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On Alternate Relaying with Improper Gaussian Signaling

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***Abstract*—In this letter, we investigate the potential benefits of adopting improper Gaussian signaling (IGS) in a two-hop alternate relaying (AR) system. Given the known benefits of using IGS in interference-limited networks, we propose to use IGS to relieve the inter-relay interference (IRI) impact on the AR system assuming no channel state information is available at the source. In this regard, we assume that the two relays use IGS and the source uses proper Gaussian signaling (PGS). Then, we optimize the degree of impropriety of the relays signal, measured by the circularity coefficient, to maximize the total achievable rate. Simulation results show that using IGS yields a significant performance improvement over PGS, especially when the first hop is a bottleneck due to weak source-relay channel gains and/or strong IRI.**

***Index Terms*—Improper Gaussian signaling, asymmetric complex signaling, alternate relaying, two-path relaying, decode-and-forward, inter-relay-interference.**

I. INTRODUCTION

Cooperative relaying has been widely recognized as a powerful coverage extension solution for next generation wireless communication systems through the implementation of multi-hop half duplex relays [1]. To compensate the spectral efficiency loss, full duplex relaying is proposed to double the spectral efficiency by allowing the relays to transmit and receive simultaneously in the same frequency band. The implementation of such systems is mainly governed by the progress of developing self-interference cancellation techniques, which are still not mature for practical implementation [2]. Alternate relaying (AR), or *two-path relaying*, is a distributed realization of full duplex relaying that can almost double the spectral efficiency without upgrading the network hardware [3]. The AR system uses two half-duplex relays that transmit and receive, alternately. As a result, the source (destination) transmits (receives) a message in every time slot similar to the full-duplex mode. The main limitation of the AR system is the so-called inter-relay interference (IRI).

Several interference mitigation techniques have been reported in the literature for decode-and-forward (DF) AR systems to partially or fully cancel the IRI [4]–[6]. Particularly, in [4], repetition codes (RC) are applied at both relays in which the relays forward the message from the source using the same codebook used at the source considering a direct link from the source to the destination. Moreover, the IRI is suppressed by successive decoding (SD) which decodes the IRI if it is stronger than the desired signal and subtracts it from the received signal. Otherwise, it performs single-user decoding (SUD) by treating the IRI as Gaussian noise. Dirty paper coding (DPC) and lattice based DPC are used in [5] and [6], respectively, in order to fully cancel the IRI. However, these schemes assume the availability of global channel state information (CSI) at the source which might be difficult to be realized in practice [7]. It is worth mentioning that the aforementioned techniques used a proper Gaussian signaling (PGS) scheme, which assumes uncorrelated real and imaginary signal components and equal power for each component. Improper Gaussian signaling (IGS) is a signaling scheme, where the aforesaid PGS characteristics are relaxed.

IGS has been shown to be beneficial in various interference-limited systems [8]–[11]. Firstly, it has been proven in [8] that by employing IGS, we achieve higher degrees of freedom for the 3-user single-input single-output interference channel (SISO-IC) than its PGS counterpart. The Pareto boundary of the achievable rate region for the 2-user SISO-IC was characterized in [9]. In cognitive radio systems, the merits of IGS were obtained when the primary user is not fully loaded and with weak primary user(s) direct channels and/or strong secondary user interference channels [10], [11].

In this letter, we employ IGS combined with DF in the AR system to relieve the interference impact on the achievable rate performance. Specifically, we investigate mitigating the IRI by adopting IGS at both relays while using PGS at the source. To this end, we tune the degree of impropriety, measured by the circularity coefficient, in order to maximize the total end-to-end achievable rate of the AR system. Then, we show that the benefits of adopting IGS can be reaped when the first hop is the bottleneck of the system performance, which can result from weak source-relay channel gains and/or strong IRI. To the best of the authors' knowledge, this is the first work to employ IGS in AR systems.

Notation: We use $|.|$ to denote the absolute value operation, $\mathbb{E}\{.\}$ to denote the statistical expectation and $\min\{x, y\}$ to denote the minimum between x and y.

II. SYSTEM MODEL

Consider a two-hop relay network consisting of one source node; S, two half duplex relay nodes; R_1 and R_2, and one destination node; D, as shown in Fig. 1. The relays transmit and receive in turn, i.e., in one time slot one relay receives and the other relay transmits, then, in the next time slot, the situation is reversed. As we use IGS throughout the paper, before proceeding further, some definitions are in order.

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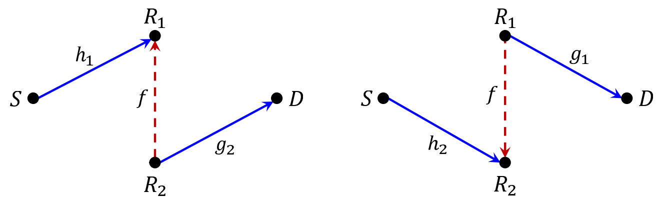

(a) Odd time slots (b) Even time slots

Fig. 1. A two-relay network based on the AR scheme. The blue solid lines represent the signal links and the red dashed lines represent the IRI links.

Definition 1. [12] A zero mean complex random variable x with variance $\sigma_x^2 = \mathbb{E}\{|x|^2\}$ and pseudo-variance $\tilde{\sigma}_x^2 = \mathbb{E}\{x^2\}$ is called proper if its pseudo-variance is equal to zero; otherwise, it is called improper.

Definition 2. [10] The circularity coefficient of the signal x is a measure of its impropriety degree and is defined as $\mathcal{C}_x = \left|\tilde{\sigma}_x^2\right|/\sigma_x^2$, where $0 \le \mathcal{C}_x \le 1$. In particular, $\mathcal{C}_x = 0$ and $\mathcal{C}_x = 1$ correspond to proper and maximally improper signals, respectively.

Let h_i and g_i, $i \in \{1,2\}$, denote the channel of the $S-R_i$ and $R_i - D$ links, respectively. The inter-relay channel is reciprocal and is represented by f. We assume that there is no direct link between S and D. Also, we assume that no CSI is available at S and hence, we use PGS at its transmitter. For simplicity and tractability, we consider a basic yet illustrative scenario by assuming the use of IGS at R_i with the same $\mathcal{C}_x$ for each relay. Furthermore, we assume that both S and R_i transmit with a fixed equal power p.

During time slot k, the received signal at R_i where $i = 2-\mathrm{mod}\ (k,2)$ is given by[1]

$$y_i[k] = \sqrt{p}h_i[k]s[k] + \sqrt{p}f[k]x_j[k] + n_i[k], \tag{1}$$

where $s[k]$ is the transmitted proper signal by S in time slot k and $n_i[k]$ is the additive white Gaussian noise (AWGN) at R_i. $x_j[k]$ is the transmitted improper signal by R_j, with circularity coefficient $\mathcal{C}_x$. The received signal at D in time slot k is given by

$$r[k] = \sqrt{p}g_i[k]x_i[k] + n[k]. \tag{2}$$

where $n[k]$ is the AWGN at D. All channels are assumed to be quasi-static block flat fading channels, and hence we drop the time index k for notational convenience. The additive noise at the receivers is modeled as a white, zero-mean, circularly symmetric, complex Gaussian with variance σ_n^2.

III. Improper Gaussian Signaling Design for Alternate Relaying

In this section, we investigate and analyze the AR system using IGS at R_i and PGS at S. First, we derive the total achievable rate of the AR system, then, we optimize the circularity coefficient in order to enhance the rate.

[1]For the rest of the letter, we assume $j \in \{1,2\}, j \neq i$.

A. Achievable Rates of the AR System with IGS

The AR system mimics a full duplex system by transferring the data through two Z-interference channels, where two transmitters (S and R_i) are broadcasting messages each intended for one of the two receivers (R_j and D) as shown in Fig. 1. Hence, as a result of using IGS at R_j and PGS at S while treating the interference as Gaussian noise, the achievable rate of the first hop of the ith path ($S - R_i$) can be expressed after some simplification steps as [9]

$$\mathcal{R}_{i,1}\left(\mathcal{C}_x\right)=\frac{1}{2}\log_2\left(1+\frac{\left(p|h_i|^2+2(p|f|^2+\sigma_n^2)\right)p|h_i|^2}{\left(1-\mathcal{C}_x^2\right)p^2|f|^4+2p|f|^2\sigma_n^2+\sigma_n^4}\right). \tag{3}$$

Similarly, the achievable rate of the second hop of the ith path ($R_i - D$) can be obtained as

$$\mathcal{R}_{i,2}\left(\mathcal{C}_x\right) = \frac{1}{2}\log_2\left(1+\frac{2p|g_i|^2}{\sigma_n^2}+\frac{p^2|g_i|^4\left(1-\mathcal{C}_x^2\right)}{\sigma_n^4}\right). \tag{4}$$

Accordingly, the total achievable rate of the AR system, for sufficiently large number of time slots, can be expressed as

$$\mathcal{R}_{\mathrm{T}}\left(\mathcal{C}_x\right) = \frac{1}{2}\sum_{i=1}^{2}\min\left\{\mathcal{R}_{i,1}\left(\mathcal{C}_x\right), \mathcal{R}_{i,2}\left(\mathcal{C}_x\right)\right\}. \tag{5}$$

One can notice that if $\mathcal{C}_x = 0$ in (5), we obtain the conventional expression for the total achievable rate of PGS.

Proposition 1. *$\mathcal{R}_{i,1}\left(\mathcal{C}_x\right)$ and $\mathcal{R}_{i,2}\left(\mathcal{C}_x\right)$ are strictly increasing and decreasing in $\mathcal{C}_x$ on the interval $0 < \mathcal{C}_x < 1$, respectively.*

Proof. The proof can be readily verified. □

From Proposition 1, if we increase $\mathcal{C}_x$, the rate of the first hop improves, and the rate of the second hop deteriorates, which creates a trade-off that can be optimized to maximize the AR total rate. As a result, the IGS can improve the AR rate if the first hop is the performance bottleneck, which occurs due to weak $S - R_i$ gains and/or strong IRI.

B. Circularity Coefficient Optimization

Here, we aim at optimizing the circularity coefficient $\mathcal{C}_x$ in order to maximize the instantaneous total achievable rate of the AR system. For this purpose, we formulate the following optimization problem

$$\begin{aligned} \mathbf{P1}: \max_{\mathcal{C}_x} \quad & \mathcal{R}_{\mathrm{T}}\left(\mathcal{C}_x\right) \\ \text{s.t.} \quad & 0 \le \mathcal{C}_x \le 1. \end{aligned} \tag{6}$$

This optimization problem is a non-convex optimization problem which makes it, in general, hard to solve optimally. However, by exploiting the properties of $\mathcal{R}_{\mathrm{T}}\left(\mathcal{C}_x\right)$, we show that the solution of the **P1** lies either on the boundaries, i.e., (0 or 1), or at the intersection of $\mathcal{R}_{i,1}\left(\mathcal{C}_x\right)$ and $\mathcal{R}_{i,2}\left(\mathcal{C}_x\right)$, or at the stationary point of the function $\mathcal{F}_{i,j}\left(\mathcal{C}_x\right) = \mathcal{R}_{i,1}\left(\mathcal{C}_x\right)+\mathcal{R}_{j,2}\left(\mathcal{C}_x\right)$, over the interval $0 < \mathcal{C}_x < 1$. Next, the intersection and stationary points, if exist, are computed from the following propositions.

Proposition 2. *There exists at most one intersection point of $\mathcal{R}_{i,1}\left(\mathcal{C}_x\right)$ and $\mathcal{R}_{i,2}\left(\mathcal{C}_x\right)$ on the interval $0 < \mathcal{C}_x < 1$. Moreover, this intersection point, if exists, is given by*

$$\mathcal{C}_i = \sqrt{1-\mu_i}, \tag{7}$$

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where

$$\mu_i = \frac{\sigma_n^4}{2p^2|g_i|^2|f|^4} \times \left(\sqrt{\Lambda_i^2 + 4\frac{|f|^4}{\sigma_n^4}\left(\Omega_i - 2p|g_i|^2\left(2p|f|^2 + \sigma_n^2\right)\right)} - \Lambda_i\right), \quad (8)$$

where $\Lambda_i = 2p\frac{|f|^2}{\sigma_n^2}\left(|g_i|^2 + |f|^2\right) + |g_i|^2$ and $\Omega_i = p^2|h_i|^4 + 2p|h_i|^2\left(p|f|^2 + \sigma_n^2\right)$.

Proof. The existence of at most one intersection point follows directly from Proposition 1. Moreover, we equate the functions $\mathcal{R}_{i,1}(\mathcal{C}_x)$ and $\mathcal{R}_{i,2}(\mathcal{C}_x)$ to form a quadratic equation in terms of $\mathcal{C}_x$ which can be solved to obtain $\mathcal{C}_i$. □

Proposition 3. *There exists at most one stationary point for $\mathcal{F}_{i,j}(\mathcal{C}_x)$ on the interval $0 < \mathcal{C}_x < 1$. Moreover, this stationary point, if exists, is given by*

$$\mathcal{C}_{\mathrm{st_i}} = \sqrt{1 - \mu_{\mathrm{st}_i}}, \quad (9)$$

where

$$\mu_{\mathrm{st}_i} = -p^2|f|^4\Psi\left(|f|^2\right) + \sqrt{p^2|f|^8\Psi^2\left(|f|^2\right)(p^2-1) + \frac{\Omega_i|f|^4\Psi\left(|g_j|^2\right)}{|g_j|^4} - \Omega_i\Psi\left(|f|^2\right)}, \quad (10)$$

and $\Psi(z) = \frac{\sigma_n^2\left(2pz+\sigma_n^2\right)}{p^2|f|^8}$.

Proof. We equate the derivative of $\mathcal{F}_{i,j}$ with respect to $\mathcal{C}_x$ to zero to form a quadratic equation in $\mathcal{C}_x$ which can be easily verified, by solving the equation, that it has only one positive root $\mathcal{C}_{\mathrm{st}_i}$. □

For notational convenience, we give the following definition.

Definition 3. *Let π denote the permutation of $\{1,2\}$ that sorts the intersection points $\mathcal{C}_i$, if exist, in an increasing order such that $\mathcal{C}_{\pi_1} \le \mathcal{C}_{\pi_2}$. Also, let $k_i(x) = \underset{a\in\{1,2\}}{\arg\min}\,\mathcal{R}_{i,a}(x)$.*

After calculating the intersection points from Proposition 2, the solution of $\mathbf{P1}$ can be computed from the following theorem.

Theorem 1. *In the AR system, where the two relays use IGS, the optimal impropriety degree, measured by the circularity coefficient, that maximizes the total achievable rate can be obtained as follows:*

Case 1: *no intersection points*

$$\mathcal{C}_x^* = \begin{cases} 0, & \text{if} \quad k_1(\mathcal{C}) = k_2(\mathcal{C}) = 2,\ 0 \le \mathcal{C} \le 1 \\ 1, & \text{if} \quad k_1(\mathcal{C}) = k_2(\mathcal{C}) = 1,\ 0 \le \mathcal{C} \le 1 \\ \underset{\mathcal{C}_x\in\{0,\mathcal{C}_{\mathrm{st}_i},1\}}{\arg\max} \mathcal{F}_{i,j}(\mathcal{C}_x), & \text{if} \quad k_1(\mathcal{C}) = i, k_2(\mathcal{C}) = j, 0 \le \mathcal{C} \le 1 \end{cases}. \quad (11)$$

Case 2: *one intersection point, $\mathcal{C}_i$*

$$\mathcal{C}_x^* = \begin{cases} \underset{\mathcal{C}_x\in\{\mathcal{C}_i,\mathcal{C}_{\mathrm{st}_j},1\}}{\arg\max} \mathcal{F}_{j,i}(\mathcal{C}_x), & \text{if} \quad k_j(\mathcal{C}) = 1,\ 0 \le \mathcal{C} \le 1 \\ \underset{\mathcal{C}_x\in\{0,\mathcal{C}_{\mathrm{st}_i},\mathcal{C}_i\}}{\arg\max} \mathcal{F}_{i,j}(\mathcal{C}_x), & \text{if} \quad k_j(\mathcal{C}) = 2,\ 0 \le \mathcal{C} \le 1 \end{cases}. \quad (12)$$

Case 3: *two intersection points, $(\mathcal{C}_{\pi_1}, \mathcal{C}_{\pi_2})$*

$$\mathcal{C}_x^* = \underset{\mathcal{C}_x\in\{\mathcal{C}_{\pi_1},\mathcal{C}_{\mathrm{st}_{\pi_2}},\mathcal{C}_{\pi_2}\}}{\arg\max} \mathcal{F}_{\pi_2,\pi_1}(\mathcal{C}_x). \quad (13)$$

Proof. For the first case, we have three different orientations for the minimum pair of rate functions for the two paths. The minimum pair is the two decreasing functions $\mathcal{R}_{i,2}(\mathcal{C}_x), \forall i$ and hence, their sum will also be decreasing and the optimal solution is $\mathcal{C}_x^* = 0$. Similar argument applies if the minimum pair is the two increasing functions yielding $\mathcal{C}_x^* = 1$. If the minimum pair is of opposite monotonicity, we need to compute the stationary point of their sum because if there is a maximum on $0 < \mathcal{C}_x < 1$, it must occur at the stationary point calculated from Proposition 3.

In the second case, the intersection point, $\mathcal{C}_i$, of the two hops rates of the ith path, divides the $\mathcal{C}_x$ range into two intervals. In the first interval $0 < \mathcal{C}_x \le \mathcal{C}_i$, the minimum rate of the ith path is $\mathcal{R}_{i,1}(\mathcal{C}_x)$, and in the second interval $\mathcal{C}_i < \mathcal{C}_x \le 1$, the minimum rate of the ith path is $\mathcal{R}_{i,2}(\mathcal{C}_x)$. For the jth path, we have two different orientations on $0 < \mathcal{C}_x < 1$, either the minimum is the first or the second hop and hence, by a similar argument as in Case 1, the result follows directly.

Finally, in the third case, we can write the total achievable rate as

$$\mathcal{R}_{\mathrm{T}}(\mathcal{C}_x) = \frac{1}{2} \times \begin{cases} \sum_{i=1}^{2} \mathcal{R}_{i,1}(\mathcal{C}_x), & \text{if} \quad 0 < \mathcal{C}_x \le \mathcal{C}_{\pi_1} \\ R_{\pi_2,1}(\mathcal{C}_x) + R_{\pi_1,2}(\mathcal{C}_x), & \text{if} \quad \mathcal{C}_{\pi_1} < \mathcal{C}_x \le \mathcal{C}_{\pi_2} \\ \sum_{i=1}^{2} \mathcal{R}_{i,2}(\mathcal{C}_x), & \text{if} \quad \mathcal{C}_{\pi_2} < \mathcal{C}_x < 1 \end{cases}. \quad (14)$$

By applying similar arguments as in the previous cases, the result follows directly and this concludes the proof. □

C. IGS-based schemes

In this work, we incorporate IGS to be implemented with different interference mitigation techniques proposing two schemes named as, IGS-SUD and IGS-[SD,SUD]. For IGS-SUD scheme, we treat the IRI as noise and design the circularity coefficient using Theorem 1. Different from the PGS-based scheme in [4] that switches between the SD and SUD techniques when the IRI is stronger than the desired signal, IGS-[SD-SUD] scheme chooses the technique that achieves higher end-to-end rate. In our combined scheme, SD technique uses PGS at both relays as it is the optimal in this scenario.

IV. Numerical Results

In this section, we numerically evaluate the average rate performance of the AR system using IGS. Throughout the simulation results, we compare our IGS-based schemes with the following benchmark techniques, PGS-SUD, PGS-[SUD, SD] reported in [4] but without the direct link, and the cut-set upper bound in [5], which can be achieved by using DPC. As for the simulation parameters, unless otherwise specified, we assume $p = 1$, $\sigma_n^2 = 1$ and statistically symmetric links with zero mean complex Gaussian distribution channels and variances of $\sigma_{h_1}^2 = \sigma_{h_2}^2 = \sigma_h^2 = 10$ dB, $\sigma_{g_1}^2 = \sigma_{g_2}^2 = \sigma_g^2 = 15$ dB and $\sigma_f^2 = 15$ dB. The nodes are assumed to be located in a two-dimensional plane where L and L_{SR} denote the distances of $S-D$ and $S-R$ links. Both relays are located symmetrically on a vertical line between S and D. We assume a shadowing of 5 dB and a path loss exponent of 2. The results are averaged over 10^5 independent channel realizations.

Firstly, we study the average rate performance of PGS and IGS with different schemes versus the average IRI link gain

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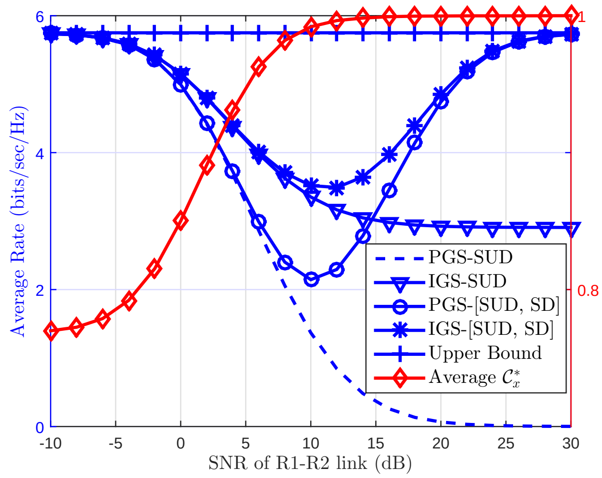

Fig. 2. The average achievable rates (blue curves) for PGS and IGS for different schemes versus the average IRI link gain σ_f^2 along with the average optimal circularity coefficient (red curve).

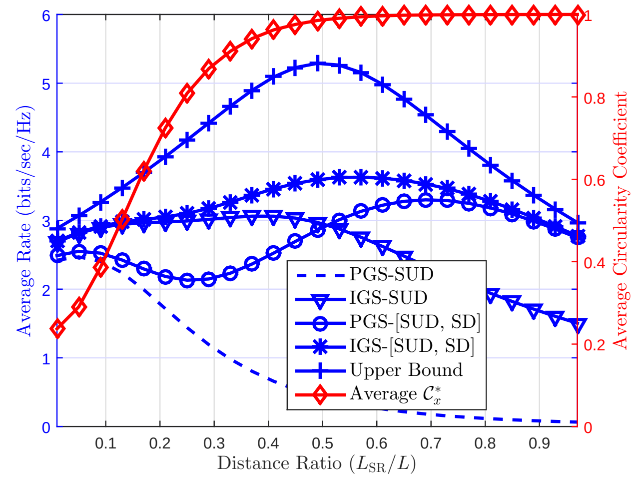

Fig. 3. The average achievable rates (blue curves) for PGS and IGS different schemes versus the distance ratio L_{SR}/L along with the average optimal circularity coefficient (red curve).

σ_f^2 as shown in Fig 2. Furthermore, we plot in the same figure the average optimal circularity coefficient. We observe that all methods coincide with the upper bound at very weak IRI as it can be neglected. In addition, only the schemes with IRI detection capability coincide with the upper bound at very strong IRI as it can be decoded perfectly. For PGS with SUD, the performance deteriorates significantly with increasing IRI, while PGS with combined SUD and SD improves at high IRI due to the expected decoding gain. On the other hand, IGS with SUD tolerates more IRI by increasing the signal impropriety and outperforms PGS-[SUD, SD] till certain IRI. Moreover, IGS-[SUD, SD] scheme achieves a significant performance improvement than its PGS counterpart. Furthermore, the switching point between treating interference as noise and decoding it shifts to the right as IGS provides higher rates than PGS. Hence, this significant advantage of IGS enables the use of the the simple strategy of treating interference as noise for a wider range of IRI values, while improving the rate gain.

Secondly, Fig. 3 shows both the average rate performance and average circularity coefficients versus the distance ratio L_{SR}/L. When R_i is close to S, the optimal signaling reduces to PGS and the $R_i - D$ link becomes the performance bottleneck. As R_i moves away from the S, the first hop quality deteriorates and the IGS boosts $\mathcal{C}_x$ to relieve the IRI impact. The IGS gain increases with higher values of $\mathcal{C}_x$ until maximally improper is reached, then the gain decreases. The performance of all schemes deteriorates as the relays get closer to D because of the continuous decrease of σ_h^2.

V. Conclusion

In this letter, we proposed the use of the IGS scheme at the relays in AR systems in order to mitigate the IRI. The degree of signal impropriety is optimally tuned to balance the trade-off between enhancing the first hop rate and deteriorating the second hop rate in order to maximize the total instantaneous achievable rate. The IGS benefits are reaped when the first hop is the bottleneck due to weak source-relay link gains or strong IRI. Moreover, these benefits do not appear only with SUD scheme but also with combined SD and SUD schemes. Future research lines include considering different circularity coefficient for each relay node.

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