

Definition 3. *Circularity coefficient $\mathcal{C}_x$ of a random variable x is defined as the ratio of the absolute pseudo-variance of x and its variance, i.e., $\mathcal{C}_x$=$\left|\tilde{\sigma}_x^2\right|/\sigma_x^2$, where $0 \leq \mathcal{C}_x \leq 1$ [25].*

The circularity coefficient measures the degree of impropriety or symmetry of x, $\mathcal{C}_x = 0$ indicates proper or symmetric signal and $\mathcal{C}_x = 1$ indicates maximally improper or maximally asymmetric signal.

Definition 4. *The complex Gaussian random variable x is described as follows: $x \sim \mathcal{CN}(m_x, \sigma_x^2, \tilde{\sigma}_x^2)$, where m_x is the mean of x, i.e., $m_x = \mathbb{E}[x]$.*

B. Distortion Model

HWD is modeled as an additive Gaussian noise at both the transmitter and the receiver [4], [9], [10], [26]–[28]. Thus, the received signals at different receiver streams are given in $\mathbf{r}$,

$$\mathbf{r} = \sqrt{p}\mathbf{h}\left(x + d_{\mathrm{T}}\right) + \mathbf{d}_{\mathrm{R}} + \mathbf{w} \tag{1}$$

where p is the transmitted power, $\mathbf{h}$ is a vector that contains the flat fading channel coefficients from the the transmit antenna to receive antennas, x is the transmitted signal, d_{T} is the HWD at transmitter side, $\mathbf{d}_{\mathrm{R}}$ is the HWD at different receiver streams and $\mathbf{w}$ is the additive thermal noise vector at the receiver streams, which is modeled as iid additive white Gaussian noise (AWGN) with $\mathbf{w} \sim \mathcal{CN}(0, \sigma_w^2 \mathbf{I}_N, 0)$. The channel coefficients in $\mathbf{h}$ is iid with $\mathbf{h} \sim \mathcal{CN}(0, \pi \mathbf{I}_N, 0)$. According to our asymmetric HWD assumption, d_{T} follows IGS distribution with $d_{\mathrm{T}} \sim \mathcal{CN}(0, \kappa_{\mathrm{tx}}, \tilde{\kappa}_{\mathrm{tx}})$. As for the HWD at the receiver, they are modeled as $\mathbf{d}_{\mathrm{R}} \sim \mathcal{CN}(0, p\mathbf{h}\mathbf{K}_{\mathrm{rx}}\mathbf{h}^{\mathrm{H}}, p\mathbf{h}\tilde{\mathbf{K}}_{\mathrm{rx}}\mathbf{h}^{\mathrm{T}})$ with $\mathbf{K} = \kappa_{\mathrm{rx}} \mathbf{I}_N$ and $\tilde{\mathbf{K}} = \tilde{\kappa}_{\mathrm{rx}} \mathbf{I}_N$ assuming common local oscillator and thus iid HWD at different receiver streams. The proposed asymmetric HWD model is a general model to the existing models in the literature, where it reduces to the HWD models in [4], [10], [27] when $\tilde{\kappa}_{\mathrm{tx}} = \tilde{\kappa}_{\mathrm{rx}} = 0$. As for the transmitted signal, we assume it to be chosen from IGS scheme with $x \sim \mathcal{CN}(0, \sigma_x^2, \tilde{\sigma}_x^2)$ since PGS scheme represents a special case of IGS, at which $\tilde{\sigma}_x^2 = 0$. By considering the aggregate effect of the HWD at both the transmitter and the receiver, we can write (1) equivalently as

$$\mathbf{r} = \sqrt{p}\mathbf{h}\left(x + d\right) + \mathbf{w}, \tag{2}$$

where d represents the aggregate effect of HWDs in the whole system and modeled as $d \sim \mathcal{CN}(0, \kappa, \tilde{\kappa})$, where $\kappa = \kappa_{\mathrm{tx}} + \kappa_{\mathrm{rx}}$ and $\tilde{\kappa} = \tilde{\kappa}_{\mathrm{tx}} + \tilde{\kappa}_{\mathrm{rx}}$ account for the aggregate distortion in each channel path. At the receiver side, the signal can be detected following different diversity techniques. In the rest of this paper, we apply our analysis on two combing techniques known as linear receiver combiner and selection combining.

1) Linear Combining Receiver: Under SIMO receive diversity, the received observations are linearly combined as $y = \varphi^{\mathrm{H}}\mathbf{r}$ in order to be efficiently used in the detection process. Given the channel state information at the receiver side, the optimal linear receiver combiner vector that maximizes the SNR for the ideal system model is $\varphi = \mathbf{h}$. Therefore, the resultant superposed signal becomes

$$y_{\mathrm{LC-Ideal}} = \sqrt{p}\mathbf{h}^{\mathrm{H}}\mathbf{h}x + \mathbf{h}^{\mathrm{H}}\mathbf{w}. \tag{3}$$

For the non-ideal hardware scenario, we use the same combining vector for non-ideal hardware to render minimal computational expenses.

$$y_{\mathrm{LC-HWD}} = \sqrt{p}\mathbf{h}^{\mathrm{H}}\mathbf{h}x + \sqrt{p}\mathbf{h}^{\mathrm{H}}\mathbf{h}d + \mathbf{h}^{\mathrm{H}}\mathbf{w} \tag{4}$$

2) Switched Combining Receiver: Linear combining scheme is efficient, however it imposes a receiver processing overhead with some computational and hardware complexity. Conversely, switched diversity or SC scheme reduces hardware complexity, power consumption, inter-channel interference, and antenna synchronization problems [29]. In this scheme, the receiver selects the receiver stream that achieves maximum performance. For ideal hardware scenario, the received signal at the $j-\mathrm{th}$ stream is expressed as

$$y_j = \sqrt{p}h_j x + w_j, \tag{5}$$

where $1 \leqslant j \leqslant N$ and w_j is AWGN thermal noise at j^{th} receiver stream with $w_j \sim \mathcal{CN}(0, \sigma_w^2, 0)$. In this scenario, the selected stream maximizes the signal-to-noise ratio (SNR) and in turn provides the maximum instantaneous achievable rate and minimal system outage probability when compared with other streams. On the other hand, for the non-ideal scenarios, the received signal at the $j-\mathrm{th}$ receiver stream is expressed as

$$y_j = \sqrt{p}h_j(x + \eta_j) + w_j, \tag{6}$$

where η_j is the aggregate HWD with $\eta_j \sim \mathcal{CN}(0, \kappa, \tilde{\kappa})$.

III. ACHIEVABLE RATE

In this section, we express the instantaneous achievable rate expressions assuming ideal and non-ideal hardware for SIMO systems under two different receiver combining schemes, LC and SC. First, under the assumption of proper Gaussian noise, the differential entropy is maximized if the transmitted signal is PGS [15]. However, according to our proposed asymmetric HWD where the aggregate noise reduces to IGS, the differential entropy is maximized when the transmitted signal is IGS [15]. The achievable rate in terms of mutual information between complex Gaussian transmitted random variable $\mathbf{x}$ and the received signal vector $\mathbf{y}$ in the presence of Gaussian interference $\mathbf{z}$, is given as

$$R = I\left(\mathbf{x};\mathbf{y}\right) = h\left(\mathbf{y}\right) - h\left(\mathbf{y}|\mathbf{x}\right) = h\left(\mathbf{y}\right) - h\left(\mathbf{z}\right) \tag{7}$$

where, $I\left(\mathbf{x};\mathbf{y}\right)$ is mutual information between two variables $\mathbf{x}$ and $\mathbf{y}$ and $h\left(.\right)$ is the differential entropy of the information signal that is given by the following lemma.

Lemma 1. *The entropy of a random complex IGS random vector $\mathbf{x}$ with dimension of N, and augmented covariance matrix $\underline{\mathbf{C}}_{\mathbf{x}}$, is in general a function of both covariance and pseudo-covariance matrices and is given as [15]*

$$h\left(\mathbf{x}\right) = \frac{1}{2}\log_2\left(\left(\pi e\right)^{2N}\left|\underline{\mathbf{C}}_{\mathbf{x}}\right|\right) \tag{8}$$

where $\underline{\mathbf{C}}_{\mathbf{x}}$ is defined as

$$\underline{\mathbf{C}}_{\mathbf{x}} \triangleq \mathrm{E}\left(\left[\begin{array}{c}\mathbf{x}\\ \mathbf{x}^*\end{array}\right]\left[\begin{array}{c}\mathbf{x}\\ \mathbf{x}^*\end{array}\right]^{\mathrm{H}}\right) = \left[\begin{array}{cc}\mathbf{C}_{\mathbf{x}} & \tilde{\mathbf{C}}_{\mathbf{x}}\\ \tilde{\mathbf{C}}_{\mathbf{x}}^* & \mathbf{C}_{\mathbf{x}}^*\end{array}\right]. \tag{9}$$

For a PGS random variable $\mathbf{x}$ with $\tilde{\mathbf{C}}_{\mathbf{x}} = 0$, the entropy reduces to a well-known expression $h\left(\mathbf{x}\right) = \log_2\left(\left(\pi e\right)^{N}\left|\mathbf{C}_{\mathbf{x}}\right|\right)$.

After applying Lemma 1 on the SIMO system using one of the suggested combiners, we obtain the achievable rate according to the following lemma.

Lemma 2. *The achievable rate expression for SIMO systems with HWD is expressed as*

$$R_{\mathrm{SIMO}} = \frac{1}{2}\log_2\left(\frac{\sigma_y^4 - \left|\tilde{\sigma}_y^2\right|^2}{\sigma_z^4 - \left|\tilde{\sigma}_z^2\right|^2}\right), \tag{10}$$

where σ_y^2 and $\tilde{\sigma}_y^2$ are the variance and the pseudo-variance of the received signal, respectively. However, σ_z^2 and $\tilde{\sigma}_z^2$ are the variance and pseudo-variance of the undesired signal components, respectively.

A. Linear Combining Receiver

In this subsection, we analyze the instantaneous achievable rate for SIMO systems that use LC and are subjected to HWD. The rate analysis is done assuming both signaling schemes PGS and IGS, besides considering the ideal hardware scenario as reference case. Moreover, we optimize the IGS scheme to maximize the achievable rate by mitigating the transceiver HWD.

1) Ideal Hardware Scenario: First, consider the LC observation of SIMO system without any HWD given in (3). We observe that the undesired signal, is only comprised of a linear combination of iid PGS thermal noise, with variance $\sigma_z^2 = \mathbf{h}^{\mathrm{H}}\mathbf{h}\sigma_w^2$. Moreover, the superposed received signal y has a variance $\sigma_y^2 = p(\mathbf{h}^{\mathrm{H}}\mathbf{h})^2\sigma_x^2 + \sigma_z^2$. The symmetric nature of the superposed signals and distortions induces zero pseudo-variances. Thus, using the achievable rate expression presented in Lemma 2 for the ideal LC system with x belonging to PGS scheme is expressed as

$$R_{\mathrm{LC-Ideal}} = \log_2\left(1 + \frac{p\left(\mathbf{h}^{\mathrm{H}}\mathbf{h}\right)\sigma_x^2}{\sigma_w^2}\right) \tag{11}$$

2) Non-Ideal Hardware with PGS Scenario: The linearly combined signal under HWD is expressed in (4) with undesired signal component $z = \sqrt{p}\mathbf{h}^{\mathrm{H}}\mathbf{h}d + \mathbf{h}^{\mathrm{H}}\mathbf{w}$. The variances of the superposed signal y and distortion signal z are found to be $\sigma_y^2 = p\left(\mathbf{h}^{\mathrm{H}}\mathbf{h}\right)^2\left(\sigma_x^2 + \kappa\right) + \mathbf{h}^{\mathrm{H}}\mathbf{h}\sigma_w^2$ and $\sigma_z^2 = p\left(\mathbf{h}^{\mathrm{H}}\mathbf{h}\right)^2\kappa + \mathbf{h}^{\mathrm{H}}\mathbf{h}\sigma_w^2$, respectively. As for the pseudo-variance, using PGS yields the same pseudo-variance where $\sigma_y^2 = \sigma_z^2 = p\mathbf{h}^{\mathrm{H}}\mathbf{h}\tilde{\kappa}$. Therefore, the instantaneous achievable rate of LC system under HWD when PGS is adopted for transmission is obtained using Lemma 2 as

$$R_{\mathrm{LC-PGS}} = \frac{1}{2}\log_2\frac{\left(p\mathbf{h}^{\mathrm{H}}\mathbf{h}\left(\sigma_x^2+\kappa\right)+\sigma_w^2\right)^2 - \left|p\mathbf{h}^{\mathrm{H}}\mathbf{h}\tilde{\kappa}\right|^2}{\left(p\mathbf{h}^{\mathrm{H}}\mathbf{h}\kappa+\sigma_w^2\right)^2 - \left|p\mathbf{h}^{\mathrm{H}}\mathbf{h}\tilde{\kappa}\right|^2}. \tag{12}$$

3) Non-Ideal Hardware with IGS Scenario: In this scenario, the received and undesired signals have the same expressions and variances as in the previous PGS scenario. The only difference is on the pseudo-variance, where $\tilde{\sigma}_y^2 = p\mathbf{h}^{\mathrm{H}}\mathbf{h}(\tilde{\sigma}_x^2 + \tilde{\kappa})$ and $\tilde{\sigma}_z^2 = p\mathbf{h}^{\mathrm{H}}\mathbf{h}\tilde{\kappa}$ respectively. As a result, the instantaneous achievable rate of LC system under HWD when IGS is adopted for transmission is given as

$$R_{\mathrm{LC-IGS}} = \frac{1}{2}\times \log_2\frac{\left(p\mathbf{h}^{\mathrm{H}}\mathbf{h}\left(\sigma_x^2+\kappa\right)+\sigma_w^2\right)^2 - \left|p\mathbf{h}^{\mathrm{H}}\mathbf{h}(\tilde{\sigma}_x^2+\tilde{\kappa})\right|^2}{\left(p\mathbf{h}^{\mathrm{H}}\mathbf{h}\kappa+\sigma_w^2\right)^2 - \left|p\mathbf{h}^{\mathrm{H}}\mathbf{h}\tilde{\kappa}\right|^2} \tag{13}$$

HWD systems employing IGS can be designed to mitigate the hardware imperfections impact by optimizing the pseudo-variance of the transmitted signal in order to maximize the achievable rate (13). The design optimization problem can be written as

$$\begin{aligned} \mathbf{P1}: \quad & \max_{\tilde{\sigma}_x^2} R_{\mathrm{LC-IGS}}\left(\sigma_x^2, \tilde{\sigma}_x^2\right) \\ & \text{s. t.} \quad 0 \le \left|\tilde{\sigma}_x^2\right| \le \sigma_x^2. \end{aligned}$$

Theorem 1. *The achievable rate of LC system that suffers from HWD is maximized, when adopting IGS scheme, by tuning the transmitted signal characteristics such that* $\tilde{\sigma}_x^2 = -\tilde{\kappa}$.

Proof: Proof is given in Appendix A. ■

The maximum achievable rate of the IGS-based scheme, $\hat{R}_{\mathrm{LC-IGS}}$, after optimizing the IGS transmitted signal is expressed as

$$\hat{R}_{\mathrm{LC-IGS}} = \frac{1}{2}\log_2\frac{\left(p\mathbf{h}^{\mathrm{H}}\mathbf{h}\left(\sigma_x^2+\kappa\right)+\sigma_w^2\right)^2}{\left(p\mathbf{h}^{\mathrm{H}}\mathbf{h}\kappa+\sigma_w^2\right)^2 - \left|p\mathbf{h}^{\mathrm{H}}\mathbf{h}\tilde{\kappa}\right|^2} \tag{14}$$

B. Switched Combining Receiver

Employing SC receivers offers a less complexity system solution with improved energy efficiency at a cost of performance loss. In the following, we study the achievable rate for ideal hardware scenario and non-ideal scenarios assuming the transmitted signal to be either PGS or IGS for the later scenarios while PGS for the former scenario.

1) Ideal Hardware Scenario: Consider the ideal hardware SIMO system using SC receiver. Thus, the achievable rate of this receiver is expressed as

$$R_{\mathrm{SC-Ideal}} = \max_j R^j_{\mathrm{SC-Ideal}}, \tag{15}$$

where $R^j_{\mathrm{SC-Ideal}}$ is the achievable rate of the $j-\mathrm{th}$ receiver stream that is expressed as

$$R^j_{\mathrm{SC-Ideal}} = \log_2\left(1 + \frac{p|h_j|^2\sigma_x^2}{\sigma_w^2}\right). \tag{16}$$

Choosing the best receiver stream according to the maximum rate is equivalent to the stream with maximum SNR as can be deduced from (15) and (16)

2) Non-Ideal Hardware with PGS Scenario: In the non-ideal hardware scenario with PGS, the achievable rate of the SC receiver is expressed as

$$R_{\mathrm{SC-PGS}} = \max_j R^j_{\mathrm{SC-PGS}}, \tag{17}$$

where $R^j_{\mathrm{SC-PGS}}$ is the achievable rate of the $j-\mathrm{th}$ receiver stream when the transmitted signal is PGS and the system suffers from asymmetric HWD at both the transmitter and the receiver. $R^j_{\mathrm{SC-PGS}}$ can be derived based on the received signal at the $j-\mathrm{th}$ stream in (6) and Lemma 2 as

$$R^j_{\mathrm{SC-PGS}} = \frac{1}{2}\log_2\frac{\left(p|h_j|^2\left(\sigma_x^2+\kappa\right)+\sigma_w^2\right)^2 - \left|p{h_j}^2\tilde{\kappa}\right|^2}{\left(p|h_j|^2\kappa+\sigma_w^2\right)^2 - \left|p{h_j}^2\tilde{\kappa}\right|^2}. \tag{18}$$

Different from the ideal hardware scenario which has a clear mapping relation between the achievable rate and the SNR, the non-ideal hardware scenario does not have such clear relation. Thus, the best receiver stream should be chosen based on the maximum rate performance.

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3) Non-Ideal Hardware with IGS Scenario: Similar to the PGS based transmission in the non-ideal hardware scenario, the IGS based SC system maximum achievable rate is expressed as

$$R_{\mathrm{SC-IGS}} = \max_{j} R^{j}_{\mathrm{SC-IGS}}, \quad (19)$$

where $R^{j}_{\mathrm{SC-IGS}}$ is the achievable rate under HWD of the $j-\mathrm{th}$ receiver stream assuming IGS transmitting scheme, which is expressed as

$$R^{j}_{\mathrm{SC-IGS}} = \frac{1}{2}\times \log_2 \frac{\left(p|h_j|^2\left(\sigma_x^2+\kappa\right)+\sigma_w^2\right)^2 - \left|p{h_j}^2\left(\tilde{\sigma}_x^2+\tilde{\kappa}\right)\right|^2}{\left(p|h_j|^2\kappa+\sigma_w^2\right)^2 - \left|p{h_j}^2\tilde{\kappa}\right|^2}. \quad (20)$$

It is worthy to emphasize that the IGS based scheme does not have a clear mapping between the SNR and the achievable rate similar to the PGS based scheme. To improve the system performance of the IGS based scheme, the pseudo-variance of the transmitted signal is optimized to increase the achievable rate performance of each receiver stream. Then, the stream with maximum rate is chosen for the detection process. The pseudo-variance is tuned according to the following optimization problem

$$\mathbf{P2}: \quad \max_{\tilde{\sigma}_x^2} R^{j}_{\mathrm{SC-IGS}}\left(\sigma_x^2, \tilde{\sigma}_x^2\right) \quad \text{s. t.} \quad 0 \le \left|\tilde{\sigma}_x^2\right| \le \sigma_x^2.$$

Theorem 2. *The achievable rate of HWD SIMO systems with SC receiver is maximized, when adopting IGS scheme, by tuning the transmitted signal characteristics such that* $\hat{\tilde{\sigma}}_x{}^2 = -\tilde{\kappa}$.

Proof: The proof can be obtained by following the similar steps as in Appendix A. ■

Therefore, the maximum achievable rate of the $j-\mathrm{th}$ receiver stream, $\hat{R}^{j}_{\mathrm{SC-IGS}}$, is expressed based on Theorem 2 as

$$\hat{R}^{j}_{\mathrm{SC-IGS}} = \frac{1}{2}\log_2 \frac{\left(p|h_j|^2\left(\sigma_x^2+\kappa\right)+\sigma_w^2\right)^2}{\left(p|h_j|^2\kappa+\sigma_w^2\right)^2 - \left|p{h_j}^2\tilde{\kappa}\right|^2}. \quad (21)$$

The maximized instantaneous achievable rate of the SIMO systems with LC or SC receivers is used in the following section is studying the outage probability analysis for the ideal and non-ideal hardware scenarios with either PGS or IGS transmission scheme.

IV. Outage Probability Analysis

In this section, we analyze the outage probability performance of the SIMO systems for the three different scenarios, ideal hardware, non-ideal hardware with PGS and non-ideal hardware with IGS. It is worth to emphasize that although studying the SNR-based outage analysis is equivalent to the rate-based outage for the ideal hardware scenario, it is not the same case for non-ideal hardware scenarios due to the existence of the asymmetric signals. Therefore, considering rate-based outage is imperative to have a fair comparison study between different scenarios.

A. Linear Combining Receiver

Based on the rate expressions obtained in the former section, the system outage probability is defined as the probability of an event when the instantaneous achievable rate R_{SIMO} falls below certain threshold target rate R_{th}.

$$P_{\mathrm{LC}} = \Pr\left\{R_{\mathrm{LC}} \leqslant R_{\mathrm{th}}\right\} \quad (22)$$

Using (22), system outage probability analysis is carried out in receive diversity systems under ideal hardware, non-ideal hardware with PGS or IGS transmission scheme.

1) Ideal Hardware Scenario: The ideal hardware scenario provides a lower bound on the outage probability performance of systems with HWD. To this end, the outage probability of Raleigh fading channel with a target rate R_{th} bits/sec/Hz is derived using the rate expression $R_{\mathrm{LC-Ideal}}$ (11) in (22)

$$P_{\mathrm{LC-Ideal}} = \Pr\left\{\frac{\mathbf{h}^{\mathrm{H}}\mathbf{h}}{\sigma_w^2} \leqslant \frac{2^{R_t}-1}{p\sigma_x^2}\right\} \quad (23)$$

Let $\psi = \frac{\mathbf{h}^{\mathrm{H}}\mathbf{h}}{\sigma_w^2}$ and since $\mathbf{h} \sim \mathcal{CN}(0, \pi\mathbf{I}_N, 0)$, then ψ follows Erlang distribution, i.e., $\psi \sim \mathrm{Erlang}(\frac{\sigma_\mathrm{w}^2}{\pi}, \mathrm{N})$. Thus, the probability density function of ψ, $f_\psi(x)$, is expressed for $x \geq 0$ as

$$f_\psi(x) = \frac{\sigma_w^{2N}}{\pi^N (N-1)!} x^{N-1} e^{-\frac{\sigma_w^2}{\pi}x} \quad \forall\{x \geqslant 0\}. \quad (24)$$

Moreover, define $\psi_0 = \frac{2^{R_\mathrm{t}}-1}{p\sigma_x^2}$, then the outage probability in (23) reduces to $P_{\mathrm{LC}} = \Pr\{\psi \leqslant \psi_0\}$. Therefore, the outage probability is expressed as

$$P_{\mathrm{LC-Ideal}} = \int_0^{\psi_0} f_\psi(x)\,dx = 1 - \frac{\sigma_w^{2N}}{\pi^N (N-1)!}{\psi_0}^N \int_1^{\infty} \frac{e^{-\frac{\sigma_w^2}{\pi}\psi_0 t}}{t^{1-N}} dt \quad (25)$$

2) Non-Ideal Hardware with PGS Scenario: The outage probability performance of the SIMO system with transceivers asymmetric HWD under PGS transmission is given by the following theorem.

Theorem 3. *In a SIMO system that adopts PGS scheme for transmission and LC receiver while suffering from asymmetric HWD, the outage probability,* $P_{\mathrm{LC-PGS}}(R_{\mathrm{th}})$*, to achieve a target rate* R_{th} bits/sec/Hz *assuming Rayleigh fading channel is given by* (26)*, where* ψ_1 *is expressed as*

$$\psi_1 = \frac{-\sigma_x^2 - \left(1-2^{2R_{\mathrm{th}}}\right)\kappa + \sqrt{2^{2R_{\mathrm{th}}}\sigma_x^4 + \left(1-2^{2R_{\mathrm{th}}}\right)^2|\tilde{\kappa}|^2}}{p\left[\left(1-2^{2R_{\mathrm{th}}}\right)\left(\kappa^2-|\tilde{\kappa}|^2\right)+2\kappa\sigma_x^2+\sigma_x^4\right]} \quad (27)$$

$$P_{\mathrm{LC-PGS}}\left(R_{\mathrm{th}}\right)=\begin{cases}1-\frac{\sigma_w^{2N}}{\pi^N(N-1)!}\psi_1{}^N\int\limits_1^\infty\frac{e^{-\frac{\sigma_w^2}{\pi}\psi_1 t}}{t^{1-N}}dt & \text{if } 0\le R_{\mathrm{th}}\le R_1\\ 1 & \text{if } R_1\le R_{\mathrm{th}}<\infty\end{cases} \tag{26}$$

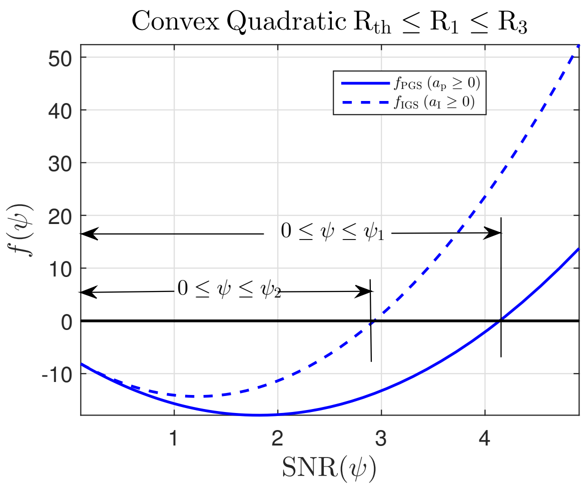

Fig. 1. Convex quadratic function

and R_1 is found from

$$R_1=\frac{1}{2}\log_2\left(\frac{\left(\sigma_x^2+\kappa\right)^2-|\tilde{\kappa}|^2}{\kappa^2-|\tilde{\kappa}|^2}\right). \tag{28}$$

Proof: Based on the LC-PGS achievable rate, the outage probability of non-ideal SIMO system under PGS is given by $\Pr\left\{R_{\mathrm{LC-PGS}}\leqslant R_{\mathrm{th}}\right\}$. Thus using the rate expression from (12), we obtain $P_{\mathrm{LC-PGS}}\left(R_{\mathrm{th}}\right)$ as a quadratic inequality.

$$P_{\mathrm{LC-PGS}}\left(R_{\mathrm{th}}\right)=\Pr\left\{a_{\mathrm{p}}\psi^2+b\psi+c\le 0\right\} \tag{29}$$

where, $a_{\mathrm{p}}=p^2\left(\left(1-2^{2R_{\mathrm{th}}}\right)\left(\kappa^2-|\tilde{\kappa}|^2\right)+2\kappa\sigma_x^2+\sigma_x^4\right),$

$$b=2p\left(\left(1-2^{2R_{\mathrm{th}}}\right)\kappa+\sigma_x^2\right),$$

$$c=\left(1-2^{2R_{\mathrm{th}}}\right)\quad\text{and}\quad\psi=\frac{\mathbf{h}^{\mathrm{H}}\mathbf{h}}{\sigma_w^2}$$

Let $f_{\mathrm{PGS}}\left(\psi\right)=a_{\mathrm{p}}\psi^2+b\psi+c$. To solve (29), we need to investigate the cases at which $f_{\mathrm{PGS}}\left(\psi\right)\le 0$. First, having $R_{\mathrm{th}}>0$ implies $c\le 0$, thus different possible cases for $f_{\mathrm{PGS}}\left(\psi\right)\le 0$ is determined based on a_{p} and b as follows

1) Convex Quadratic ($a_{\mathrm{p}}\ge 0$): In this case, $f_{\mathrm{PGS}}\left(\psi\right)$ represents convex function in ψ and the equality $f_{\mathrm{PGS}}\left(\psi\right)=0$ possesses only one non-negative zero ψ_1, that is given by (27), irrespective of the value of b, as illustrated in Fig 1. Therefore, we can find the outage probability by finding the probability of event when $0\le\psi\le\psi_1$, which is given as follows

$$\begin{aligned}P_{\mathrm{LC-PGS}}&=\int\limits_0^{\psi_1}f_\psi\left(x\right)dx\\&=1-\frac{\sigma_w^{2N}}{\pi^N(N-1)!}\psi_1{}^N\int\limits_1^\infty\frac{e^{-\frac{\sigma_w^2}{\pi}\psi_1 t}}{t^{1-N}}dt.\end{aligned} \tag{30}$$

The aforementioned expression is valid for $a_{\mathrm{p}}\ge 0$, which reduces to $R_{\mathrm{th}}\le R_1$, where R_1 is given by (28) after some mathematical manipulations.

2) Concave Quadratic ($a_{\mathrm{p}}\le 0$, $b\le 0$): In this case, $f_{\mathrm{PGS}}\left(\psi\right)<0$ is always valid, which reduces to outage probability equal to 1. Moreover, the condition ($a_{\mathrm{p}}\le 0$, $b\le 0$ and $c\le 0$) implies that R_{th} should be greater than R_1 to incur complete system outage.

3) Concave Quadratic ($a_{\mathrm{p}}\le 0, b\ge 0$): In this case, $a_{\mathrm{p}}\le 0$ reduces to $R_{\mathrm{th}}\ge R_1$, while $b\ge 0$ reduces to $R_{\mathrm{th}}\le R_2$, which is expressed as

$$R_2=\frac{1}{2}\log_2\left(1+\frac{\sigma_x^2}{\kappa}\right). \tag{31}$$

Appendix B demonstrates $R_2<R_1$, that results in contradicting conditions. Consequently, this case does not contribute in the overall outage probability expression.

Encompassing all the possible cases of outage occurrence with PGS scheme, which is summarized in Table I, completes the proof of Theorem 3. ■

3) Non-Ideal Hardware with IGS Scenario: The outage probability of the LC SIMO system under HWD when the system uses IGS scheme for transmission can be found from evaluated according to

$$P_{\mathrm{LC-IGS}}\left(R_{\mathrm{th}}\right)=\Pr\left\{\hat{R}_{\mathrm{LC-IGS}}<R_{\mathrm{th}}\right\}, \tag{32}$$

and is given by the following theorem.

Theorem 4. *In a SIMO system that uses LC receiver and IGS scheme for transmission and is subjected to asymmetric HWD, the outage probability of achieving a target rate* R_{th} bits/sec/Hz *assuming Rayleigh fading channel is given by* (33)*, where ψ_2 is defined as*

$$\psi_2=\frac{-\sigma_x^2-\left(1-2^{2R_{\mathrm{th}}}\right)\kappa+2^{R_{\mathrm{th}}}\sqrt{\sigma_x^4-\left(1-2^{2R_{\mathrm{th}}}\right)|\tilde{\kappa}|^2}}{p\left[\left(\sigma_x^2+\kappa\right)^2-2^{2R_{\mathrm{th}}}\left(\kappa^2-|\tilde{\kappa}|^2\right)\right]} \tag{34}$$

and R_3 is defined as

$$R_3=\frac{1}{2}\log_2\left(\frac{\left(\sigma_x^2+\kappa\right)^2}{\kappa^2-|\tilde{\kappa}|^2}\right) \tag{35}$$

$$P_{\mathrm{LC-IGS}}\left(R_{\mathrm{th}}\right)=\begin{cases}1-\frac{\sigma_w^{2N}}{\pi^{\ N}(N-1)!}{\psi_2}^N\int\limits_1^\infty \frac{e^{-\frac{\sigma_w^2}{\pi}\psi_2 t}}{t^{1-N}}dt & \text{if } 0\le R_{\mathrm{t}}\le R_3\\ 1 & \text{if } R_3\le R_{\mathrm{th}}<\infty\end{cases} \tag{33}$$

Proof: By substituting the optimized rate expression $\hat{R}_{\mathrm{LC-IGS}}$ from (14) in (32), yields outage probability as given by (36).

$$P_{\mathrm{LC-IGS}}\left(R_{\mathrm{th}}\right)=\Pr\left\{a_{\mathrm{I}}\psi^2+b\psi+c\le 0\right\} \tag{36}$$

where, $a_{\mathrm{I}}=p^2\left(\left(1-2^{2R_{\mathrm{th}}}\right)\kappa^2+2^{2R_{\mathrm{th}}}|\tilde{\kappa}|^2+2\kappa\sigma_x^2+\sigma_x^4\right)$. Let $f_{\mathrm{IGS}}\left(\psi\right)=a_{\mathrm{I}}\psi^2+b\psi+c$. To solve (36), detailed investigation has been carried out to enumerate all cases at which $f_{\mathrm{IGS}}\left(\psi\right)\le 0$. It is important to highlight that the coefficients b and c remain the same for both PGS and IGS schemes. Given, $R_{\mathrm{th}}>0$ implies $c\le 0$, thus different possible cases for $f_{\mathrm{IGS}}\left(\psi\right)\le 0$ are analyzed based on a_{I} and b as follows

1) Convex Quadratic ($a_{\mathrm{I}}\ge 0$): In this case, $f_{\mathrm{IGS}}\left(\psi\right)$ is evidently a convex function in ψ as shown by dotted line in Fig 1. Regardless of the value of b, the equality $f_{\mathrm{IGS}}\left(\psi\right)=0$ possesses only one non-negative zero ψ_2, that is given by (34). Consequently, we can find the outage probability $P_{\mathrm{LC-IGS}}$ by finding the probability of event when $0\le\psi\le\psi_2$, which is given in closed-form expression as

$$\begin{aligned}P_{\mathrm{LC-IGS}}&=\int\limits_0^{\psi_2} f_\psi\left(x\right)dx\\ &=1-\frac{\sigma_w^{2N}}{\pi^N(N-1)!}{\psi_2}^{\mathrm{N}}\int\limits_1^\infty\frac{e^{-\frac{\sigma_w^2}{\pi}\psi_2 t}}{t^{1-N}}dt\end{aligned} \tag{37}$$

The above stated expression is valid for $a_{\mathrm{I}}\ge 0$, which reduces to $R_{\mathrm{th}}\le R_3$. R_3 stated by (35), is a function of signal power and HWI characteristics.

2) Concave Quadratic ($a_{\mathrm{I}}\le 0,\ b\le 0$): In this case, $f_{\mathrm{IGS}}\left(\psi\right)$ <0 is always valid $\forall\psi\ge 0$, which imparts complete system outage i.e. $P_{\mathrm{LC-IGS}}=1$. Furthermore, the condition ($a_{\mathrm{I}}\le 0,\ b\le 0$ and $c\le 0$) implies that R_{t} should be greater than R_3 to result in overall system outage.

3) Concave Quadratic ($a_{\mathrm{I}}\le 0,\ b\ge 0$): This case implies two constraints on the target achievable rate. Firstly, $a_{\mathrm{I}}\le 0$ reduces to $R_{\mathrm{th}}\ge R_3$, whereas $b\ge 0$ reduces to $R_{\mathrm{th}}\le R_2$, where R_2 is given by (31).
Following the similar steps as in Appendix B, one can prove that $R_2<R_3$. This results in contradicting conditions. Therefore, this case does not participate in the overall outage probability expression for $P_{\mathrm{LC-IGS}}$.

Encompassing all the possible cases of outage occurrence with IGS scheme, completes the proof of Theorem 4. ∎

B. Switched Combining Receiver

In this section, we analyze the outage probability performance of a SIMO system with SC receiver. In this system, the outage is declared when the maximum instantaneous achievable rate $\hat{R}_{\mathrm{SC}}^j$ falls below a target rate R_{th}.

$$P_{\mathrm{SC}}\left(R_{\mathrm{th}}\right)=\Pr\left\{\max_j \hat{R}_{\mathrm{SC}}^j<R_{\mathrm{th}}\right\} \tag{38}$$

1) Ideal Hardware Scenario: Ideal hardware scenario of SIMO system with SC receiver sets a lower bound of the outage probability for SC with HWD. Substituting $R_{\mathrm{SC-Ideal}}^j$ form (16) in (38), yields the outage probability of Raleigh fading channel with a target rate R_{th} $\mathrm{bits/sec/Hz}$

$$P_{\mathrm{SC-Ideal}}\left(R_{\mathrm{th}}\right)\ =\Pr\left\{\max_j\ |h_j|^2\leqslant\frac{\sigma_w^2\left(2^{R_{\mathrm{th}}}-1\right)}{p\sigma_x^2}\right\}. \tag{39}$$

Let $\zeta_j=|h_j|^2$, which has iid exponential distribution, i.e., $\zeta_j\sim\exp(\pi)\ \ \forall 1\le j\le N$. Moreover, assume $\zeta_0=\frac{\sigma_w^2\left(2^{R_{\mathrm{th}}}-1\right)}{p\sigma_x^2}$ and $\zeta=\max_j\zeta_j$, thus the probability density function of ζ is given by

$$f_\zeta\left(z\right)=N\left(\frac{1}{\pi}e^{-\frac{z}{\pi}}\right)\left(1-e^{-\frac{z}{\pi}}\right)^{N-1};\quad z\geqslant 0. \tag{40}$$

Therefore, the outage probability reduces to the probability of the event when $0\leqslant\zeta\leqslant\zeta_0$. Thus integrating the probability density function for this range yields the following outage probability

$$P_{\mathrm{SC-Ideal}}\left(R_{\mathrm{th}}\right)=\int\limits_0^{\zeta_0} f_\zeta\left(z\right)dz=\left(1-e^{-\frac{\zeta_0}{\pi}}\right)^N. \tag{41}$$

2) Non-Ideal Hardware with PGS: The outage probability of a SIMO system with SC receiver under asymmetric HWD with PGS transmission scheme is introduced through the following theorem.

Theorem 5. *In s SIMO system under HWD and uses PGS transmission with a SC receiver for detection, the outage probability assuming Rayleigh fading channel with a target rate* R_{th} $\mathrm{bits/sec/Hz}$ *is given by*

$$P_{\mathrm{SC-PGS}}\left(R_{\mathrm{th}}\right)=\begin{cases}\left(1-e^{-\frac{\sigma_w^2\psi_1}{\pi}}\right)^N & \text{if } 0\le R_{\mathrm{th}}\le R_1\\ 1 & \text{if } R_1\le R_{\mathrm{th}}<\infty\end{cases} \tag{42}$$

where ψ_1 *and* R_1 *are the same as expressed in* (27) *and* (28) *respectively.*

Proof: Consider the achievable rate expression for the $j-\mathrm{th}$ receiver stream of HWD SIMO system with SC receiver

TABLE I. OUTAGE PROBABILITY PERFORMANCE OF HWD SIMO SYSTEMS USING PGS AND IGS FOR VARIOUS TARGET RATE REGIONS

Signaling	Cases	Target Rate Region	P_{LC}	P_{SC}
PGS	Convex ($a_{\text{p}} \geq 0$)	$0 \leq R_{\text{th}} \leq R_1$	$1-\frac{\sigma_w^{2N}}{\pi^N(N-1)!}{\psi_1}^N\int_1^\infty \frac{e^{-\frac{\sigma_w^2}{\pi}\psi_1 t}}{t^{1-N}}dt$	$\left(1-e^{-\frac{\sigma_w^2\psi_1}{\pi}}\right)^N$
	Concave ($a_{\text{p}} \leq 0, b \leq 0$)	$R_1 \leq R_{\text{th}} < \infty$	1	1
	Concave ($a_{\text{p}} \leq 0, b \geq 0$)	$R_{\text{th}} \leq R_2$ & $R_{\text{th}} \geq R_1$	Infeasible	Infeasible
IGS	Convex ($a_{\text{I}} \geq 0$)	$0 \leq R_{\text{th}} \leq R_3$	$1-\frac{\sigma_w^{2N}}{\pi^N(N-1)!}{\psi_2}^N\int_1^\infty \frac{e^{-\frac{\sigma_w^2}{\pi}\psi_2 t}}{t^{1-N}}dt$	$\left(1-e^{-\frac{\sigma_w^2\psi_2}{\pi}}\right)^N$
	Concave ($a_{\text{I}} \leq 0, b \leq 0$)	$R_3 \leq R_{\text{th}} < \infty$	1	1
	Concave ($a_{\text{I}} \leq 0, b \geq 0$)	$R_{\text{th}} \leq R_2$ & $R_{\text{th}} \geq R_3$	Infeasible	Infeasible

system employing PGS in (18) and substituting it in the system outage probability expression (38), we obtain

$$P_{\text{SC}-\text{PGS}}\left(R_{\text{th}}\right) = \Pr\left\{\max_j \; f_{\text{PGS}}(\zeta_j) \leqslant 0\right\} \tag{43}$$

where $f_{\text{PGS}}(\zeta_j) = a_{\text{p}}{\zeta_j}^2 + b\zeta_j + c$ and $\zeta_j = \frac{|h_j|^2}{\sigma_w^2}$ and the coefficients a_{p},b and c are given by (29). Given iid exponentially distributed $\zeta_j \sim \exp(\frac{\pi}{\sigma_w^2}) \quad \forall 1 \leq j \leq N$, the outage probability in (43) reduces to

$$P_{\text{SC}-\text{PGS}}\left(R_{\text{th}}\right) = \left(\Pr\left\{f_{\text{PGS}}(\zeta_j) \leqslant 0\right\}\right)^N \tag{44}$$

where $\Pr\left\{f_{\text{PGS}}(\zeta_j) \leqslant 0\right\} = \Pr\left\{a_{\text{p}}{\zeta_j}^2 + b\zeta_j + c \leqslant 0\right\}$, takes the similar form as in (36) with same coefficients and thus follow a similar case breakdown structure which lead to the event when quadratic $f_{\text{PGS}}(\zeta_j) \leqslant 0$. Furthermore, iid exponential distribution of ζ_j implies

$$\Pr\left\{f_{\text{PGS}}(\zeta_j) \leqslant 0\right\} = 1 - e^{-\frac{\sigma_w^2}{\pi}\psi_1} \quad \forall R_{\text{th}} \leqslant R_1 \tag{45}$$

Therefore, using this expression in (44), yields system outage probability for HWD SIMO SC receiver with PGS transmission

$$P_{\text{SC}-\text{PGS}}\left(R_{\text{th}}\right) = \left(1 - e^{-\frac{\sigma_w^2}{\pi}\psi_1}\right)^N \quad \forall R_t \leqslant R_1 \tag{46}$$

However, for target rates $R_{\text{th}} \geqslant R_1$, the system suffers complete outage with outage probability $P_{\text{SC}-\text{PGS}}\left(R_{\text{th}}\right) = 1$, as proved in Theorem 3. Combining both rate regions and their corresponding outage probabilities proves Theorem 5. ■

3) Non-Ideal Hardware with IGS: To study the impact of adopting IGS transmission scheme on a SIMO system under asymmetric HWD with SC receiver, we introduce the following theorem to analyze the outage probability.

Theorem 6. *In a SIMO system under asymmetric transceiver HWD and uses IGS for transmission and a SC receiver, the outage probability with a target rate* $R_{\text{t}}\,\text{bits/sec/Hz}$ *assuming Rayleigh fading channel is given by*

$$P_{\text{SC}-\text{IGS}}\left(R_{\text{th}}\right) = \begin{cases} \left(1 - e^{-\frac{\sigma_w^2\psi_2}{\pi}}\right)^N & \text{if } 0 \leq R_{\text{th}} \leq R_3 \\ 1 & \text{if } R_3 \leq R_{\text{th}} < \infty \end{cases} \tag{47}$$

where ψ_2 *and* R_3 *are the same as expressed in* (34) *and* (35) *respectively.*

Proof: Substituting the optimized instantaneous achievable rate expression from (21) in (38), we obtain

$$P_{\text{SC}-\text{IGS}}\left(R_{\text{th}}\right) = \Pr\left\{\max_j \; f_{\text{SIMO-IGS}}(\zeta_j) \leqslant 0\right\} \tag{48}$$

where $f_{\text{IGS}}(\zeta_j) = a_{\text{I}}{\zeta_j}^2 + b\zeta_j + c$ and $\zeta_j = \frac{p|h_j|^2}{\sigma_w^2}$ and the coefficients a_{I}, b and c are given by (29) and (36). Exploiting the fact that given $\max_j \; f_{\text{IGS}}(\zeta_j) \leqslant 0$ implies $f_{\text{IGS}}(\zeta_j) \leqslant 0 \forall j$ and statistically independent fading channels, the outage probability in (48) becomes

$$\begin{aligned} P_{\text{SC}-\text{IGS}}\left(R_{\text{th}}\right) &= \Pr\left\{\max_j \; f_{\text{IGS}}(\zeta_j) \leqslant 0\right\} \\ &= \prod_{j=1}^{N} \Pr\left\{f_{\text{IGS}}(\zeta_j) \leqslant 0\right\} \end{aligned} \tag{49}$$

Under the assumption of iid exponentially distributed, i.e., $\zeta_j \sim \exp(\frac{\pi}{\sigma_w^2}) \quad \forall 1 \leq j \leq N$, $\Pr\left\{f_{\text{IGS}}(\zeta_j) \leqslant 0\right\}$ for the j^{th} receiver stream is exactly the similar form as given by (36). Thus following all the feasible cases leading to the event $0 \leqslant \zeta_j \leqslant \psi_1$ given $R_{\text{th}} \leqslant R_3$, we obtain

$$P_{\text{SC}-\text{IGS}}\left(R_{\text{th}}\right) = \left(1 - e^{-\frac{\sigma_w^2}{\pi}\psi_2}\right)^N \quad \forall R_t \leqslant R_3 \tag{50}$$

However, for target rates $R_{\text{th}} \geqslant R_3$, the system suffers complete outage with outage probability $P_{\text{SC}-\text{IGS}}\left(R_{\text{th}}\right) = 1$, as proved in Theorem . Combining both rate regions and their corresponding outage probabilities proves Theorem 6.

■

All the presented cases, respective rate regions and outage probabilities for the HWI LC and SC systems, employing proper Gaussian and improper Gaussian transmission signaling, are summarized in Table I.

V. NUMERICAL AND SIMULATION RESULTS

In this section, we numerically investigate the degradation in the system performance caused by the asymmetric HWD. These distortions can severely degrade system performance compared with the ideal hardware system. We consider both the achievable rate and the outage probability to investigate the deteriorating effect on system performance due to HWD. The simulation results consider both combing receivers, LC and SC. We investigate also the single-input single-output (SISO) system configurations, which is the special case derived from

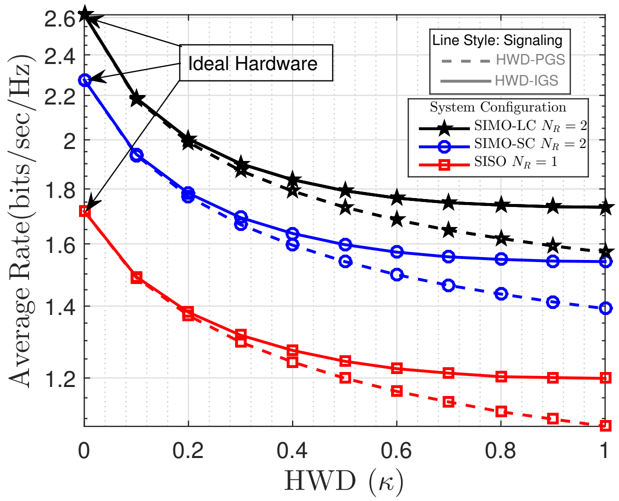

Fig. 2. Average achievable rate vs. HWI level in SISO and SIMO transceivers

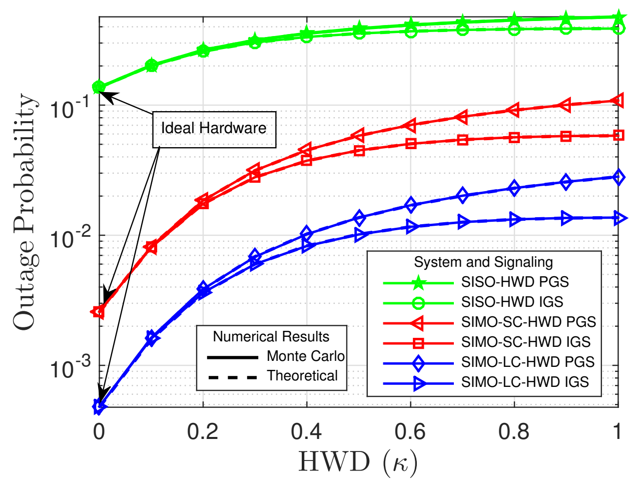

Fig. 3. Outage probability vs. HWD level in SISO and SIMO transceivers

SC, to quantify the system performance losses in each configuration under HWD. Furthermore, we examine the benefits reaped by adopting IGS in mitigating the HWD compared to the PGS scheme.

As for the system parameters, we assume, unless otherwise specified, $P = 1W$, $\sigma_x^2 = 1$ and the AWGN noise has $\sigma_w^2 = 1$. When adopting IGS, it is optimized to maximize the achievable rate as given in Theorem 1 and 2. As for the HWD, we assume unit variance across all transceivers pertaining to the employment of common oscillator for modulation and demodulation to the same carrier frequency in a spatially diverse SIMO system. Moreover, the HWD signal is assumed to be maximally improper with $|\tilde{\kappa}| = 1$. In the rest of this section, we study the average achievable rate and outage probability performance against average SNR, HWD levels, target achievable rate and number of receiver streams for the three aforementioned scenarios: ideal and non-ideal hardware with both the PGS and IGS schemes. In the outage probability numerical examples, we include both theoretical results and Monte Carlo simulations to validate our results. Different line formats and markers are used to distinguish between system configurations employed signaling scheme, analytical and simulated results.

A. *Effect of Hardware Impairment Level*

Firstly, We investigate the average achievable rate performance versus distortion level κ, ranging from 0 to 1, for SISO, SIMO-LC and SIMO-SC systems in Fig 2. We assume $\mathrm{N_R} = 2$ for LC and SC systems and a highly fading channel rendering 5dB SNR. Ideal hardware scenario results is seen at $\kappa = 0$, while the maximally non-ideal hardware is seen at $\kappa = 1$. According to Fig 2, we first observe that the average achievable rate decreases with the increase of impairment level for all adopted systems. Furthermore, we note that LC systems offer relatively higher average rate as compared to the SC schemes. In addition, receive diversity enhances system performance relative to the SISO system, where employing two receive streams yields almost 48% increase in the average rate. Fig 2 shows that the deteriorating effect of HWD is considerably mitigated in the presence of IGS as compared to it's counterpart PGS for all adopted systems, especially for highly hardware-impaired systems.

Secondly, we evaluate the system outage probability performance with the increase of HWD level κ from 0 to 1, in Fig 3. Simulation and theoretical results have been analyzed for SISO and SIMO system with two receiver combining schemes. Under the assumption of a moderate noise level, we choose the SNR to be 15 dB and a target rate of $R_{\mathrm{t}} = 2.5\,\mathrm{bits/sec/Hz}$. Evidently, the system outage probability increases with the increase of distortion level, as we move away from ideal system configuration. SIMO systems with LC receivers shows lower outage probabilities than the ones with SC and SISO at a given distortion level. Eventually, it is worth noting that IGS scheme reaps a huge gain in terms of system outage probability by highly mitigating the effects of HWD, especially for high transceiver distortion levels. SIMO systems reap the maximum benefit of deploying IGS transmission relative to the SISO system configuration. Finally, we find that our theoretical results are in perfect accordance with the Monte-Carlo simulations, which validates the presented analysis.

B. *Effect of Signal-to-Noise Ratio*

In this numerical example, we study the average achievable rate versus the SNR hardware impaired SISO and SIMO systems with LC and SC schemes with $\mathrm{N_R} = 3$. The numerical results are carried out with SNR ranging from 0 dB to 25 dB in Fig 4. Ideal hardware system is chosen to be the benchmark to quantify the performance loss under HWD. The highly degrading effect of HWD in non-ideal scenario relative to the ideal assumption rendering almost 50% decrease in average achievable rate. Furthermore, the deteriorating effect of HWD is considerably mitigated in the presence of IGS as compared to it's counterpart PGS. LC provides higher rate being the

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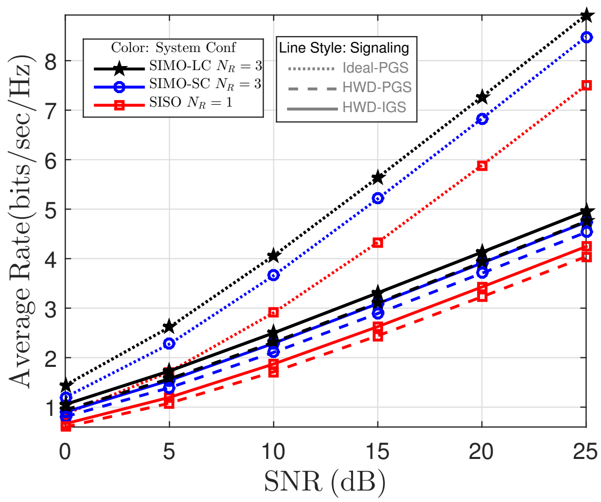

Fig. 4. Average achievable rate vs. SNR for adopted systems

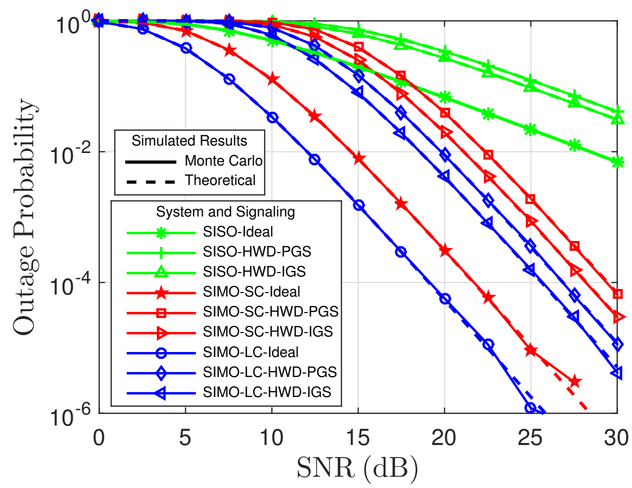

Fig. 5. Outage probability vs. SNR for adopted systems

optimal combining scheme relative to SC at a cost of increased complexity.

As for the outage probability performance, the theoretical and simulation are met in different systems of SISO and SIMO syst versus SNR from 0 dB to 30 dB, as shown in Fig 5. We assumed $N_{\mathrm{R}} = 3$ receive streams and a target rate of $R_{\mathrm{t}} = 3\,\mathrm{bits/sec/Hz}$. Evidently, outage probabilities decreases with the increase in SNR. However, ideal and non-ideal outage probabilities are relatively close to each other for low SNR values, whereas this difference becomes significant from moderate to high SNR values. Therefore, adopting IGS offers a significant signaling mitigation technique to lower outage probability compared to the PGS outage probability performance. The benefit of employing IGS is more dominant in case of receive diversity relative to the single link communication system. SIMO system with linear combining scheme offers the least outage probability despite of all HWD degradation. The difference between ideal and non-ideal system outage probability, signifies the impact of HWD.

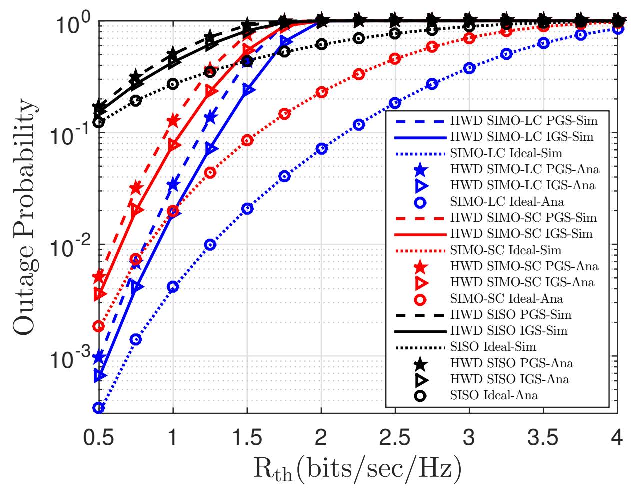

Fig. 6. Outage probability vs. target achievable rates

C. Effect of Target Achievable Rate and Receiver Streams

Fig 6 depicts the result of both simulated and analytic outage probability versus various target rates (R_{th}) in case of ideal and non-ideal hardware. As for the ideal hardware performance, it is evident from Fig 6 that there is no severe outage till a very large target rate value. On the other hand, HWD significantly degrades the system performance and results in more severe degradation effects at higher target rates. SISO system depicts lower outage probability relative to other receive diversity systems. The IGS scheme succeeds to lessen the HWD and improve the system performance compared with the PGS especially at high target rate R_{th} values. It is important to note that the analytical expressions are well in accordance with the simulated results for all scenarios.

Finally, the effect of receiver RF front-ends is studied to investigate the required number of receiver streams to achieve a target outage probability in Fig. 6 in a highly fading channel with 5dB SNR level. Under these circumstances, we declare system outage when instantaneous achievable rate falls below the target rate of $R_{\mathrm{th}} = 2, 3\,\mathrm{bits/sec/Hz}$. We vary N_{R} from 1 to 6, and study the benefits obtained by increasing receiver spatial diversity. HWD significantly degrades the system performance, however these effects can be dampened by utilizing more receiver streams and employ IGS transmission. HWD results in more severe degradation effects for lower values of N_{R} and higher target rates. The IGS scheme succeeds to lessen the HWD and improves the system performance compared with the PGS especially for large N_{R} streams. As expected, LC outperforms SC for a given target rate and their analytical expressions are well in accordance with the simulated results.

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VI. Conclusion

In this paper, we studied the performance of receive diversity systems under asymmetric HWD assuming linear and switched combining receivers. The proposed model generalizes the existing nonlinear HWD models and consider a wider distortion categories. Moreover, the proposed asymmetric model motivated us to employ IGS transmission scheme to mitigate the deterioration caused by the transceivers hardware imperfections by tuning the pseudo-variance of the transmitted signal in order to maximize the achievable rate. Based on the optimized signaling scheme, we derived closed form expressions for the outage probability in case of non-ideal hardware with PGS and IGS transmission schemes for SC and LC receivers. Furthermore, numerical results demonstrated the benefit of IGS scheme in non-ideal hardware scenario for higher distortion levels, higher number of receive streams and different target rates. Therefore, asymmetric signaling is a strong candidate to improve achievable rates and decrease system outage in the presence of HWD, which makes it appropriate for the next generation of wireless networks.

Appendix A
Proof of Theorem 1

Assuming unconstrained optimization problem, to maximize the achievable rate expression in case of non-ideal hardware employing IGS scheme. It is proven to be strictly concave with respect to $\tilde{\sigma}_x^2$. Furthermore, maximizing $R_{\text{SIMO-IGS}}$ is equivalent to maximizing $f = 2^{2R_{\text{SIMO-IGS}}}$. Therefore, the proof of strict concavity is as follows:

$$\frac{\partial f}{\partial \tilde{\sigma}_x^2} = \frac{-2p^2 \left|\mathbf{h}^{\mathrm{H}}\mathbf{h}\right|^2 \left(\tilde{\sigma}_x^2 + \tilde{\kappa}\right)}{\left(p\mathbf{h}^{\mathrm{H}}\mathbf{h}\kappa + \sigma_w^2\right)^2 - \left|p\mathbf{h}^{\mathrm{H}}\mathbf{h}\tilde{\kappa}\right|^2} \tag{51}$$

Taking second derivative with respect to $\tilde{\sigma}_x^2$ yields,

$$\frac{\partial^2 f}{\partial {\tilde{\sigma}_x^2}^2} = \frac{-2p^2 \left|\mathbf{h}^{\mathrm{H}}\mathbf{h}\right|^2}{\left(p\mathbf{h}^{\mathrm{H}}\mathbf{h}\kappa + \sigma_w^2\right)^2 - \left|p\mathbf{h}^{\mathrm{H}}\mathbf{h}\tilde{\kappa}\right|^2} < 0 \tag{52}$$

Hence it is proved that given $p > 0$, the objective function is strictly concave in $\tilde{\sigma}_x^2$. and the corresponding optimal solution is obtained by setting $\frac{\partial f}{\partial \tilde{\sigma}_x^2} = 0$, as the stationary point is necessary and sufficient KKT condition, for optimality, in the given scenario. This yields $\tilde{\sigma}_x^2 = -\tilde{\kappa}$. Predominantly, $|\tilde{\kappa}| \leq \sigma_x^2$, hence the presented constraint is automatically satisfied.

Appendix B
Proof of $R_2 < R_1$ and $R_2 < R_3$

The proof of $R_2 \leq R_1$ is equivalent to the proof of $\psi_2 \leq \psi_1$ where $\psi_k = 2^{2R_k}$ for $k \in (1, 2)$. To prove this, first we will show that ψ_1 is an increasing function in $\left|\tilde{\sigma}_\eta^2\right|^2$. By taking the first derivative of ψ_1, we obtain

$$\frac{d\psi_1}{d\left|\tilde{\sigma}_\eta^2\right|^2} = \frac{\sigma_x^2\left(\sigma_x^2 + 2\kappa\right)}{\left(\kappa^2 - \left|\tilde{\sigma}_\eta^2\right|^2\right)^2} \geq 0 \quad \forall\, 0 \leq \left|\tilde{\sigma}_\eta^2\right|^2 \leq \kappa^2 \tag{53}$$

which proves that ψ_1 is an increasing function in $\left|\tilde{\sigma}_\eta^2\right|^2$ for the given range.

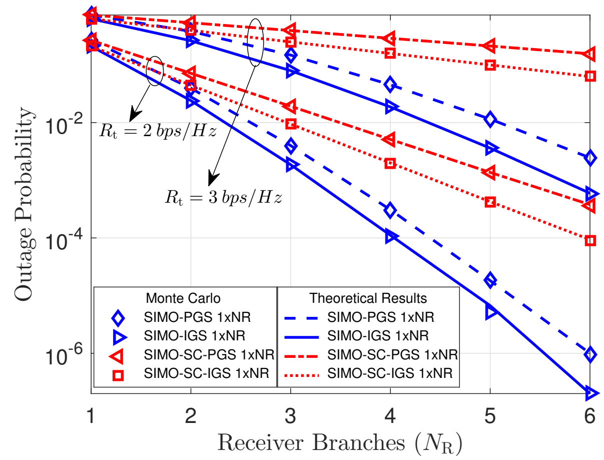

Fig. 7. Outage probability vs. number of receiver streams in HWD receive diversity system.

Therefore, using (27) the minimum value of ψ_1 as a function of $\left|\tilde{\sigma}_\eta^2\right|^2$ occurs at $\left|\tilde{\sigma}_\eta^2\right|^2 = 0$ and is given by

$$\tilde{\psi}_1 = \left(1 + \frac{\sigma_x^2}{\kappa}\right)^2 = \psi_2^2. \tag{54}$$

Moreover, to prove $\psi_2 \leq \psi_1$, it is sufficient to prove that $\psi_2 \leq \tilde{\psi}_1$ as $\psi_2 \leq \tilde{\psi}_1 \leq \psi_1$. Given $\psi_2 \geq 1$, (54) proves that $\psi_2 \leq \tilde{\psi}_1 \leq \psi_1$ which in turn implies that $R_2 \leq R_1$. Given non-zero signal power and finite HWD levels, R_2 is strictly less than R_1.

Similarly, it can also be proved that $R_2 < R_3$ by following similar steps.

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