Supplementary Materials
Spin Swapping Transport and Torques in Ultrathin Magnetic Bilayers

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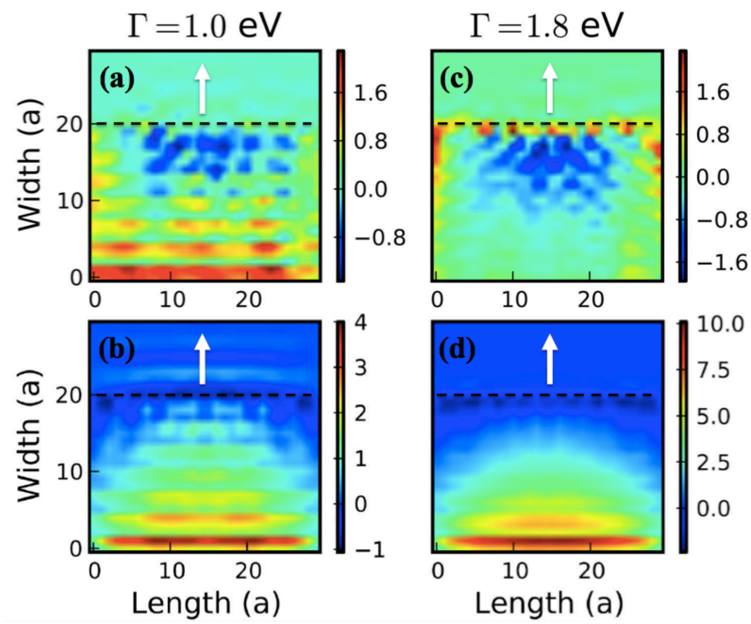

FIG. 1. (Color online) Two-dimensional mapping of the spin density components, (a,c) δS_x and (b,d) δS_y, in (a,b) weak and (c,d) strong disordered regimes. The dashed line represents the interface between the normal metal and the ferromagnet and the white arrow indicates the direction of the magnetization. The parameters are the same as in Fig. 2 in the main text.

TWO DIMENSIONAL MAPPING OF THE SPIN DENSITY

Figure 1 displays the two dimensional mapping of the spin density, (a,c) δS_x and (b,d) δS_y in weak (a,b) and strong disorder regimes (c,d). One can notice that in the weak disorder regime, δS_x, attributed to spin swapping, expands over the thickness of the normal metal, thereby injecting a substantial spin current into the ferromagnet. In contrast, in the strong disorder regime, δS_x remains confined close to the interface, resulting in a small spin current injection.

DERIVATION OF THE DRIFT-DIFFUSION MODEL

We consider a bilayer composed of a normal metal N and a ferromagnet F deposited along the z-direction. The normal metal extends from $z = -d_{\rm N}$ to $z = 0$, and the ferromagnet extends from $z = 0$ to $z = d_{\rm F}$. The spin-orbit coupled spin transport in the normal metal can be modeled using the spin diffusion equation developed in Ref. 1 in the 1^{st} Born approximation

$$e\mathbf{j}_e/\sigma_{\rm N} = -\boldsymbol{\nabla}\mu_c + \frac{\alpha_{\rm sh}}{2}\boldsymbol{\nabla}\times\boldsymbol{\mu}, \tag{1}$$

$$e^2\mathcal{J}^i_{{\rm s},j}/\sigma_{\rm N} = -\nabla_j\frac{\mu_i}{2} + \alpha_{\rm sh}(\mathbf{e}_i\times\boldsymbol{\nabla})\cdot\mathbf{e}_j\mu_c - \frac{\alpha_{\rm sw}}{2}[\mathbf{e}_i\times(\boldsymbol{\nabla}\times\boldsymbol{\mu})]\cdot\mathbf{e}_j, \tag{2}$$

where $\sigma_{\rm N}$ is the bulk conductivity, $\alpha_{\rm sh} = \alpha/\lambda k_{\rm F}$ is the Hall angle from side jump scattering (within 1^{st} Born approximation, skew scattering is absent) and $\alpha_{\rm sw} = 2\alpha/3$ is the spin swapping coefficient. λ and $k_{\rm F}$ are the mean free path and Fermi wavevector, respectively. μ_c and $\boldsymbol{\mu}$ are the spin-independent and spin-dependent electrochemical potentials, related to the charge and spin accumulation by $\mu_c = n/\mathcal{N}$ and $\boldsymbol{\mu} = \delta\mathbf{S}/\mathcal{N}$, and $\mathcal{N}$ is the density of state. Note that $\mathbf{j}_e$ is the current density vector whereas $\mathcal{J}_{\rm s}$ is the spin current density tensor and $\mathcal{J}^i_{{\rm s},j}$ is the i-th spin component of the spin current propagating along the j-th direction. This set of equations is combined with the spin and charge accumulation continuity equations $\boldsymbol{\nabla}\cdot\mathbf{j_c} = 0$ and $\boldsymbol{\nabla}^2\boldsymbol{\mu} = \boldsymbol{\mu}/\lambda^{\rm N}_{\rm sf}$ where $\lambda^{\rm N}_{\rm sf} = \sqrt{\sigma_{\rm N}\tau_{\rm sf}/e^2\mathcal{N}}$ is the spin relaxation length.

In the ferromagnet, the spin density is assumed aligned on the magnetization (see below) and therefore, the drift-diffusion equations read

$$e\mathbf{j}_e/\sigma_{\rm F} = -\boldsymbol{\nabla}\mu_c + \frac{\beta}{2}\boldsymbol{\nabla}\boldsymbol{\mu}\cdot\mathbf{m} \tag{3}$$

$$e^2\mathcal{J}^{\parallel}_{{\rm s},j}/\sigma_{\rm F} = -\nabla_j\frac{\mu_{\parallel}}{2} + \beta\nabla_j\mu_c, \tag{4}$$

where β is the polarization of the ferromagnet and the index $\parallel$ refers to the projection along the magnetization direction $\mathbf{m}$. In the following, the spin-dependent electrochemical potential and spin current are written

$$\boldsymbol{\mu} = \mu_{\parallel}\mathbf{m} + \mu_{\rm op}\mathbf{m}\times\mathbf{y} + \mu_{\rm ip}\mathbf{m}\times(\mathbf{y}\times\mathbf{m}), \tag{5}$$

$$\mathcal{J}_s = \mathcal{J}^{\parallel}_s\mathbf{m} + \mathcal{J}^{\rm op}_s\mathbf{m}\times\mathbf{y} + \mathcal{J}^{\rm ip}_s\mathbf{m}\times(\mathbf{y}\times\mathbf{m}), \tag{6}$$

where the superscripts "ip" and "op" refer to components in and out of the ($\mathbf{m}$,$\mathbf{y}$) plane.

To model the torque exerted on the ferromagnet, we assume that the spin dephasing in the magnetic layer is so short that the incoming spin current is entirely absorbed within a few monolayers from the interface. The boundary conditions along the direction z are then written [2]

$$j_{e,z} = 2g\Delta\mu_c + 2\gamma g\Delta\mu_{\parallel},\ \ \mathcal{J}^{\parallel}_{s,z} = 2\gamma g\Delta\mu_c + 2g\Delta\mu_{\parallel}, \tag{7}$$

$$\mathcal{J}^{\perp}_{s,z} = 2g_r^{\uparrow\downarrow}\mathbf{m}\times(\Delta\boldsymbol{\mu}\times\mathbf{m}) - 2g_i^{\uparrow\downarrow}\mathbf{m}\times\Delta\boldsymbol{\mu}. \tag{8}$$

Here $\mathcal{J}^{\perp}_{s,z} = \mathcal{J}_{s,z} - \mathcal{J}^{\parallel}_{s,z} \cdot \mathbf{m}$ is the spin current transverse to the magnetization $\mathbf{m}$ and $\Delta\mu_c = \mu_c^{\rm N} - \mu_c^{\rm F}$, $\Delta\boldsymbol{\mu} = \boldsymbol{\mu}^{\rm N} - \boldsymbol{\mu}^{\rm F}$. We define $g = (g_\uparrow + g_\downarrow)/2$ and $\gamma = (g_\uparrow - g_\downarrow)/2g$, g_s being the interfacial conductance for spin s and $g^{\uparrow\downarrow} = g_r^{\uparrow\downarrow} + ig_i^{\uparrow\downarrow}$ is the (complex) mixing conductance [2]. In the normal metal, the spin-dependent electrochemical potential has the general form

$$\mu^{\rm N}_{\parallel,{\rm ip},{\rm op}} = A^{\rm N}_{\parallel,{\rm ip},{\rm op}} \cosh z/\lambda^{\rm N}_{\rm sf} + B^{\rm N}_{\parallel,{\rm ip},{\rm op}} \sinh z/\lambda^{\rm N}_{\rm sf}, \tag{9}$$

and in the ferromagnet

$$\mu^{\rm F}_{\parallel} = A^{\rm F}_{\parallel} \cosh z/\lambda^{\rm F}_{\rm sf} + B^{\rm F}_{\parallel} \sinh z/\lambda^{\rm F}_{\rm sf},\ \mu^{\rm F}_{\rm ip,op} = 0. \tag{10}$$

Imposing that no spin current flows out of the structure (i.e. $\mathcal{J}_{s,z}|_{z=-\sigma_{\rm N}} = \mathcal{J}_{s,z}|_{z=d_{\rm F}} = 0$), we obtain

$$\mu^{\rm N}_{\parallel} = A^{\rm N}_{\parallel}(\cosh z/\lambda^{\rm N}_{\rm sf} + \tanh d_{\rm N}/\lambda^{\rm N}_{\rm sf} \sinh z/\lambda^{\rm N}_{\rm sf}) - m_y \frac{\sinh z/\lambda^{\rm N}_{\rm sf}}{\cosh d_{\rm N}/\lambda^{\rm N}_{\rm sf}} 2\frac{\alpha_{\rm sh}}{1+\alpha_{\rm sw}} \partial_x \mu_c, \tag{11}$$

$$\mu^{\rm N}_{\rm ip} = A^{\rm N}_{\rm ip}(\cosh z/\lambda^{\rm N}_{\rm sf} + \tanh d_{\rm N}/\lambda^{\rm N}_{\rm sf} \sinh z/\lambda^{\rm N}_{\rm sf}) - \frac{\sinh z/\lambda^{\rm N}_{\rm sf}}{\cosh d_{\rm N}/\lambda^{\rm N}_{\rm sf}} 2\frac{\alpha_{\rm sh}}{1+\alpha_{\rm sw}} \partial_x \mu_c, \tag{12}$$

$$\mu^{\rm N}_{\rm op} = A^{\rm N}_{\rm op}(\cosh z/\lambda^{\rm N}_{\rm sf} + \tanh d_{\rm N}/\lambda^{\rm N}_{\rm sf} \sinh z/\lambda^{\rm N}_{\rm sf}) \tag{13}$$

$$\mu^{\rm F}_{\parallel} = A^{\rm F}_{\rm op}(\cosh z/\lambda^{\rm F}_{\rm sf} - \tanh d_{\rm F}/\lambda^{\rm F}_{\rm sf} \sinh z/\lambda^{\rm F}_{\rm sf}). \tag{14}$$

We now need to determine four unknowns, $A^{\rm N,F}_{\parallel,{\rm ip},{\rm op}}$, and apply the boundary conditions Eqs. (7)-(8). At the interface, $z = 0$, the spin-dependent electrochemical potential and spin current components read

$$\mathcal{J}^{\parallel,{\rm N}}_{s,z} = -\frac{\sigma_{\rm N}}{2\tilde{\lambda}^{\rm N}_{\rm sf}} \left((1+\alpha_{\rm sw}(1-m_z^2))A^{\rm N}_{\parallel} + \alpha_{\rm sw} m_y m_z^2 A^{\rm N}_{\rm ip} - \alpha_{\rm sw} m_x m_z A^{\rm N}_{\rm op}\right) - m_y \sigma_{\rm N} \tilde{\alpha}_{\rm sh} \partial_x \mu_c, \tag{15}$$

$$\mathcal{J}^{{\rm ip},{\rm N}}_{s,z} = -\frac{\sigma_{\rm N}}{2\tilde{\lambda}^{\rm N}_{\rm sf}} \left(\alpha_{\rm sw}\frac{m_y m_z^2}{1-m_y^2} A^{\rm N}_{\parallel} + (1+\alpha_{\rm sw}(1-\frac{m_y^2 m_z^2}{1-m_y^2}))A^{\rm N}_{\rm ip} + \alpha_{\rm sw}\frac{m_x m_y m_z}{1-m_y^2} A^{\rm N}_{\rm op}\right) - \sigma_{\rm N} \tilde{\alpha}_{\rm sh} \partial_x \mu_c, \tag{16}$$

$$\mathcal{J}^{{\rm op},{\rm N}}_{s,z} = -\frac{\sigma_{\rm N}}{2\tilde{\lambda}^{\rm N}_{\rm sf}} \left(-\alpha_{\rm sw}\frac{m_x m_z}{1-m_y^2} A^{\rm N}_{\parallel} + \alpha_{\rm sw}\frac{m_x m_y m_z}{1-m_y^2} A^{\rm N}_{\rm ip} + (1+\alpha_{\rm sw}(1-\frac{m_x^2}{1-m_y^2}))A^{\rm N}_{\rm op}\right), \tag{17}$$

$$\mathcal{J}^{\parallel,{\rm F}}_{s,z} = \frac{\sigma_{\rm F}}{2\tilde{\lambda}^{\rm F}_{\rm sf}} A^{\rm F}_{\parallel}, \tag{18}$$

$$\mu^{\rm N}_{\parallel,{\rm ip},{\rm op}} = A^{\rm N}_{\parallel,{\rm ip},{\rm op}},\ \mu^{\rm F}_{\parallel} = A^{\rm F}_{\parallel},\ \mu^{\rm F}_{\rm ip,op} = 0. \tag{19}$$

Here, we defined the reduced spin relaxation length and reduced spin Hall angle, $\tilde{\lambda}^i_{\rm sf} = \lambda_i/\tanh d_i/\lambda^i_{\rm sf}$ $(i = N, F)$ and $\tilde{\alpha}_{\rm sh} = \alpha_{\rm sh}(1 - \cosh^{-1} d_{\rm N}/\lambda^{\rm N}_{\rm sf})$, respectively. The boundary conditions, Eqs. (7)-(8), reduce to

$$\mu_c^{\rm N} - \mu_c^{\rm F} = -\gamma(\mu_\parallel^{\rm N} - \mu_\parallel^{\rm F}), \tag{20}$$

$$\mathcal{J}_{s,z}^{\parallel,\rm N} = \mathcal{J}_{s,z}^{\parallel,\rm F} = 2g(1-\gamma^2)(\mu_\parallel^{\rm N} - \mu_\parallel^{\rm F}), \tag{21}$$

$$\mathcal{J}_{s,z}^{\rm ip,N} = 2g_r^{\uparrow\downarrow}\mu_{\rm ip}^{\rm N} + 2g_i^{\uparrow\downarrow}\mu_{\rm op}^{\rm N}, \tag{22}$$

$$\mathcal{J}_{s,z}^{\rm op,N} = -2g_i^{\uparrow\downarrow}\mu_{\rm ip}^{\rm N} + 2g_r^{\uparrow\downarrow}\mu_{\rm op}^{\rm N}. \tag{23}$$

Injecting Eqs. (15)-(19) into Eqs. (21)-(23) produces the following linear system

$$\alpha_{\rm sw}m_xm_zA_\parallel^{\rm N} - (\alpha_{\rm sw}m_xm_ym_z - (1-m_y^2)\tilde{g}_i^{\uparrow\downarrow})A_{\rm ip}^{\rm N} - ((1-m_y^2)(1+\tilde{g}_r^{\uparrow\downarrow}) + \alpha_{\rm sw}m_z^2)A_{\rm op}^{\rm N} = 0, \tag{24}$$

$$(1+\eta+\alpha_{\rm sw}(1-m_z^2))A_\parallel^{\rm N} + \alpha_{\rm sw}m_ym_z^2A_{\rm ip}^{\rm N} - \alpha_{\rm sw}m_xm_zA_{\rm op}^{\rm N} = -2m_y\tilde{\alpha}_{\rm sh}\tilde{\lambda}_{\rm sf}^{\rm N}\partial_x\mu_{\rm c}, \tag{25}$$

$$\alpha_{\rm sw}\frac{m_ym_z^2}{1-m_y^2}A_\parallel^{\rm N} + ((1+\alpha_{\rm sw}+\tilde{g}_r^{\uparrow\downarrow}) - \alpha_{\rm sw}\frac{m_y^2m_z^2}{1-m_y^2})A_{\rm ip}^{\rm N} + (\tilde{g}_i^{\uparrow\downarrow} + \alpha_{\rm sw}\frac{m_xm_ym_z}{1-m_y^2})A_{\rm op}^{\rm N} = -2\tilde{\alpha}_{\rm sh}\tilde{\lambda}_{\rm sf}^{\rm N}\partial_x\mu_{\rm c}. \tag{26}$$

Here, we defined the reduced spin mixing conductance $\tilde{g}_{r,i}^{\uparrow\downarrow} = 4\tilde{\lambda}_{\rm sf}^{\rm N}g_{r,i}^{\uparrow\downarrow}/\sigma_{\rm N}$ and the transparency coefficient $\eta = \frac{4(1-\gamma^2)g\tilde{\lambda}_{\rm sf}^{\rm N}/\sigma_{\rm N}}{1+4(1-\gamma^2)g\tilde{\lambda}_{\rm F}/\sigma_{\rm F}}$. Solving the linear system given above is cumbersome but does not present any difficulty. We obtain

$$A_\parallel^{\rm N} = -([(\tilde{g}_i^{\uparrow\downarrow})^2 + (1+\tilde{g}_r^{\uparrow\downarrow})^2]m_y + \alpha_{\rm sw}(\tilde{g}_i^{\uparrow\downarrow}m_xm_z + (1+\tilde{g}_r^{\uparrow\downarrow})m_y))\frac{2\tilde{\lambda}_{\rm sf}^{\rm N}\tilde{\alpha}_{\rm sh}\partial_x\mu_{\rm c}}{D_\theta(1-m_y^2)}, \tag{27}$$

$$A_{\rm ip}^{\rm N} = -((1+\eta)(1+\tilde{g}_r^{\uparrow\downarrow})(1-m_y^2) + \alpha_{\rm sw}((1+\eta)m_z^2 + m_x^2(1+\tilde{g}_r^{\uparrow\downarrow}) - \tilde{g}_i^{\uparrow\downarrow}m_xm_ym_z))\frac{2\tilde{\lambda}_{\rm sf}^{\rm N}\tilde{\alpha}_{\rm sh}\partial_x\mu_{\rm c}}{D_\theta(1-m_y^2)}, \tag{28}$$

$$A_{\rm op}^{\rm N} = -((1+\eta)(1-m_y^2)\tilde{g}_i^{\uparrow\downarrow} + \alpha_{\rm sw}(\tilde{g}_i^{\uparrow\downarrow}m_x^2 + (\tilde{g}_r^{\uparrow\downarrow}-\eta)m_xm_ym_z))\frac{2\tilde{\lambda}_{\rm sf}^{\rm N}\tilde{\alpha}_{\rm sh}\partial_x\mu_{\rm c}}{D_\theta(1-m_y^2)}, \tag{29}$$

$$D_\theta = (1+\eta+\alpha_{\rm sw})[(\tilde{g}_i^{\uparrow\downarrow})^2 + (1+\tilde{g}_r^{\uparrow\downarrow})(1+\tilde{g}_r^{\uparrow\downarrow}+\alpha_{\rm sw})] - \alpha_{\rm sw}m_z^2[(\tilde{g}_i^{\uparrow\downarrow})^2 + (\tilde{g}_r^{\uparrow\downarrow}-\eta)(1+\tilde{g}_r^{\uparrow\downarrow}+\alpha_{\rm sw})]. \tag{30}$$

Using Eqs. (22)-(22), we can derive the interfacial spin current and thereby, the spin transfer torque. Noticing that $j_{\rm N} = \sigma_{\rm N}\partial_x\mu_{\rm c}$ and

$$(1-m_y^2)\mathbf{m}\times(\mathbf{x}\times\mathbf{m}) = -m_z\mathbf{m}\times\mathbf{y} - m_xm_y\mathbf{m}\times(\mathbf{y}\times\mathbf{m}), \tag{31}$$

$$(1-m_y^2)\mathbf{m}\times\mathbf{x} = -m_xm_y\mathbf{m}\times\mathbf{y} + m_z\mathbf{m}\times(\mathbf{y}\times\mathbf{m}), \tag{32}$$

we obtain the total torque, $\boldsymbol{\tau} = \boldsymbol{\tau}_{\rm sh} + \boldsymbol{\tau}_{\rm sw}$, with

$$\boldsymbol{\tau}_{\rm sh} = \frac{\tilde{\alpha}_{\rm sh} j_{\rm N}}{D_\theta}(1+\eta+\alpha_{\rm sw})\left[-(\tilde{g}_r^{\uparrow\downarrow} + |\tilde{g}^{\uparrow\downarrow}|^2)\mathbf{m}\times(\mathbf{y}\times\mathbf{m}) + \tilde{g}_i^{\uparrow\downarrow}\mathbf{m}\times\mathbf{y}\right], \tag{33}$$

$$\boldsymbol{\tau}_{\rm sw} = \alpha_{\rm sw} m_z \frac{\tilde{\alpha}_{\rm sh} j_{\rm N}}{D_\theta}\left[(|\tilde{g}^{\uparrow\downarrow}|^2 - \eta\tilde{g}_r^{\uparrow\downarrow})\mathbf{m}\times\mathbf{x} + \eta\tilde{g}_i^{\uparrow\downarrow}\mathbf{m}\times(\mathbf{x}\times\mathbf{m})\right], \tag{34}$$

By noting $\eta_0 = 1+\eta+\alpha_{\rm sw}$, we obtain the equations of the main text.

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