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BLMA: A Blind Matching Algorithm with Application to Cognitive Radio Networks

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***Abstract*—We consider a two-sided one-to-one abstract matching problem with a defined notion of pairwise stability. The formulated problem is shown to encompass the ordinal and cardinal utility markets. We propose a distributed blind matching algorithm (BLMA) to solve the problem. BLMA is characterized by random activations of agents, and by generic negotiation and aspiration (utility) update processes. We prove the solution produced by BLMA will converge to an ϵ-pairwise stable, equivalently ϵ-core, outcome with probability one. We then consider three BLMA applications in cognitive radio networks. We propose a simple BLMA negotiation and aspiration update dynamic to produce an ϵ-pairwise stable solution for the case of quasi-convex and quasi-concave utilities. In the case of more general utility forms, we show another BLMA process to provide equilibrium. We also consider the use of BLMA in an ordinal utility market.In all applications of the BLMA, we impose a limited information exchange in the network so that agents can only calculate their own utilities, but no information is available about the utilities of any other user in the network.**

***Index Terms*—Decentralized matching, generalized assignment games, one-to-one matching, ϵ-pairwise stability, ϵ-core, cognitive radio.**

I. Introduction

Starting with the marriage market model in [1], two-sided matching theory has evolved into a solid framework with applications in many areas [2]. The theory asks a simple question: Given two finite and disjoint sets of users and given the preferences of each agent on individuals on the other side of the market, is there a way to form partnerships between the disjoint sets of users such that there do not exist agents from each side of the market who prefer each other to their assigned match?

The seminal work of Gale and Shapley in [1] provided an affirmative answer to the above question considering one-to-one matchings (marriages) of two disjoint sets (men and women). Gale and Shapley used the term "stability" to describe the desired matching outcome, i.e., a stable matching is a matching in which pairs of agents from each side of the market have no incentive to deviate. Since then, many variations of the basic matching problem have emerged with an array of applications in areas as wide as labor markets, housing markets, college admissions programs, and communication networks.

Also of interest is a model for a two-sided matching market with transferable linear utilities [3]. In the (linear) assignment game model, matchings have an associated value, and utility transfers occur at a one-to-one rate from one player to any other [4]. An important solution concept in such games is the core. The core of any assignment game is the set of feasible payoff vectors that cannot be improved upon by any coalition of players [4]. Shapley and Shubik showed that the core of the linear assignment game is not empty. In such settings, payoff vectors belonging to the core are equivalently *pairwise stable* [3][1].

The case of general nonlinear utilities induces another class of games called generalized assignment games [2][2]. The model was first described by Demange and Gale in [6] and also shown to have a non empty core in [7].

Our focus in this paper will be the latter generalized assignment games. Specifically, we will consider one-to-one matching problems with *possibly* general nonlinear utilities[3]. Our goal will be to formulate a generic matching problem which encompasses both the ordinal and cardinal utility markets given market and information decentralization. Market decentralization means we do not stipulate a particular order on the encounters of agents in our model and do not need a central authority to monitor the convergence to stability. Information decentralization means that agents have knowledge only of their utilities but no information is available about the

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[1]Thereafter, we will use the terms "stability" and "pairwise-stability" interchangeably.

[2]As argued in [5], in this case social welfare (i.e., maximizing the sum of the utilities) is not well defined since different agents' utilities are not measured in the same units and are therefore non-transferable.

[3]As argued in [8], the main difference between marriage problems (or matching problems) and assignment games is that agents in the former have ordinal preferences over agents on the opposite side of the market and thus this game is said to be a nontransferable utility (NTU) cooperative game, while in linear assignment games, on the other hand, agents have cardinal utilities which they can transfer to agents on the other side of the market. Such games are called transferable utility (TU) cooperative games. Assignment games with nonlinear utilities, however, fall under the class of NTU games since different agents' utilities are not measured in the same units and are therefore non-transferable [5], [9]. Bearing all this in mind, we will thereafter refer to our problem, which encompasses both marriage, linear, and generalized assignment games, as a matching problem.

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utilities of other agents in the market and no entity in the network has such knowledge[4].

The motivation for requiring market and information decentralization is anticipating the need for this decentralization in future wireless networks and applications such as in cognitive radio networks, small cell networks and large scale device-to-device communications. As argued in [11], the need for self-organizing solutions to manage the scarce spectral resources is a prevalent theme in many emerging wireless systems. Distributed matching algorithms with market decentralization and limited information exchange are certainly a key enabler to this goal.

Towards this end, we formulate a context-free matching problem. In our model, agents maintain aspiration levels which are an abstraction of their potential utility from a match. An agreement function indicates to two agents on the opposite side of the market whether their current aspiration levels are agreeable and hence if a match is possible. The matching problem is solely defined by the two agents' sets and these agreement functions. We demonstrate that our model is general enough to encompass the marriage, linear, and generalized assignment game frameworks[5]. Our solution concept is a modified notion of pairwise stability such that the payoff vectors, the aspiration levels, cannot be ϵ-improved upon by any pair of players. We show that the ϵ-pairwise stable solutions coincide with the ϵ-core solutions in our model.

Having defined our problem, we propose a blind matching algorithm (BLMA) to produce a matching and two vectors of stable aspiration levels. The algorithm proceeds by making random encounters between pairs of agents from each side of the market (market decentralization). If the agents' current aspirations are agreeable, they match with a positive probability and possibly increase their aspiration levels[6]. Otherwise, single agents lower their aspiration levels and wait for their next random activation instance. We show that this simple dynamic will converge to a state satisfying our defined notion of pairwise stability, and hence the ϵ-core, with probability one.

In its abstract form, the proposed BLMA does not specify how agents negotiate their way into a matching. Neither does the algorithm specify how agents eventually update their aspirations if they match. All that is required for assured convergence is that agents have a mechanism of knowing if they have agreeable matches and if there exists such agreement of two agents' aspirations, that their match occurs with positive probability. It is in that sense that the algorithm is both blind and non-deterministic as the details of all the aforementioned concerns are left to the specific context of the matching problem.

Next, we show how information decentralization can be achieved using the BLMA in the context of cognitive radios. We formulate a matching problem where secondary users (SUs) relay primary user (PU) data in exchange for spectrum access time. Agents' utilities are nonlinear in the time and power resources. We consider two cases for the agents' utilities. In the first case, the PU's utility is quasi-convex while the SU's utility is quasi-concave[7]. Exploiting quasi-convexity, we propose a negotiation and aspiration update processes, BLMA1, to achieve ϵ-pairwise stability. In the second case, we further drop the quasi-convex assumption and propose a second process, BLMA2, to achieve stability. In both cases, we only make the mild assumption that agents can calculate their utilities, but no information is available about the utilities of other agents in the network. We finally make a remark on the applicability of our algorithm to other existing matching problems such as an ordinal utility market.

To summarize, the main contributions of our work are as follows:

- We formulate a generic matching problem that is shown to encompass ordinal and cardinal utility market models. The elements of our matching model are the two agents' sets and the agreement functions.
- We define a new notion of stability called ϵ-pairwise stability and show it coincides with the ϵ-core in our model.
- We propose a simple algorithm, BLMA, to pair agents in the formulated matching problem and prove the algorithm converges to ϵ-pairwise stable solutions with probability one despite market decentralization.
- Considering the specific context of cognitive radio networks, we propose two negotiation dynamics to ascertain agreement of aspiration levels in the BLMA process. The dynamics only make the assumption that agents can calculate their own utilities but do not require agents to have knowledge of the utilities of other agents in the network. This result serves our purpose of providing a simple learning process which is guaranteed to reach a stable solution despite market and information decentralization.
- We further elaborate on possible applications of BLMA to other cognitive radio network models as long as the studied market can be cast within our generic matching framework.

The rest of this paper is organized as follows. Section II reviews the related literature on matching markets

[4]We use the terms market decentralization and information decentralization in the sense defined in [10]. However, our formulation is more general as it allows agents to have nonlinear utilities and it is also possible to use our algorithm to find equilibrium in ordinal utility markets.

[5]We use the term "marriage game" to refer to the original ordinal utility game proposed by Gale and Shapley in [1].

[6]As we will demonstrate through our sample applications, the agents need not have knowledge of the agreement functions to ascertain whether their aspiration levels are agreeable.

[7]Where clear and for convenience, we will thereafter use the term quasi-convex to refer to both the quasi-convex and quasi-concave properties.

and applications of matching theory to cognitive radio networks. Section III introduces the context-free matching problem, the proposed BLMA and proof of convergence. Section IV considers an application of the BLMA to cognitive radio networks assuming quasi-convex utilities. Section V presents another application for the BLMA for general nonlinear utilities. Section VI presents some numerical examples to illustrate our results and to compare with existing work in the literature. Finally, Section VII concludes this paper.

II. Related Literature

A. Related Work in Matching Theory

The linear assignment problem was introduced in [12] in 1957. Shapley and Shubik later formulated the linear assignment game in coalitional form in a housing market. They showed that there always exists pairwise stable solutions in the assignment game. These stable solutions coincide with the core of the game [3] [8]. Core solutions enjoy a special mathematical structure called a lattice. This means that there is an extreme point in the core that is seller-optimal (also buyer-pessimal) and another extreme point that is buyer-optimal (also seller-pessimal) [3]. In between these two extremes are payoff vectors with varying degree of satisfaction for the users on the two sides of the market. The authors in [6] showed that the generalized assignment model carried over the lattice structure from the linear case.

The outcome of the assignment game is a specification of utilities to agents together with a matching such that every agent receives his most preferred match at the announced utilities [5]. In the linear model in [3], generally, only one optimal matching exists that maximizes the sum of the utilities, and it is compatible with infinitely-many stable payoffs. In the generalized model, possibly many optimal matchings exist. The set of stable payoff vectors compatible with a given matching need not be connected, although the whole set of stable outcomes (the core) is connected [2].

Numerous techniques exist on finding equilibrium, i.e., a matching supported by a stable payoff, in linear assignment games under various information assumptions [8], [13], [14]. Of particular interest to us is the work of Nax and Pradelski in [8]. This work proposed a decentralized algorithm to find a pairwise stable solution in the linear assignment game involving matching workers and firms. The authors considered a limited information scenario so that market participants know nothing about other players' utilities, nor can they deduce such information from prior rounds of play. Agents have aspiration levels that they adjust from time to time based on their experienced payoffs. The algorithm proceeds by making random encounters between agents on both sides of the market. Agents then make offers to each other compatible with their current aspirations. If they both find their offers profitable, they match; otherwise they return to their old match or lower their aspirations if they were single. This dynamic learning algorithm is shown to converge to pairwise stable (core) solutions in finite time with probability one.

Building on this idea, we consider a similar dynamic for our matching problem which encompasses the ordinal, linear, and nonlinear utility cases and show that this dynamic converges to ϵ-pairwise stable solutions with probability one. In addition to the generalized nature of our matching formulation, our problem is enriched by the presence of many degrees of freedom that control the value of the utility. This feature also enlarges the search space for pairwise stable solutions. We show however that such issues can be significantly suppressed by carefully designing the agents' negotiation mechanisms.

B. Application of Matching theory in Cognitive Radio Networks

In addition to the recent publication of [15], a growing body of literature is using matching theory to solve resource allocation problems in wireless communications, e.g. see [16] for a comprehensive literature review, [11] for a tutorial, and [17]–[20] and the references therein for various applications of matching theory to wireless networks.

The closest work to ours is [17] where the scenario of multiple PUs and SUs is considered. SUs use part of their power to relay the PU messages in exchange for spectrum access time. The problem formulation permits the classification of SUs based on their so called type information, which is a compact representation of an SU's private information such as transmitter power. The paper proposes a distributed algorithm to solve the matching problem under two assumptions on the available information. In the partially incomplete information scenario, PUs have knowledge of the types of all SUs connected to them, while in the incomplete information scenario, a PU knows the set of SUs' types connected to itself, but does not know the exact type of each SU. Note that knowledge of type information permits a PU to know the utility of an SU for a given time and power allocation.

We consider a similar setting to [17] where SUs relay PU data in exchange for spectrum access time. However, we adopt less restrictive information assumptions. In our formulation, agents, whether PUs or SUs, have enough information to calculate their own utilities. No knowledge is available about the utilities of other users, whether on the same side or the opposite side of the market.

Furthermore, to demonstrate the wide applicability of our algorithm to other matching problems, we also provide a comparison with the work in [19]. The authors in [19] propose an algorithm to assign SUs to orthogonal PU channels. The proposed algorithm relies on heavy information exchange between the agents and requires

[8] Pairwise stable solutions are immune to deviations by pairs of users, while core solutions are immune to deviations by any coalition of players

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a coordinator, while all this exchange is not needed for our decentralized algorithm.

III. Context-free BLMA Algorithm

A. Setup

We consider a two-sided matching problem constructed as follows. There are two disjoint sets of agents, $\mathcal{K} = \{1, 2, ..., K\}$ and $\mathcal{L} = \{1, 2, ..., L\}$, that form two sides of a matching market. We exclusively will use k and ℓ to denote a representative element of $\mathcal{K}$ and $\mathcal{L}$, respectively, and sometimes use j to denote a representative element of $\mathcal{K} \cup \mathcal{L}$.

Definition 1. *A* ***matching*** *is a mapping*

$$\mu : \mathcal{K} \cup \mathcal{L} \to \mathcal{K} \cup \mathcal{L} \cup \{\emptyset\}$$

such that for any $k \in \mathcal{K}$ and $\ell \in \mathcal{L}$:

- $\mu(k) \in \mathcal{L} \cup \{\emptyset\}$.
- $\mu(\ell) \in \mathcal{K} \cup \{\emptyset\}$.
- $\ell = \mu(k) \Leftrightarrow k = \mu(\ell)$.

If $\ell = \mu(k)$, then k and ℓ are said to be *matched*. In our model, an agent can be matched to at most one agent on the opposite side of the market. If $\mu(j) = \emptyset$, then agent $j \in \mathcal{K} \cup \mathcal{L}$ is said to be *single*.

A matching, μ, can be characterized by a $K \times L$ matrix, $\mathbf{M}_\mu$, with elements in $\{0, 1\}$, such that

$$\mathbf{M}_\mu(k, \ell) = \begin{cases} 1, & \mu(k) = \ell; \\ 0, & \text{otherwise.} \end{cases}$$

Let $\mathcal{M}$ denote the set of all feasible matching matrices induced by some matching, μ. For any $\mathbf{M} \in \mathcal{M}$, let $\mu_{\mathbf{M}}$ denote the matching consistent with $\mathbf{M}$.

In order to describe the preferences of agents, we introduce *aspiration levels* that abstractly represent the potential utility to be derived from a match. If a_k is the aspiration level of agent $k \in \mathcal{K}$ and b_ℓ is the aspiration level of agent $\ell \in \mathcal{L}$, then agents k and ℓ are willing to be matched if the matching can produce utilities of at least a_k and b_ℓ, respectively.

More formally, we introduce the notion of an agreement function as follows.

Definition 2. *An* ***agreement function*** *is a mapping*

$$\mathcal{A} : \mathbb{R}_+ \times \mathbb{R}_+ \to \{0, 1\}$$

such that

1) *If $\mathcal{A}(a, b) = 0$, then $\mathcal{A}(a', b') = 0$ for all $a' \geq a$ and $b' \geq b$.*
2) *There exists a $\gamma > 0$ such that $\mathcal{A}(a, b) = 0$ if $a \geq \gamma$ or $b \geq \gamma$.*

We associate $\mathcal{A}(a, b) = 1$ to mean that the aspiration levels a and b are agreeable. Accordingly, condition 1 defines a monotonicity property for aspiration levels: once aspirations are not agreeable, further increases in aspiration levels also are not agreeable. Condition 2 defines a boundedness property for agreeable aspiration levels.

Definition 3. *A* ***matching problem*** *is a collection of agreement functions, $\mathcal{A}_{k\ell}$, indexed by $k \in \mathcal{K}$ and $\ell \in \mathcal{L}$.*

We are interested in defining the notion of a stable outcome of a matching problem specified by a set of agreement functions. Towards this end, we will consider vectors of aspiration levels $\mathbf{a} \in \mathbb{R}_+^K$ and $\mathbf{b} \in \mathbb{R}_+^L$, with elements denoted by a_k and b_ℓ, respectively. In the same way that j denotes a representative element of $\mathcal{K} \cup \mathcal{L}$, we will use c_j to denote the associated aspiration level.

The specific stability notion of interest here will be ϵ-pairwise stability defined as follows.

Definition 4. *For $\epsilon > 0$, the matching μ and aspiration levels $\mathbf{a}$ and $\mathbf{b}$ form an* ***ϵ-pairwise stable*** *solution to a matching problem if:*

1) *For all (k, ℓ) such that $\ell = \mu(k)$,*
$$\mathcal{A}_{k\ell}(a_k, b_\ell) = 1.$$
2) *For all (k, ℓ),*
$$\mathcal{A}_{k\ell}(a_k + \epsilon, b_\ell + \epsilon) = 0.$$
3) *For all $j \in \mathcal{K} \cup \mathcal{L}$ with $\mu(j) = \emptyset$, $c_j = 0$.*

In words, condition 1 states that aspiration levels between matched pairs are agreeable. Condition 2 implies that no pair of agents have agreeable ϵ-improvement aspiration levels. Note that condition 2 also applies to agents that are matched (i.e., even if $\ell = \mu(k)$). Condition 3 implies that single agents must have zero aspiration levels at an ϵ-pairwise stable solution.

We are further interested in relating the ϵ-pairwise stable matching outcome with the core of this matching market. Let

$$z \in \mathbb{R}_+^K \times \mathbb{R}_+^L \times \mathcal{M},$$

be the state containing agents' aspiration levels and matching.

Definition 5. *A state z is* ***agreeable*** *if for all (k, ℓ) such that $\ell = \mu(k)$,*

$$\mathcal{A}_{k\ell}(a_k, b_\ell) = 1.$$

Furthermore, let $S \subseteq \mathcal{K} \cup \mathcal{L}$ be a coalition of players[9] and let $z^S = [\mathbf{a}^S \ \mathbf{b}^S \ \mathbf{M}^S]$ be the associated state of coalition S, i.e., the state containing the aspiration levels of coalition members and their matching. Now, we are in a position to define the ϵ-core similar to other definitions of the core in the literature [21].

Definition 6. *A state z is in the* ***ϵ-core*** *of the game if there exists no coalition $S \subseteq \mathcal{K} \cup \mathcal{L}$ with an agreeable state z^S such that $\mathbf{a}_k + \epsilon < \mathbf{a}_k^S$ for every player $k \in S$ and $\mathbf{b}_\ell + \epsilon < \mathbf{b}_\ell^S$ for every player $\ell \in S$. Such a coalition is called a blocking coalition.*

Naturally, core allocations imply ϵ-core allocations with equality when $\epsilon = 0$ [22]. It was shown in [2], [6], [23] that the core, and hence the ϵ-core, of the one-to-one generalized assignment game like the one described

[9]By the nature of our problem, only mixed-pair coalitions are relevant.

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by our matching problem is non-empty. We also relate the ϵ-pairwise stable solutions and the ϵ-core solutions in the following proposition:

Proposition 1. *ϵ-pairwise stable states equal ϵ-core states.*

Proof: The proof follows the approach used in [2]. If z is ϵ-pairwise unstable via some k and ℓ, then it is ϵ-dominated via the coalition $S = \{k, \ell\}$ by any matching which makes $\mu(k) = \ell$ with agreeable aspiration levels $\mathbf{a}_k^S > \mathbf{a}_k + \epsilon$ and $\mathbf{b}_\ell^S > \mathbf{b}_\ell + \epsilon$.

In the other direction, if z is not in the ϵ-core, then it is ϵ-dominated by some coalition S and an agreeable state z^S. Since z^S is agreeable, there exists a k and ℓ in S with $\mathbf{a}_k{}^S > \mathbf{a}_k + \epsilon$ and $\mathbf{b}_\ell{}^S > \mathbf{b}_\ell + \epsilon$. Hence z is ϵ-pairwise unstable.

Here are three examples of matching problems using the above formulation.

- *Matching market with transferable utility:* The sets $\mathcal{K}$ and $\mathcal{L}$ represent firms and workers. For each pair, $k \in \mathcal{K}$ and $\ell \in \mathcal{L}$, the value $p_{k\ell}$ is the maximum salary firm k is willing to pay worker ℓ. Similarly, $q_{k\ell}$ is the minimum salary worker ℓ is willing to take to work for firm k. Suppose firm k has aspiration level a_k and worker ℓ has aspiration level b_ℓ. Then a match is agreeable if $\mathcal{A}_{k\ell}(a_k, b_\ell) = 1$, where

$$\mathcal{A}_{k\ell}(a_k, b_\ell) = \begin{cases} 1, & p_{k\ell} - a_k \geq q_{k\ell} + b_\ell; \\ 0, & \text{otherwise.} \end{cases}$$

  The agreement function $\mathcal{A}_{k\ell}$ is fully characterized by parameters $p_{k\ell}$ and $q_{k\ell}$.
- *Matching market with non-transferable utility:* There are two commodities, G and H. An agent $k \in \mathcal{K}$ has an initial endowment of $g_k > 0$ of commodity G, whereas an agent $\ell \in \mathcal{L}$ has an initial endowment of $h_\ell > 0$ of commodity H. Every agent $k \in \mathcal{K}$ has an indexed collection of utility functions,

$$u_{k\ell}(g, h) : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}$$

  that expresses how much it values g of its own commodity G with h of commodity H from agent ℓ. Likewise, every agent $\ell \in \mathcal{L}$ has an indexed collection of utility functions,

$$v_{k\ell}(g, h) : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}$$

  that expresses how much it values h of its own commodity H with g of commodity G from agent k. We assume that all utility functions are strictly increasing in both arguments. Suppose agent k has aspiration level a_k and agent ℓ has aspiration level b_ℓ. Define the set

$$S_{k\ell}(a_k, b_\ell) = \{(g, h) \mid g \leq g_k; h \leq h_\ell; \\ u_{k\ell}(g_k - g, h) \geq a_k; v_{k\ell}(g, h_\ell - h) \geq b_\ell\}. \quad (1)$$

  In words, this set describes all possible exchanges of g from agent k to agent ℓ in return for h from agent ℓ to agent k such that their utilities meet the specified aspiration levels. Then a match is agreeable if $\mathcal{A}_{k\ell}(a_k, b_\ell) = 1$, where

$$\mathcal{A}(a_k, b_\ell) = \begin{cases} 1, & S_{k\ell}(a_k, b_\ell) \neq \{\emptyset\}; \\ 0, & \text{otherwise.} \end{cases} \quad (2)$$

- *Ordinal utility market:* Consider the classical stable marriage problem of [1]. The sets $\mathcal{K}$ and $\mathcal{L}$ represent women and men, respectively. Each k in $\mathcal{K}$ has a preference list, P_k, over the agents in $\mathcal{L}$. For simplicity, let the ordering be strict, i.e., k is never indifferent between two agents on the opposite side of the market. For example $P_k = \{l_1, l_4, l_7, k\}$ means k prefers being matched to l_1, then l_4, then l_7 over being single. For convenience, let us transform the ordering to a decreasing order utility $U_k = \{u_{k1}, u_{k4}, u_{k7}, 0\}$ with $u_{k1} > u_{k4} > u_{k7} > 0$[10]. Likewise define P_ℓ and V_ℓ as the preference and decreasing order utility for ℓ, respectively. Suppose woman k has aspiration level a_k and man ℓ has aspiration level b_ℓ. Then a match is agreeable if $\mathcal{A}_{k\ell}(a_k, b_\ell) = 1$, where

$$\mathcal{A}(a_k, b_\ell) = \begin{cases} 1, & u_{k\ell} > a_k \text{ and } v_{k\ell} > b_\ell; \\ 0, & \text{otherwise.} \end{cases} \quad (3)$$

B. BLMA Algorithm

We now present an algorithm that leads to an ϵ-pairwise stable solution. The algorithm is inspired by the recent work of [8] on transferable utility assignment games. Informally, the algorithm proceeds as follows:

- Aspirations levels, $\mathbf{a}(t)$ and $\mathbf{b}(t)$, as well as a matchings characterized by a matching matrix, $\mathbf{M}(t)$, evolve over stages $t = 0, 1, 2, ...$
- At stage t, a pair of agents, (k, ℓ), are activated at random.
- If the increased aspiration levels $a_k(t) + \epsilon$ and $b_\ell(t) + \epsilon$ are agreeable, i.e.,

$$\mathcal{A}_{k\ell}(a_k(t) + \epsilon, b_\ell(t) + \epsilon) = 1,$$

  then agents k and ℓ become matched with a positive probability η and break previous matches, if any. The new aspiration levels $a_k(t+1)$ and $b_\ell(t+1)$, as well as matching matrix, $\mathbf{M}(t+1)$, are updated accordingly.
- If the increased aspiration levels $a_k(t) + \epsilon$ and $b_\ell(t) + \epsilon$ are not agreeable, i.e.,

$$\mathcal{A}_{k\ell}(a_k(t) + \epsilon, b_\ell(t) + \epsilon) = 0,$$

  then
  - The matching matrix remains unchanged, i.e., $\mathbf{M}(t+1) = \mathbf{M}(t)$.

[10]One may think of the elements of U_k as dowries being offered by the $\mathcal{L}$ side to k. The dowries are merely indexed by the identities of the matched pair but are not subject to optimization.

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- If either agent k or ℓ is single, then that agent reduces its aspiration by δ, and the new aspiration levels $a_k(t+1)$ and/or $b_\ell(t+1)$ are updated accordingly.

Algorithm 1 presents pseudo code for the BLMA. Here, the aforementioned "stages" are executions of the main loop. The time indexing of $t = 0, 1, 2, ...$ is suppressed for clarity of presentation. The notation "$\text{RAND}[0, 1]$" (Line 5) means an i.i.d. sample of a uniformly distributed random variable over the interval $[0, 1]$. Also $[x]^+ = \max(0, x)$ (Line 16).

BLMA is "blind" in the sense that potential matches between agents k and ℓ are outcomes of bilateral negotiations that only depend on the agreement function $\mathcal{A}_{k\ell}$. The negotiation process is abstracted through the randomized outcome determined by $\text{RAND}[0, 1] \geq \eta$. An agent need not know the details behind another agent's acceptance or rejection. Furthermore, since such outcomes can be randomized, it may be difficult to make deterministic conclusions from a rejected offer. The main point is that all such issues are suppressed, with the specifics to depend on the actual context. Also, BLMA is non-deterministic in that revised aspiration levels (Lines 9–11) are not fully specified. Again, how this selection occurs will depend on the specific context.

Algorithm 1 BLMA

Require: $\epsilon > \delta > 0$ and $\eta \in (0, 1]$.
1: Initialize $\mathbf{a} \geq 0$, $\mathbf{b} \geq 0$, $\mathbf{M} = \mathbf{0}$.
2: **loop**
3: Activate a pair of agents uniformly at random, $(k, \ell) \in \mathcal{K} \times \mathcal{L}$.
4: **if** $\mathcal{A}_{kl}(a_k + \epsilon, b_\ell + \epsilon) = 1$ **then**
5: **if** $\text{RAND}[0, 1] \geq \eta$ **then**
6: $\mathbf{M}(k, \ell) \leftarrow 1$
7: $\mathbf{M}(k, \ell') \leftarrow 0, \forall \ell' \neq \ell$
8: $\mathbf{M}(k', \ell) \leftarrow 0, \forall k' \neq k$
9: Select arbitrary $a' \geq a_k + \epsilon$ and $b' \geq b_\ell + \epsilon$ such that $\mathcal{A}_{k\ell}(a', b') = 1$.
10: $a_k \leftarrow a'$
11: $b_\ell \leftarrow b'$
12: **end if**
13: **else**
14: **for** $j \in \{k, \ell\}$ **do**
15: **if** $\mu_{\mathbf{M}}(j) = \emptyset$, **then**
16: $c_j \leftarrow [c_j - \delta]^+$
17: **end if**
18: **end for**
19: **end if**
20: **end loop**

We now state the main result.

Theorem 1. *Using BLMA, from any initial $\mathbf{a} \geq 0$, $\mathbf{b} \geq 0$, and $\mathbf{M} = 0$, the matching, $\mu_{\mathbf{M}}$, and aspiration levels, $\mathbf{a}$ and $\mathbf{b}$, converge to an ϵ-pairwise stable matching with probability one.*

C. Proof

This subsection is devoted to the proof of Theorem 1. We first introduce some specialized notation and terminology. Recall that we use z to denote the state of the algorithm. A state is a triplet $z \in \mathbb{R}_+^K \times \mathbb{R}_+^L \times \mathcal{M}$, which is a combination of aspiration levels and a matching matrix. Each execution of the main loop results in an update in the state, e.g., $z \leftarrow z_{\text{new}}$. A state is *reachable* if it can be realized in a finite number of main loop executions.

A state, $z = (\mathbf{a}, \mathbf{b}, \mathbf{M})$, will be called **pre-stable** if

1) For all (k, ℓ) such that $\ell = \mu(k)$,
$$\mathcal{A}_{k\ell}(a_k, b_\ell) = 1.$$
2) For all (k, ℓ),
$$\mathcal{A}_{k\ell}(a_k + \epsilon, b_\ell + \epsilon) = 0.$$

Strictly speaking, this definition depends on ϵ and accordingly could be called ϵ-pre-stable. We will suppress this dependence for clarity of presentation. Note that the conditions for a pre-stable state are the first two conditions for an ϵ-pairwise stable state. The only distinction is that a pre-stable state may have single agents with non-zero aspiration levels.

Claim 1. *From any reachable state, z, there exists a finite sequence of admissible transitions to a state, z', that is pre-stable.*

Proof: In the main loop, activate and match any pair of agents (k, ℓ) with
$$\mathcal{A}_{kl}(a_k + \epsilon, b_\ell + \epsilon) = 1.$$
Continue to do so until there are no remaining such pairs. This process must terminate because of the boundedness property of the agreement functions and the algorithmic property that, in such a sequence, no agents are reducing their aspiration levels. Upon termination, the resulting state, z', must be pre-stable by construction. □

Given any state $z = (\mathbf{a}, \mathbf{b}, \mathbf{M})$, let $\text{SNZ}(z)$ denote the set of all single agents with non-zero aspiration levels, i.e.,
$$\text{SNZ}(z) = \{j \in \mathcal{K} \cup \mathcal{L} \mid \text{(i) } \mu(j) = \emptyset, \text{(ii) } c_j > 0\}.$$

A pre-stable state, z, is called **tight** for any agent $j^* \in \text{SNZ}(z)$ with aspiration level c_{j^*}, if the new state, $z' = (\mathbf{a}', \mathbf{b}', \mathbf{M}')$, defined by:
$$\mathbf{M}' = \mathbf{M}$$
$$c'_j = \begin{cases} c_j, & j \neq j^*; \\ [c_j - \delta]^+, & j = j^*, \end{cases}$$
is *not* pre-stable. The implication here is that a state, z, is pre-stable and tight if (i) there are no ϵ-improvement agreeable matches at current aspirations levels, but (ii) there will exist an ϵ-improvement agreeable match after any agent in $\text{SNZ}(z)$ lowers its aspiration level by δ.

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Claim 2. *From any pre-stable state, z, there exists a finite sequence of admissible transitions to a state, z', that is either (i) pre-stable and tight or (ii) ϵ-pairwise stable.*

Proof: Let z be a pre-stable state. If $\mathrm{SNZ}(z)$ is empty, then z is already ϵ-pairwise stable. Otherwise, select an arbitrary $j^* \in \mathrm{SNZ}(z)$. In the main loop, let j^* be activated with a matched agent. Since z is pre-stable, the proposed match is not ϵ-improvement agreeable, and so the new match is not accepted. Accordingly, the aspiration level c_{j^*} is reduced by δ. Repeat this sequence with the same agent j^* until either c_{j^*} is within a single δ reduction of admitting an ϵ-improvement match (with some unspecified agent) or $c_{j^*} = 0$. Let z^+ be the resulting state. The only difference between z and z^+ is in the aspiration level of j^*. Now select a different $j^{**} \in \mathrm{SNZ}(z)$ and repeat accordingly. Upon visiting all of the agents in $\mathrm{SNZ}(z)$, the resulting state z' is pre-stable and tight by construction. □

Claim 3. *From any pre-stable and tight state, z, with $|SNZ(z)| \neq 0$, there exists a finite sequence of admissible transitions to a pre-stable state z' with $|SNZ(z')| < |SNZ(z)|$, i.e., a strict reduction in the number of single agents with non-zero aspiration levels.*

Proof: Let z be a state that is pre-stable and tight. Let us assume without loss of generality that there exists a $k^* \in \mathrm{SNZ}(z)$, i.e., some agent in $\mathcal{K}$ is single with non-zero aspiration levels. (Analogous arguments hold if the selected agent is in $\mathcal{L}$.) Let us call k^* the token holding agent. The token is not released until a new state, z', is reached with the desired reduction in cardinality.

Activate (the token holding) k^* with any matched agent $\ell \in \mathcal{L}$. Since z is pre-stable, a new match does not occur, and the state is updated so that agent k^* has an aspiration level of $a_{k^*} - \delta$.[11] Furthermore, since z was tight, there exists an agent ℓ^* for which

$$\mathcal{A}_{k^*\ell^*}(a_{k^*} - \delta + \epsilon, b_{\ell^*} + \epsilon) = 1.$$

Let the match between k^* and ℓ^* occur. There are three possible scenarios:

- *Scenario A. $\ell^* \in SNZ(z)$:* Agent ℓ^* was also single with non-zero aspiration. Let z^+ denote the resulting state, and $a^+_{k^*}$ and $b^+_{\ell^*}$ be the revised aspiration levels of agents k^* and ℓ^*, respectively. Then

$$a^+_{k^*} \geq a_{k^*} - \delta + \epsilon,$$
$$b^+_{\ell^*} \geq b_{\ell^*} + \epsilon.$$

Since $\epsilon > \delta$, the aspiration levels of z^+ are greater than the aspiration levels of z. Accordingly, z^+ is pre-stable. Now apply the procedure of Claim 2 to produce a state z' that is pre-stable and tight. By construction, the number of single agents with non-zero aspiration has been reduced by at least two, i.e.,

$$|\mathrm{SNZ}(z')| \leq |\mathrm{SNZ}(z)| - 2.$$

Accordingly, the token is released.
- *Scenario B. $\mu(\ell^*) = \emptyset$ and $b_{\ell^*} = 0$:* Agent ℓ^* is also single, but with zero aspiration level. Proceed in a similar manner to Scenario A to construct a state z' where the number of single agents with non-zero aspiration has been reduced by at least one, i.e.,

$$|\mathrm{SNZ}(z')| \leq |\mathrm{SNZ}(z)| - 1.$$

Accordingly, the token is released.
- *Scenario C. $\mu(\ell^*) = k^{**}$:* Agent ℓ^* is matched to another agent, namely k^{**}, on the $\mathcal{K}$ side of the market. Increase the aspiration levels of newly matched agents k^* and ℓ^* as required. Furthermore, reassign the token to the newly single agent, k^{**}. At this stage, the number of single agents with non-zero aspiration levels has not changed, and hence the token has not been released, but rather reassigned. The new state would be pre-stable and tight except for the aspiration level of the new token holding agent, k^{**}. Accordingly, through a series of executions of the main loop, reduce the aspiration level of k^{**} until either $c_{k^{**}} = 0$ or the realized state is pre-stable and tight. In the former case, the number of single agents with non-zero aspiration levels has been reduced by one, as desired, and the token is released. In the latter case, since the realized state is pre-stable and tight, one can now invoke the aforementioned procedures in the proof of Claim 3 *while selecting k^{**} as the token holding agent* (i.e., without reassigning the token). If the outcome is Scenario A or B, then the number of single agents with non-zero aspirations levels has been reduced, and the token is released. Otherwise, the token is passed to yet another agent, e.g., k^{***}, and the process is repeated. Note that whenever the token is reassigned, it stays on the same side, $\mathcal{K}$, of the market. Furthermore, with each reassignment, the sum of the aspiration levels of the $\mathcal{L}$ side of the market strictly increases. Such increase cannot continue indefinitely because of the boundedness of the agreement functions, and so eventually the token must be released with the number of single non-zero agents reduced. Now apply the procedure of Claim 2 to assure that the exiting state z' is pre-stable and tight.

□

With Claims 1–3 in place, we are now in a position to prove Theorem 1. From any reachable state, there exists a finite sequence of admissible transitions that leads to a pre-stable state (Claim 1) followed by a finite sequence of admissible transitions that leads to a pre-stable and tight state (Claim 2). By a repeated application of Claim 3, there exists a finite sequence of

[11] Here, we assume for convenience that $a_{k^*} - \delta > 0$. Similar arguments hold in case $[a_{k^*} - \delta]^+ = 0$.

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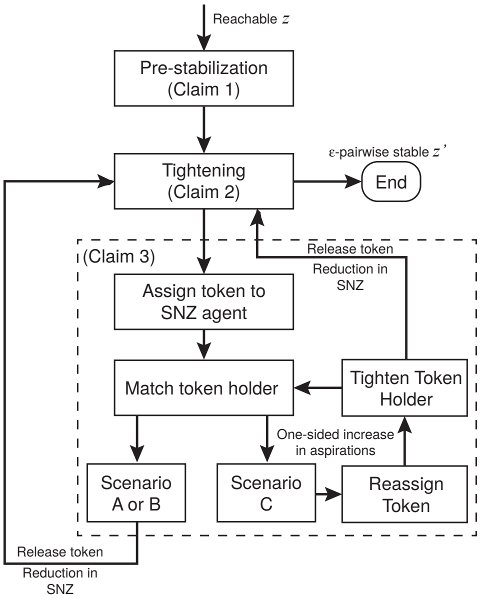

Fig. 1: Illustration of the combined effects of Claims 1–3 to transition from any reachable state, z, to an ϵ-pairwise stable state ("End").

admissible transitions to an ϵ-pairwise stable state.

Fig. 1 illustrates the combined effect of Claims 1–3. A state is first made to be pre-stable and then pre-stable and tight. At that point, the procedure behind Claim 3 is executed. There are two types of loops in Figure 1. The first type of loop involves a *releasing* of the token and a return to the "Tightening" procedure. This loop results in a reduction in the number of single agents with non-zero aspiration levels, and so there can only be a finite number of such iterations. The second type of loop involves a *reassignment* of the token. In this loop, the number of single agents with non-zero aspiration levels remains constant. However, this loop results in an increase in the sum of aspiration levels on one side of the market (namely, on the side opposite to the token), and so this loop must eventually be exited because of the boundedness assumption on agreement functions. Ultimately, the process much reach the "End" state, which is ϵ-pairwise stable.

Note that Figure 1 is intended to illustrate a feasible sequence of admissible transitions to an ϵ-pairwise stable state, and as such a *positive probability* flow of the BLMA algorithm. Putting all this together, the conclusion is that from any reachable state, there is a positive probability of a finite sequence of admissible transitions eventually leading to an ϵ-pairwise stable state. Given the finite cardinality of $\mathcal{K}$ and $\mathcal{L}$, and the boundedness property of agreement functions, there exists some finite number of executions of the main loop, say T, such that from any reachable state, z, the probability of reaching an ϵ-pairwise stable state after T executions of the main loop is at least $p(\epsilon, \delta) > 0$, where both $p(\epsilon, \delta)$ and T do not depend on z. Accordingly, the probability of not terminating after T executions is at most $(1 - p(\epsilon, \delta))$. Likewise, the probability of not terminating after mT executions is at most $(1 - p(\epsilon, \delta))^m$. Let E_i be the event of not terminating after iT executions. Since the sum $\sum_{i=1}^{\infty} E_i$ is finite, we can subsequently conclude that the algorithm reaches an ϵ-pairwise stable state with probability one by the Borel-Cantelli lemma [24].

D. A Note on Convergence Rates

While Theorem 1 provides a proof of convergence for the BLMA, a question remains as to the actual convergence rates. Previous work in the matching literature provided results on convergence rates. For example in [25], the authors showed that uncoordinated two-sided marriage markets can take exponentially long to converge and proposed variations on the market setup to improve convergence. The work in [26], considers decentralized local dynamics to balanced outcomes in a linear assignment market[12]. The market is defined on a weighted graph where nodes represent players and where the weight of an edge represents the dollar amount to be shared between two agents. By converting the network bargaining game into a random-turn game, the authors prove their decentralized dynamic converges to balanced outcomes in polynomial time. The matching however is fixed and the convergence rate is that of players' payoffs.

The work of [10] presents the most relevant result to our work. The setting is a linear assignment game with market and information decentralization reminiscent of our modeling. Firms and workers match, break up, and re-match in search for better opportunities. The behavior of agents fluctuates according to a random variable called market sentiment[13]. Given some further assumption on the match values (or the weights of edges), the author proves polynomial time convergence occurs in this linear model. The best bound obtained in [10] is that the dynamic converges in $\mathcal{O}(N^4)$, where N is the maximum number of agents on either side of the market. Studying actual convergence rates for our abstract and generalized algorithm is interesting and is subject to ongoing investigation. We remark however that for the linear and nonlinear assignment games, convergence in our simulations results does not exceed the $\mathcal{O}(N^4)$ bound of [10].

IV. Application: Cognitive Radio Market with Quasi-convex Utilities

We adopt the system model of [17] in this section, formulate a matching problem, and provide a solution

[12]A balanced outcome is a stronger notion of equilibrium in linear assignment markets than a stable outcome. An outcome is balanced if it is stable and if matched players divide the surplus equally among themselves. Furthermore, balancedness provides a generalization of the Nash bargaining solution to exchange networks.

[13]This market sentiment random variable is where the dynamic in [10] departs from the original work in [8]. However, the author in [10] shows this exogenous random variable is needed to establish polynomial time convergence.

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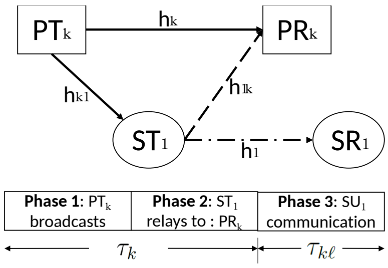

Fig. 2: Interaction between PU_k and SU_ℓ during the three transmission phases.

to this problem using the BLMA with significantly less information assumptions. Consider a cognitive radio network comprised of a set $\mathcal{K} = \{1, 2, ..., K\}$ of PUs and a set $\mathcal{L} = \{1, 2, ..., L\}$ of SUs. Each network node is made up of a transmitter-receiver pair. Suppose that PU_k is matched with SU_ℓ with agreed upon time, $\tau_{k\ell} \in [0, 1]$, and power, $P_{k\ell} > 0$. The PU_k's transmission time, τ_k, is fixed while τ_{kl} is the time allocated by PU_k to SU_ℓ's communications. There are three phases of communication as follows:

- During the first $\frac{\tau_k}{2}$ part of the time slot, PU's transmitter, PT_k, broadcasts its data. The data is received by the PU's receiver, PR_k, and by SU's transmitter, ST_ℓ, contingent on $h_{k\ell} \geq h_k$, where $h_{k\ell}$ is PT_k to ST_ℓ channel gain, while h_k is PT_k to PR_k direct link channel gain.
- During the second $\frac{\tau_k}{2}$ part of the time slot, ST_ℓ decodes the data received in phase 1 and relays PU_k's message to PR_k using power $P_{k\ell}$.
- A third time phase, $\tau_{k\ell}$, is allocated by PU_k for SU_ℓ's own communication.

The above communication structure is shown in Fig. 2 with the other relevant channel gains. Once matched, PU_k gives up time, $\tau_{k\ell}$, for SU_ℓ's spectrum access, in exchange for relaying help from SU_ℓ. Moreover, SU_ℓ uses power, $P_{k\ell}$, for relaying PU_k's message in exchange for spectrum access.

A. Utility Functions and Matching Problem Formulation

Suppose that specific time and power allocations have been agreed upon by PU_k and SU_ℓ. Assuming, without loss of generality, that $\tau_k = 1$, then the average achievable PU_k data rate over the entire time when matched with SU_ℓ is

$$R^{\text{P}}_{k\ell}(\tau_{k\ell}, P_{k\ell}) = \begin{cases} \frac{1}{2(1+\tau_{k\ell})}\left[R_k^{\text{P,dl}} + \log_2\left(1 + \frac{P_{k\ell}}{\sigma^2}\right)\right] & \\ \qquad\qquad \text{if } h_{k\ell} \geq h_k, & \\ 0 \qquad\qquad \text{otherwise}, & \end{cases} \tag{4}$$

where the above rate expression can be achieved using a decode-and-forward protocol with parallel channel coding [27]. The requirement $h_{k\ell} \geq h_k$ is the condition that ST_ℓ is in the decoding set of PU_k [28]. $R_k^{\text{P,dl}}$ is PU_k's direct link rate which can be calculated as $R_k^{\text{P,dl}} = \log_2\left(1 + \frac{P_k|h_k|^2}{\sigma^2}\right)$, where P_k is PU_k's transmission power. Equation (4) is comprised of two terms. The first term, $R_k^{\text{P,dl}}$, refers to phase one of the communication, while the second term reflects the rate accrued due to relaying by SU_ℓ in the second communication phase. Then, the associated utility for PU_k when matched with SU_ℓ is

$$u_{k\ell}(\tau_{k\ell}, P_{k\ell}) = [R^{\text{P}}_{k\ell}(\tau_{k\ell}, P_{k\ell}) - R_k^{\text{P,dl}}]^+, \tag{5}$$

where $u_{k\ell} \geq 0$ reflects the fact that a matching with any SU must at least provide the same rate as if PU_k were single. We note in (4) that the matched SU offers the PU a fixed rate; i.e., SU_ℓ uses power $P_{k\ell}/|h_{\ell k}|^2$ to send PU_k's packets so that the rate enhancement to PU_k is kept constant at $\frac{1}{2(1+\tau_{k\ell})}\log_2\left(1 + \frac{P_{k\ell}}{\sigma^2}\right)$.

On the other side, SU_ℓ gains utility from the spectrum access opportunity when matched with PU_k, while paying a relaying *price*, in terms of power used to help the primary link. The secondary utility can be written as

$$v_{k\ell}(\tau_{k\ell}, P_{k\ell}) = \left[\tau_{k\ell}\log_2\left(1+\frac{P_\ell|h_\ell|^2}{\sigma^2}\right) - c_\ell\left(\frac{P_{k\ell}}{2|h_{lk}|^2} + \tau_{kl}P_\ell\right)\right]^+, \tag{6}$$

where P_ℓ is the power used for SU_ℓ's own communication and c_ℓ is SU_ℓ's sensitivity for unit power consumption. The problem of interest is to pair the PUs and the SUs with the appropriate time and power allocations. We will cast this problem in the matching problem formulation.

Recall the matching market with non-transferable utility introduced in the previous section. The two commodities here are time and power. Assume that any PU can allocate a maximum of $\tau_{k\ell} \leq 1$ for all $\ell \in \mathcal{L}$. Given equation (6) and a specified $\tau_{k\ell}$ value, the SU can allocate a maximum power of $\overline{P}_{k\ell}(\tau_{k\ell})$. This is the $P_{k\ell}$ value which yields zero utility for SU_ℓ in (6). Since we chose to write the utilities in terms of $\tau_{k\ell}$, i.e., the time allotted by $\text{PU}_{k\ell}$ to $SU_{k\ell}$ and $P_{k\ell}$, the power granted by SU_ℓ to PU_k, the set $S_{k\ell}(a_k, b_\ell)$ in (1) can now be redefined as

$$S_{k\ell}(a_k, b_\ell) = \big\{(\tau_{k\ell}, P_{k\ell}) \mid \tau_{k\ell} \leq 1; P_{k\ell} \leq \overline{P}_{k\ell}(\tau_{k\ell}); \\ u_{k\ell}(\tau_{k\ell}, P_{k\ell}) \geq a_k; v_{k\ell}(\tau_{k\ell}, P_{k\ell}) \geq b_\ell\big\}. \tag{7}$$

The agreement functions $\mathcal{A}_{k\ell}$ are defined as in (2) and the matching problem formulation follows accordingly.

B. BLMA Realization

Since the problem of assigning PUs to SUs with agreed upon time and power allocations is a matching problem in the sense defined in Section III, we will be able to find an ϵ-pairwise stable matching solution using

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the BLMA. We illustrate here the details. Note first from (5) that $u_{k\ell}$ is decreasing in $\tau_{k\ell}$ and increasing in $P_{k\ell}$, while the opposite is true for $v_{k\ell}$. Furthermore, we can easily verify the utility $u_{k\ell}$ is quasi-convex and the utility $v_{k\ell}$ is quasi-concave[14][15]. Consider the following two procedures:

1) *BLMA1: The PU-SU negotiation process*
   - Active agents make offers compatible with their aspirations levels.
     a) Let PU_k pick, uniformly at random, an offer $(\tau^{\text{P}}_{k\ell}, P^{\text{P}}_{k\ell})$ such that $a_k + \epsilon = u_{k\ell}(\tau^{\text{P}}_{k\ell}, P^{\text{P}}_{k\ell})$.
     b) Let SU_ℓ pick, uniformly at random, an offer $(\tau^{\text{S}}_{k\ell}, P^{\text{S}}_{k\ell})$ such that $b_\ell + \epsilon = v_{k\ell}(\tau^{\text{S}}_{k\ell}, P^{\text{S}}_{k\ell})$.
   - **If**
     a) $\lfloor u_{k\ell}(\tau^{\text{S}}_{k\ell}, P^{\text{S}}_{k\ell})\rfloor_\delta \geq a_k + \epsilon$, and
     b) $\lfloor v_{k\ell}(\tau^{\text{P}}_{k\ell}, P^{\text{P}}_{k\ell})\rfloor_\delta \geq b_\ell + \epsilon$

     Then
     $\mathcal{A}_{k\ell}(a_k + \epsilon, b_\ell + \epsilon) = 1.$
     End If
2) *BLMA1: Updating the aspiration levels*
   - Select point $(\tau_{k\ell}, P_{k\ell})$ uniformly at random on the line segment connecting $(\tau^{\text{P}}_{k\ell}, P^{\text{P}}_{k\ell})$ and $(\tau^{\text{S}}_{k\ell}, P^{\text{S}}_{k\ell})$.
     a) Update $a_k \leftarrow \lfloor u^{\text{P}}_{k\ell}(\tau_{k\ell}, P_{k\ell})\rfloor_\delta$.
     b) Update $b_\ell \leftarrow \lfloor u^{\text{S}}_{k\ell}(\tau_{k\ell}, P_{k\ell})\rfloor_\delta$.

To compare all the forthcoming realizations of the BLMA, we will collectively refer to the above two procedures as BLMA1. Note that $(\tau^{\text{P}}_{k\ell}, P^{\text{P}}_{k\ell})$ is the time and power offer of PU_k, $(\tau^{\text{S}}_{k\ell}, P^{\text{S}}_{k\ell})$ is the time and power offer of SU_ℓ, and $\lfloor x\rfloor_\delta = \max\{m\delta \mid m\delta \leq x \text{ for } m \in \mathbb{Z}_+\}$. Considering the above two procedures, we have the following result:

Proposition 2. *Given quasi-convex utility $u_{k\ell}$ and quasi-concave utility $v_{k\ell}$ and $\epsilon = q\delta$ for some $\{q \mid q > 1$ and $q \in \mathbb{Z}_+\}$, BLMA1 converges to an ϵ-pairwise stable state with probability one.*

Proof: Read the negotiation process above as Line 4 in the BLMA. PU_k with aspiration level a_k and SU_ℓ with aspiration level b_ℓ will only declare their match agreeable when $\lfloor u_{k\ell}(\tau^{\text{S}}_{k\ell}, P^{\text{S}}_{k\ell})\rfloor_\delta \geq a_k + \epsilon = u_{k\ell}(\tau^{\text{P}}_{k\ell}, P^{\text{P}}_{k\ell})$, and $\lfloor v_{k\ell}(\tau^{\text{P}}_{k\ell}, P^{\text{P}}_{k\ell})\rfloor_\delta \geq b_\ell + \epsilon = v_{k\ell}(\tau^{\text{S}}_{k\ell}, P^{\text{S}}_{k\ell})$. Read the aspiration update above as Line 9 in the BLMA. Quasi-convexity of the utilities ensures that any point on the line segment connecting $(\tau^{\text{P}}_{k\ell}, P^{\text{P}}_{k\ell})$ and $(\tau^{\text{S}}_{k\ell}, P^{\text{S}}_{k\ell})$ is agreeable. The revised aspiration levels will be $a^+_k \geq a_k + \epsilon > a_k$ and $b^+_\ell \geq b_\ell + \epsilon > b_\ell$ since $\epsilon > 0$. The process cannot continue indefinitely by the boundedness of the aspiration levels and we will reach a pre-stable state, hence Claim 1 is established. Fig. 3 provides an illustration[16].

Claim 2 is established by Lines 15 and 16 and hence is not changed. Considering a pre-stable and tight state z, let SU_ℓ be any single in $\text{SNZ}(z)$ with aspiration b_ℓ. Activate SU_ℓ with any agent PU_k with aspiration a_k. There is no agreement since the state is pre-stable and tight. Since SU_ℓ is single, it lowers its aspiration level by δ. Since the aspirations were tight, we now have an agreement. The two agents will match with revised aspirations $a^+_k \geq a_k + \epsilon > a_k$ and $b^+_\ell \geq b_\ell - \delta + \epsilon \geq b_\ell$ since $\epsilon > \delta > 0$. Since the aspiration levels were pre-stable and tight and agents PU_k and SU_ℓ increased their aspiration levels, we are again at a pre-stable state. We now consider the recursive application of Claim 3 coupled with the tightening process of Claim 2 to reach an ϵ-pairwise stable state.

Finally, we make a note that $a_k,\ b_\ell \in \{0, \delta, 2\delta, ..., \gamma\}$ by the requirement $\epsilon = q\delta$ and the "flooring" procedure of the aspiration update (line 9 in the BLMA). This confinement of the aspiration levels to the discrete grid of δ steps and the requirement of ϵ-improvement agreements ensure that as the algorithm progresses and the aspiration levels pre-stabilize with agents squeezing out all available resources that there still exists some $\eta > 0$ probability of making the match (Line 5 in the BLMA).

□

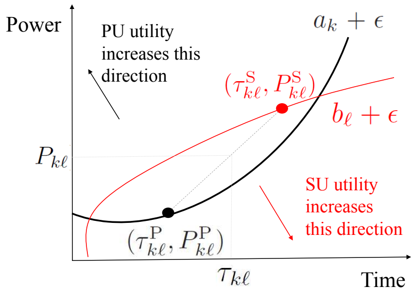

Fig. 3: Figure shows contours for agents' aspiration levels given equations (5) and (6). An agreement of aspiration levels $a_k+\epsilon$ and $b_\ell+\epsilon$ implies that the $b_\ell+\epsilon$ curve must have some segment above the $a_k+\epsilon$ curve. PU_k chooses a random point on this $a_k+\epsilon$ contour, $(\tau^{\text{P}}_{k\ell}, P^{\text{P}}_{k\ell})$, as its offer to SU_ℓ. SU_ℓ also chooses a random point on its $b_\ell+\epsilon$ contour, $(\tau^{\text{S}}_{k\ell}, P^{\text{S}}_{k\ell})$, as its offer to PU_k. The match point $(\tau_{k\ell}, P_{k\ell})$ is chosen uniformly at random on the line connecting these two offers.

[14] A function $f : \mathcal{K} \to \mathbb{R}$ over a convex set, $\mathcal{K}$, is quasi-concave (quasi-convex) if super-level (sub-level) sets , $\{x : f(x) \geq \rho\}$ ($\{x : f(x) \leq \rho\}$), are convex [29].

[15] In fact $v_{k\ell}$ is only linear in $t_{k\ell}$ and $P_{k\ell}$ as can be verified from equation (6) but we use the more general "quasi-concave" term to emphasize that the modified algorithm will still function under this less-strict assumption.

[16] Although $v_{k\ell}$ is linear in its variables, we plot SU_ℓ's aspiration contour as being quasi-concave function in Fig. 3 to assert that BLMA1 will still work for quasi-concave functions also.

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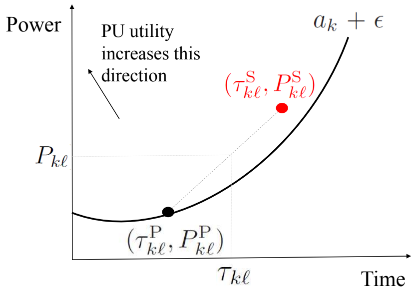

Fig. 4: Information available at PU_k given SU_ℓ's offer.

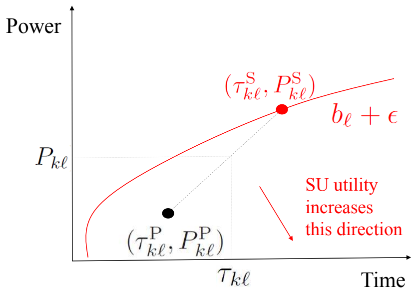

Fig. 5: Information available at SU_ℓ given PU_k's offer.

C. *Limited Information Scenario*

Herein, we motivate how our choices for the PU-SU negotiation process and aspiration level update serve the goal of limiting information exchange in the network. Besides quasi-convexity, we have only assumed that agents have all the information needed to calculate their own utility but no information is available about the utilities of other users on either side of the market. Figures 4 and 5 illustrates the information users have about each other. Under the assumption of quasi-convex utility functions, the straight line connecting the two offers guarantees an increase in utility for both users. Given a time and power offer from SU_ℓ, PU_k only has information of its aspiration contour a_k and SU_ℓ's offer $(\tau^{\mathrm{S}}_{k\ell}, P^{\mathrm{S}}_{k\ell})$. PU_k has no knowledge of any other points on the b_ℓ contour. Points on the line connecting the two offers $(\tau^{\mathrm{P}}_{k\ell}, P^{\mathrm{P}}_{k\ell})$ and $(\tau^{\mathrm{S}}_{k\ell}, P^{\mathrm{S}}_{k\ell})$ are agreeable from PU_k's perspective given the quasi-convexity. A similar statement can be made about SU_ℓ.

We contrast this framework, for example, with the work in [17], wherein equation (6) is normalized and its terms re-arranged so that the SU_ℓ's utility is of the form

$$v^{\mathrm{alt}}_{k\ell} = \tau_{k\ell}\theta_{k\ell} - P_{k\ell}, \tag{8}$$

and $\theta_{k\ell}$ is a compact representation of SU_ℓ's private information, or so called type, when paired with PU_k. The SU's type contains information as: 1) The relaying capability of SU_ℓ with PU_k, 2) the channel coefficients $h_{\ell k}$ and h_ℓ, and 3) SU_ℓ's sensitivity for unit power consumption. The work in [17] considers two types of limited information scenarios. Under the partially incomplete information scenario, a PU knows the type of each SU connected to itself, but not to other PUs. Under the incomplete information scenario, a PU knows the values of all the SU types but has no way of associating a particular type with a specific SU[17]. Given the above discussion, it is clear that we consider a more restrictive incomplete information scenario. It is also in that sense, that we name our algorithm a blind matching algorithm since very little information is available about other users in the market, yet we still converge to a stable outcome. We will contrast our approach with the one in [17] in the numerical results section.

V. BLMA Application to a Cognitive Radio Network with General Utilities

In this section, we modify the system model slightly to show another application where the utilities of agents on the two sides of the market show opposite trends but are not necessarily quasi-convex[18]. We continue to consider a limited information scenario so that agents can only calculate their utilities but no information is available about the utilities of other users in the network. Consider a modified cooperative relaying scheme such that PU_k's time slot when matched with SU_ℓ is now divided as follows:

- During the first $\frac{\tau_k(1-\tau_{k\ell})}{2}$ part of the time slot, PT_k broadcasts its data. The data is received by the PU's receiver, PR_k, and by SU's transmitter, ST_ℓ, contingent on $h_{k\ell} \geq h_k$.
- During the second $\frac{\tau_k(1-\tau_{k\ell})}{2}$ part of the time slot, ST_ℓ decodes the data received in phase 1 and relays PU_k's message to PR_k using power $P_{k\ell}$.
- A third time phase, $\tau_{k\ell}\tau_k$, is allocated by PU_k for SU_ℓ's own communication.

Now PU_k's time slot is fixed at τ_k and a portion of that time slot is dedicated for SU_ℓ's communication[19]. This changes the agents' utilities so that the achievable PU_k rate when matched with SU_ℓ is

$$R^{\mathrm{P}}_{k\ell}(\tau_{k\ell}, P_{k\ell}) = \begin{cases} \frac{1-\tau_{k\ell}}{2}\left[R^{\mathrm{P,dl}}_k + \log_2\left(1+\frac{P_{k\ell}}{\sigma^2}\right)\right] & \text{if } h_{k\ell} \geq h_k, \\ 0 & \text{otherwise.} \end{cases} \tag{9}$$

The associated utility for PU_k when matched with SU_ℓ will still be calculated as in (5). We modify SU_ℓ's utility so that,

$$v_{k\ell}(P_{k\ell}, \tau_{k\ell}) = \tau_{k\ell}\log_2\left(1+\frac{\hat{P}_{k\ell}|h_\ell|^2}{\sigma^2}\right), \tag{10}$$

[17]The authors in [17] do not motivate how such an incomplete information scenario can be realized.

[18]The realization suggested in this section naturally include the more stringent case of quasi-convex utilities.

[19]We will thereafter continue to assume $\tau_k = 1$ w.l.o.g.

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where $\hat{P}_{k\ell}$ is the power remaining for SU$_\ell$'s own communication after relaying for PU$_k$. We now include a total energy constraint so that,

$$\frac{1-\tau_{k\ell}}{2}\frac{P_{k\ell}}{|h_{\ell k}|^2}+\tau_{k\ell}\hat{P}_{k\ell}=P_\ell^{\mathrm{T}}, \tag{11}$$

where P_ℓ^{T} is SU$_\ell$'s total power. We have now accounted for SU$_\ell$'s total power budget directly into its achievable rate in (10). We note that the secondary utility form in (10) is no longer quasi-concave. However $u_{k\ell}(\tau_{k\ell}, P_{k\ell})$ is still decreasing in $\tau_{k\ell}$ and increasing in $P_{k\ell}$, while the opposite is true for $v_{k\ell}(\tau_{k\ell}, P_{k\ell})$[20]. Let us consider another negotiation and aspiration update process in this context.

A. BLMA2

Similar to the previous section, we can show that the problem of assigning PUs to SUs with the utility (5) and the modified utility (10) can be formulated as a matching problem. We now illustrate the negotiation process and aspiration update details. We will collectively refer to these two procedures as BLMA2. Previously, due to quasi-convexity, by connecting the line between the two agents' offers, we were assured the randomly chosen matching point provides an improvement for both agents. This is no longer true. However, we will bypass this difficulty by focusing on one dimension at any given instant while only requiring the less stringent assumption that the interests of the agents on the two sides of the market are opposed.

1) *BLMA2: The PU-SU negotiation process*
   - **Initialize** (τ_{kl}, P_{kl})
   - **If** $\textsc{rand}[0,1]>\frac{1}{2}$,
     a) Flag$=1$.
     b) Calculate $P_{k\ell}^{\mathrm{P}}$ such that $a_k+\epsilon=u_{kl}(P_{k\ell}^{\mathrm{P}}, \tau_{kl})$,
     c) Calculate $P_{k\ell}^{\mathrm{S}}$ such that $b_\ell+\epsilon=v_{kl}(P_{k\ell}^{\mathrm{S}}, \tau_{kl})$,

     Else
     a) Flag$=0$.
     b) Calculate $\tau_{k\ell}^{\mathrm{P}}$ such that $a_k+\epsilon=u_{kl}(P_{k\ell}, \tau_{kl}^{\mathrm{P}})$,
     c) Calculate $\tau_{k\ell}^{\mathrm{S}}$ such that $b_\ell+\epsilon=v_{kl}(P_{k\ell}, \tau_{kl}^{\mathrm{S}})$,

     End If
   - **If** $\lfloor u_{k\ell}(\tau_{k\ell}, P_{k\ell}^{\mathrm{S}})\rfloor_\delta \geq a_k+\epsilon$ **and** $\lfloor v_{k\ell}(\tau_{k\ell}, P_{k\ell}^{\mathrm{P}})\rfloor_\delta \geq b_\ell+\epsilon$
     OR
     $\lfloor u_{k\ell}(\tau_{k\ell}^{\mathrm{S}}, P_{k\ell})\rfloor_\delta \geq a_k+\epsilon$ **and** $\lfloor v_{k\ell}(\tau_{k\ell}^{\mathrm{P}}, P_{k\ell})\rfloor_\delta \geq b_\ell+\epsilon$
     Then $\mathcal{A}_{k\ell}(a_k+\epsilon, b_\ell+\epsilon)=1$.
     End If

2) *BLMA2: Aspiration Update*
   - **If** Flag$=1$,
     Choose P_{kl} uniformly at random in $[P_{k\ell}^{\mathrm{P}}, P_{k\ell}^{\mathrm{S}}]$.
     Else
     Choose τ_{kl} uniformly at random in $[\tau_{k\ell}^{\mathrm{S}}, \tau_{k\ell}^{\mathrm{P}}]$.
     End If
   - Update $a_k \leftarrow \lfloor u_{kl}(P_{k\ell}, \tau_{kl})\rfloor_\delta$,
   - Update $b_\ell \leftarrow \lfloor v_{kl}(P_{k\ell}, \tau_{kl})\rfloor_\delta$,

This time, users initialize with arbitrary time and power offers (τ_{kl}, P_{kl}). Agents then flip a fair coin. If the outcome is heads, they calculate their aspiration based power levels given the existing time. Otherwise they calculate their aspiration based time request/offer given the existing power.

Proposition 3. *Given $u_{k\ell}(\tau_{k\ell}, P_{k\ell})$ and $v_{k\ell}(\tau_{k\ell}, P_{k\ell})$ with opposing trends in $\tau_{k\ell}$ and $P_{k\ell}$, and $\epsilon=q\delta>0$ for some $\{q \mid q>1$ and $q\in\mathbb{Z}\}$, BLMA2 will converge to an ϵ-pairwise stable state with probability one.*

Proof:
The proof proceeds as we did in Proposition 2. We can no longer connect any line segment between agents' time and power offers, and assume that any point in between these two offers will be agreeable. We can, however, make use of the fact that users have opposing interests. Fixing time, if $\lfloor u_{k\ell}(\tau_{k\ell}, P_{k\ell}^{\mathrm{S}})\rfloor_\delta \geq a_k+\epsilon$ and $\lfloor v_{k\ell}(\tau_{k\ell}, P_{k\ell}^{\mathrm{P}})\rfloor_\delta \geq b_\ell+\epsilon$, then any point on the vertical line connecting $P_{k\ell}^{\mathrm{P}}$ and $P_{k\ell}^{\mathrm{S}}$ must be agreeable. A similar statement can be said about choosing any point on the horizontal line connecting the time offers $\tau_{k\ell}^{\mathrm{P}}$ and $\tau_{k\ell}^{\mathrm{S}}$. Figures 6 and 7 illustrate the process of choosing the match point in BLMA2 for the sample case of the SU utility not being quasi-concave. Once agreement is established, the proof follows as in Proposition 2. □

B. Ordinal Utility BLMA Application

In this subsection, we highlight another application of the BLMA. In [19], it is required to assign SUs to orthogonal PU channels[21]. To make access decisions, SUs sense the primary channels to detect primary activities. Sensing is erroneous and characterized by the false alarm and detection probabilities. The SUs and PUs gain utility for this assignment such that the PU

[20]While it is obvious $v_{k\ell}(\tau_{k\ell}, P_{k\ell})$ is decreasing in $P_{k\ell}$, it is not immediately clear whether it is increasing in $\tau_{k\ell}$. However taking the derivative of $v_{k\ell}(\tau_{k\ell}, P_{k\ell})$ with respect to $\tau_{k\ell}$ yields $\frac{\partial v_{k\ell}(\tau_{k\ell},P_{k\ell})}{\partial \tau_{k\ell}}=\frac{P_{k\ell}|h_\ell|^2/\sigma^2}{(\ln 2)(1+x)}+\frac{1}{\ln 2}\left(\ln(1+x)-\frac{x}{1+x}\right)$, where $x=\hat{P}_{k\ell}|h_\ell|^2/\sigma^2$. Now let $f(x)=\ln(1+x)-\frac{x}{1+x}$. Note that $x\geq 0$ in our case as it is a power term. Also note that $f(0)=0$ while $\frac{\partial f(x)}{\partial x}=\frac{x}{(1+x)^2}\geq 0$. Since $f(0)=0$ and $f(x)$ is a monotonically increasing function of x, we conclude that $f(x)\geq 0$, and hence $v_{k\ell}(\tau_{k\ell}, P_{k\ell})$ is an increasing function of $\tau_{k\ell}$.

[21]In the model of [19], a many-to-one matching scenario is considered with the possibility of assigning many PUs to a given SU. We will consider the case of one-to-one matching to compare with our work.

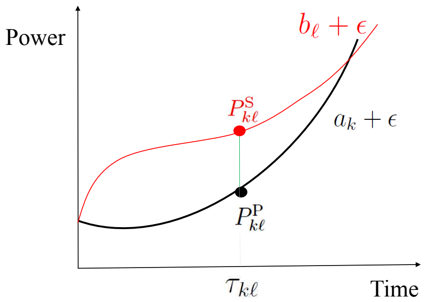

Fig. 6: If $\textsc{rand}[0,1] \geq \frac{1}{2}$, PU_k and SU_ℓ fix the time offer and search for agreements in the power offers.

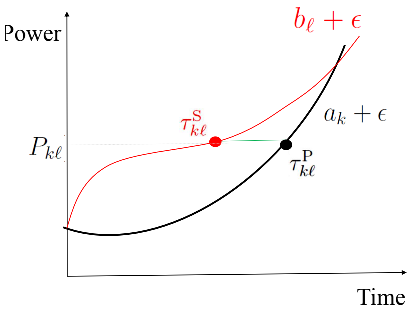

Fig. 7: If $\textsc{rand}[0,1] < \frac{1}{2}$, PU_k and SU_ℓ fix the power offer and search for agreeable matches in the time offers.

utility is:

$$u_{k\ell} = \theta_k d_{k\ell} \log_2\left(1 + \frac{P_k|h_k|^2}{\sigma^2}\right) + \theta_k(1 - d_{k\ell}) \log_2\left(1 + \frac{P_k|h_k|^2}{\sigma^2 + P_\ell|h_{\ell k}|^2}\right), \quad (12)$$

where $\theta_{k\ell}$ is PU_k's transmission probability, $d_{k\ell}$ is the detection probability, and h_{ij} is the interference channel from agent i to agent j. The SU utility on the other hand is written as:

$$v_{k\ell} = (1 - \theta_k)\,(1 - f_{k\ell}) \log_2\left(1 + \frac{P_\ell|h_\ell|^2}{\sigma^2}\right) + \theta_k\,(1 - d_{k\ell}) \log_2\left(1 + \frac{P_\ell|h_\ell|^2}{\sigma^2 + P_k|h_{k\ell}|^2}\right), \quad (13)$$

where $f_{k\ell}$ is the false alarm probability. The desired outcome of the problem is the assignment of SUs to the PUs such that the matching is pairwise stable. All the power, access and sensing parameters are assumed known and constant. We can see that this fits the ordinal utility model of Section III.

The authors in [19] propose an algorithm to produce the stable matching. In an initialization phase, SUs calculate the utilities with all the available PUs. Furthermore, it is assumed that the SUs can calculate the primary utility for all PUs and forward such information to a coordinator which responds on behalf of the PUs. The algorithm then proceeds in a similar fashion to the deferred acceptance algorithm of [1] with the SUs making offers to the coordinator starting with their most preferred PU. If a minimum PU rate requirement is met, the coordinator approves the move and sends the necessary information to dissolve any previous matching and enforce the new one. Furthermore, the coordinator sends messages to disqualify SUs who either do not meet the minimum PU rate requirement or offer the PU a utility less than its current value.

In the numerical results section, we also compare the algorithm in [19] with the BLMA as applied to the above setting. We show comparable convergence performance of the two algorithms despite the significantly less information available at agents about each other and despite the lack of a coordinator to monitor the matching. We also note that the BLMA can be applied to the spectrum allocation problem of [30] which also uses an ordinal utility formulation to arrive at a stable matching.

VI. Numerical Results

In this section, we evaluate the performance of the BLMA in cognitive radio networks. We randomly place the PUs and the SUs in a 1×1 square km area. We consider large scale fading so that the channel coefficients are computed as the inverse of the distances between the transmitting and receiving nodes. The PU transmit power $P_k = 0.01$ for all PUs and $P^{\mathrm{T}}_\ell = 1$ for all SUs and $\sigma^2 = 1$. We also take $\delta = 0.05$ and $\epsilon = 3\delta$.

A. Validating Proof technique

In Fig. 8, we plot the performance of the BLMA1 used to solve the matching problem of Section IV where we assumed quasi-convexity. In this particular figure, we plot the index of the SU matched to a particular PU and the utilities of the secondary users. Since the market is unbalanced, there are 3 PUs and 5 SUs, two SUs (SU_1 and SU_2 in this example) are left unmatched with aspiration levels zero. This run shows a sample realization that is close to our proof technique. First, agents' aspiration levels steadily increase in the so-called pre-stable phase. This stage happens fast, in about 24 steps. Then agents compete over matches, SU_1 and SU_2 compete over PU_3 while SU_4 steadily reduces it aspiration level as it is single. SU_4 eventually "wins" at around 1400 steps. The matching matrix does not change beyond this point. Single agents then steadily decrease their aspirations until ϵ-pairwise stability is reached which happens in this realization at around 1800 steps. It is possible to achieve faster convergence if we use an adaptive δ scheme similar to the one suggested in [31]. We also note in Fig. 8 that convergence occurs in $\mathcal{O}(N^3)$, where $N = 5$ is the maximum number of agents on one side of the market.

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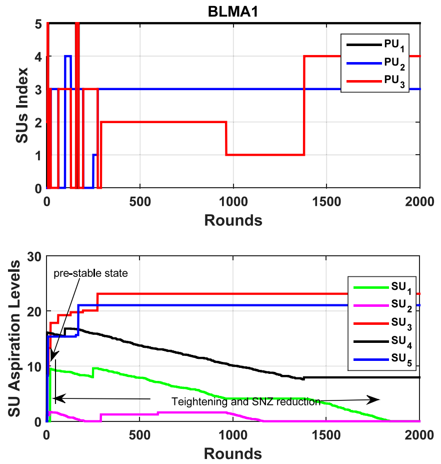

Fig. 8: Number of negotiation rounds till stable matching occurs for BLMA1 in a 3 PUs $\times$ 5 SUs case.

B. A Complete Information Reference Scheme

Next, we contrast our results with the scenario proposed in [17]. As mentioned earlier, the authors consider two scenarios regarding assumptions about the $\theta_{k\ell}$'s. We will contrast with the partially incomplete information scenario where PUs know the type of all SUs connected to them. This is the stronger information assumption in the work of [17]. Once this information is available, a PU can calculate the SU utility for a given time and power offer. It is in that sense that we refer to the scheme of [17] as a complete information benchmark since in our work agents know nothing about the utilities of other users in the network even if the time and power offers are known.

Fig. 9 shows the resulting PU utility using BLMA1 and the algorithm in [17] denoted as Feng et al. The algorithm in that work resembles an ascending auction, PUs start with the time and power offers that give the least utility to SUs, and gradually increase the utility offers to SUs until the first stable allocation of time and power is reached. It is clear that the first stable matching occurring using this algorithm is the best from the PUs perspective. Since there is only one sequence in which events can happen in that algorithm, there is no need to average the algorithm results for a given channels' allocation. In our algorithm, on the other hand, the agents randomly encounter each other and so we average our results over 1000 runs of the algorithm to account for different ways in which convergence may occur. For both algorithms, we plot the resulting PUs' utilities for the cases of a balanced (3 PUs $\times$ 3 SUs) and an unbalanced market (3 PUs $\times$ 5 SUs). In the case of a balanced market, BLMA1 converges in an average of 1464 rounds and converges in 5338 rounds in the unbalanced case. Convergence takes longer in unbalanced markets since all singles' aspiration levels must be zero for stability to occur. The algorithm in

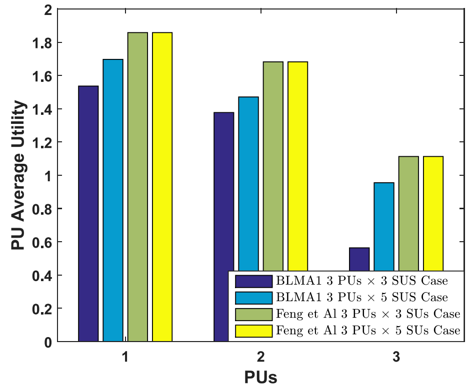

Fig. 9: PUs' utility using BLMA1 and the perfect information approach in [17] for the case of balanced and unbalanced markets.

Algorithm in [19]		BLMA	
PU sum utility	SU sum utility	PU sum utility	SU sum utility
11.1718	19.8026	11.2291	19.7323

TABLE I: Comparison of the performance of BLMA and the algorithm of [19] for the case of 20 PUs and 10 SUs.

[17] converges in 1782 rounds.

For this simulation example, whether the market is balanced is irrelevant to the algorithm in [17] and subsequently there is no difference in the achievable utility in both cases. This is the case since agents know the types of SUs connected to them. Stability occurs before any offers can be made to "weak" agents. Due to lack of information about other agents, this is not the case in our algorithm and we note an appreciable increase in the PUs' utilities from the balanced to the unbalanced market case. The performance of BLMA1 in the unbalanced market case is clearly close to the perfect information scheme of [17]. If we compare with the weaker information assumption of [17], we expect to find instances when our algorithm performs better despite the limited information restriction.

C. Comparison with an ordinal utility market

We implement the same system model as in [19] using the BLMA. Agents randomly encounter each other and if their pairing is agreeable they match, otherwise they lower their aspiration if they were single. Agents have no prior knowledge of preference lists or of utilities they can gain and learn these with each encounter with a new agent. Table I summarizes the performance of the two algorithms.

We note that the two algorithms perform closely despite the more information available to SU agents in the model of [19] and the initialization phase. Furthermore,

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the algorithm in [19] provides the best possible match for SUs. The random encounters of our model allows agents to visit the possibly multiple stable matchings that exist which are known to exhibit a lattice structure [2]. This means that our algorithm can visit the PU-optimal match and the SU-optimal match and any other matches in between. This explains the discrepancy between the values attained by our algorithm and the algorithm in [19] which are averaged over 10^4 runs of both algorithms.

Finally, in [19], the authors bound the maximum number of bits between any SU and the coordinator by $|\mathcal{K}|^2 + |\mathcal{K}| + \sum_{k=1}^{\mathcal{K}} \log_2(k)$. So, for the case of 10 SUs, the total number of bits exchanged are bounded by roughly 5k bits. Our algorithm converged, on average, in 1000 rounds. After the initial encounter, where agents learn the utility they can get with each other, agents only need to identify themselves to each other to see if they are currently agreeable. For the case of 20 PUs and 10 SUs, this takes on average 5.2 bits to accomplish. Hence, our algorithm took an average of 5.2k bits to converge which is comparable with the algorithm in [19].

VII. Conclusion

We considered a context-free matching problem defined by agents' agreement functions. We proposed an abstract algorithm, BLMA, to pair agents in this generic two-sided market without specifying detailed negotiation mechanisms or the actual allocation of utilities among users. The BLMA was shown to converge to ϵ-pairwise stable solutions with probability one.

Next, we applied the BLMA to cognitive radio networks. We showed three examples. For the quasi-linear utility case, we showed a procedure for the BLMA, BLMA1, that exploits users' convex level sets to obtain stable solutions. For the case of nonlinear utilities that still retained users' opposed interests in the optimization variables, we provided another procedure, BLMA2, to specify a negotiation mechanism and a way to update users' aspirations without the need for quasi-convexity. In a third application, we compared with an ordinal utility market. In all applications, we stipulated a minimum information exchange so that users could only calculate their utilities, but no information is available about the utilities of other users in the network.

Our approach does not preclude the possibility of adding some *structure* to the algorithm realizations, for practical implementation and for improved convergence. For example, PUs maybe activated in a round-robin fashion or the δ step reduction in aspirations can be made adaptive as the algorithm progresses. We may also make it possible to activate multiple players at once, as long as there remains a positive probability that a single player is activated. In all such cases, our results still follow as long as we can guarantee that agents have a positive probability of making a match if it exists. In the end, the BLMA process is simple. Agents with agreeable functions, match with positive probability, otherwise, they lower their aspirations by a small step and wait for their next random activation round.

While we considered a general matching market with possibly ordinal or cardinal utilities and proposed an algorithm to find equilibrium despite market and information decentralization, some further aspects of the matching market remain to be considered. Further extensions include generalizing our result to the case of many-to-one and many-to-many matching markets. Such extensions are not trivial since, for example, many-to-many matching markets may have an empty core or an empty set of pairwise stable solutions, or the pairwise stable solutions and core solutions may not coincide [32]. Hence, a new or modified solution concept is needed.

Another possible extension is considering matching markets with externalities. In markets with externalities, the utility of a user depends not only on the shared resources with its matched user but also generally depends on the existing matching [11]. The main difficulty in dealing with externalities lies in the fact that the core of a cooperative game might be empty in the presence of externalities [33]. Furthermore, the very concept of stability and of blocking pairs is not well defined. Several works in the literature, e.g. [16], [34], [35], propose solutions to circumvent these difficulties. However, it is interesting to see if it is possible to find a matching formulation with the same spirit of BLMA: general enough to accommodate different utility forms, lacking any central authority, and with minimum information exchange.

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