Supplementary Material

Enhanced spin-orbit torque via modulation of spin current absorption

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S1. Sample properties: sheet conductance, perpendicular magnetic anisotropy, pinning field and saturation magnetization

Sheet conductance

The sheet resistance, $R_{\text{tot}}$, of the Ru capped samples are measured by the standard four probe method. As can be seen from Fig. S1, the sheet conductance ($1/R_{\text{tot}}$) shows an overall linear correlation with Ru thickness, which suggests that the Ru layer is continuous in our films.

![Fig. S1. The sheet conductance (1/sheet resistance) vs. $t_{\text{Ru}}$ for the first series of devices with the film structure of substrate/2 MgO/4 Pt/0.4 Co/0.3 Ni/0.1 Co/$t_{\text{Ru}}$ Ru/1.2 MgO/3 SiO$_2$ (thicknesses are in nm).](image)

Perpendicular anisotropy

With an external magnetic field, the equilibrium direction of the magnetization can be described as

$$H \sin(\theta - \theta_H) - H_{\text{an}} \sin\theta \cos\theta = 0$$

where $H$, $\theta$, $\theta_H$, and $H_{\text{an}}$ are the external magnetic field, magnetization direction, external magnetic field direction, and perpendicular magnetic anisotropy field, respectively. As the anomalous Hall resistance ($R_H$) represents the $z$ component of the magnetization and $R_H = R_{H0} \sin\theta$, where $R_{H0}$ is the anomalous Hall resistance with the magnetization aligned along the normal direction of the film plane, the perpendicular magnetic anisotropy field ($H_{\text{an}}$) can be obtained by fitting the anomalous Hall loop with Eq. (1). The representative fittings for the 4 Pt/0.4 Co/0.3 Ni/0.1 Co/$t_{\text{Ru}}$ Ru/1.2 MgO/3 SiO$_2$ (all thicknesses are in nm) devices
with \( t_{Ru} = 0 \) and 0.6 nm are shown in Fig. S2b,c. \( H_{an} = 5.8 \) and 3.8 kOe are obtained for the device with \( t_{Ru} = 0 \) and 0.6 nm, respectively. Figure S1d shows \( H_{an} \) for the devices with different \( t_{Ru} \). \( H_{an} \) of the devices with different ferromagnet thicknesses are also shown in Fig. S3.

**Fig. S2.** a, Schematic of the magnetization and external magnetic field. Anomalous Hall loop of the device with \( t_{Ru} = 0 \) (b), and 0.6 nm (c). The external magnetic field is applied at \( \theta_H = 4^\circ \). Red solid lines show the fits. d, \( H_{an} \) vs. \( t_{Ru} \) for the first series samples (Fig. 1).

**Fig. S3.** \( H_{an} \) of the samples with different ferromagnet thicknesses (2\(^{nd}\) and 3\(^{rd}\) series samples in Fig. 2).
Pinning field

In perpendicular magnetized multilayers, the magnetic reversal, either by external magnetic field or SOT, is generally via domain nucleation and expansion processes [25,40,41]. Thus the pinning field is of essential importance in SOT driven magnetic reversal.

To study the pinning field and magnetic reversal behavior in our samples, the anomalous Hall loops are measured at various directions. Fig. S4a shows the out-of-plane anomalous Hall loops. Fig. S4b shows the angular dependence of coercivity $H_C(\theta)$. All the samples exhibit that $H_C(\theta)$ increases when $\theta$ approaches 90°, which is as predicted by the domain wall depinning model [49]. Furthermore, the angular dependence of coercivity is fitted by $H_C(\theta) = H_{P0}/\cos\theta$ to obtain the average pinning field $H_{P0}$. Fig. S4c summarizes $H_{C0}$ and $H_{P0}$ vs. $t_{Ru}$. For 0.2 nm < $t_{Ru}$ < 1 nm, $H_{C0}$ and $H_{P0}$ reduce to minimal values and are almost invariant to $t_{Ru}$. By comparing Fig. S4c with Fig. 1(c-d), one can conclude that the enhancement of SOT by Ru capping is not caused by the change of pinning field of the samples.

Saturation magnetization

The saturation magnetization ($M_S$) of the samples is measured by the vibrating sample magnetometer (VSM). As shown in Fig. S5a, $M_S$ shows a non-monotonic change with $t_{Ru}$. The result can be related to the Ru induced interfacial spin polarization. It is reported that Ru strongly modifies the density of states at the Ru/FM interface and forms quantum well states in the bulk, which consequently results in an oscillated spin polarization [42,43]. The negative spin polarization reduces the total $M_S$ and a minimal $M_S$ is observed with $t_{Ru} = 0.4$–0.6 nm.

The difference in $M_S$ for the samples capped with 0.6 nm Ru and 1.2 nm MgO decreases with increasing the FM thickness (Fig. S5b). At $t_{FM} = 1.4$ nm, $M_S$ of the Ru capped sample is still noticeably smaller than that of the MgO capping one.
Fig. S4. a, Out-of-plane anomalous Hall loops for the samples with different Ru capping thicknesses (1st series samples in Fig. 1). b, The angular dependence of coercive field $H_c(\theta)$. The solid lines are the fits of $H_p/cos\theta$. $\theta$ is the angle between the applied magnetic field and film normal directions. c, $H_{C0}$ and $H_{P0}$ vs. $t_{Ru}$. 
Fig. S5. $M_S$ of the samples with different Ru capping thicknesses (1st series samples in Fig. 1) (a), and ferromagnet thicknesses (2nd and 3rd series samples in Fig. 2) (b).

S2. Harmonic analysis of current induced spin-orbit effective magnetic fields

Current induced spin-orbit effective fields are evaluated by the harmonic measurements [17,37,38]. A sinusoidal ac current $I_{ac}$ with an amplitude of 3.5 mA at a frequency of 13.7 Hz is applied to the devices, the first and second harmonic Hall voltages ($V_{2f,\parallel}$ or $V_{2f,\perp}$, where the $\parallel$ or $\perp$ denotes the signal measured in the longitudinal or transverse geometry, respectively) are measured by two lock-in amplifiers [38]. Changes in harmonic Hall signals are observed for the samples with different Ru thicknesses, as can be seen from Fig. S6a,b. From the harmonic measurement data, the current induced spin-orbit effective field components, $H_L$ and $H_T$, are extracted from the fitting considering the planar Hall effect (Fig. S6a,b) [38]. The deviation between the data and fit in the magnetic field region away from the peak/dip is due to the angular dependence of spin-orbit effective fields [37,38,50], however our method is still capable of extracting the strength of spin-orbit effective fields. The extracted values are consistent with the spin-orbit switching measurements, where the device with larger spin-orbit effective fields shows a larger SOT switching efficiency ($\eta$).
Fig. S6. The second harmonic voltages measured in longitudinal (||) and transverse (⊥) geometries and the corresponding fittings for the devices with $t_{Ru} = 0$ (a), and 0.6 nm (b). The obtained $H_L$ and $H_T$ values are indicated in the graphs.

**S3. Rh capping samples**

Rh has one more additional 4$d$ electron compared to Ru. Thus, the Rh capping samples are also fabricated to study the influence of Rh on spin-orbit torques. The samples are in the structure of Si substrate with native oxide/2 MgO/4 Pt/0.4 Co/0.3 Ni/0.1 Co/$t_{Rh}$ Rh/1.2 MgO/3 SiO$_2$ (all thicknesses are in nm), where $t_{Rh}$ is varied from 0 to 3 nm. The SOT induced magnetization switching, perpendicular anisotropy field, and current induced spin-orbit effective fields are measured for the Rh capping samples. As can be seen from Fig. S7, $J_S$, $H_{an}$, and $M_S$ show minimal values at $t_{Rh} = 0.6$ nm, but both the SOT switching efficiency and effective fields are maximized at $t_{Rh} \sim 0.2$ nm, which is different from the Ru case.
Fig. S7. Rh capping samples. **a**, The SOT switching current density $J_S$. The inset of (a) shows perpendicular anisotropy field $H_{an}$. **b**, SOT switching efficiency $\eta$. **c**, Saturation magnetization $M_S$. **d**, Current induced spin-orbit effective fields for the Rh capping samples.

S4. Cu capping samples

Cu is known to possess weak spin orbit coupling and small interfacial spin scattering. To elucidate the role of interfacial spin scattering on the enhancement of SOT, the Cu capping samples are investigated. The samples are in the structure of Si substrate with native oxide/2 MgO/4 Pt/0.4 Co/0.3 Ni/0.1 Co/t$_{cu}$/1.2 MgO/3 SiO$_2$ (all thicknesses are in nm), where $t_{cu}$ is varied from 0 to 1.2 nm. The samples show perpendicular anisotropy. The SOT induced magnetization switching and perpendicular anisotropy field are measured. Figure S8a shows the SOT induced magnetization switching curves with different Cu capping thicknesses, and no significant change is observed compared to Ru and Rh capping cases. The switching current density, perpendicular anisotropy field, SOT switching efficiency, and the saturation magnetization are summarized in Fig. S8b-d. Overall the SOT decreases and the $M_S$ increases with increasing Cu thickness, which show Cu capping does not enhance SOT and the interfacial negative polarization is negligible.
Fig. S8. Cu capping samples. a, $R_H$ (offset for clarity) as a function of in-plane pulsed currents for the devices with different Cu thicknesses ($t_{Cu}$). A 1000 Oe external magnetic field is applied along the current direction. $t_{Cu}$ is indicated for each curve. b, The SOT switching current density $J_S$, and perpendicular anisotropy field $H_{an}$ (inset of b). c, SOT switching efficiency $\eta$ vs. $t_{Cu}$. d, Saturation magnetization $M_S$ vs. $t_{Cu}$. e, Illustration of Cu capping effect.

S5. Discussion of $M_S$ versus spin-orbit torques

With a charge current flowing in a heavy metal (HM) layer, HM exerts an effective magnetic field onto the magnetization of the adjacent magnetic layer. Based on the simple spin Hall effect model, the current induced effective field $H_L$ is [26,38,51]:

$$H_L = \hbar \theta_{SH} |I_e|/(2|e|M_{S|FM})$$  \hspace{1cm} (2)
where $\theta_{SH}$, $M_s$, and $t_{FM}$ are the spin Hall angle, magnetization, and ferromagnet thickness, respectively. Equation (2) shows that $H_L$ is inversely proportional to $M_s$. Figure S3a shows that the $M_s$ of the Ru capped samples changes noticeably. Moreover, $M_s$ becomes a minimum value at $t_{Ru} = 0.6$ nm where SOT displays a maximum value, which suggests that a change in $M_s$ could be a plausible reason for the modulation of SOT.

However, a change in $M_s$ is not likely to be the underlying origin of the observed SOT modulation. First, the modulation of SOT by Ru vanishes at $t_{FM} > 1.2$ nm (Fig. 2c-e), while $M_s$ still shows a difference (Fig. S5b). Second, the SOT modulation is not correlated with the $M_s$ change in the Rh series samples (Fig. S7).

**S6. Field modulated ferromagnetic resonance measurements**

Field-modulated ferromagnetic resonance (FM-FMR) measurements are performed in order to probe the modulation effect on the transverse spin current. A microwave power of 9 dBm is supplied from an Agilent E8257D microwave generator and sent through a coplanar waveguide between the poles of an electromagnet capable of generating a perpendicular magnetic field up to 1.8 T. The sample is placed faced down on top of the coplanar waveguide, absorbing the microwave power at resonance conditions. To obtain the FMR signal, the microwave signal is measured by a 75VA50 Anritsu power detector.

The deposited stacks are Si substrate with native oxide/3 Ta/2 Cu/[0.3 Co/0.6 Ni]$_3$/0.1 Co/2.4 Cu/0.4 Co/0.3 Ni/0.1 Co/$t_{Ru}$ Ru/1.2 MgO/3 SiO$_2$ (all thicknesses are in nm). The Cu spacer allows decoupling of the bottom thick FM and the top thin FM. The Ru thickness ($t_{Ru}$) is varied from 0 to 1.4 nm. A microwave signal of frequency ranging 10-30 GHz is applied to the thick FM through the coplanar waveguide. The derivative of the absorbed power versus external field is obtained by lock-in detection of the output power while sweeping the external perpendicular field with a modulation of this field of near 30 Oe, applied by additional coils at a frequency of 103 Hz. The background obtained from an empty waveguide without any film is subtracted. The full width at half-maximum (FWHM) of the resonance peaks is obtained from a fit as the sum of one symmetric and one antisymmetric Lorentzian functions with three parameters such as the peak amplitude, the resonance field $H_{res}$, and the FWHM.
The FWHM dependence on frequency is given by the contributions of the inhomogeneous broadening \( \Delta H_0 \) and damping \( \alpha \) [52], \( \text{FWHM} = \Delta H_0 + 4\pi\alpha f/(\sqrt{3}\gamma) \). The damping \( \alpha \) is obtained from a linear fit of the FWHM data, after obtaining the gyromagnetic ratio \( \gamma \) from fitting \( H_{\text{res}} \) to \( H_{\text{res}} = 4\pi M_{\text{eff}} + 2\pi f/\gamma \), where \( 4\pi M_{\text{eff}} \) is the effective perpendicular magnetic anisotropy. The same procedure is performed for a reference sample without thin FM, Si substrate/3 Ta/2 Cu/[0.3 Co/0.6 Ni]3/0.1 Co/2.4 Cu/1.2 MgO/3 SiO\(_2\) (all thicknesses are in nm) in order to obtain \( \alpha_{\text{ref}} \).

**S7. Spin-torque ferromagnetic resonance measurements on Ru and Pt**

Spin torque ferromagnetic resonance (ST-FMR) measurements are performed to evaluate the spin Hall angle of Ru and Pt. The ST-FMR device is in the stack structure of Si substrate with native oxide/4 Co/4 Ru or Pt/1.2 MgO/3 SiO\(_2\) (all thicknesses are in nm). The device fabrication and measurement are similar to our previous reports [53]. An oscillating radio frequency current \( I_{RF} \) (8 GHz and a nominal power of 16 dBm) is applied to the device, with an external magnetic field in the film plane at a 35° angle with respect to the current direction. The oscillating current in the normal metal layer (Ru or Pt) generates an oscillating spin current that flows perpendicular to the sample plane and exerts an oscillating spin torque on the magnetic moment of the Co layer. When the frequency of the current and the magnitude of the bias magnetic field meet the condition for ferromagnetic resonance, the amplitude of magnetization precession is maximized. Mixing of \( I_{RF} \) and the oscillating anisotropic magnetoresistance of the Co results in a dc voltage \( V_{\text{mix}} \). The symmetric component of the \( V_{\text{mix}} \) arises from the spin torque, whereas the antisymmetric peak is a consequence of the torque generated by the Oersted field. The measurement results are shown in Fig. S9. By extracting the symmetric and antisymmetric components (\( V_s \) and \( V_a \), respectively) of \( V_{\text{mix}} \), the spin Hall angle is obtained by [39,53] \( \theta_{\text{SH}} = (V_s/V_a)(e\mu_0 M_t d/h)[1+(4\pi M_{\text{eff}}/H_{\text{ext}})]^{1/2} \). The spin Hall angle of Ru and Pt are determined to be 0.033 and 0.151, respectively. Although the SOT by Ru compensates the Pt SOT due to the same sign of spin Hall angle, the SOT generated by bulk Ru can be ruled out as the origin of observed SOT enhancement, because Ru has a much smaller spin Hall angle compared to that of Pt.
**S8. Theoretical modelling**

**System definition**

Let us consider a trilayer, composed of a ferromagnet FM, embedded between two normal metals, referred to as T (top) and B (bottom), as shown in Fig. S10. Only the bottom normal metal possesses the spin Hall effect. The current spinors at the top and bottom interfaces can be expressed through the transmission matrix across the ferromagnetic layer

\[
\hat{I}_T = \sum_{nm} t^{BT}_{nm} \hat{f}_B (t^{BT}_{nm})^+ - \hat{f}_T \delta_{nm} + r^T_{nm} \hat{f}_T (r^T_{nm})^+ \tag{1}
\]

\[
\hat{I}_B = -\sum_{nm} t^{TB}_{nm} \hat{f}_T (t^{TB}_{nm})^+ - \hat{f}_B \delta_{nm} + r^B_{nm} \hat{f}_B (r^B_{nm})^+ \tag{2}
\]

where \( \hat{f}_{B,T} = n_{B,T} + \hat{\sigma} \cdot \hat{s}_{B,T} \) is the particle density in layer B,T in the spinor form, \( r_{nm}^i \) is the reflection matrix element from mode \( n \) to mode \( m \) in layer \( i \), and \( t^{ij}_{nm} \) is the transmission matrix element from mode \( n \) in layer \( i \) to mode \( m \) in layer \( j \). Both the reflection and transmission coefficients are \( 2 \times 2 \) matrices in spin space,

\[
\hat{r} = t + (t^t - t^i) \hat{\sigma} \cdot \hat{m} \tag{3}
\]

\[
\hat{r} = r + (r^t - r^i) \hat{\sigma} \cdot \hat{m} \tag{4}
\]
Then, the spinor current at the top and bottom interfaces reads explicitly

\[ \mathbf{J}^T |_B = \left( (G^{1,7B} + G^{1,7T}) (n_s \hat{\mathbf{\sigma}} \cdot \hat{\mathbf{m}} (\hat{\mathbf{s}}_B \cdot \hat{\mathbf{m}}) - n_s \hat{\mathbf{\sigma}} \cdot \hat{\mathbf{m}} (\hat{\mathbf{s}}_T \cdot \hat{\mathbf{m}})) + (G^{1,7B} - G^{1,7T}) \right) \left( \hat{\mathbf{m}} \cdot \hat{\mathbf{s}}_B + n_s \hat{\mathbf{\sigma}} \cdot \hat{\mathbf{m}} - \hat{\mathbf{m}} \cdot \hat{\mathbf{s}}_T - n_s \hat{\mathbf{\sigma}} \cdot \hat{\mathbf{m}} \right) \\
+ 2 \text{Im} G^{1,7B} \hat{\mathbf{\sigma}} \cdot \hat{\mathbf{m}} \times \hat{\mathbf{s}}_T - 2 \text{Re} G^{1,7B} \hat{\mathbf{\sigma}} \cdot \hat{\mathbf{m}} \times \hat{\mathbf{s}}_T - 2 \text{Im} G^{1,7T} \hat{\mathbf{\sigma}} \cdot \hat{\mathbf{m}} \times \hat{\mathbf{s}}_B + 2 \text{Re} G^{1,7T} \hat{\mathbf{\sigma}} \cdot \hat{\mathbf{m}} \times \hat{\mathbf{s}}_B \right) \]

\[ \mathbf{J}^T |_T = \left( (G^{1,7T} + G^{1,7B}) (n_s \hat{\mathbf{\sigma}} \cdot \hat{\mathbf{m}} (\hat{\mathbf{s}}_B \cdot \hat{\mathbf{m}}) - n_s \hat{\mathbf{\sigma}} \cdot \hat{\mathbf{m}} (\hat{\mathbf{s}}_T \cdot \hat{\mathbf{m}})) + (G^{1,7T} - G^{1,7B}) \right) \left( \hat{\mathbf{m}} \cdot \hat{\mathbf{s}}_B + n_s \hat{\mathbf{\sigma}} \cdot \hat{\mathbf{m}} - \hat{\mathbf{m}} \cdot \hat{\mathbf{s}}_T - n_s \hat{\mathbf{\sigma}} \cdot \hat{\mathbf{m}} \right) \\
- 2 \text{Im} G^{1,7T} \hat{\mathbf{\sigma}} \cdot \hat{\mathbf{m}} \times \hat{\mathbf{s}}_T + 2 \text{Re} G^{1,7T} \hat{\mathbf{\sigma}} \cdot \hat{\mathbf{m}} \times \hat{\mathbf{s}}_T + 2 \text{Im} G^{1,7T} \hat{\mathbf{\sigma}} \cdot \hat{\mathbf{m}} \times \hat{\mathbf{s}}_B - 2 \text{Re} G^{1,7T} \hat{\mathbf{\sigma}} \cdot \hat{\mathbf{m}} \times \hat{\mathbf{s}}_B \right) \]

where \( \mathbf{J}^T |_B = \mathbf{\hat{J}}(z = 0) \) and \( \mathbf{J}^T |_T = \mathbf{\hat{J}}(z = d_F) \) and

\[
G^\sigma = \left( e^2 / h \right) \sum_{nm} \delta_{nm} \left( \tau^\sigma \right)_{nm}^\sigma = \left( e^2 / h \right) \sum_{nm} \delta_{nm} - \tau^\sigma_{nm} \frac{r^\sigma_{nm}}{r^\sigma_{nm} + r^\sigma_{nm}} \\
G^{\uparrow\downarrow} = \left( e^2 / h \right) \sum_{nm} \delta_{nm} - \tau_{nm}^{\uparrow\downarrow} \frac{r_{nm}^{\uparrow\downarrow}}{r_{nm}^{\uparrow\downarrow} + r_{nm}^{\uparrow\downarrow}} \\
G_{\uparrow \downarrow} = \left( e^2 / h \right) \sum_{nm} \delta_{nm} - \tau_{nm}^{\uparrow \downarrow} \frac{r_{nm}^{\uparrow \downarrow}}{r_{nm}^{\uparrow \downarrow} + r_{nm}^{\uparrow \downarrow}}
\]

\( G^{\uparrow\downarrow} \) is called the reflected mixing conductance and \( G_{\uparrow \downarrow} \) is the transmitted mixing conductance.

**Drift-diffusion equation**

The spin and charge currents in the normal metal \( N (=T,B) \) fulfill the following drift-diffusion equations

\[
e\mathbf{\hat{J}}_c = -\sigma_N \nabla \mathbf{\mu}_c + \sigma_N \frac{\theta_N}{2} \nabla \times \hat{\mathbf{\mu}} \\
\mathbf{\hat{J}}_s = -\sigma_N \nabla \otimes \hat{\mathbf{\mu}} + \sigma_N \theta_N \hat{\mathbf{\sigma}} \times \nabla \mathbf{\mu}_c \\
\nabla^2 \hat{\mathbf{\mu}} = \hat{\mathbf{\mu}} / \lambda_N^2, \nabla^2 \mathbf{\mu}_c = 0
\]

where \( \sigma_N \), \( \theta_N \), and \( \lambda_N \) are the conductivity, Hall angle, and spin diffusion length of the normal metal, respectively. \( \mathbf{\hat{J}}_c (\mathbf{\hat{J}}_s) \) and \( \mathbf{\mu}_c (\hat{\mathbf{\mu}}) \) are the charge (spin) current and chemical potential, such that \( \mathbf{\hat{J}} = \mathbf{\hat{J}}_c + \hat{\mathbf{\sigma}} \cdot \mathbf{\hat{J}}_s \) and \( \hat{f}/eN = \mathbf{\mu}_c + \hat{\mathbf{\sigma}} \cdot \hat{\mathbf{\mu}} / 2 \).
Analytical derivation

We now solve the drift-diffusion equations given above taking $\theta_T = 0$ (no spin Hall effect in the top layer) and assuming the boundary conditions Eqs. (5-6) and $\hat{J}(-d_B) = \hat{J}(d_f + d_f) = 0$. In this way, one can obtain the interfacial spin currents $\hat{J}_{s,T}, \hat{J}_{s,B}$ from which one extracts the spin Hall torque. The derivation is summarized below.

To simplify the derivation, the spin current and chemical potential are written in the form

$$\tilde{\mu} = \mu T \tilde{m} + \mu_{op} \hat{\tilde{m}} \times (\tilde{y} \times \tilde{m}) + \mu_{op} \hat{\tilde{m}} \times \tilde{y}$$

$$\hat{\tilde{J}} = \hat{J}_{s,T} \otimes \hat{\tilde{m}} + \hat{J}_{ip} \otimes \hat{\tilde{m}} \times (\tilde{y} \times \hat{\tilde{m}}) + \hat{J}_{op} \otimes \hat{\tilde{m}} \times \tilde{y}.$$  

In the top and bottom layers, the spin current and chemical potential read

$$\mu_{ip,op}^N = A_{ip,op}^N \cosh \frac{z}{\lambda_N} + B_{ip,op}^N \sinh \frac{z}{\lambda_N}$$

$$J_{s,z}^N = -\frac{\sigma_N}{2} \nabla_z \mu_{ip}^N - \sigma_N \theta_N m_y \nabla_x \mu_c$$

$$J_{ip}^N = -\frac{\sigma_N}{2} \nabla_z \mu_{ip}^N - \sigma_N \theta_N m_y \nabla_x \mu_c$$

$$J_{op}^N = -\frac{\sigma_N}{2} \nabla_z \mu_{op}^N$$

Here, $A_{ip,op}^{T,B}$ are the constants to be determined. Applying the boundary conditions $\hat{J}(-d_B) = \hat{J}(d_f + d_f) = 0$, we get the spin-dependent chemical potential and spin current in the top layer

$$\mu_{ip,op}^T = A_{ip,op}^T \left( \cosh \frac{z}{\lambda_{T,B}} - \tanh \frac{d_f + d_f}{\lambda_T} \sinh \frac{z}{\lambda_{T,B}} \right) J_{s,z}^T = -\frac{\sigma_T}{2} \left( \sinh \frac{z}{\lambda_T} - \tanh \frac{d_f + d_f}{\lambda_T} \cosh \frac{z}{\lambda_{T,B}} \right) A_{ip,op}^T.$$  

In the bottom layer, we get
Now, we need to solve Eqs. (5-6). Since we are interested in the spin transfer torque, we only aim at calculating the interfacial spin current transverse to the magnetization. Hence, the only useful relations are

\[ J_{ip} = -\frac{\sigma_B}{2\lambda_B} A^B \left( \sinh \frac{z}{\lambda_B} + \frac{d_B}{\lambda_B} \cosh \frac{z}{\lambda_B} \right) - \sigma_B \theta_B \left[ \cosh \frac{z}{\lambda_B} - \frac{d_B}{\lambda_B} \right] m_x \nabla_x \mu_c \]

To allow for an analytically tractable result, we neglect the imaginary part of the mixing conductance. In this case, we obtain
\[
A_{op}^T = A_{op}^B = 0
\]
\[
A_{ip}^T = \frac{g_r^{\uparrow, BT} g_r^{\downarrow, BT}}{g_r^{\uparrow, BT} g_r^{\downarrow, BT} - (1 + g_r^{\uparrow, BT}) (1 + g_r^{\downarrow, BT})} \frac{2 \lambda_B \theta_B}{\tanh \frac{d_B}{\lambda_B}} \left[ 1 - \cosh^{-1} \frac{d_B}{\lambda_B} \right] \nabla \mu_c
\]
\[
A_{ip}^B = \frac{g_r^{\uparrow, LT} g_r^{\downarrow, LT}}{g_r^{\uparrow, LT} g_r^{\downarrow, LT} - (1 + g_r^{\uparrow, LT}) (1 + g_r^{\downarrow, LT})} \frac{2 \lambda_B \theta_B}{\tanh \frac{d_B}{\lambda_B}} \left[ 1 - \cosh^{-1} \frac{d_B}{\lambda_B} \right] \nabla \mu_c
\]

where we define the unitless mixing conductances

\[
g_{i,LT}^{\uparrow,LT} = \frac{4 \hat{\lambda}_B \text{Re} G_i^{\uparrow,LT} \cosh \frac{d_T}{\lambda_T}}{\sigma_T \sinh \frac{d_T}{\lambda_T}} g_{i,LT}^{\downarrow,LT} = \frac{4 \hat{\lambda}_T \text{Re} G_i^{\downarrow,LT} \cosh \frac{d_T}{\lambda_T}}{\sigma_T \sinh \frac{d_T}{\lambda_T}}, \quad g_{i,LT}^{\downarrow,LT} = \frac{4 \hat{\lambda}_N \text{Re} G_i^{\downarrow,LT}}{\sigma_N \sinh \frac{d_N}{\lambda_N}}, \quad \tilde{\lambda}_N = \lambda_N / \tanh \frac{d_N}{\lambda_N}.
\]

As a result, the interfacial spin currents read

\[
J_{\nu}^{\uparrow} = \frac{g_{i,LT}^{\uparrow,LT} (1 + g_{i,LT}^{\uparrow,LT})}{g_{i,LT}^{\uparrow,LT} (1 + g_{i,LT}^{\uparrow,LT})} \cosh \frac{d_T}{\lambda_T} \cosh \frac{d_T}{\lambda_T} \left[ 1 - \cosh^{-1} \frac{d_B}{\lambda_B} \right] \nabla \mu_c
\]
\[
J_{\nu}^{\downarrow} = \frac{g_{i,LT}^{\uparrow,LT} (1 + g_{i,LT}^{\uparrow,LT})}{g_{i,LT}^{\uparrow,LT} (1 + g_{i,LT}^{\uparrow,LT})} \cosh \frac{d_T}{\lambda_T} \cosh \frac{d_T}{\lambda_T} \left[ 1 - \cosh^{-1} \frac{d_B}{\lambda_B} \right] \nabla \mu_c
\]

**Spin Hall torque**

The spin Hall torque is defined as the difference between the spin current at the two interfaces \( \tau = -\int \vec{\nabla} \cdot \vec{J} \, dz = J_{\nu}^{\downarrow} - J_{\nu}^{\uparrow} \). Explicitly

\[
\tau = -4 \hat{\lambda}_B \left[ \frac{\text{Re} G_i^{\uparrow} \text{Re} G_i^{\downarrow} \cosh \frac{d_T}{\lambda_T}}{\sigma_T + 4 \hat{\lambda}_T (\text{Re} G_i^{\uparrow} \text{Re} G_i^{\downarrow} - \text{Re} G_i^{\uparrow,LT} \text{Re} G_i^{\downarrow,LT})} \sigma_T \theta_B \left[ 1 - \cosh^{-1} \frac{d_B}{\lambda_B} \right] \nabla \mu_c \right]
\]

Let us consider a realistic system composed of transition metal thin films, such as Pt/Co/Ru or Pt/Co/Cu. The reflected and transmitted spin mixing conductance are in the order of \( \text{Re} G_i^{\uparrow} \approx 0.5 \times 10^{15} \Omega^{-1} \cdot \text{m}^{-2}, \text{Re} G_i^{\downarrow} \approx 0.1 \times 10^{15} \Omega^{-1} \cdot \text{m}^{-2} \) [54], so that terms at the second order in the transmitted mixing conductance can be neglected. Hence, the simplified expression for the torque reads

\[
\tau = -4 \hat{\lambda}_B \left[ \text{Re} G_i^{\uparrow,LT} - \eta_T \text{Re} G_i^{\downarrow} \right] \eta_B \theta_B \left[ 1 - \cosh^{-1} \frac{d_B}{\lambda_B} \right] \nabla \mu_c
\]
where $\eta_N = \frac{\sigma_N}{\sigma_N + 4 \lambda_N \text{Re} G_{r}^{\uparrow \downarrow \uparrow \downarrow}}$ is the spin transparency at the N/FM interface. Note that the definition of the spin transparency given here is different from the spin transparency ($T$) from recent reports by Chen et al. [55] and W. Zhang et al. [28]. Typically, for $\sigma_N \approx 6 \times 10^6 \Omega^{-1} \cdot m^{-1}$, $\text{Re} G_{r}^{\uparrow \downarrow} \approx 0.5 \times 10^{15} \Omega^{-1} \cdot m^{-2}$ and $\tilde{\lambda}_T \approx 2 - 10 nm$, we obtain $\eta_N \approx 0.5 - 0.8$. Finally, defining $\nabla \cdot \mu = j_c / \sigma$, where $j_c$ is the current density, $\sigma$ is the conductivity of the multilayer, we obtain the formula given in the main text,

$$\tau = -\left(\frac{j_c}{\sigma}\right) 4 \lambda_B \eta_B \theta_B \frac{1 - \cosh^{-1} d_B / \lambda_B}{\tanh d_B / \lambda_B} \left[\text{Re} G_{r}^{\uparrow \downarrow \uparrow \downarrow} - \eta_T \text{Re} G_{r}^{\uparrow \downarrow} \right].$$

Notice that the spin diffusion length ($\lambda_N$) is a property of the bulk normal metal, while the interfacial properties are contained in the spin mixing conductance. Therefore, if strong spin flip is present in the system such as Pt/FM/Ru, $\text{Re} G_{r}^{\uparrow \downarrow \uparrow \downarrow}$ vanishes together with the associated interfacial spin torque at the top interface, leading to an enhancement of SOTs.