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Decentralized SINR Balancing in Cognitive Radio Networks

Oussama Dhifallah, *Student Member, IEEE*, Hayssam Dahrouj, *Senior Member, IEEE*, Tareq Y.Al-Naffouri, *Member, IEEE*, Mohamed-Slim Alouini, *Fellow, IEEE*

***Abstract*—This paper considers the downlink of a cognitive radio (CR) network formed by multiple primary and secondary transmitters, where each multi-antenna transmitter serves a pre-known set of single-antenna users. The paper assumes that the secondary and primary transmitters can transmit simultaneously their data over the same frequency bands, so as to achieve a high system spectrum efficiency. The paper considers the downlink balancing problem of maximizing the minimum signal-to-interference-plus noise ratio (SINR) of the secondary transmitters subject to both total power constraint of the secondary transmitters, and maximum interference constraint at each primary user due to secondary transmissions. The paper proposes solving the problem using the alternating direction method of multipliers (ADMM), which leads to a distributed implementation through limited information exchange across the coupled secondary transmitters. The paper additionally proposes a solution that guarantees feasibility at each iteration. Simulation results demonstrate that the proposed solution converges to the centralized solution in a reasonable number of iterations.**

I. Introduction

Cognitive radio (CR) are considered as a promising network architecture to effectively exploit the limited radio spectrum resources. In a spectrum sharing-based CR network, unlicensed secondary users can transmit their data using the bandwidth allocated to the licensed primary users [1]. Two main spectrum sharing paradigms are commonly considered in the literature: overlay and underlay. Overlay spectrum sharing architecture exploits the unused portions of the licensed bands to transmit the data of the secondary users [2]. On the other hand, underlay spectrum sharing architecture allows the unlicensed secondary and licensed primary users to transmit simultaneously over the same frequency bands, as long as the interference seen by the primary users due to secondary transmissions is below a pre-specified threshold [3].

This paper considers the downlink of an underlay spectrum sharing-based CR network where licensed primary transmitters and unlicensed secondary transmitters can transmit simultaneously their data over the same frequency bands. Each multi-antenna transmitter communicates with a pre-known set of single-antenna receivers. In order to avoid the disruption of the licensed primary communication, the unlicensed secondary transmitter adapts its power to maintain its interference with the licensed primary transmitter below a desirable level. Therefore, the performance of the considered secondary network becomes a function of the secondary inter-cell and intra-cell interference and the interference with the primary transmitters.

A. Contributions

The main focus of this paper is to determine the beamforming vector by maximizing the minimum SINR while satisfying the total power constraint of the secondary transmitters and guaranteeing that the maximum interference seen by each primary user due to secondary transmissions is below a pre-specified level. The paper proposes a distributed algorithm based on ADMM that can be implemented in a distributed fashion at each secondary transmitter by allowing reasonable amount of information exchange between the coupled secondary transmitters. The proposed algorithm additionally guarantees a feasible set of beamforming vectors to the original problem at each iteration at the cost of solving additional feasibility problems. Simulation results show that the proposed distributed SINR balancing algorithm converges to the centralized solution in a reasonable number of iterations.

B. Related Work

Spectrum sharing-based CR network has been widely investigated in recent literature. The work in [4] and [5] addresses the problem of maximizing the minimum SINR for the robust and non-robust scenarios respectively. The references [6] and [7] consider the robust and multicast beamforming design problems for spectrum sharing-based CR network. All the aforementioned references, however, provide centralized solution which are impractical, since otherwise, the secondary transmitters would require joint signal-processing and high signaling overhead. This paper is more related to reference [8] which provides a distributed algorithm to solve the rate balancing problem by applying the ADMM approach [9] to its equivalent power minimization problem. However, this algorithm is generally impractical since it may require large number of iterations to converge. To address this problem, our paper considers the SINR balancing problem from a different perspective. It proposes a distributed algorithm to directly solve the considered optimization problem. Further, the problem considered in this paper is related to [10] which provides distributed algorithm to solve the multi-cell SINR balancing problem in conventional cellular networks. The work in [11] addresses the sum power minimization problem for a CR network and provides a distributed algorithm to solve this problem using a primal decomposition approach.

C. Organization

The rest of this paper is organized as follows, Section II presents the system model and formulates the SINR balancing optimization problem. Then, Section III presents the proposed SINR balancing algorithm. Numerical examples illustrating the performance of the proposed algorithm are given in Section IV. Finally, Section V concludes this paper.

II. System Model and Problem Formulation

A. System Model

Consider the downlink of a spectrum sharing-based CR network with B_p primary transmitters and B_s secondary transmitters. Assume that each transmitter b is equipped with N_b antennas. Further, assume that the network comprises U_p single-antenna primary receivers and U_s single-antenna secondary receivers where each primary transmitter b serves U_{pb} primary users and each secondary transmitter b serves U_{sb} secondary users. The channel state information is assumed to be perfectly known by the secondary

O. Dhifallah and M.-S. Alouini are with the Division of Computer, Electrical and Mathematical Sciences and Engineering, King Abdullah University of Science and Technology, Thuwal 23955-6900, Saudi Arabia (e-mail: {oussama.dhifallah, slim.alouini}@kaust.edu.sa).

H. Dahrouj is with the Department of Electrical and Computer Engineering, Effat University, Jeddah 22332, Saudi Arabia (e-mail: hayssam.dahrouj@gmail.com).

T. Y. Al-Naffouri is with King Abdullah University of Science and Technology, Thuwal 23955-6900, Saudi Arabia, and also with King Fahd University of Petroleum and Minerals, Dhahran 31261, Saudi Arabia (e-mail: tareq.alnaffouri@kaust.edu.sa).

transmitters and the user association is assumed to be predefined. Figure 1 illustrates an example of the considered network with two secondary transmitters and one primary receiver. Let $\mathbf{w}_{bu} \in \mathbb{C}^{N_b}$

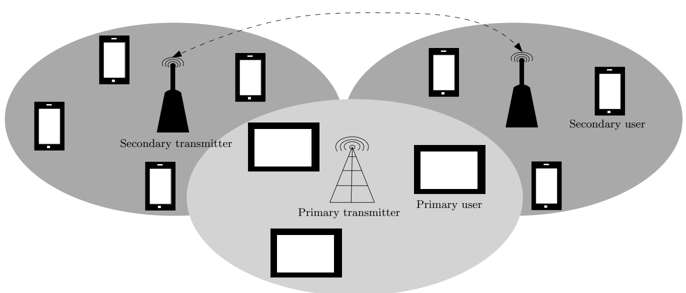

Figure 1. The considered cognitive radio network architecture.

be the beamforming vector from transmitter b to user u, and let $\mathbf{h}_{bu} \in \mathbb{C}^{N_b}$ be the channel vector from transmitter b to user u. Let b_u denotes the transmitter serving user u and $\bar{\mathcal{B}}_{su}$ denotes the set of all the secondary transmitters except for the secondary transmitter serving user u. The received signal $y_u \in \mathbb{C}$ at the secondary user u served by the secondary transmitter b can be written as follows

$$y_u = \mathbf{h}_{bu}^H \mathbf{w}_{bu} q_u + \sum_{u' \in \mathcal{U}_{sbu}} \mathbf{h}_{bu}^H \mathbf{w}_{bu'} q_{u'} + \sum_{b' \neq b} \sum_{u' \in \mathcal{U}_{sb'}} \mathbf{h}_{b'u}^H \mathbf{w}_{b'u'} q_{u'} + \sum_{b' \in \mathcal{B}_p} \sum_{u' \in \mathcal{U}_{pb'}} \mathbf{h}_{b'u}^H \mathbf{w}_{b'u'} q_{u'} + n_u, \tag{1}$$

where $\mathcal{U}_{sbu} = \mathcal{U}_{sb} \setminus \{u\}$, $\mathcal{U}_{sb'}$ denotes the set of the secondary receivers served by the secondary transmitter b', $\mathcal{U}_{pb'}$ denotes the set of the primary receivers served by the primary transmitter b' and $\mathcal{B}_p$ denotes the set of primary transmitters, and where q_u is a complex scalar denoting the data symbol for the secondary user u and $n_u \sim \mathcal{CN}(0, \sigma_u^2)$ represents the additive white Gaussian noise which is assumed to be independent from the transmitted data symbols q_u.

B. Problem Formulation

Based on the introduced signal model (1), the SINR of the secondary user u served by the secondary transmitter b can be expressed as

$$\Gamma_u = \frac{\left|\mathbf{h}_{bu}^H \mathbf{w}_{bu}\right|^2}{\sum\limits_{u' \in \mathcal{U}_{sbu}} \left|\mathbf{h}_{bu}^H \mathbf{w}_{bu'}\right|^2 + \sum\limits_{b' \neq b} \sum\limits_{u' \in \mathcal{U}_{sb'}} \left|\mathbf{h}_{b'u}^H \mathbf{w}_{b'u'}\right|^2 + \theta_u^2 + \sigma_u^2},$$

where we assume that the transmitted data symbols q_u for the user u have unit power, i.e. $E(|q_u|^2) = 1$, and independent from each other. Besides, $\theta_u^2 = \sum_{b' \in \mathcal{B}_p} \sum_{u' \in \mathcal{U}_{pb'}} |\mathbf{h}_{b'u}^H \mathbf{w}_{b'u'}|^2$ denotes the interference seen by the secondary user u due to primary transmission which is assumed to be known by the secondary transmitters.

One of the constraints of the optimization problem studied in this paper is the total transmit power constraint, given by

$$\sum_{b \in \mathcal{B}_s} \sum_{u \in \mathcal{U}_{sb}} ||\mathbf{w}_{bu}||_{\ell_2}^2 \ \leq \ P, \tag{2}$$

where P is a given nominal maximum total transmit power. Besides, we assume that the interference seen by the primary user due to secondary transmission can not exceed a fixed threshold $\beta_{u'}$ which can be translated to the following constraints

$$\sum_{b \in \mathcal{B}_s} \sum_{u \in \mathcal{U}_{sb}} |\mathbf{h}_{bu'}^H \mathbf{w}_{bu}|^2 \ \leq \ \beta_{u'}, \ \forall \ u' \in \mathcal{U}_p, \tag{3}$$

where $\mathcal{B}_s$ denotes the set of secondary transmitters.

The optimization problem studied in this paper considers the maximization of the minimum SINR subject to the total power constraint of the secondary transmitters and the maximum interference seen by each primary user due to secondary transmissions. Specifically, the considered optimization problem can be formulated as follows

$$\begin{aligned} \max_{\mathbf{w}} \quad & \min_{u} \ \Gamma_u \\ s.t. \quad & \sum_{b \in \mathcal{B}_s} \sum_{u \in \mathcal{U}_{sb}} ||\mathbf{w}_{bu}||_{\ell_2}^2 \ \leq \ P \\ & \sum_{b \in \mathcal{B}_s} \sum_{u \in \mathcal{U}_{sb}} |\mathbf{h}_{bu'}^H \mathbf{w}_{bu}|^2 \ \leq \ \beta_{u'}, \ \forall \ u' \in \mathcal{U}_p, \end{aligned} \tag{4}$$

where the optimization is over the beamforming vectors $\mathbf{w} = [\mathbf{w}_{bu}^T; \ \forall (b,u) \in (\mathcal{B}_s \times \mathcal{U}_{sb})]^T \in \mathbb{C}^N$, where $N = \sum_{b=1}^{B_s} U_{sb} N_b$, and where $\beta_{u'}$ denotes the maximum interference that can be tolerated by the primary user u'. The above problem (4) can be easily solved using the bisection method [12]. However, centralized solutions to solve the SINR balancing optimization problem are impractical, since otherwise, the secondary transmitters would require joint signal-processing and high signaling overhead. Thus, a distributed algorithm between the secondary transmitters is proposed using the ADMM approach and by allowing a limited information exchange between the secondary transmitters. Further, a feasible solution of the original problem is estimated locally at each iteration. Simulation results show that the proposed decentralized algorithm converges to the centralized solution in a reasonable number of iterations.

III. Distributed Max–Min SINR Beamforming

A. Problem Relaxation

First, define the secondary inter-cell interference terms ${\psi_{b'u}}^2$ from an interfering secondary transmitter b' to the secondary user u served by any secondary transmitter different than b' as follows

$${\psi_{b'u}}^2 = \sum_{u' \in \mathcal{U}_{sb'}} \left|\mathbf{h}_{b'u}^H \mathbf{w}_{b'u'}\right|^2. \tag{5}$$

Second, define the primary inter-cell interference terms ${\chi_{b'u'}}^2$ from an interfering secondary transmitter b' to the primary user u' served by any primary transmitter as follows

$${\chi_{b'u'}}^2 = \sum_{u \in \mathcal{U}_{sb'}} \left|\mathbf{h}_{b'u'}^H \mathbf{w}_{b'u}\right|^2. \tag{6}$$

Finally, define the local power consumption terms ${\kappa_{b'}}^2$ of the secondary transmitter b' as follows

$${\kappa_{b'}}^2 = \sum_{u' \in \mathcal{U}_{sb'}} ||\mathbf{w}_{b'u'}||_{\ell_2}^2. \tag{7}$$

By relaxing the equalities in (5), (6) and (7) into inequalities, the SINR balancing problem (4) can be reformulated as

$$\begin{aligned} \max_{\mathbf{w}, \boldsymbol{\psi}, \boldsymbol{\chi}, \boldsymbol{\kappa}} \ \min_{u} \ \Gamma_u = & \frac{|\mathbf{h}_{bu}^H \mathbf{w}_{bu}|^2}{\sum\limits_{u' \in \mathcal{U}_{sbu}} |\mathbf{h}_{bu}^H \mathbf{w}_{bu'}|^2 + \sum\limits_{b' \neq b} {\psi_{b'u}}^2 + \theta_u^2 + \sigma_u^2} \\ s.t. \quad & \sum_{u \in \mathcal{U}_{sb}} ||\mathbf{w}_{bu}||_{\ell_2}^2 + \sum_{b' \neq b} {\kappa_{b'}}^2 \ \leq \ P \\ & \sum_{u \in \mathcal{U}_{sb}} |\mathbf{h}_{bu'}^H \mathbf{w}_{bu}|_{\ell_2}^2 + \sum_{b' \neq b} {\chi_{b'u'}}^2 \ \leq \ \beta_{u'}, \ \forall \ u' \in \mathcal{U}_p \\ & {\psi_{b'u}}^2 \geq \sum_{u' \in \mathcal{U}_{sb'}} \left|\mathbf{h}_{b'u}^H \mathbf{w}_{b'u'}\right|^2, \ \forall u \notin \mathcal{U}_{b'}, \ \forall b' \in \mathcal{B}_s \\ & {\chi_{b'u'}}^2 \geq \sum_{u \in \mathcal{U}_{sb'}} \left|\mathbf{h}_{b'u'}^H \mathbf{w}_{b'u}\right|^2, \ \forall \ u' \in \mathcal{U}_p, \ \forall b' \in \mathcal{B}_s \\ & {\kappa_{b'}}^2 \geq \sum_{u' \in \mathcal{U}_{sb'}} ||\mathbf{w}_{b'u'}||_{\ell_2}^2, \ \forall b' \in \mathcal{B}_s, \end{aligned} \tag{8}$$

where the optimization is over the beamforming vectors, the secondary inter-cell interference vector $\boldsymbol{\psi} = [\psi_{b'u}; \ \forall u \notin \mathcal{U}_{sb'}, \forall b' \in \mathcal{B}_s]^T \in \mathbb{R}^{U'}$, the primary inter-cell interference vector $\boldsymbol{\chi} = [\chi_{b'u'}; \ \forall u' \in \mathcal{U}_p, \forall b' \in \mathcal{B}_s]^T \in \mathbb{R}^{U_p B_s}$ and the local

$$\mathcal{F}_b = \Bigg\{ \mathbf{w}_b, \boldsymbol{\xi}^{(b)}, \boldsymbol{\phi}^{(b)}, \boldsymbol{\varrho}^{(b)}, \boldsymbol{\delta}^{(b)} \Bigg| \frac{|\mathbf{h}_{bu}^H \mathbf{w}_{bu}|^2}{\sum\limits_{u'' \in \mathcal{U}_{sbu}} |\mathbf{h}_{bu}^H \mathbf{w}_{bu''}|^2 + \sum\limits_{b' \neq b} {\xi_{b'u}^{(b)}}^2 + \theta_u^2 + \sigma_u^2} \geq \delta^{(b)}, \forall u \in \mathcal{U}_{sb};\ {\varrho_b^{(b)}}^2 \geq \sum_{u'' \in \mathcal{U}_{sb}} ||\mathbf{w}_{bu''}||_{\ell_2}^2,$$
$${\xi_{bu}^{(b)}}^2 \geq \sum_{u'' \in \mathcal{U}_{sb}} \left|\mathbf{h}_{bu}^H \mathbf{w}_{bu''}\right|^2, {\phi_{bu'}^{(b)}}^2 \geq \sum_{u'' \in \mathcal{U}_{sb}} \left|\mathbf{h}_{bu'}^H \mathbf{w}_{bu''}\right|^2, \sum_{u'' \in \mathcal{U}_{sb}} |\mathbf{h}_{bu'}^H \mathbf{w}_{bu''}|^2 + \sum_{b' \neq b} {\phi_{b'u'}^{(b)}}^2 \leq \beta_{u'}, \forall u \notin \mathcal{U}_{sb'}, \forall u' \in \mathcal{U}_p;$$
$$\sum_{u'' \in \mathcal{U}_{sb}} ||\mathbf{w}_{bu''}||_{\ell_2}^2 + \sum_{b' \neq b} {\varrho_{b'}^{(b)}}^2 \leq P \Bigg\}. \tag{9}$$

power consumption vector $\boldsymbol{\kappa} = [\kappa_{b'};\ \forall b' \in \mathcal{B}_s]^T \in \mathbb{R}^{B_s}$, and where $U' = \sum_{b=1}^{B_s} \sum_{b' \neq b} U_{b'}$. Relaxing the secondary inter-cell interference, the primary inter-cell interference and the local power consumption constraints with inequality in the reformulated problem (8) is, in general, suboptimal as compared to (4).

Proposition 1. *The original optimization problem (4) and the relaxed optimization problem (8) have the same optimal solution.*

The proof of the above proposition follows by first showing by contradiction that the third constraints of the optimization problem (8) are tight for the user with the minimum SINR. Then, we can easily conclude the equivalence between the two optimization problems. The detailed proof of proposition 1 is omitted due to space limitations.

B. Distributed Solution via the Alternating Direction Method of Multipliers

This section proposes a distributed algorithm to solve the SINR balancing optimization problem (4), using the ADMM approach, first introduced in the context of multi-cell systems [10]. To this end, we start by introducing local auxiliary variables and equality constraints in order to decouple the SINR constraints, the total power constraint and the primary interference constraints. Then, the ADMM algorithm [9] is used to achieve a distributed solution to the considered optimization problem. Specifically, problem (8) can be reformulated as

$$\begin{aligned}
&\max_{\mathbf{w}, \boldsymbol{\psi}, \boldsymbol{\chi}, \boldsymbol{\kappa}, \boldsymbol{\xi}, \boldsymbol{\phi}, \boldsymbol{\varrho}} \quad \frac{1}{B_s} \sum_{b \in \mathcal{B}_s} \delta^{(b)} \\
&s.t. \quad \left\{ \mathbf{w}_b, \boldsymbol{\xi}^{(b)}, \boldsymbol{\phi}^{(b)}, \boldsymbol{\varrho}^{(b)}, \boldsymbol{\delta}^{(b)} \right\} \in \mathcal{F}_b,\ \forall\, b \in \mathcal{B}_s \\
&\qquad \xi_{b'u}^{(b')} = \psi_{b'u},\ \xi_{b'u}^{(b_u)} = \psi_{b'u},\ \forall\, (u, b') \in (\mathcal{U}_s \times \bar{\mathcal{B}}_{su}) \\
&\qquad \phi_{b'u'}^{(b)} = \chi_{b'u'},\ \forall\, (u', b, b') \in (\mathcal{U}_p \times \mathcal{B}_s \times \mathcal{B}_s) \\
&\qquad \varrho_{b'}^{(b)} = \kappa_{b'},\ \forall\, (b, b') \in (\mathcal{B}_s \times \mathcal{B}_s) \\
&\qquad \delta^{(b)} = \gamma,\ \forall\, b \in \mathcal{B}_s,
\end{aligned} \tag{10}$$

where the vector $\boldsymbol{\xi} = [\xi_{b'u}^{(b)};\ \forall b \in \{b', b_u\}\ \forall u \notin \mathcal{U}_{b'}\ \forall b' \in \mathcal{B}_s]^T \in \mathbb{R}^{2U'}$, vector $\boldsymbol{\phi} = [\phi_{b'u'}^{(b)};\ \forall b \in \mathcal{B}_s\ \forall u \in \mathcal{U}_p\ \forall b' \in \mathcal{B}_s]^T \in \mathbb{R}^{{B_s}^2 U_p}$ and vector $\boldsymbol{\varrho} = [\varrho_{b'}^{(b)};\ \forall b \in \mathcal{B}_s\ \forall b' \in \mathcal{B}_s]^T \in \mathbb{R}^{{B^s}^2}$ and where $\mathcal{F}^b$ is given in (9). Note that introducing the equality constraints in (10) decouples the SINR constraints, the total power constraint and the maximum interference caused by the secondary transmission constraints. To derive a distributed solution to the optimization problem (10), we start by defining the following indicator functions $\forall b \in \mathcal{B}_s$

$$I_b(\mathbf{w}_b, \boldsymbol{\xi}^{(b)}, \boldsymbol{\phi}^{(b)}, \boldsymbol{\varrho}^{(b)}, \boldsymbol{\delta}^{(b)}) = \begin{cases} 0 & \text{if } \{\mathbf{w}_b, \boldsymbol{\xi}^{(b)}, \boldsymbol{\phi}^{(b)}, \boldsymbol{\varrho}^{(b)}, \boldsymbol{\delta}^{(b)}\} \in \mathcal{F}_b \\ +\infty & \text{Otherwise} \end{cases}. \tag{11}$$

Therefore, the augmented Lagrangian of the optimization problem (10) can be expressed as in (12), where the vectors $\boldsymbol{\lambda}^{(b)} = [\lambda_{bu'}^{(b)};\ \forall u' \notin \mathcal{U}_{sb},\ \lambda_{b'u}^{(b)};\ \forall b' \neq b\ \forall u \in \mathcal{U}_{sb}]^T$ and $\boldsymbol{\xi}^{(b)} = [\xi_{bu'}^{(b)};\ \forall u' \notin \mathcal{U}_b,\ \xi_{b'u}^{(b)};\ \forall b' \neq b\ \forall u \in \mathcal{U}_b]^T$ represent respectively the local dual vector and the introduced local vector associated with the secondary inter-cell interference vector $\boldsymbol{\psi}^{(b)} = [\psi_{bu'};\ \forall u' \notin \mathcal{U}_b,\ \psi_{b'u};\ \forall b' \neq b\ \forall u \in \mathcal{U}_b]^T$. Further, the vectors $\boldsymbol{\mu}^{(b)} = [\mu_{b'u'}^{(b)};\ \forall u' \in \mathcal{U}_p\ \forall b' \in \mathcal{B}_s]^T$ and $\boldsymbol{\phi}^{(b)} = [\phi_{b'u'}^{(b)};\ \forall u' \in \mathcal{U}_p\ \forall b' \in \mathcal{B}_s]^T$ represent respectively the local dual vector and the introduced local vector associated with the primary inter-cell interference vector $\boldsymbol{\chi}^{(b)} = [\chi_{b'u'};\ \forall u' \in \mathcal{U}_p\ \forall b' \in \mathcal{B}_s]^T$. Besides, the vectors $\boldsymbol{\vartheta}^{(b)} = [\vartheta_{b'}^{(b)};\ \forall b' \in \mathcal{B}_s]^T$ and $\boldsymbol{\varrho}^{(b)} = [\varrho_{b'}^{(b)};\ \forall b' \in \mathcal{B}_s]^T$ represent respectively the local dual vector and the introduced local vector associated with the local power consumption vector $\boldsymbol{\kappa}^{(b)} = [\kappa_{b'};\ \forall b' \in \mathcal{B}_s]^T$. Finally, $\nu^{(b)}$ denotes the dual variable associated with the introduced local variable $\delta^{(b)}$. The ADMM algorithm solves the relaxed optimization problem (10) iteratively by performing two primal minimization steps and a dual variable update at each iteration.

1) First Step Minimization: The first step of the ADMM approach consists of minimizing the augmented Lagrangian over the variables $\mathbf{w}$, $\boldsymbol{\xi}$, $\boldsymbol{\phi}$, $\boldsymbol{\varrho}$ and $\boldsymbol{\delta}$ while all the other variables are fixed at their current values. Specifically, the first step can be formulated as follows

$$\min_{\mathbf{w}, \boldsymbol{\xi}, \boldsymbol{\phi}, \boldsymbol{\varrho}, \boldsymbol{\delta}} \mathcal{L}_\rho(\mathbf{w}, \boldsymbol{\xi}, \boldsymbol{\phi}, \boldsymbol{\varrho}, \boldsymbol{\delta}, \{\boldsymbol{\psi}, \boldsymbol{\chi}, \boldsymbol{\kappa}, \gamma\}^k, \{\boldsymbol{\lambda}, \boldsymbol{\mu}, \boldsymbol{\vartheta}, \boldsymbol{\nu}\}^k). \tag{13}$$

The above optimization problem is fully separable between the secondary transmitters. Specifically, each secondary transmitter b can update its corresponding beamforming vector and the introduced local vector independently by solving the following optimization problem

$$\begin{aligned}
\min_{\substack{\mathbf{w}_b, \boldsymbol{\xi}^{(b)}, \delta^{(b)} \\ \boldsymbol{\phi}^{(b)}, \boldsymbol{\varrho}^{(b)}}} \; & -\frac{\delta^{(b)}}{B_s} + \frac{\rho}{2}\, g_b\left(\boldsymbol{\xi}^{(b)}, \boldsymbol{\phi}^{(b)}, \boldsymbol{\varrho}^{(b)}\right) \\
& + \frac{\rho}{2} \left| \delta^{(b)} - \gamma^k + \frac{1}{\rho} {\nu^{(b)}}^k \right|^2 \\
s.t. \; & \left\{ \mathbf{w}_b, \boldsymbol{\xi}^{(b)}, \boldsymbol{\phi}^{(b)}, \boldsymbol{\varrho}^{(b)}, \boldsymbol{\delta}^{(b)} \right\} \in \mathcal{F}_b,
\end{aligned} \tag{14}$$

where the function $g_b\left(\boldsymbol{\xi}^{(b)}, \boldsymbol{\phi}^{(b)}, \boldsymbol{\varrho}^{(b)}\right)$ is given in (15), and where the optimization is over the local beamforming vector $\mathbf{w}_b$ and the local variables $\boldsymbol{\xi}^{(b)}$, $\boldsymbol{\phi}^{(b)}$, $\boldsymbol{\varrho}^{(b)}$ and $\delta^{(b)}$ of the secondary transmitter b. The above optimization problem is not convex due to the SINR constraints. However, for a fixed $\delta^{(b)}$, the optimization problem (14) can be easily reformulated as a second-order cone programming (SOCP). Thus, it can be solved using efficient numerical algorithms [13].

Proposition 2. *The following function is unimodal on an interval* $[0\ \delta_{max}^{(b)}]$, $\forall b \in \mathcal{B}_s$

$$h\left(\delta^{(b)}\right) = -\frac{\delta^{(b)}}{B_s} + \frac{\rho}{2} \left| \delta^{(b)} - \gamma^k + \frac{1}{\rho} {\nu^{(b)}}^k \right|^2 + \frac{\rho}{2} g_b^*, \tag{16}$$

where g_b^* *is the optimal objective value of (14) for a fixed* $\delta^{(b)}$.

The proof of the above proposition is omitted in this paper, as it mirror the proof used in [10]. Using the above proposition, the optimization problem (14) can be easily solved using the golden search method with any predefined accuracy $\epsilon > 0$.

$$\mathcal{L}_\rho(\mathbf{w},\boldsymbol{\xi},\boldsymbol{\phi},\boldsymbol{\varrho},\boldsymbol{\delta},\boldsymbol{\psi},\boldsymbol{\chi},\boldsymbol{\kappa},\gamma,\boldsymbol{\lambda},\boldsymbol{\mu},\boldsymbol{\vartheta},\boldsymbol{\nu}) = -\frac{1}{B_s}\sum_{b\in\mathcal{B}_s}\delta^{(b)} + \sum_{b\in\mathcal{B}_s} I_b(\mathbf{w}_b,\boldsymbol{\xi}^{(b)},\boldsymbol{\phi}^{(b)},\boldsymbol{\varrho}^{(b)},,\delta^{(b)}) + \frac{\rho}{2}\sum_{b\in\mathcal{B}_s}\left\|\boldsymbol{\xi}^{(b)}-\boldsymbol{\psi}^{(b)}+\frac{1}{\rho}\boldsymbol{\lambda}^{(b)}\right\|_{\ell_2}^2 + \frac{\rho}{2}\sum_{b\in\mathcal{B}_s}\left\|\boldsymbol{\phi}^{(b)}-\boldsymbol{\chi}^{(b)}+\frac{1}{\rho}\boldsymbol{\mu}^{(b)}\right\|_{\ell_2}^2 + \frac{\rho}{2}\sum_{b\in\mathcal{B}_s}\left\|\boldsymbol{\varrho}^{(b)}-\boldsymbol{\kappa}^{(b)}+\frac{1}{\rho}\boldsymbol{\vartheta}^{(b)}\right\|_{\ell_2}^2 + \frac{\rho}{2}\sum_{b\in\mathcal{B}_s}\left|\delta^{(b)}-\gamma+\frac{1}{\rho}\nu^{(b)}\right|^2. \quad (12)$$

$$g_b\left(\boldsymbol{\xi}^{(b)},\boldsymbol{\phi}^{(b)},\boldsymbol{\varrho}^{(b)}\right) = \left\|\boldsymbol{\xi}^{(b)}-\boldsymbol{\psi}^{(b)^k}+\frac{1}{\rho}\boldsymbol{\lambda}^{(b)^k}\right\|_{\ell_2}^2 + \left\|\boldsymbol{\phi}^{(b)}-\boldsymbol{\chi}^{(b)^k}+\frac{1}{\rho}\boldsymbol{\mu}^{(b)^k}\right\|_{\ell_2}^2 + \left\|\boldsymbol{\varrho}^{(b)}-\boldsymbol{\kappa}^{(b)^k}+\frac{1}{\rho}\boldsymbol{\vartheta}^{(b)^k}\right\|_{\ell_2}^2. \quad (15)$$

2) Second Step Minimization: The second step of the ADMM approach consists of minimizing the augmented Lagrangian over the variables $\boldsymbol{\psi}$, $\boldsymbol{\chi}$, $\boldsymbol{\kappa}$ and γ while all the other variables are fixed at their current values. Specifically, the second step can be formulated as follows

$$\min_{\boldsymbol{\psi},\boldsymbol{\chi},\boldsymbol{\kappa},\gamma} \mathcal{L}_\rho(\{\mathbf{w},\boldsymbol{\xi},\boldsymbol{\phi},\boldsymbol{\varrho},\boldsymbol{\delta}\}^{k+1},\boldsymbol{\psi},\boldsymbol{\chi},\boldsymbol{\kappa},\gamma,\{\boldsymbol{\lambda},\boldsymbol{\mu},\boldsymbol{\vartheta},\boldsymbol{\nu}\}^k). \quad (17)$$

The optimization problem is convex and separable on the variables $\boldsymbol{\psi}$, $\boldsymbol{\chi}$, $\boldsymbol{\kappa}$ and γ. After computing the gradient of the cost function in (17) with respect to $\boldsymbol{\psi}$, $\boldsymbol{\chi}$, $\boldsymbol{\kappa}$ and γ and setting it to zero, respectively, the secondary inter-cell interference terms ψ_{bu} $\forall b \in \mathcal{B}_s$ $\forall u \notin \mathcal{U}_{sb}$, the primary inter-cell interference terms $\chi_{bu'}$ $\forall b \in \mathcal{B}_s$ $\forall u' \in \mathcal{U}_p$, the local power consumption terms κ_b $\forall b \in \mathcal{B}_s$ and γ are respectively updated as follows

$$\begin{cases} {\psi_{bu}}^{k+1} = \frac{\left(\xi_{bu}^{(b)^{k+1}}+\xi_{bu}^{(b_u)^{k+1}}\right)}{2} + \frac{\left(\lambda_{bu}^{(b)^k}+\lambda_{bu}^{(b_u)^k}\right)}{2\rho} \\ {\chi_{bu'}}^{k+1} = \frac{1}{B_s}\sum\limits_{b'\in\mathcal{B}_s}\left(\phi_{bu'}^{(b')^{k+1}}+\frac{1}{\rho}\mu_{bu'}^{(b')^k}\right) \\ {\kappa_b}^{k+1} = \frac{1}{B_s}\sum\limits_{b'\in\mathcal{B}_s}\left(\varrho_b^{(b')^{k+1}}+\frac{1}{\rho}\vartheta_b^{(b')^k}\right) \\ \gamma^{k+1} = \frac{1}{B_s}\sum\limits_{b'\in\mathcal{B}_s}\left(\delta^{(b')^{k+1}}+\frac{1}{\rho}\nu^{(b')^k}\right) \end{cases}. \quad (18)$$

Note that the above updates can be performed locally at each secondary transmitter by allowing the exchange of the introduced local variables and the dual variables between the coupled secondary transmitters.

3) Dual Variable Update: The last step of the ADMM algorithm consists of updating the dual variables associated with the equality constraints in (10). The dual variables $\lambda_{bu}^{(b')}$ $\forall b' \in (b, b_u)$ $\forall u \notin \mathcal{U}_{sb}$ $\forall b \in \mathcal{B}_s$; $\mu_{bu'}^{(b')}$ $\forall u \in \mathcal{U}_p$ $\forall b' \in \mathcal{B}_s$ $\forall b \in \mathcal{B}_s$; $\vartheta_b^{(b')}$ $\forall b' \in \mathcal{B}_s$ $\forall b \in \mathcal{B}_s$ and $\nu^{(b)}$ $\forall$ $b \in \mathcal{B}_s$ are respectively updated as follows

$$\begin{cases} \lambda_{bu}^{(b')^{k+1}} = \lambda_{bu}^{(b')^k} + \rho\left(\xi_{bu}^{(b')^{k+1}} - {\psi_{bu}}^{k+1}\right) \\ \mu_{bu'}^{(b')^{k+1}} = \mu_{bu'}^{(b')^k} + \rho\left(\phi_{bu'}^{(b')^{k+1}} - {\chi_{bu'}}^{k+1}\right) \\ \vartheta_b^{(b')^{k+1}} = \vartheta_b^{(b')^k} + \rho\left(\varrho_b^{(b')^{k+1}} - {\kappa_b}^{k+1}\right) \\ {\nu^{(b)}}^{k+1} = {\nu^{(b)}}^k + \rho\left({\delta^{(b)}}^{k+1} - \gamma^{k+1}\right) \end{cases}. \quad (19)$$

It can be noticed that the dual variable update rules can be performed locally at each secondary transmitter.

4) Feasible Primal Solution: The proposed decentralized solution consists of iterating between the first step minimization, the second step minimization and the dual variable update rules. Clearly, the introduced auxiliary variables may not lead to a feasible solution for the original optimization problem (4) at each iteration of the ADMM algorithm, because the equality constraints in (10) are not satisfied at the intermediate iterations. However, a feasible solution can be reached at each iteration by first fixing the introduced local variables associated with the secondary inter-cell interference terms, the primary inter-cell interference terms and the local power consumption terms as follows

$$\begin{cases} \xi_{bu}^{(b)} = \frac{1}{3}\left(\xi_{bu}^{(b)} + \xi_{bu}^{(b_u)} + \psi_{bu}\right), \forall u \notin \mathcal{U}_b, b \in \mathcal{B}_s \\ \xi_{bu}^{(b_u)} = \frac{1}{3}\left(\xi_{bu}^{(b_u)} + \xi_{bu}^{(b)} + \psi_{bu}\right), \forall u \notin \mathcal{U}_b, b \in \mathcal{B}_s \\ \phi_{b'u'}^{(b)} = \frac{1}{B_s+1}\left(\sum\limits_{b''\in\mathcal{B}_s}\phi_{b'u'}^{(b'')} + \chi_{b'u'}\right), \forall (u,b',b) \in (\mathcal{U}_b,\mathcal{B}_s,\mathcal{B}_s) \\ \vartheta_{b'}^{(b)} = \frac{1}{B_s+1}\left(\sum\limits_{b''\in\mathcal{B}_s}\vartheta_{b'}^{(b'')} + \varrho_{b'}\right), \forall (b',b) \in (\mathcal{B}_s,\mathcal{B}_s) \\ \delta^{(b)} = \min\left([\delta^{(b)}, \forall b \in \mathcal{B}_s, \gamma]\right), \forall b \in \mathcal{B}_s \end{cases}$$

Then, resolve the optimization problem (14) with the above fixed variables. Specifically, the following feasibility problem is solved $\forall b \in \mathcal{B}_s$

$$\begin{aligned} &\text{Find} && \mathbf{w}_{bu},\ \forall u \in \mathcal{U}_b \\ &s.t. && \mathbf{w}_b \in \mathcal{F}_b. \end{aligned} \quad (20)$$

Finally, if the above problem is feasible $\forall b \in \mathcal{B}_s$, the feasible solution to the original problem (4) will be

$$\delta^{feas} = \frac{1}{B_s}\sum_{b\in\mathcal{B}_s}\delta^{(b)}. \quad (21)$$

Otherwise, decrease $\delta^{(b)}$ $\forall b \in \mathcal{B}_s$ and resolve the above feasibility problem.

5) Iterative Decentralized Algorithm: The proposed decentralized solution requires to iterate between three levels. At the first level, the optimization problems (14) are solved locally and independently at each secondary transmitter b using the golden search method in order to determine the local beamforming vector $\mathbf{w}_b$, the introduced local variables $\boldsymbol{\xi}^{(b)}$, $\delta^{(b)}$, $\boldsymbol{\phi}^{(b)}$ and $\boldsymbol{\varrho}^{(b)}$. Then, each secondary transmitter b broadcasts the optimal introduced local variables $\boldsymbol{\xi}^{(b)}$, $\delta^{(b)}$, $\boldsymbol{\phi}^{(b)}$ and $\boldsymbol{\varrho}^{(b)}$ to the coupled secondary transmitters. Given the knowledge of the optimal introduced local variables, each transmitter updates its corresponding secondary inter-cell interference terms, primary inter-cell interference terms and local power consumption terms locally as in (18) at the second level. Then, each secondary transmitter b updates its corresponding dual variables locally as in (19). Alternatively, the considered problem can be implemented in a distributed fashion using primal or dual decomposition approaches [14]. Specifically, the primal or dual decomposition is used to solve the power minimization problem as an intermediate step of the SINR balancing problem. However, this method is generally impractical since the primal or dual decomposition algorithm is controlled by a bisection search. Therefore, this approach needs a large number of iteration to converge which cause high signaling overhead.

IV. Simulation Results

This section evaluates the performance of the proposed decentralized algorithm. Consider two secondary transmitters, i.e. $\mathcal{B}_s = 2$. Further, assume that each transmitter serves two single-antenna receivers and each transmitter is equipped with three antennas. In the first secondary cell, the receivers are assumed to be uniformly and independently distributed in the square region $[0\ 500]\times[0\ 500]$ meters and the transmitter is assumed to be located at $(250, 250)$ meters. In the second secondary cell, the receivers are assumed to be uniformly and independently distributed in the square region $[500\ 1000] \times [0\ 500]$ meters and the transmitter is located at $(750, 250)$ meters. In the primary cell, the receivers are assumed to be uniformly and independently distributed in the square region $[1000\ 1500] \times [0\ 500]$ meters. The channel model is assumed to be formed by a distance-dependent path loss $L(d_{bu}) = 128.1 + 37.6\log_{10}(d_{bu})$, and Rayleigh fading component, where d_{bu} denotes the distance between transmitter b and receiver u in kilometers. The noise power spectral density is $\sigma_u^2 = -96$ dBm/Hz $\forall u$. We set the initial secondary inter-cell interference, the primary inter-cell interference, the local power consumption and all the dual variables to 0.01. Besides, we assume that $\beta_{u'} = -90$ dBm/Hz $\forall u' \in \mathcal{U}_p$.

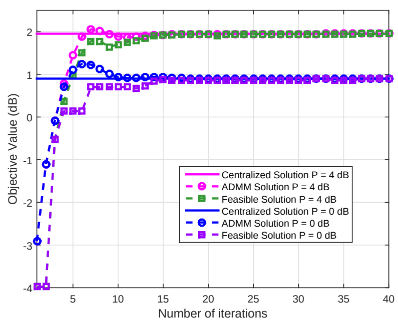

Figure 2. Convergence behavior of the proposed algorithm for different maximum total transmit power.

First, we consider different maximum total transmit power values and we set the accuracy $\epsilon = 10^{-3}$. Figure 2 illustrates the convergence behavior of the proposed algorithm. It can be noticed that our proposed decentralized solution converges to the centralized solution [12] in a reasonable number of iterations for different maximum total transmit power.

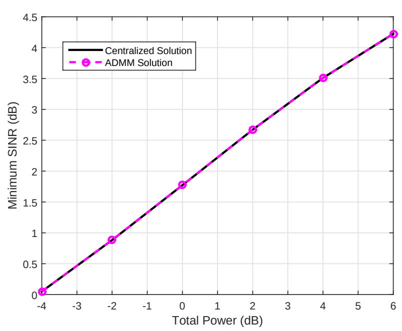

Figure 3. Minimum SINR (dB) versus the maximum total transmit power (dB).

Then, we compare the optimal minimum SINR obtained using the centralized algorithm and the proposed distributed algorithm. Figure 3 plots the optimal minimum SINR as a function of the maximum total transmit power. The figure clearly shows that the proposed decentralized solution achieves the same performance as the centralized solution for all values of the maximum total transmit power.

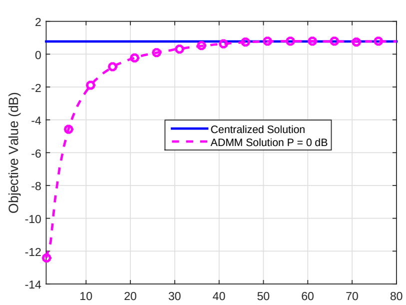

Figure 4. Convergence behavior of the proposed algorithm.

Finally, to simulate our algorithm in a larger network, assume that each secondary transmitter serves 14 single-antenna secondary receivers and each transmitter is equipped with 18 antennas. Further, assume that the network is formed by 16 single-antenna primary receivers. Figure 4 illustrates the convergence behavior of the proposed algorithm for target power $P = 0$ dB and penalty parameter $\rho = 0.5$. It can be noticed that the proposed ADMM-based distributed solution converges to the centralized solution in a reasonable number of iterations.

V. Conclusion

In this paper, the downlink of a spectrum sharing-based CR is considered. Using ADMM, the paper solves the SINR balancing optimization problem subject to the total power constraint of the secondary transmitters and the maximum interference seen by each primary user due to secondary transmissions constraints. The proposed solution leads to decentralized implementation by allowing a limited information exchange between the coupled secondary transmitters. Simulation results demonstrate that the proposed distributed SINR balancing algorithm converges to the centralized solution in a reasonable number of iterations.

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