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## Dynamic Circuit Model for Spintronic Devices

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### Abstract

In this work we propose a finite-difference scheme based circuit model of a general spintronic device and benchmark it with other models proposed for spintronic switching devices. Our model is based on the four-component spin circuit theory and utilizes the widely used coupled stochastic magnetization dynamics/spin transport framework. In addition to the steady-state analysis, this work offers a transient analysis of carrier transport. By discretizing the temporal and spatial derivatives to generate a linear system of equations, we derive new and simple finite-difference conductance matrices that can, to the first order, capture both static and dynamic behaviors of a spintronic device. We also discuss an extension of the spin modified nodal analysis (SMNA) for time-dependent situations based on the proposed scheme.

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### 1. Introduction

With complementary metal-oxide semiconductor (CMOS) miniaturization and energy-efficiency rapidly approaching an inevitable limit, a search for an alternative technology is accelerating. Spintronic devices [1] are one of the prime candidates to tackle the intractable physics of CMOS. In spintronics, information is stored in the alignment of spins. The quantum, mechanically digitized angular momentum allows the spin to be viewed as a binary entity (e.g., '1' = spin up, '0' = spin down) and thus can carry and process magnetic information in a digital manner to function for logic and memory. In addition to providing zero stand-by power (due to non-volatility), spintronic devices also consume low switching energies, and thus offer an unprecedented chance for further technological progress.

To assess the feasibility of spintronic devices as new generation switches and memory elements, equivalent circuit modeling is critical for rapid computer aided design and verification. An accurate model allows the designer to evaluate the impact of new materials and phenomena and predict important device parameters such as switching current,

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switching time, and switching energy-delay product; the latter being of utmost importance to address the scaling trends of the device studied.

Previous circuit models of spintronic devices have been successfully developed. Examples are the elemental modules [2] and the compact circuit models [3-5]. Although these models can capture most of the device physics and are compatible with CMOS design tools, each still lack or neglect one of several important aspects such as the dynamics of carrier transport, stochastic thermal effects in the nanomagnets, or the ability of the models to be augmented with full micromagnetic simulations and inclusion of other interesting phenomena like spin Hall effect (SHE), spin-orbit torque (SOT), and topological insulators (TI).

This paper presents a circuit model of diffusive spintronic devices based on simple discretization schemes to capture the main device characteristics. The analysis is generalization and a more compact treatment of our prior work [6, 7] that still builds on the notion of the four-component circuit theory [3] where the total voltage drop is related to the total current flow through the conductance tensor as  $\mathbf{I} = G\mathbf{V}$ , which is Ohm’s law for spin circuits [4].

## 2. Theory of Carrier Transport

The transport of electrons in diffusive nonmagnetic (NM) metals, i.e. those for which the transport length satisfies  $L > \lambda$ , where  $\lambda$  is the mean-free path between elastic scattering events, is adequately described by the drift-diffusion equation. Such equation can be obtained from the classical Boltzmann equation in the limit where the density and external potentials are slowly varying on the scale of  $\lambda$ . In such formalism, high enough temperatures are usually assumed to dispel the question of diffusion. In addition, spin-orbit (SO) coupling is ignored, or otherwise a full quantum approach like the non-equilibrium Green’s function (NEGF) or scattering theory has to be applied.

The one-dimensional transport equations for charge and spin densities are given by

$$\frac{\partial n}{\partial t} - \frac{1}{e} \frac{\partial J_c}{\partial z} = 0 \qquad J_c = eD \frac{\partial n}{\partial z} + e\tilde{\mu}nE \qquad (1a, b)$$

$$\frac{\partial \mathbf{s}}{\partial t} - \frac{1}{e} \frac{\partial \mathbf{J}_s}{\partial z} = -\frac{\delta \mathbf{s}}{\tau_{sf}} \qquad \mathbf{J}_s = eD \frac{\partial \delta \mathbf{s}}{\partial z} + e\tilde{\mu}E\mathbf{s} \qquad (2a, b)$$

where  $n$  is the electron density,  $J_c$  is the electric current density,  $D$  is the diffusion coefficient,  $\tilde{\mu}$  is the mobility,  $E$  is the electric field,  $\mathbf{s}$  is the spin density,  $\delta \mathbf{s}$  is the spin accumulation, and  $\mathbf{J}_s$  is the spin current density.

The previous equations can be written in a form more relevant for circuit modeling by introducing the circuit variables  $I_c$ ,  $\mathbf{I}_s$ ,  $V_c$ , and  $\mathbf{V}_s$ . These variables are usually introduced through the relations  $I_c = J_c A$ ,  $\mathbf{E} = -\nabla V_c$ ,  $\mathbf{I}_s = \mathbf{J}_s A$ , and  $\tilde{\mu}_{S,k} = eV_{S,k}$ , where  $\tilde{\mu}_{S,k}$  is the k-th component of the vector spin quasicheical potential (the half-difference of the up and down spins chemical potentials) and  $A$  is the cross-section area. In addition to the previous circuit variables, one can introduce the per unit length electrostatic capacitance  $C_\ell$  and the per volume quantum capacitance  $C_q$ , which are usually obtained from the relations  $\rho_\ell = C_\ell V_c$  and  $e s_k = C_q V_{S,k}$ . Using these ideas, one can show [5-7]

$$\frac{\partial I_c}{\partial z} = -C_\ell \frac{\partial V_c}{\partial t} \qquad I_c = -\sigma A \frac{\partial V_c}{\partial z} \qquad (3a, b)$$

$$\frac{\partial \mathbf{I}_s}{\partial z} = C_q A \frac{\partial \mathbf{V}_s}{\partial t} + C_q A \frac{\partial \mathbf{V}_s}{\tau_{sf}} \qquad \mathbf{I}_s = \frac{\tilde{\mu} I_c}{D} \mathbf{V}_s + \sigma A \frac{\partial \mathbf{V}_s}{\partial z} \qquad (4a, b)$$

where  $\sigma = e\tilde{\mu}n$  is the conductivity. Here, we neglected the diffusion of charge (and not spin) using the fact that any charge imbalance is effectively screened out over a microscopic length known as the screening length  $\delta = \left(\frac{e^2}{\epsilon} \chi_c\right)^{-1/2}$ , where  $\chi_c$  is compressibility factor. This assumption has to be revised in the case of semiconductors since electron-hole packets are charge-neutral and can therefore be long-lived resulting in density inhomogeneity.

### Circuit Modeling

#### 2.1. Spatial Discretization

In this subsection, we follow [5] and show how a compact circuit model can be found for a finite NM section of length  $\Delta z$ . Employing first-order finite-difference for the spatial derivatives in (3) and (4), we obtain the pair of equations

$$I_C(z) - I_C(z + \Delta z) = C_e \Delta z \frac{\partial V_C}{\partial t} \quad I_C = \frac{\sigma A}{\Delta z} [V_C(z) - V_C(z + \Delta z)] \quad (5a, b)$$

$$I_{S,k}(z) - I_{S,k}(z + \Delta z) - C_q A \Delta z \frac{V_{S,k}}{\tau_{sf}} = C_q A \Delta z \frac{\partial V_{S,k}}{\partial t} \quad I_{S,k} = \frac{\tilde{\mu} I_C}{D} V_{S,k} + \frac{\sigma A}{\Delta z} [V_{S,k}(z) - V_{S,k}(z + \Delta z)] \quad (6a, b)$$

which yields the typical RC  $\pi$ - and  $T$ -networks in Fig. 1.

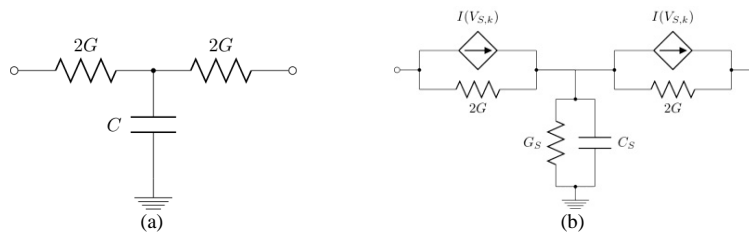


Fig. 1. Distributed RC networks for charge and spin transport in a nonmagnetic (NM) wire. (a)  $T$ -model for charge transport where  $G = \sigma A / \Delta z$  and  $C = C_e \Delta z$ . (b)  $T$ -model for spin transport  $G = \sigma A / \Delta z$ ,  $G_S = C_q A \Delta z / \tau_{sf}$ ,  $C_S = C_q A \Delta z$ , and  $I(V_{S,k}) = \tilde{\mu} I_C V_{S,k} / D$ .

#### 2.2. Temporal Discretization

The presence of the capacitors allows one to describe the dynamics (i.e., the charging and discharging character). However, for us to be able to use the generalized Ohm’s relation  $\mathbf{I} = G\mathbf{V}$ , we must linearize the transport equations. One way to do that is to use, for example, the backward Euler (BE) and trapezoidal rule (TR) to linearize the capacitor relation  $I = C dV / dt$  to obtain [6, 7] the capacitor finite-difference model shown below.

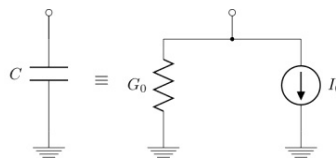


Fig. 2. Finite-difference model for a capacitor.

The last idea, when used in (5a) and (6a) allows us to represent a NM wire with a  $T$ -network with the following new series and shunt finite-difference conductance matrices along with the initial condition vector current [6, 7]

$$G_{NM,se} = \begin{bmatrix} G_{C,se} & 0 & 0 & 0 \\ 0 & G_{S,se} & 0 & 0 \\ 0 & 0 & G_{S,se} & 0 \\ 0 & 0 & 0 & G_{S,se} \end{bmatrix} \quad G_{NM,sh} = \begin{bmatrix} G_{C,sh} & 0 & 0 & 0 \\ 0 & G_{S,sh} & 0 & 0 \\ 0 & 0 & G_{S,sh} & 0 \\ 0 & 0 & 0 & G_{S,sh} \end{bmatrix} \quad \mathbf{I}_{NM,0}(\mathbf{V}^n) = \begin{bmatrix} I_{C,0}(V_C^n) \\ I_{S,x,0}(V_{S,x}^n) \\ I_{S,y,0}(V_{S,y}^n) \\ I_{S,z,0}(V_{S,z}^n) \end{bmatrix}$$

where the components are given by, for BE (left), and TR (right) as [6, 7]

$$\begin{aligned}
 G_{C,se} &= G_{S,se} = \frac{2\sigma A}{\Delta z} & G_{C,se} &= G_{S,se} = \frac{2\sigma A}{\Delta z} \\
 G_{C,sh} &= \frac{C_l \Delta z}{\Delta t} & G_{C,sh} &= \frac{2C_l \Delta z}{\Delta t} \\
 G_{S,sh} &= C_q A \Delta z \left( \frac{1}{\Delta t} + \frac{1}{\tau_{sf}} \right) & G_{S,sh} &= C_q A \Delta z \left( \frac{2}{\Delta t} + \frac{1}{\tau_{sf}} \right) \\
 I_{C,0}(V_C^n) &= -\frac{C_l \Delta z}{\Delta t} V_C^n & I_{C,0}(V_C^n) &= -\frac{2C_l \Delta z}{\Delta t} V_C^n - [I_C^n(z) - I_C^n(z + \Delta z)] \\
 I_{S,k}(V_{S,k}^n) &= -\frac{C_q A \Delta z}{\Delta t} V_{S,k}^n & I_{S,k}(V_{S,k}^n) &= -C_q A \Delta z \left( \frac{2}{\Delta t} - \frac{1}{\tau_{sf}} \right) V_{S,k}^n - [I_C^n(z) - I_C^n(z + \Delta z)]
 \end{aligned}$$

### 3. Simulations

Here we benchmark our model for the recently proposed device known as all-spin logic (ASL) [8] shown in Fig. 3a. Using the above model, for example through the BE approximation, we numerically integrate the stochastic Landau-Lifschitz-Gilbert-Slonczewski (LLGS) equation using the generalized Heun predictor-corrector scheme and the device parameters given in [6, 7]. In addition, here, we generalize the spin modified nodal analysis (SMNA) [4, 7] to time-dependent situations for the scheme presented above to an algorithm of the form  $A\mathbf{x}^{n+1} = \mathbf{b}$ ,  $n = 0, 1, \dots, N$ , given the initial condition  $\mathbf{x}_0$  and a suitable integrating time-step.

We plot the transient response of the spin currents in Figs. 3b,c for two values of channel capacitance:  $C = 0$  pF, and  $C = 100$  pF, which are extreme limiting values to study the contribution of the capacitances on the dynamics. The capacitance can present in general due to the multilayer thin film deposition, rough surfaces, etc. [6, 7]. From the figures, it is apparent that the model adequately captures the introduced dynamics and are consistent in the steady state limit with the results presented in [4].

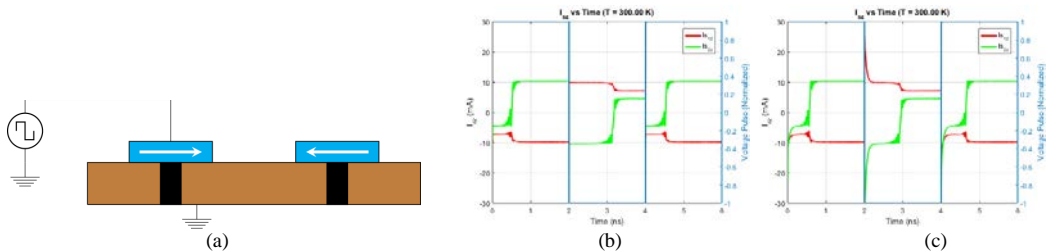


Fig. 3. (a) All-spin logic (ASL) device. (b) Transient spin current vs time, for  $C = 0$  pF. (c) Transient spin current vs time, for  $C = 100$  pF

### 4. Conclusion

In this paper we presented a finite-difference model of nonmagnetic sections in diffusive spintronic devices that can capture the dynamics occurring in the spin conducting channels. Given the finite-difference model presented, we generalized the spin modified nodal analysis (SMNA) to time-dependent situations in a simple algorithmic way. The model has been benchmarked on the device called all-spin logic (ASL) and the results are consistent with the ones reported in literature.

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