Performance Limits of Communication with Energy Harvesting

Thesis by
Mohamed Ridha Znaidi

In Partial Fulfillment of the Requirements

For the Degree of

Master of Science

King Abdullah University of Science and Technology, Thuwal,
Kingdom of Saudi Arabia

April, 2016

The thesis of Mohamed Ridha Znaidi is approved by the examination committee

---

Committee Chairperson: Dr. Mohamed-Slim Alouini

Committee Member: Dr. Zouheir Rezki

Committee Member: Dr. Taous-Meriem Laleg-Kirati

Committee Member: Dr. Basem Shihada

ABSTRACT

Performance Limits of Communication with Energy Harvesting

Mohamed Ridha ZNAIDI

In energy harvesting communications, the transmitters have to adapt transmission to the availability of energy harvested during communication. The performance of the transmission depends on the channel conditions which vary randomly due to mobility and environmental changes. During this work, we consider the problem of power allocation taking into account the energy arrivals over time and the quality of channel state information (CSI) available at the transmitter, in order to maximize the throughput. Differently from previous work, the CSI at the transmitter is not perfect and may include estimation errors. We solve this problem with respect to the energy harvesting constraints. Assuming a perfect knowledge of the CSI at the receiver, we determine the optimal power policy for different models of the energy arrival process (offline and online model). Indeed, we obtain the power allocation scheme when the transmitter has either perfect CSI or no CSI. We also investigate of utmost interest the case of fading channels with imperfect CSI. Moreover, a study of the asymptotic behavior of the communication system is proposed. Specifically, we analyze of the average throughput in a system where the average recharge rate goes asymptotically to zero and when it is very high.

Key words

Average recharge rate, asymptotic average throughput, channel state information, channel estimation, dynamic programming, energy harvesting, optimal power allocation, throughput maximization.

ACKNOWLEDGEMENTS

I would first like to thank Allah, Almighty who has helped me, guided me and gave me the strength to complete this work. Next, I would like to gratefully acknowledge the guidance of my advisor, Prof. Mohamed-Slim ALOUINI for his tremendous support and advice throughout my Master degree at KAUST. I also thank him profusely for immeasurable support extended, life lessons that he has imparted and the confidence he has shown in me.

I would like to express my deepest gratitude to Dr. Zouheir REZKI, senior research scientist at KAUST, for his excellent guidance, assistance, sincere advice in both academic and non-academic issues and insight during the development of our work. His invaluable encouragement, support, constructive criticism and friendly attitude motivated me to complete this report.

I would take this opportunity to acknowledge my professors at KAUST, for priceless knowledge. They have been a source of great inspiration for me. I would like to extend gratitude to all my colleagues and friends at KAUST for help and a nice environment to work in. Special thanks to KAUST for providing me this opportunity to enrich myself.

Last but not the least, I would like to thank my parents, whose sincere prayers for the sake of my goodness always make me feel safe. Their everlasting encouragement has always been a source of tremendous support for me. I owe to them all my achievements. I would present my special thanks to my brother and my sister for their support and company throughout life.

April, 2016 - KAUST, Saudi Arabia　　　　　　Mohamed Ridha ZNAIDI

TABLE OF CONTENTS

LIST OF ABBREVIATIONS

The following acronyms and terms are used within this report.

AT	Average Throughput
ARR	Average Recharge Rate
AWGN	Additive White Gaussian Noise
CSI	Channel State Information
CSI-T	Channel State Information at the Transmitter
CSI-R	Channel State Information at the Receiver
EH	Energy Harvesting
KKT	Karush-Kuhn-Tucker
MMSE	Minimum Mean Square Error
PDF	Probability Distribution Function
RF	Radio Frequency
RV	Random Variable
RX	Receiver
TS	Time Slot
TX	Transmitter
WSN	Wireless Sensors Network

LIST OF SYMBOLS

The following mathematical symbols are used in this report.

Latin Symbols

B_i	Amount of energy in the battery during TS i
e_i	Epoch i
E_i	Energy harvested just before epoch i
E_{max}	Capacity of battery
$\mathcal{E}$	The set of possible energy states
h	Channel fading
$\widehat{h}$	Channel estimation
$\tilde{h}$	Channel estimation error
L_i	duration of an epoch
$\mathcal{L}$	Lagrangian
$\mathbf{M}$	The matrix of the transition probabilities
n	Noise
N	The number of TSs
N_s	The cardinal of the Markov chain
N_0	Noise variance
p_i	Power transmitted during the TS i
$P_{j,k}$	The transition probability from state j to state k
R_i	The reward function during TS i
S_i	The state at TS i
$\mathcal{S}^i$	The set of possible states in TS i given the state S_i
T	The deadline of transmission
t_i^e	Time of energy arrival
t_i^c	Time of fading change
W	Bandwidth

Greek Symbols

α	MMSE error variance
βi	Lagrangian multiplier associated to the causality constraint
ηi	Lagrangian multiplier associated to the non-overflow constraint
γ	Fading level
$\hat{\gamma}$	Fading level estimation
$\tilde{\gamma}$	Fading level estimation error
λ_i	Lagrangian multiplier associated to the positiveness constraint
π^*	The steady state probability
σ^2	Channel variance

Notations

$\mathbb{E}_E(\cdot)$	The expectation over the energy arrival
$\mathbb{E}_\gamma(\cdot)$	The expectation over the fading
$f_X(\cdot)$	The probability density function
$M_n(\mathbb{R})$	The set of real matrices $n \times n$
$\mathbb{P}(A\|B)$	The probability of A given B
$X(dB)$	$= 10 \log 10(X)$
$X \sim \mathcal{CN}(0, \sigma^2)$	The random variable X has a complex normal distribution with mean 0 and variance σ^2

Definitions

We define:

- The average throughput (AT): the throughput of the communication system per second. i.e.,

$$AT = \frac{1}{T} \sum_{i=1}^{N} T_i, \tag{1}$$

where T_i denotes the throughput at epoch i.

In this work, we use these notations for AT:

- AT_{Stat}: AT when the channel is static.
- $AT_{P.CSI-T}$: AT when CSI-T is perfectly known.
- $AT_{N.CSI-T}$: AT when CSI-T is unavailable.
- $AT_{I.CSI-T,\alpha}$: AT when CSI-T is imperfect.

- The average recharge rate (ARR): the average of energy harvested over the deadline T. i.e.,

$$ARR = \frac{1}{T}\mathbb{E}_{\underline{E}}\left(\sum_{i=0}^{N-1} E_i\right), \tag{2}$$

where E_i denotes the amount of energy harvested at epoch i and the averaging is over the distribution of energy arrivals.

LIST OF FIGURES

Chapter 1

Introduction

1.1 Motivation

Today, with the era of advances in the wireless communications field, as the demand for high performance of the wireless communication systems is increasing in many industries, the energy consumption in the communication process between nodes is becoming excessive. In fact, some studies estimate that the energy of communication networks is rising each year [1]. As a result, the energy efficiency consumption has been an active research area during last decades. With the emergence of battery limited wireless devices, i.e., sensor nodes, cell phones, etc., designing an efficient system is becoming a crucial issue in the design of wireless sensor networks (WSNs) architecture. In order to solve the energy inefficiency problem, alternative power sources should be deployed. Therefore, Energy Harvesting (EH) technique is recognized as a promising technology. Indeed, recent improvements in EH technique have made the recharge process feasible for wireless devices with battery limited by harvesting ambient energy from different sources such as the wind, vibrations, the sun, etc.

1.2 Applications

EH technology is becoming a relevant aspect of the wireless communication development. This technology has caught considerable interest as an environmentally friendly provision of energy for the communication systems. This technology used for green communication promises a potentially infinite lifetime of wireless systems [2]. The EH sensors are fast emerging as viable options for many applications.

Specifically, solar energy is considered one of the most promising energy sources and typical environmental monitoring applications that are running with accessing to solar energy. In addition, converting mechanical energy to electrical energy using piezoelectric materials has been the choice for many EH applications. So, this technique can be considered as a solution to the limited resources of fossil fuel. As known, by using this technology, batteries are solely considered as energy buffers and not as primary energy sources as suggested in the conventional systems. So that, the size, weight and cost of batteries can be minimized significantly. Last but not the least, the EH concept has attracted the interest of the communications community to enhance the energy efficiency of a WSN. Also, different path is taken in communication powered by energy harvester nodes where the researchers introduce the concept of energy cooperation in EH communications [3–6]. In order to enhance the performance of EH communication systems, energy transfer is considered in the context of EH networks. This idea is a promising solution to the problem of energy inefficiency. In fact, radio-frequency (RF) signals are presented not only as information signals but also as sources of energy that enhance the performance of the communication system [7–9]. Thus, the receivers (RXs) can harvest energy from RF signals. Besides, the concept of energy transfer presented via EH technology has been introduced in the cognitive radio framework to solve the problem of spectrum inefficiency, e.g., [10]. Using this technique, EH communication system can improve the reliability of the communication extensively. Consequently, the EH paradigm is considered a breakthrough achievement in the design of green communication systems, it provides an exciting solution to the problem of energy inefficiency and spectrum inefficiency.

1.3 Project framework

1.3.1 Objectives

In the EH communication system, the energy required for the transmission, is extracting from outside sources and becomes available at the buffer of the battery during the time of transmission. The constraints induced by the harvested energy plays an important role in the design of efficient communication systems. In fact, the performance of wireless communication systems, powered by energy harvested transmitters (TXs), is determined by adapting the transmission power to the energy availability. My work during this project focuses on the optimal design of transmission policy for EH wireless communication systems. In fact, my task is to develop an optimal policy for an EH-TX broadcasting data to RX through a wireless channel under EH constraints for different cases of channel state information at TX (CSI-T) availability. Also, the aim of this project is to study the asymptotic performance of such system, in order to provide analysis for the applications that are running in the low and high energy regime.

1.3.2 Related works

The performance of wireless system, powered by energy harvested TXs is determined by adapting the transmission power to energy availability. The analysis of this performance leads to several transmission methods and power allocation policies that were presented in recent works. In this work, we focus on the point-to-point communication where TX is powered by energy harvester node. Such framework was introduced in [11–20]. These works are classified into two categories depending on the energy arrival model: the offline case, considered for instance in [11–14]; the online case, addressed in [15–20]. In the first model (offline), TX has a perfect and causal knowledge of the energy arrivals, whereas in the second scenario (online) a random EH model is considered. In [11], an EH system is considered assuming that the CSI-T and the CSI-R is perfect. Achieving the optimal data rate requires efficient energy storage devices that can provide the anticipated power policy. Nevertheless, the storage devices, in practice, have storage losses particularly leakage that has an impact on the optimal power policy. A battery with energy leakage is considered for the problem of the offline power scheduling for EH systems in [12]. The transmission completion time minimization problem is solved in [13, 14]. In [14], the authors solve the problem taking into account both the arrivals of the harvested energy and the availability of the data over time. Considering the throughput maximization problem, the optimal transmission in the case where the EH model is random was studied in [15]. The challenge in EH systems is how to allocate the energy resources. Thus, increasing the lifetime of the energy storage device has been proposed in [16] where the authors proposed a dynamic power management policy to stabilize data queues. In order to prolong lifetime of wireless network, optimal transmission policies taking into account both transmission and processing energy costs are studied in [17]. Reference [18] suggested a sub-optimal power policies for perfect and imperfect knowledge of the energy. In [19], a Markov process was proposed to solve the throughput maximization problem with 1-bit channel feedback. The authors in [20] proposed a data-driven stochastic EH model to solve the throughput maximization problem.

1.3.3 Contributions

As known in practical wireless communication systems, the fading channel changes randomly. Naturally, the fading level is not perfectly known at TX due to the noise in the feedback link and other limitations [21]. Due to effects of fading, the strength of the channel can fluctuate during the transmission. A solution to resist to this effect is through a dynamic strategy of power allocation during data transmission. The design of the optimal power policy during transmission for different degree of CSI is important to enhance the performance of our system. Also, deployment of practical EH communication systems does not seem feasible without assessing the loss incurred by imperfect CSI on the performance [22], [23], [24] and [25].

The framework of the present project is to study the optimal power management for system powered by energy harvester node. Specifically, our work focuses on practical wireless systems where the CSI-T is imperfect and studies the optimal power policy. Namely, we assume that TX has an estimated version of the actual channel gain obtained, for instance, through a minimum mean square error (MMSE) filter of a feedback link. This model of CSI-T is widely accepted by the community and has been extensively adopted, e.g, [22] and [23].

In this work, we have started by investigating the optimal offline scheduling for maximum throughput by a deadline T under the assumption of imperfect CSI-T, where a perfect knowledge of the amounts of energy harvested is provided prior the data transmission and we assume a statistical knowledge about the channel. Then, we have focused on more practical scenario by developing an online model that takes into account the randomness of the amounts of energy harvested. In fact, we have designed a stochastic model to capture the dynamics of the EH communication system in the settings where the energy arrival process is not perfectly known at TX under the assumption of imperfect CSI-T. Furthermore, during this work, we have also studied of utmost interest the performance of EH systems by analyzing the asymptotic behavior of the average throughput (AT) where the energy harvested amounts are very small and when they are very high for different cases of CSI-T.

1.3.4 Report Overview

The report is organized into five chapters, including this introductory chapter. Chapter 2 focus on EH concept. We start by presenting advantages, challenges and sources of energy that can be harvested. Afterwards, we detailed some properties of communication system powered by EH technology by dealing with the constraints and the properties of power policies in such context.

In Chapter 3, we analyze the fundamental communication limits of EH wireless sensor networks in the offline case, when the energy arrival is known before communication. In fact, we investigate the optimal offline policy for different degree of CSI-T availability. Moreover, we compare the asymptotic performance of the communication system even at low and high values of the average recharge rate in each case.

In Chapter 4, we study the communication based on EH technology in the online case when we modeled the energy arrival by random Markov model. In fact, we propose a model that can take into account the randomness of the energy arrival. We provide a solution for solution for the optimization using dynamic programming.

Chapter 5 concludes this report.

Chapter 2

Energy Harvesting

2.1 Introduction

As discussed in the previous chapter, today the EH technique has emerged many applications. Specifically, this technology is considered important and relevant for WSNs to maintain the independence of a deployed infrastructure. Also, in some scenarios, replacing or recharging nodes' batteries is not practical due to the difficulty of the accessibility as well as the large number of the sensor nodes. Thus, EH is considered recognized as a solution to enhance the performance of sensor networks. Indeed, Using nodes powered by energy transducers, the generated energy increases significantly the autonomy of the nodes.

2.2 Advantages

The rise of energy consumption has been considered an important issue in the design of network architecture. Therefore, WSNs which are harvesting energy from their environment have been developed in the last decade. Specifically, scavenging energy via solar energy is considered as one of the most important energy sources for different applications based on this kind of energy. Also, equipment powered with photo-voltaic cells, can be performed and operation that executed by the equipment becomes possible without replacing and recharging the batteries, frequently. In addition, EH technology allows batteries with limited capacity to reduce the cost of sensor nodes.

2.3 Challenges

EH needs hardware as well as software control. Thus, adaptive software solutions for EH systems must be designed in order to control the power and avoid wasting energy. Indeed, simple and low-complexity solutions are needed for sensor nodes that sense and transmit data. As a result, the lifetime of the network can be optimized by managing the harvested energy [27].
Subsequently, the power scheduling decides on how to allocate energy based on the energy harvested profile. In addition, the sensor node has to adapt the scheme of power to the energy arrival by deciding when to use that energy at that moment. Therefore, this adaptation has the goal to maximize the utility of an application in a long-term perspective. As shown, the performance of a system powered by EH nodes depends mainly on the energy profile, so there are many challenges to deal with such as:

- How the sensor node should manage the time between transmitting data and recharging battery.
- What is the optimal dimension for the battery to fit the variations of energy.
- If the battery is empty or full, respectively, how can the system avoid the energy underflow or overflow, respectively.
- How can the harvester node uses the energy harvested for measurements, calculation and data transmission to improve optimally its performance, with respect to the hardware constraints.

2.4 Environmental energy harvesting sources

In this part, an overview about potential EH sources available on the market is presented. Generally, the source and amount of the harvested energy is determined by the application and the environment.

Vibrational Energy

Vibration is a source of mechanical energy which can be easily obtained from ambient environment and directly converted into electrical energy through various transducers such as the electromagnetic, electrostatic or piezoelectric transducer. These transducers are widely employed in the vibration energy harvesters due to their simplicity in use and high power generation capacity. These vibrations caused by some aerodynamic instability phenomena were considered recently as unwanted and even destructive. But several investigations and studies carried out on these aerodynamic instabilities showed that the recuperated oscillations can be exploited as a source of self-powered renewable energy for small energy harvesters. In addition, human activity can be the source of vibrational energy using shoes which harvests energy from human walking activity.

Thermal Energy

As known, the presence of a gradient of temperature between two different semiconductors causes a voltage. Many commercial products, exploiting the thermoelectric conversion, have been developed in order to convert heat into electricity which can be used for different application areas such as bio-medical implants, electronic systems as well as the WSNs. In fact, there is a great challenge to design performed systems functioning with this conversion, since it is characterized by low efficiency. As a matter of fact, to design materials that run in the high temperature regime, and that can conduct electricity well without considering heat is, really, considered a trade-off. Achieving this goal can increase efficiency of the discussed conversion.

Solar Energy

Solar energy is becoming one of the most powerful EH sources because it is among the most promising due its higher power density. Moreover, another reason why solar energy is considered a crucial one is that solar energy is the most energy that can be predicted. As a result, the prediction of energy can enhance the performance of systems by designing a planning of the future energy consumption. Recently, multiple solar powered prototype have been designed and performed more efficient energy conversion [26].
Using these environmental forms of EH, WSNs has grown dramatically in the last two decades, a considerable effort that is focused on the development of producing energy from sources discussed previously. The challenging fact is that the power distribution along time is not available. For that reason, resolving the power allocation is crucial for systems powered by energy harvested nodes.

2.5 EH for communication systems

EH is a promising technology for green communication which has the potential to provide a powerful solution to the problem of limited lifetime of batteries. Employing systems powered by energy harvester nodes introduces new challenges related to performance of these systems. An important factor that determines this performance is the EH profile, which models the variation of the harvested energy with time. Therefore, the utilization of the harvested energy is constrained by the EH profile that determines a feasible energy consumption domain containing the feasible policies adapted to energy arrivals.

To understand fundamental performance limits of communications with EH, we consider during this work the point-to-point communication link during a fixed deadline. In fact, we solve the throughput maximization problem subject to the constraints required by the EH profile in order to derive the optimal power allocation taking into account the energy arrivals during the deadline T.
Hence, we considered a system where a node transmits data continuously over time. Its rate can be evaluated via power control. Indeed, at instant t, TX chooses to transmit with power $p(t)$ in order to achieve a rate $r(t)$. The rate is a non-negative, increasing and strictly concave function: power-rate function. During all the work, we referred to Shannon capacity relation between power and the rate for AWGN channel:

$$r(p(t)) = \frac{1}{2}\log_2(1 + p(t)). \tag{2.1}$$

Remark 1.

According to properties of the power-rate function, we note that an necessary condition for power allocation to be optimal is that the transmit power must kept constant between energy arrivals.
During this work, we considered an EH system with a finite capacity battery. The battery can support up to energy E_{max} units of energy. This capacity is assumed to be constant throughout the lifetime of the battery. We define energy feasibility as the property to keep the battery energy non-negative and below its capacity, i.e. the amount of energy stored in the battery at each instant must be in the interval $[0, E_{max}]$. The energy recharge process is modeled as a discrete process with packets of energy harvested amounts arriving at different time instants as shown in 2.1.

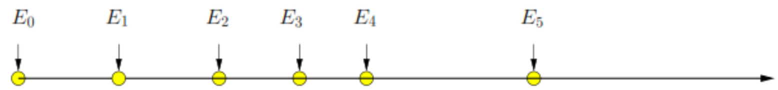

Figure 2.1: Energy harvesting model.

We note that the instantaneous power can not take any values, only allowed amounts by system definition and energy profile are valid. Thus, the amounts of energy available at any time is constrained, either because sufficient amount may not yet be harvested or cannot be stored in the battery. As a result, there is an energy feasibility domain on the transmission policy.

2.5.1 EH constraints on the transmission policy

As discussed in Section 2.5 the transmission of data under the deadline T is constrained by the energy profile. So, if we denote the power management policy $p(t), \forall t \in [0, T]$, there are two constraints on $p(t)$. The causality constraint which states that energy that has not arrived yet cannot be used. Moreover, due to the finite battery storage capacity, it is crucial to make sure that energy level in the battery never exceeds E_{max}.

Thus, these constraints define a feasible energy tunnel that limits power policies can be adopted during communication. In fact, the energy consumption profile must reside in the energy feasibility tunnel shown in Fig. 2.2.

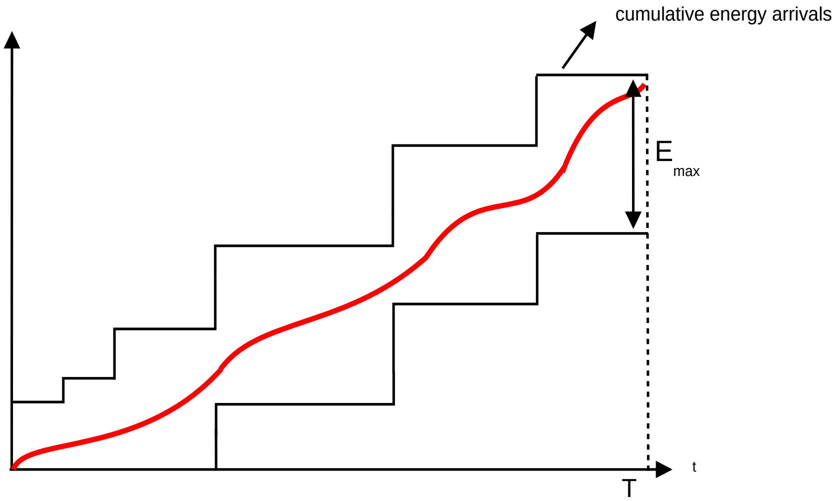

Figure 2.2: Energy feasibility tunnel.

2.5.2 Properties of the optimal policy

Optimal power policy $E_{max} = \infty$

When we consider an infinite battery storage capacity, the optimal power management policy has some properties as shown in [28, 29].

Property 1: Transmit power should remain constant between energy harvests.

Property 2: The transmission rates cannot changes unless the buffer is completely empty.

Property 3: Transmit powers increase monotonically.

These properties highlight necessary conditions for a power policy to be optimal, so as to find the tightest curve under the cumulative energy arrival staircase.

Optimal power policy $E_{max} \leq \infty$

Property 1: Transmit power should remain constant between energy harvests.

Property 2: Transmit power cannot increase if the battery is full.

These properties define the space of all feasible policies in the energy feasibility tunnel. In addition, to deploy an optimal power policy we suppose that all energy harvested during communication is used only for data transmission.

2.6 Conclusion

In this chapter, the general field of the project, EH, was presented by dealing with the WSNs in the beginning of this chapter then detailing the advantages, the challenges and the mechanisms of the concept of EH that is rapidly developing in recent researches. Finally, we present the constraints of transmission of data under EH constraints and some properties of an optimal power policy in such context.

Chapter 3

The Offline Analysis

3.1 Introduction

In this chapter, we consider an EH communication system, where energy needed for communication is harvested by TX during the course of communication. This chapter focuses on the optimal design of transmission scheduling policy for such system taking into account the energy arrivals over time and the quality of the CSI-T available. In fact, we consider an offline model where TX has a perfect knowledge about the energy profile and the instants of the arrivals prior to transmission scheduling. Also, this model suggests that TX disposes information about the instants when the fading level changes. Then, we determine the asymptotic performance of the suggested system regarding the amounts of energy arrivals.

3.2 System model

3.2.1 System description

We consider a point-to-point data transmission, during a deadline T. We assume that the energy needed for transmission is harvested by TX during the

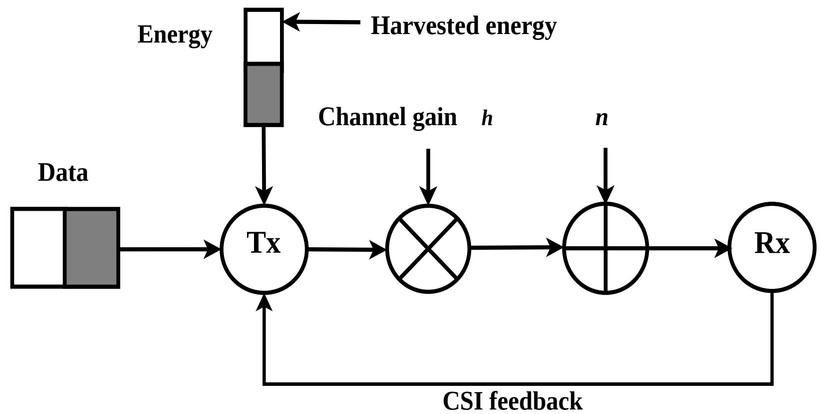

Figure 3.1: An energy harvesting communication system model with CSI feedback.

communication, as shown in Fig.3.1. The data buffer is not empty at any time $t \in [0, T]$. We assume that the transmission is performed in time slots (TSs). The channel between TX and RX experiences fading. The base band received signal y over a bandwidth W, is given by:

$$y = hx + n, \tag{3.1}$$

where $h \sim \mathcal{CN}(0, \sigma^2)$ is a zero mean circularly symmetric complex variable with variance σ^2, x is the channel input, and n is a zero-mean AWGN with spectral density N_0, and is independent of h. We define γ as $\gamma = |h|^2$. The energy available in the battery determines the bits that can be transmitted during each TS of duration L. During each epoch, TX encodes the bits to be sent as data symbols, where the block length of each symbol is assumed to be large enough so that we can guarantee the reliability of the decoding process. A feedback link is considered between the RX and TX, the CSI feedback is sent to TX from RX and it is required only at the beginning of TSs.

We consider the throughput maximization, schedule and allocate power in a system comprising energy harvester node. Scheduling decisions are made according to the amount of energy harvested and channel conditions. Depending on the quality of the CSI-T, we propose an analysis of the performance of the proposed

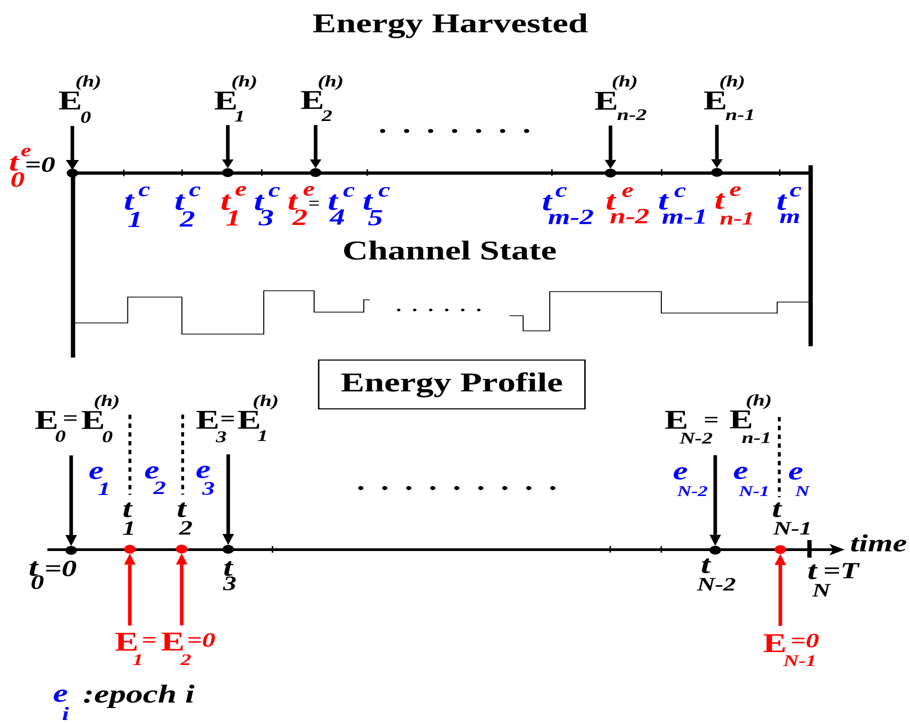

Figure 3.2: Energy arrival and fading channel during a deadline T for the offline model.

communication system.

3.2.2 Channel Model

The channel experiences fading during the transmission of data, in some special cases, the channel is considered static. It varies randomly due to the environment. In most operating scenarios, the wireless channel varies slowly over time, so the fading level is assumed to be constant at each epoch during the transmission of data. That is why, we can consider a change in the channel fading level in a discrete time $t_1^c, t_2^c, ..., t_m^c$, marked with blue color in Fig.3.2, and the fading level in $[t_i^c, t_{i+1}^c]$ is h_i. We assume that throughout the communication, perfect CSI-R is available. Different scenarios for the availability of the CSI-T are studied in this work. In the case where the CSI-T is perfect, we consider that changes in fading are known before the beginning of the transmission. On the other hand, if TX does not know perfectly the instantaneous CSI due to different sources of errors in the estimation process, TX estimates the actual channel and its estimate follows a linear model given by:

$$h = \sqrt{1-\alpha}\,\widehat{h} + \sqrt{\alpha}\,\tilde{h}, \tag{3.2}$$

where α is the error variance, $\alpha \in [0, 1]$. TX has knowledge only about $\hat{h}$ which designates the actual channel estimation which $\mathcal{CN}(0, \sigma^2)$ independent of the channel estimation error denoted as $\tilde{h}$ which is also $\mathcal{CN}(0, \sigma^2)$.

3.2.3 Energy Model

We consider the case where energy arrives at TX in a discrete time $t_0^e, t_1^e, ..., t_{n-1}^e$, marked with red color in Fig.3.2. At t_i^e, $E_i^{(h)}$ units of energy arrive, and the battery is assumed to be empty initially. The transmission begins at $t_0^e = 0$ when the energy $E_0^{(h)}$ arrives at TX. The energy is used only for data transmission, the unused energy is stored in the battery which has a limited capacity equal to E_{max} units of energy. The energy consumed for sensing at each TS, channel estimation and data compression is assumed negligible. In this model, we assume that the amount of energy arrival is a random variable with a density function $f_E(.)$, and the instants of energy arrivals are known before the communication starts.

Let us define the number of events by N, i.e., $N = m + n - k$, where m designates the number of times the channel state changes, n designates the number of times the energy arrives and k is the number of times when the channel changes and the energy arrives at the same time. We denote E_i the amount of energy arrival at each event $i \in [\![1, N]\!]$. To simplify the expressions and to have the same number of epochs for different scenarios studied in this work, we denote $E_i = 0$ if the event i is associated to times related to channel state change only. We denote by $\{t_0, t_1, ..., t_{N-1}\}$ the times of an event i.e., either a change in the fading or an energy arrival and $t_N = T$. Hence, we have N epochs denoted by $e_i; i = 1, \ldots, N$, marked with blue color in Fig.3.2, during the transmission.

3.2.4 Energy Harvesting constraints:

As mentioned in the previous chapter, in order to obtain the power policy the consumption of the energy harvested is constrained by the energy profile. Therefore, we assume the constraints existing in EH systems [11]. Accordingly, we consider the causality constraint which means that TX can not use, at the current time, energy that has not arrived yet. Following recent works [11] and [13], we assume that the transmitted power must be kept constant in each epoch. We denote the power consumed in each epoch i by p_i. So, the causality constraint can be expressed as:

$$\sum_{j=1}^{i} L_j p_j \leq \sum_{j=0}^{i-1} E_j, \forall i \in [\![1, N]\!]. \tag{3.3}$$

Because the energy arrives randomly at different time points, it is crucial to ensure that the energy level in the battery never exceeds E_{max}. This outlines the energy storage constraint on the power policy which can be expressed as:

$$\sum_{j=0}^{i} E_j - \sum_{j=1}^{i} L_j p_j \leq E_{max}, \forall i \in [\![1, N-1]\!]. \tag{3.4}$$

These constraints design a limited region of possible power policies that are feasible.

3.3 Optimal offline power policy in static channel

In this section, we explore the offline point-to-point throughput maximization problem subject to the different constraints when the channel is static, i.e., there is no fading, during the data transmission is defined as follows:

$$
(P):\begin{cases}\max\limits_{p_i\geq 0}\ \sum\limits_{i=1}^{N}\dfrac{L_i}{2}\log(1+\overline{\gamma}p_i) & \text{(3.5a)}\\[2ex] \text{s.t.}\begin{cases}\sum\limits_{j=1}^{i}L_jp_j\leq\sum\limits_{j=0}^{i-1}E_j,\forall i\in[\![1,N]\!]\\ \sum\limits_{j=0}^{i}E_j-\sum\limits_{j=1}^{i}L_jp_j\leq E_{max},\forall i\in[\![1,N-1]\!].\end{cases} & \text{(3.5b)}\end{cases}
$$

It can be shown that (3.5) is equivalent to a convex optimization problem, where the objective function is concave and the different constraints are linear. Thus, this problem has a unique maximum and it can be solved using standard techniques. We define the Lagrangian function for this problem for $\beta_i \geq 0$ and $\eta_i \geq 0$:

$$
\begin{aligned}\mathcal{L}=-\sum_{i=1}^{N}\frac{L_i}{2}\log(1+\overline{\gamma}p_i)+\sum_{i=1}^{N}\beta_i\left(\sum_{j=1}^{i}L_jp_j-\sum_{j=0}^{i-1}E_j\right)-\sum_{i=1}^{N}\lambda_ip_i\\ +\sum_{i=1}^{N-1}\eta_i\left(\sum_{j=0}^{i}E_j-\sum_{j=1}^{i}L_jp_j-E_{max}\right),\end{aligned}\tag{3.6}
$$

with the additional complementary slackness conditions as:

$$
\beta_i\left(\sum_{j=1}^{i}L_jp_j-\sum_{j=0}^{i-1}E_j\right)=0,\forall i\in[\![1,N]\!],\tag{3.7}
$$

$$
\eta_i\left(\sum_{j=0}^{i}E_j-\sum_{j=1}^{i}L_jp_j-E_{max}\right)=0,\forall i\in[\![1,N-1]\!],\tag{3.8}
$$

$$
\lambda_ip_i=0,\forall i\in[\![1,N]\!],\tag{3.9}
$$

where β_i's and η_i's are the Lagrange multipliers related to the causality and the no-energy-overflow constraints, respectively.

Hence, if $p_i > 0$ the KKT conditions for this problem are:

$$\frac{1}{1+\overline{\gamma}p_i^*} = 2\left(\sum_{j=i}^{N}\beta_j - \sum_{j=i}^{N-1}\eta_j\right), \forall i \in [\![1, N-1]\!]. \tag{3.10}$$

From (3.10) we get,

$$p_i^* = \left[\frac{1}{2\left(\sum\limits_{j=i}^{N}\beta_j - \sum\limits_{j=i}^{N-1}\eta_j\right)} - \frac{1}{\overline{\gamma}}\right]^+, \forall i \in [\![1, N-1]\!], \tag{3.11}$$

and,

$$p_N^* = \left[\frac{1}{2\ \beta_N} - \frac{1}{\overline{\gamma}}\right]^+. \tag{3.12}$$

This solution can be interpreted as a *Water-Filling* algorithm.

Proposition 1. The optimal power policy suggests that the causality constraints must be satisfied with equality for the last epoch, i.e.,

$$\sum_{j=1}^{N} L_j p_j^* = \sum_{j=0}^{N-1} E_j. \tag{3.13}$$

Proof. The proof is immediate and thus omitted. □

Remark 2. It is clear form the objective function that the optimal powers do not depend on the channel gain $\overline{\gamma}$. The optimal solution depends only on the energies harvested along time.

It is obvious that the Lagrange multipliers β_i and η_i depend only on the energy profile. So, if the amount of the energy arrived at epoch e_i is null. This epoch can be associated to time of change in the fading and no energy arrival at the same moment as explained previously, the power at the next epoch is kept constant. i.e., if $E_i = 0$, we have: $p_i^* = p_{i+1}^*$.

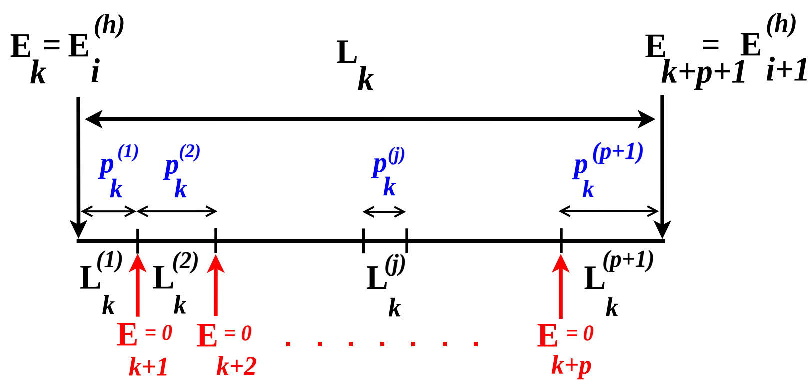

Figure 3.3: An epoch with static channel between two energy arrivals.

Hence, between two successive energy arrivals $E_i^{(h)}$ and $E_{i+1}^{(h)}$ if we suppose that the channel state change p times in the case where we have fading, the power is constant in each epoch between these energy arrivals as shown in Fig.3.3. i.e.,

$$p_k^{(j)*} = p_k^*. \qquad \forall j \in [\![1, p+1]\!] \tag{3.14}$$

So, the energy consumed during the epochs $(k+1, ..., k+p+1)$ between the two energy arrivals is:

$$\sum_{j=1}^{p+1} L_k^{(j)} p_k^* = p_k^* \sum_{j=1}^{p} L_k^{(j)} = p_k^* L_k. \tag{3.15}$$

Next, we focused on the analysis of system's performance in these assumptions. We studied the asymptotic behavior of the average throughput as the average recharge rate decreases to zero or has a high value depending on the availability of the energy that can be harvested.

3.3.1 Low ARR Regime

Now, we investigate of utmost interest the optimal power policy in the case where the energy arrivals amounts are very small. A key-motivation of studying this asymptotic regime is that many applications are working in such regime. Then, this targets applications where the energy harvested is very small at each epoch, i.e., $E_i \to 0$, $\forall i$. In this regime, since the battery capacity E_{max} is greater than the energy harvested, constraint (3.4) is always inactive. Hence, our optimization problem can be written as follows:

$$(P_1): \begin{cases} \max\limits_{p_i \geq 0} \sum\limits_{i=1}^{N} \frac{L_i}{2} \log(1+\overline{\gamma} p_i) & (3.16a) \\ \text{s.t.} \sum\limits_{j=1}^{i} L_j p_j \leq \sum\limits_{j=0}^{i-1} E_j, \forall i \in [\![1, N]\!]. & (3.16b) \end{cases}$$

As developed previously, the optimal solution is given by a *Water-Filling* algorithm. In the regime of interest, the optimal has a simple closed form in the sense that it is exempt of Lagrange multipliers as formalized by Theorem 1.

Theorem 1. *The optimal power allocation, denoted by* $[p_1^*, p_2^*, ..., p_N^*]$, *of problem* (P_1) *when the energy arrivals are all asymptotically small is obtained as follows. Denoting* $i_0 = 0$,

$$i_k = \underset{i \in [\![i_{k-1}+1, N]\!]}{\text{argmin}} \left\{ \frac{\sum\limits_{j=i_{k-1}}^{i-1} E_j}{\sum\limits_{j=i_{k-1}+1}^{i} L_j} \right\}, k \geq 1.$$

The optimal power allocation for epochs $i_{k-1}+1$,$i_{k-1}+2$,...,i_k $k \geq 1$ *is given by:*

$$p_i^* \approx \frac{\sum\limits_{j=i_{k-1}}^{i_k-1} E_j}{\sum\limits_{j=i_{k-1}+1}^{i_k} L_j}, i \in [\![i_{k-1}+1, i_k]\!]. \qquad (3.17)$$

Proof. See Appendix A.1. □

Remark 3. • The optimal power policy in the low *ARR* suggests that the powers are very low.

• We note that the power policy given by Theorem 1 is also optimal as long as $E_{max} \to \infty$ without the need of $E_i \to 0$. This can be shown along similar lines of the proof. In fact, this power allocation is optimal whenever constraint (3.4) is inactive. In that case, the asymptotic equality in (3.17) ($\approx$) is substituted by an equality ($=$).

• The epochs i_k can be seen as the transition slots where the water level changes after these slots.

As a direct consequence of Theorem 1, one can obtain a simple expression of AT_{Stat} in the regime of interest as stated in the following Corollary.

Corollary 1. Provided that all harvested energies are asymptotically low, AT grows linearly with the ARR, i.e.,

$$AT_{Stat}(ARR) \approx \frac{\overline{\gamma}}{2} ARR. \tag{3.18}$$

Proof. In the low ARR regime, all powers are very low as argued in Remark 3. Hence, we have:

$$\begin{aligned} AT_{Stat}(ARR) &= \mathbb{E}_{\underline{E}} \left\{ \sum_{i=1}^{N} \frac{L_i}{2T} \left(\log(1 + \overline{\gamma} p_i^*) \right) \right\} \approx \mathbb{E}_{\underline{E}} \left\{ \frac{\overline{\gamma}}{2} \sum_{i=1}^{N} \frac{L_i p_i^*}{T} \right\} \\ &\approx \mathbb{E}_{\underline{E}} \left\{ \frac{\overline{\gamma}}{2} \sum_{k} \sum_{j=i_{k-1}+1}^{i_k} L_j \frac{\sum_{j=i_{k-1}}^{i_k-1} E_j}{\sum_{j=i_{k-1}+1}^{i_k} L_j} \right\} = \mathbb{E}_{\underline{E}} \left\{ \frac{\overline{\gamma}}{2} \sum_{k} \sum_{j=i_{k-1}}^{i_k-1} E_j \right\} \\ &= \mathbb{E}_{\underline{E}} \left\{ \frac{\overline{\gamma}}{2} \sum_{i=0}^{N-1} \frac{E_i}{T} \right\} = \frac{\overline{\gamma}}{2} ARR. \end{aligned}$$

□

3.3.2 High ARR Regime

In this regime we assume that the energy harvested amounts are abundant. In nature, the energy harvester is installed in an area where the amounts of energy are important. In the high ARR regime, we have:

Proposition 2. Provided that all powers are very high. The AT is as follows:

$$\lim_{ARR \to +\infty} \frac{AT_{Stat}(ARR)}{\log(ARR)} \leq \frac{1}{2}. \tag{3.19}$$

Proof. In the high ARR regime, all powers are very high, so:

$$AT_{Stat}(ARR) = \mathbb{E}_{\underline{E}} \left\{ \sum_{i=1}^{N} \frac{L_i}{2T} \log(1 + \overline{\gamma} p_i^*) \right\} \tag{3.20}$$

$$= \frac{1}{2} \mathbb{E}_{\underline{E}} \left\{ \sum_{i=1}^{N} \frac{L_i \log(1 + \overline{\gamma} p_i^*)}{T} \right\}. \tag{3.21}$$

Using Jensen inequality,

$$AT_{Stat}(ARR) \leq \frac{1}{2} \mathbb{E}_{\underline{E}} \left\{ \log \left(1 + \overline{\gamma} \sum_{i=1}^{N} \frac{L_i p_i^*}{T} \right) \right\} \tag{3.22}$$

$$\leq \frac{1}{2} \log \left(1 + \overline{\gamma} \mathbb{E}_{\underline{E}} \left\{ \sum_{i=0}^{N-1} \frac{E_i}{T} \right\} \right) \tag{3.23}$$

$$= \frac{\log(1 + \overline{\gamma} ARR)}{2}. \tag{3.24}$$

Dividing both sides of (3.24) by $\log(ARR)$ and taking the limits as $ARR \to \infty$ yields the result in Proposition 2. □

Remark 4. The upper bound found in Proposition 2 is not always achievable even if the ARR is very high. Achieving this upper bound depends on the distribution of the energy arrivals which has an impact on the optimal power policy. In fact, if the amounts of energy arrivals at the beginning of the transmission are less than those arriving later, TX can not achieve the upper bound. Below, we provide two examples where the ARR is high and yet the upper bound is or is not achieved, respectively. For these examples, We consider a unit fading level $\overline{\gamma} = 1$ and uniform length over all time slots.

Example 1. If we consider the following energy profile:

$$E = [0.35, 0.25, 0.22, 0.55, 0.05, 0.17, 0.35, 0.27, 0.33, 0.06] \ kJ.$$

We have an ARR equal to $0.26kJ/sec$, the optimal transmit powers in this case is:

$$P = [0.27, 0.27, 0.27, 0.25, 0.25, 0.25, 0.25, 0.25, 0.25, 0.25] \ kW.$$

Using this power policy, we obtain an AT equal to $4.01Mbits/sec$, and we have $\frac{\log_2(ARR)}{2} = 4.01Mbits/sec$. Thus, we have $AT_{Stat} = \frac{\log_2(ARR)}{2}$ in this case.

Example 2. If we consider the amounts of energy arrival:

$$E = [0.06, 0.05, 0.22, 0.17, 0.55, 0.35, 0.27, 0.35, 0.25, 0.33] \ kJ.$$

We have the same ARR equal to $0.26kJ/sec$, the optimal transmit powers in this case is:

$$P = [0.055, 0.055, 0.195, 0.195, 0.35, 0.35, 0.35, 0.35, 0.35, 0.35] \ kW.$$

So, the AT in this case is equal to $AT_{Stat} = 3.87Mbits/sec$. Hence, $AT_{Stat} \leq \frac{\log_2(ARR)}{2}$.

3.4 Optimal offline power policy without CSI-T

In this section, we assume that TX does not have information about the fading level of the channel. In realistic scenario, the perfect CSI-T is unavailable because of fast channel dynamic, or because of a very noisy feedback. Only the PDF $f_h(.)$ is known in this case, so $f_\gamma(.)$ is known. Consequently, the throughput maximization problem can be expressed as:

$$(P_2): \begin{cases} \max\limits_{p_i \geq 0} \sum\limits_{i=1}^{N} \frac{L_i}{2} \mathbb{E}_\gamma \left\{ \log(1+\gamma p_i) \right\} & (3.25a) \\ \text{s.t.} \begin{cases} \sum\limits_{j=1}^{i} L_j p_j \leq \sum\limits_{j=0}^{i-1} E_j, \forall i \in [\![1, N]\!] \\ \sum\limits_{j=0}^{i} E_j - \sum\limits_{j=1}^{i} L_j p_j \leq E_{max}, \forall i \in [\![1, N-1]\!]. \end{cases} & (3.25b) \end{cases}$$

The related optimal solution of problem (P_2) is given below.

Theorem 2. *Given an energy harvested profile, the optimal policy when the CSI is unavailable at TX is the same as the power policy in the case when the channel is static during communication.*

Proof. We consider $(p_i^*)_i$ the optimal powers found in the case where the channel is static (3.11). The problem in (3.25) is:

$$\max_{p_i \geq 0} \mathbb{E}_\gamma \left\{ \sum_{i=1}^{N} \frac{L_i}{2} \log(1+\gamma p_i) \right\}. \tag{3.26}$$

Since $(p_i^*)_i$ is the optimal power policy when the channel is static, and by Remark 2, $(p_i^*)_i$ does not depend on the static channel gain, then we have :

$$\sum_{i=1}^{N} \frac{L_i}{2} \log(1+\gamma p_i) \leq \sum_{i=1}^{N} \frac{L_i}{2} \log(1+\gamma p_i^*), \quad \forall \gamma \tag{3.27}$$

$$\mathbb{E}_\gamma \left\{ \sum_{i=1}^{N} \frac{L_i}{2} \log(1+\gamma p_i) \right\} \leq \mathbb{E}_\gamma \left\{ \sum_{i=1}^{N} \frac{L_i}{2} \log(1+\gamma p_i^*) \right\} \tag{3.28}$$

$$\max_{p_i \geq 0} \mathbb{E}_\gamma \left\{ \sum_{i=1}^{N} \frac{L_i}{2} \log(1+\gamma p_i) \right\} \leq \mathbb{E}_\gamma \left\{ \sum_{i=1}^{N} \frac{L_i}{2} \log(1+\gamma p_i^*) \right\}. \tag{3.29}$$

So, $\mathbb{E}_\gamma \left\{ \sum\limits_{i=1}^{N} \frac{L_i}{2} \log(1+\gamma p_i^*) \right\}$ is an upper bound for the problem (3.25) that can be achieved by the power policy $(p_i^*)_i$, also this policy is feasible because it satisfies the constraints of the problem (3.25). The problem (3.25) is convex and the different constraints are linear, so the solution is unique. Consequently, the optimal power policy is $(p_i)_i^*$. □

The interpretation of Theorem 2 is that when the CSI-T is not available, TX cannot make a trade-off between the epoch when the channel is relevant for the transmission and the epoch when the fading level is very low. So the transmission in this case will be blind and independent of the fading channel. Therefore, the throughput maximization is related only to the energy arrival amounts without any adaptation to the fading level. This is why the power policies are the same when the channel is static and when the CSI feedback is unavailable at TX.

Low ARR regime:

Proposition 3. Given that all energies are very low. The AT grows linearly with ARR as ARR decreases to zero, i.e.,

$$AT_{N.CSI-T}(ARR) \approx \frac{\mathbb{E}_{\tilde{\gamma}}(\tilde{\gamma})}{2} ARR. \tag{3.30}$$

Proof. In the low ARR regime, we have all powers are very low, so:

$$AT_{N.CSI-T}(ARR) = \mathbb{E}_{\underline{E}} \left\{ \sum_{i=1}^{N} \frac{L_i}{2T} \mathbb{E}_{\tilde{\gamma}} \left\{ \log(1 + \tilde{\gamma} p_i^*) \right\} \right\} \tag{3.31}$$

$$\approx \frac{1}{2} \mathbb{E}_{\underline{E}} \left\{ \sum_{i=1}^{N} \frac{L_i}{T} p_i^* \mathbb{E}_{\tilde{\gamma}}(\tilde{\gamma}) \right\} \tag{3.32}$$

$$= \frac{1}{2} \mathbb{E}_{\underline{E}} \left\{ \sum_{i=1}^{N} \frac{L_i p_i^*}{T} \right\} \mathbb{E}_{\tilde{\gamma}}(\tilde{\gamma}). \tag{3.33}$$

Since the causality constraint (3.13) must be satisfied in the last epoch, then we have:

$$AT_{N.CSI-T}(ARR) = \frac{\mathbb{E}_{\tilde{\gamma}}(\tilde{\gamma})}{2} \mathbb{E}_{\underline{E}} \left\{ \sum_{i=0}^{N-1} \frac{E_i}{T} \right\} = \frac{\mathbb{E}_{\tilde{\gamma}}(\tilde{\gamma})}{2} \, ARR. \tag{3.34}$$

□

Since the achievable AT_{Stat} is asymptotically equal to $\frac{\overline{\gamma}}{2}ARR$ as per Corollary 1, then (3.30) asserts that with no CSI-T, no gain is provided by fading in terms of AT.

3.5 Optimal offline power policy with perfect CSI-T

Unlike the previous section, we consider a fading channel which changes randomly at each epoch γ_i, and it is known perfectly at TX before the communication starts. Hence, the throughput maximization problem in this case is expressed as:

$$(P_3): \begin{cases} \max\limits_{p_i \geq 0} \sum\limits_{i=1}^{N} \frac{L_i}{2} \log(1+\gamma_i p_i) & \text{(3.35a)} \\ \text{s.t.} \begin{cases} \sum\limits_{j=1}^{i} L_j p_j \leq \sum\limits_{j=0}^{i-1} E_j, \forall i \in [\![1, N]\!] \\ \sum\limits_{j=0}^{i} E_j - \sum\limits_{j=1}^{i} L_j p_j \leq E_{max}, \forall i \in [\![1, N-1]\!]. \end{cases} & \text{(3.35b)} \end{cases}$$

We define the Lagrangian function for this problem:

$$\begin{aligned} \mathcal{L} = &-\sum_{i=1}^{N} \frac{L_i}{2} \log(1+\gamma_i p_i) + \sum_{i=1}^{N} \beta_i \left(\sum_{j=1}^{i} L_j p_j - \sum_{j=0}^{i-1} E_j \right) \\ &+ \sum_{i=1}^{N-1} \eta_i \left(\sum_{j=0}^{i} E_j - \sum_{j=1}^{i} L_j p_j - E_{max} \right) - \sum_{i=1}^{N} \lambda_i p_i. \end{aligned} \tag{3.36}$$

The KKT conditions for this problem are:

$$\frac{\gamma_i}{1+\gamma_i p_i^*} = 2 \left(\sum_{j=i}^{N} \beta_j - \sum_{j=i}^{N-1} \eta_j \right), \forall i \in [\![1, N-1]\!]. \tag{3.37}$$

Hence, from (3.37) the optimal powers are given by:

$$p_i^* = \left[\frac{1}{2\left(\sum_{j=i}^{N} \beta_j - \sum_{j=i}^{N-1} \eta_j\right)} - \frac{1}{\gamma_i} \right]^+ , \forall i \in [\![1, N-1]\!], \tag{3.38}$$

and,

$$p_{N+1}^* = \left[\frac{1}{2\ \beta_N} - \frac{1}{\gamma_N} \right]^+ . \tag{3.39}$$

Next, we present our asymptotic results at low ARR regime. In fact, we derive the optimal power scheme in terms of energy harvested without any Lagrange multiplier.

The low ARR regime: In this regime, the energy storage constraint is always satisfied. In fact, the battery capacity E_{max} is greater than the energy harvested amounts. Therefore, the throughput maximization problem can be written as follows:

$$(P_4) : \begin{cases} \max\limits_{p_i \geq 0} \sum_{i=1}^{N} \frac{L_i}{2} \log(1 + \gamma_i p_i) & \text{(3.40a)} \\ \text{s.t.} \sum_{j=1}^{i} L_j p_j \leq \sum_{j=0}^{i-1} E_j, \forall i \in [\![1, N]\!]. & \text{(3.40b)} \end{cases}$$

The optimal solution for this problem is given by the following theorem.

Theorem 3. *Given a channel fading level at the different epochs denoted by the vector* $\underline{\gamma} = (\gamma_1, \gamma_2, ..., \gamma_N)$*. The optimal power allocation, denoted by* $[p_1^*(\underline{\gamma}), p_2^*(\underline{\gamma}), ..., p_N^*(\underline{\gamma})]$*, of problem* (P_3) *when the energy arrivals are all asymptotically small is obtained as follows. Denoting* $i_0 = 0$*,*

$$i_k = \underset{i \in [\![i_{k-1}+1, N]\!]}{\operatorname{argmax}} \{\gamma_i\}, k \geq 1.$$

The optimal power allocation for epochs $i_{k-1}+1$,$i_{k-1}+2$,...,i_k $k \geq 1$ *is given by*

$$p_i^*(\underline{\gamma}) = 0, \quad i \in [\![i_{k-1}+1, i_k-1]\!] \tag{3.41}$$

$$p_{i_k}^*(\underline{\gamma}) \approx \frac{\sum_{j=i_{k-1}}^{i_k-1} E_j}{L_{i_k}}. \tag{3.42}$$

Proof. A proof of this theorem follows directly from that of Theorem 4 below by setting the error variance $\alpha = 0$. □

Conjecture 1. In the low *ARR* regime, we have observed numerically that, the average throughput increases linearly as the the average recharge rate, we have:

$$AT_{P.CSI-T}(ARR) \approx \frac{3}{2}\mathbb{E}_\gamma(\gamma)ARR. \tag{3.43}$$

A direct consequence of our numerical observation is to evaluate the enhancement, in terms of data transmission rate, provided by the CST-T regarding the static channel. The following corollary illustrates this gain.

Corollary 2. The *Directional Water-Filling* algorithm in the case where the channel is perfectly known at TX has a linear gain over the case where channel is static. This gain depends on the the statistical property of the fading distribution, i.e.,

$$AT_{P.CSI-T}(ARR) - AT_{Stat}(ARR) \approx \mathbb{E}_\gamma(\gamma)ARR. \tag{3.44}$$

3.6 Optimal offline power policy with imperfect CSI-T

In this section, we analyze the performance of TX when he knows partially the CSI. For this case, we consider the channel estimation model explained in (3.2). TX uses minimum mean square error (MMSE) estimation to obtain $\widehat{h_i} = \mathbb{E}\left[h_i|\tilde{h_i} = \tilde{h_i}'\right]$. In order to find the optimal power policy, we must calculate the conditional PDF of $\gamma = |h|^2$ given $\hat{\gamma} = |\hat{h}|^2$, it can be expressed as:

$$f_{\gamma|\widehat{\gamma}}(t) = \begin{cases} \frac{1}{\alpha}\ e^{\frac{t+(1-\alpha)\widehat{\gamma}}{\alpha}}\ I_0\left(2\sqrt{\frac{(1-\alpha)\widehat{\gamma}t}{\alpha^2}}\right) & t > 0 \\ 0 & \text{otherwise} \end{cases} \tag{3.45}$$

where α designates the error variance of the estimation process and I_0 is the modified Bessel function of the first order.

Using (3.45), we define the offline point-to-point throughput maximization problem subject to constraints (3.3) and (3.4) of an EH system as:

$$(P_5) : \begin{cases} \max\limits_{p_i \geq 0} \sum\limits_{i=1}^{N} \frac{L_i}{2} \mathbb{E}_{\gamma|\hat{\gamma}} \left(\log(1+\gamma p_i)\right) & \text{(3.46a)} \\ \text{s.t.} \begin{cases} \sum\limits_{j=1}^{i} L_j p_j \leq \sum\limits_{j=0}^{i-1} E_j, \forall i \in [\![1, N]\!] \\ \sum\limits_{j=0}^{i} E_j - \sum\limits_{j=1}^{i} L_j p_j \leq E_{max}, \forall i \in [\![1, N-1]\!]. \end{cases} & \text{(3.46b)} \end{cases}$$

We note first that (P_5) is different from (P_4) in the sense that the expectation in the objective function of (3.46) is conditioned on knowledge of $\hat{\gamma}$. It can be shown that (3.46) is a convex optimization problem, since the objective function is concave in p_i and the constraints are linear. Hence, the Karush-Kuhn-Tucker (KKT) conditions provide necessary and sufficient optimality equations. The KKT conditions for problem (3.46) are:

$$\mathbb{E}_{\gamma|\hat{\gamma}}\left(\frac{\gamma}{1+\gamma p_i^*}\right) = 2\left(\sum_{j=i}^{N}\beta_j - \sum_{j=i}^{N-1}\eta_j\right), i \in [\![1, N-1]\!] \tag{3.47}$$

$$\mathbb{E}_{\gamma|\hat{\gamma}}\left(\frac{\gamma}{1+\gamma\ p_N^*}\right) = 2\ \beta_N, \tag{3.48}$$

with $p_i^* \geq 0$. The solution for this problem is formalized in Proposition 4.

Proposition 4. Let the function $G_{\hat{\gamma}}(\cdot)$ be defined on $[0, +\infty)$ by

$$G_{\hat{\gamma}}(x) = \mathbb{E}_{\gamma|\hat{\gamma}}\left(\frac{\gamma}{1+\gamma x}\right). \tag{3.49}$$

Let C_i be the constant defined by: $C_i = 2\left(\sum_{j=i}^{N}\beta_j - \sum_{j=i}^{N-1}\eta_j\right)$, $\forall i \in [\![1, N-1]\!]$ and $C_N = 2\alpha_N$. Then, the optimal solution of problem (3.46) is given by:

$$\forall i,\ p_i^*(\hat{\gamma}) = \begin{cases} G_{\hat{\gamma}}^{-1}(C_i) & 0 < C_i < \mathbb{E}_{\gamma|\hat{\gamma}}(\gamma) \\ 0 & \text{otherwise,} \end{cases} \tag{3.50}$$

where $G_{\hat{\gamma}}^{-1}(\cdot)$ is the inverse function of $G_{\hat{\gamma}}(\cdot)$.

Proof. First, note that $G_{\hat{\gamma}}(\cdot)$ is a continuous and strictly decreasing function in $p_i \in [0, +\infty)$, then $G_{\hat{\gamma}}^{(-1)}(\cdot)$ exists. Thus, the solution of the problem is unique. Also, since $\forall p_i \in [0, \infty)$, we have:

$$\lim_{p_i \to +\infty} G_{\hat{\gamma}}(p_i) = 0 < G_{\hat{\gamma}}(p_i) \leq G_{\hat{\gamma}}(0) = \mathbb{E}_{\gamma|\hat{\gamma}}(\gamma), \tag{3.51}$$

then (3.50) follows immediately. □

In the above optimization problem, the Lagrange multipliers β_i's and η_i's are determined by the complementary slackness conditions:

$$\beta_i \left(\sum_{j=1}^{i} L_j p_j^* - \sum_{j=0}^{i-1} E_j \right) = 0, \forall i \in [\![1, N]\!] \tag{3.52}$$

$$\eta_i \left(\sum_{j=0}^{i} E_j - \sum_{j=1}^{i} L_j p_j^* - E_{max} \right) = 0, \forall i \in [\![1, N-1]\!], \tag{3.53}$$

where p_j^* is given in Proposition 4.

We note that our result in Proposition 4 captures the cases no CSI-T and perfect CSI-T as special cases. Hence, our framework may be viewed as a generalization of the previous results. To see this, one can set $f_{\gamma|\hat{\gamma}}(\gamma) = \delta(\gamma - \hat{\gamma})$ to retrieve results corresponding to pefrect CSI-T. In this case,

$$G_{\hat{\gamma}}^{-1}(x) \begin{cases} \frac{1}{x} - \frac{1}{\hat{\gamma}} & 0 < x < \hat{\gamma} \\ 0 & \text{otherwise.} \end{cases} \tag{3.54}$$

The power $p_i^*(\hat{\gamma})$ in (3.50) coincides with the one derived in the previous section. On the other hand, setting $f_{\gamma|\hat{\gamma}}(\gamma) = f_\gamma(\gamma)$ yields the same results as the one reported in the previous section.

Next, we study the asymptotic behavior of the AT as the ARR converges to 0 or to $+\infty$. This provides an insight on the AT when the harvested energy is scarce or abundant, respectively.

3.6.1 Low ARR regime

In order to evaluate the AT when the energy harvested amounts are scarce, we investigate the optimal power policy in this regime when the CSI-T includes estimation errors. Again, in a such regime, the energy storage constraint is always satisfied. In fact, the battery capacity E_{max} is greater than the energy harvested amounts. Therefore, the throughput maximization problem can be written as follows:

$$
(P_6):\begin{cases}\max\limits_{p_i\geq 0}\sum\limits_{i=1}^{N}\frac{L_i}{2}\mathbb{E}_{\gamma|\hat{\gamma}}\left(\log(1+\gamma p_i)\right) & (3.55a)\\ \text{s.t.}\sum\limits_{j=1}^{i}L_jp_j\leq\sum\limits_{j=0}^{i-1}E_j,\forall i\in[\![1,N]\!]. & (3.55b)\end{cases}
$$

The optimal solution for problem (P_6) is given by the following theorem.

Theorem 4. *Given a channel fading level at the different epochs denoted by the vector $\underline{\hat{\gamma}}=(\hat{\gamma}_1,\hat{\gamma}_2,...,\hat{\gamma}_N)$. The optimal power allocation, denoted by $[p_1^*(\underline{\hat{\gamma}}),p_2^*(\underline{\hat{\gamma}}),...,p_N^*(\underline{\hat{\gamma}})]$, when the energy arrivals are all asymptotically small is obtained as follows. Denoting $i_0=0$,*

$$
i_k=\underset{i\in[\![i_{k-1}+1,N]\!]}{\operatorname{argmax}}\{\underline{\hat{\gamma}_i}\},k\geq 1.
$$

The optimal power allocation for epochs $i_{k-1}+1$,$i_{k-1}+2$,...,i_k $k\geq 1$ is given by

$$
p_i^*(\underline{\hat{\gamma}})=0,\quad i\in[\![i_{k-1}+1,i_k-1]\!] \tag{3.56}
$$

$$
p_{i_k}^*(\underline{\hat{\gamma}})\approx\frac{\sum\limits_{j=i_{k-1}}^{i_k-1}E_j}{L_{i_k}}. \tag{3.57}
$$

Proof. See Appendix A.2 □

Remark 5. • For communications with EH constraints, the optimal power policy in the low ARR regime, derived previously, has an interesting structure "save then transmit". TX equipped with an energy harvester node saves the harvested amounts of energy then selects the appropriate estimate of the fading gain to transmit data.

• The above theorem states a low-complexity power allocation algorithm for throughput maximization, in the low ARR regime, in the case where the CSI-T is imperfect.

Theorem 5. *Given an error variance α, the AT in the low ARR regime is a combination of the AT when the fading level of the channel is known perfectly and the AT when the CSI-T is unavailable, i.e.,*

$$AT_{I.CSI-T,\alpha}(ARR) \approx (1-\alpha)\ AT_{I.CSI-T,0}(ARR) + \alpha\ AT_{I.CSI-T,1}(ARR). \quad (3.58)$$

Proof. See Appendix A.3. □

3.6.2 High ARR regime

We investigate also the AT in the high ARR regime. Therefore, we have:

Proposition 5. Provided that all powers are very high, then the AT in the imperfect case increases as follows:

$$AT_{I.CSI-T,\alpha}(ARR) - AT_{Stat}(ARR) \approx \frac{\mathbb{E}\left[\log(\gamma)\right]}{2}, \quad \forall \alpha \in [0,1]. \quad (3.59)$$

Proof. See Appendix A.4 □

The interpretation of this result is that, at high ARR, the AT when the CSI-T is imperfect increases similarly as the AT in the case where the channel is static over all epochs, irrespective of the value of α. According to the previous result, the gap between $AT_{I.CSI-T}$ and AT_{Stat} is constant at high ARR, and the loss depends on the property of the fading distribution. Consequently, knowing the channel state when the ARR is high has no gain. In fact, TX can achieve the same performance in terms of data transmitted regardless of its channel knowledge. So, the *Water-Filling* algorithm does not provide a throughput gain at high ARR. Hence, TX can utilize during the communication the same power profile as the worst case ($\alpha = 1$) to achieve a similar performance as the best case ($\alpha = 0$).

3.7 Numerical Results

In this subsection, we present some selected numerical results that illustrate our theoretical analysis. Indeed, we consider a point to point communication system in order to compare the average throughput of this system, for different scenarios of CSI-T availability, during the transmission of data under EH constraints.

3.7.1 Model for Simulation Results

In order to obtain the numerical results, the deadline of the transmission is fixed $T = 10$ sec, also we consider a band-limited AWGN channel, with bandwidth W chosen $W = 1$ MHZ for simulations and the noise power spectral density is $N_0 = 10^{-19}$ W/Hz, with $h, \hat{h}$ and $\tilde{h} \sim \mathcal{CN}(0, \sigma^2)$. Consequently, γ, $\hat{\gamma}$ and $\tilde{\gamma}$ have an exponential distributions with mean equal to σ^2. All the simulations are performed for 5×10^4 realizations of channels and 2×10^4 realizations of energy. As mentioned, in our considered model, the data rate sent to RX at TS i is calculated as $C(h_i, p_i) = \frac{W}{2} \log_2(1 + \frac{\gamma_i p_i}{N_0 W})$ bits/sec. As mentioned, in our considered model, TX knows perfectly the realization of energy before starting the transmission of data. We assume that the times of energy arrivals are fixed and the amounts of the energy is a uniform random variable with mean equal to $\overline{E}$.

3.7.2 Characteristics of Optimal Policy

In Fig.3.4, we analyze the performances of the optimal offline policies for different scenarios depending on the availability of the CSI-T. Fig.3.4 shows the AT of the communication system against the ARR. We consider the scenario of static channel in the first step then we study the case where we have the channel estimation noise with error variance equal to α. For a given ARR, as α decreases from 1 to 0, the AT increases. Similarly, the increase of ARR for a given α furnishes the

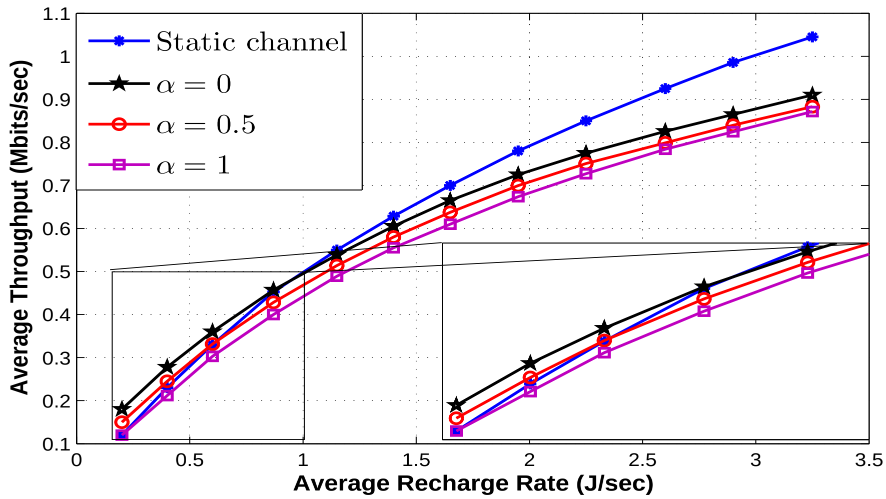

Figure 3.4: Performances of the offline policies, $E_{max} = 20$ J and $\sigma^2 = 1$.

communication system with further data rate. From Fig.3.4, we can observe that the EH system with static channel performs better than the one experiencing fading when ARR is relatively high. However, at low ARR, the fading channel has better performance than the static channel as shown in Fig.3.4 (zoomed part). This calls for studying of utmost interest the asymptotic AT for each scenario to evaluate this improvements of data transmission rate at low ARR shown in Fig.3.5. In fact, combining results derived in the low ARR regime stated in Theorem 5, Conjecture 1 and Proposition 3, one can write :

$$AT_{I.CSI-T,\alpha}(ARR) \approx \left(\frac{3(1-\alpha)}{2}\mathbb{E}(\hat{\gamma}) + \frac{\alpha}{2}\mathbb{E}(\tilde{\gamma})\right) ARR, \qquad \forall \alpha \in [0,1]. \tag{3.60}$$

Fig.3.5 shows the asymptotic performance of the offline policies at low ARR. Simulations in Fig.3.5 are performed for channel variance $\sigma^2 = 1$ when the channel experiences fading and $\overline{\gamma} = 1$ when the channel considered constant. Based on the system parameters considered in Fig.3.5, 3.60 can be simplified as follows:

$$AT_{I.CSI-T,\alpha}(ARR) \approx (3-2\alpha)AT_{Stat}(ARR), \qquad \forall \alpha \in [0,1]. \tag{3.61}$$

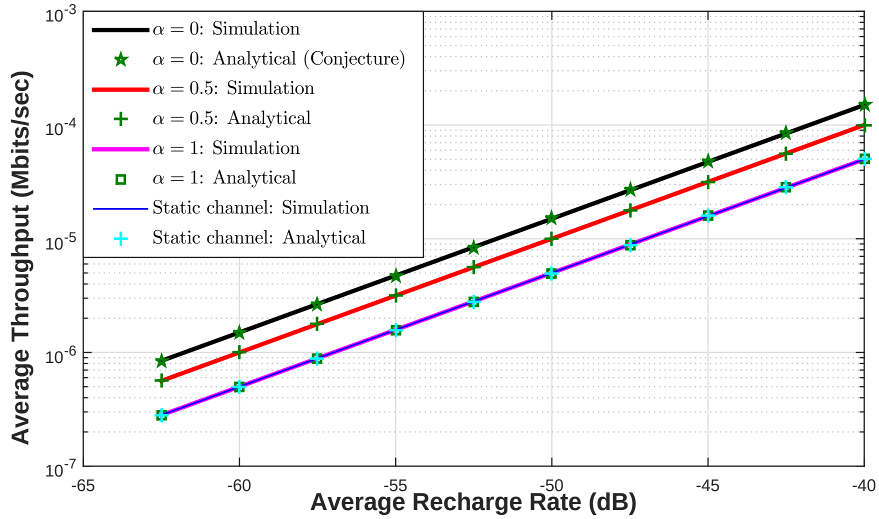

Figure 3.5: Performances of the offline policies at low ARR regime, $E_{max} = 20$ J and $\sigma^2 = 1$.

This highlights the gain provided by the CSI-T over the static case, at asymptotically low ARR which shown in Fig.3.5. Indeed, at low ARR the EH system take advantage from the partial knowledge of the channel to enhance its performance. As shown in Fig.3.5, the gain is constant relative to ARR which depends only on the error variance. Also in Fig.3.5, we can observe that the communication system, in the case where the channel is static has the same performance as the case where the channel experiences fading without CSI-T. This result is quite consistent with our theoretical results derived in the previous sections. Moreover, we can observe that the analytic analysis, in the low ARR, matches perfectly with the numerical analysis.

Fig.3.6 shows the AT of the communication system versus the ARR, considering the scenario when the CSI-T is perfect. We observe that the proposed conjecture stated in Conjecture 1, fits well with the simulations results. It shows that, by increasing the channel variance σ^2 our system reaches better performance. this result is intuitive since we consider better channel conditions.

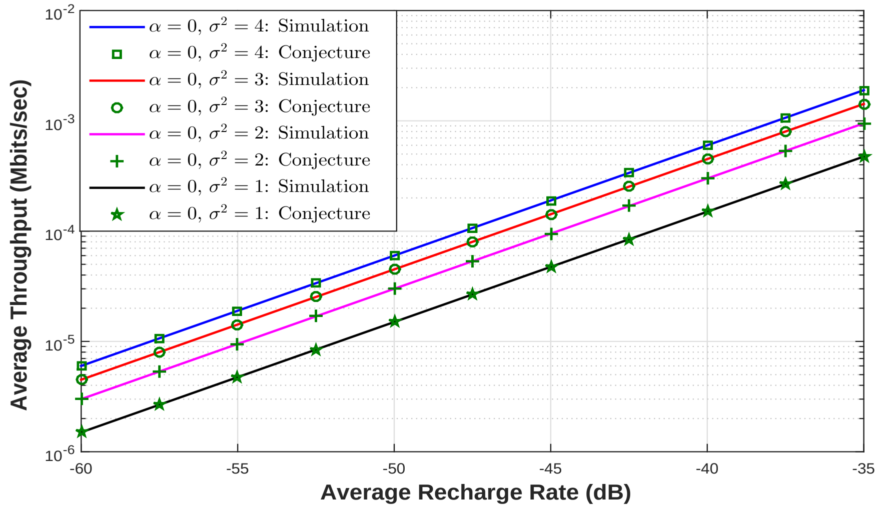

Figure 3.6: Performances of the offline policy under perfect CSI-T, at low ARR, $E_{max} = 20$ J.

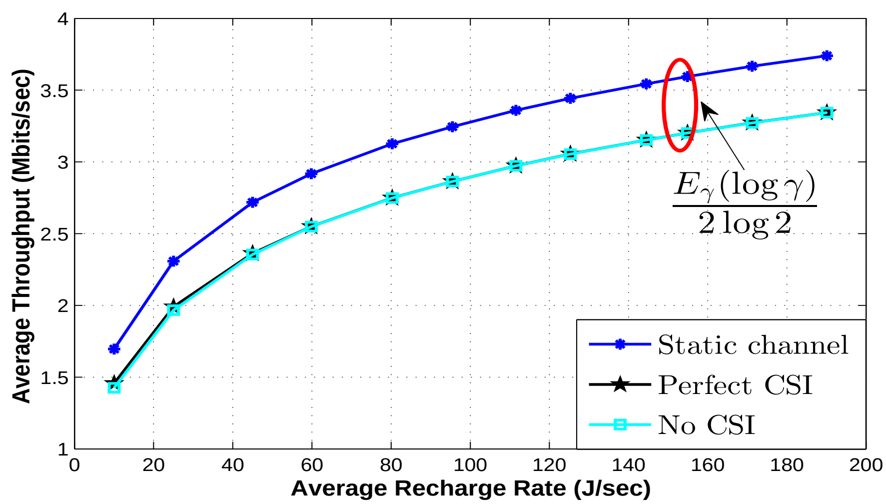

Figure 3.7: Performances of the offline policies at high ARR regime, $E_{max} = 500$ J and $\sigma^2 = 1$.

In Fig.3.7, we investigate the performance of our system when the ARR is very high. We point out that the curve of the AT when the CSI-T is perfect matches with the curve of the AT when no CSI-T is available, this is quite consistent with the theoretical results stated in Proposition 5. This means that having a perfect knowledge about the channel state, likewise a partial knowledge, or not has no impact on the performance at high ARR regime. Also, we show that, in the very high ARR regime, the gap between the data transmission rate when the channel is static and when there is fading is constant depending on the property of the fading distribution as shown in Theorem 5. In our case, the gap is equal to $\frac{\mathbb{E}_\gamma\{\log(\gamma)\}}{2\log 2} = -0.4162$.

3.8 Conclusion

The goal of this chapter was to study the performance of the point-to-point EH communication system. We solved the throughput maximization problem for an offline settings where the energy and the channel process are assumed to be perfectly known prior the data transmission. Indeed, we obtained the optimal power policy for different degree of CSI-T availability. Also an asymptotic analysis of the performance of the communication system at low and high regime of the ARR has been provided.

Chapter 4

The Online Analysis

4.1 Introduction

Studying the offline transmission policies under EH constraints is crucial to be compared with the conventional communication. However, in nature we cannot have a deterministic values of energy arrivals before starting the transmission. That is why we tackle more practical scenario in this chapter, where we model the energy arrival by random process in order to analyze the performance of system powered by energy harvester nodes.

In this chapter, according to [30], we proposed to model the energy arrival by random first-order Markov model that depends on the past random variables.

4.2 System model

4.2.1 System description

In this section, we recall the system model considered in our study in the previous chapter. Indeed, a SISO communication system where TX is powered by an EH sensor as shown in Fig.3.1. The data transmission is performed during a deadline T. We assume that there is always data available, in the buffer, for transmission. The channel between TX and RX is considered a fading channel. The base band received signal y over a bandwidth W, is given by:

$$y = hx + n, \tag{4.1}$$

where $h \sim \mathcal{CN}(0, \sigma^2)$ is a zero mean circularly symmetric complex variable with variance σ^2, x is the channel input, and n is a zero-mean AWGN with spectral density N_0, and is independent of h. The energy available in the battery determines the feasible bits that can be transmitted during each TS. During each TS, TX encodes the bits to be sent as data symbols, where the block length of each symbol is assumed to be large enough so that we can guarantee the reliability of the decoding process. A feedback link is considered between the RX and TX, the CSI feedback is sent, at the beginning of TSs, from RX to TX.

4.2.2 Energy Model

Differently from the energy model considered in Chapter 3, in this chapter we take into account the randomness and the uncertainty of the amounts of the energy harvested. In order to capture applications based on EH networks where TX has not a deterministic knowledge as suggested previously, we design a more sophisticated online model where we develop a Markov process that takes into consideration the unpredictable amount of the energy harvested. We assume that TX is equipped by a single EH sensor node. This node senses periodically in a discrete time, during the deadline T, the amount of energy available in the environment at beginning of TS $i \in \{1, 2, ..., N\}$. At t_i, E_i units of energy is harvested, and the battery is assumed to be empty initially. The transmission begins at $t_0 = 0$ when the energy E_0 is scavenged by TX, see Fig.4.1. Since the sensing is considered periodic, the length of TSs is assumed to be constant, i.e., $L_i = t_{i+1} - t_i = L = \frac{T}{N}$.

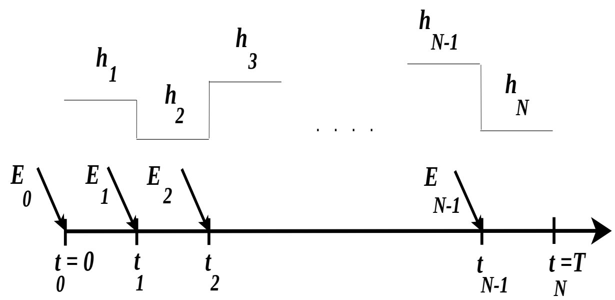

Figure 4.1: Energy arrival and fading channel during a period T for the online model.

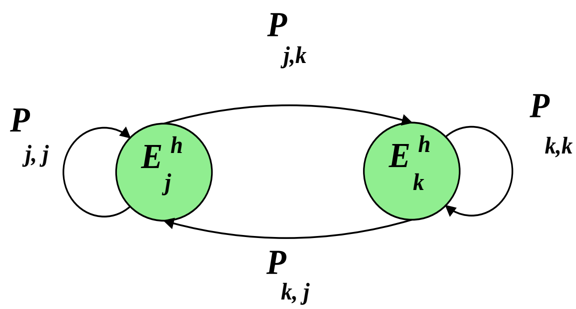

Figure 4.2: Two states from the Markov chain.

Different from most previous work that consider a deterministic EH process, we propose a first-order stationary Markov chain for the energy arrival process. In our Markov model, we suggest a finite states Markov chain are defined and the possible transitions that may occur depending on their probabilities. Each state of the model depends mainly on the environment and the amount of the energy harvested in the past. Therefore, TX disposes a statistical knowledge of energy arrival amounts. This statistical knowledge can be learned from system history or measurements, for instance.

The energy harvested amount E_i takes values in a finite set of states $\mathcal{E} = \{E_1^h = 0, E_2^h, ..., E_{N_s}^h\}$. In Fig.4.2, we show a model for two states chosen randomly from the N_s states. In fact, the states contains the zero state, i.e., $E_1^h = 0$ which corresponds to the case where no energy available in environment. The transition probabilities between states are denoted as follows:

$$\mathbb{P}(E_j^h \rightarrow E_k^h) = P_{j,k}, \forall j,k \in [\![1, N_s]\!]. \tag{4.2}$$

The matrix of the transition probabilities is denoted $\mathbf{M} \in M_{N_s}(\mathbb{R})$, defined by $M_{jk} = P_{j,k}$. The steady state probability in this case is given by $\pi^* = [\mathbb{P}_{E_1^h} ... \mathbb{P}_{E_{N_s}^h}]$. Since the EH process is modeled as first-order stationary Markov chain, the transition probability of this random variable is defined as follows:

$$\mathbb{P}(E_i | E_0, E_1, ..., E_{i-1}) = \mathbb{P}(E_i | E_{i-1}), \forall i \in [\![1, N-1]\!]. \tag{4.3}$$

Also one can write :

$$\mathbb{P}(E_i, E_{i-1}, ..., E_1 | E_0) = \prod_{t=1}^{i} \mathbb{P}(E_t | E_{t-1}), \forall i \in [\![1, N-1]\!]. \tag{4.4}$$

Based on the statistical knowledge provided as a look-up table to TX and the amount of energy harvested in the previous TS, TX has to adapt its transmission schemes to allocate optimally the energy across time in order to maximize the throughput.

4.2.3 Channel Model

As developed previously, we consider the throughput maximization problem over a deadline T. Optimal powers are made according to the channel conditions. Depending on the availability of the information about these conditions, we propose an analysis of the performance of the proposed communication system.

The transmission hop experiences complex channel fading gain. As argued in the literature, the wireless channel varies slowly over time, so the fading level is assumed to be constant at each TS during the transmission of data. That is, the fading changes in discrete time instants $t_1, t_2, ..., t_{N-1}$ which represent the sensing periods of the energy harvester node as shown in Fig.4.1. The fading level during

TS i, is assumed to be equal to h_i. We assume that throughout the communication, perfect CSI-R is available. As presented in Chapter 3, we assume a noisy CSI-T and its estimate follows the same linear model suggested in the offline analysis. This model is given by:

$$h = \sqrt{1-\alpha}\ \widehat{h} + \sqrt{\alpha}\ \tilde{h}, \tag{4.5}$$

where α is the error variance, $\alpha \in [0, 1]$. TX estimates the actual channel estimation as $\hat{h}$ which is $\mathcal{CN}(0, \sigma^2)$ independent of the channel estimation error denoted as $\tilde{h}$ which is also $\mathcal{CN}(0, \sigma^2)$.

4.2.4 Battery Model

During the epoch i of length L, TX sends the data symbol i amplified by the power p_i. Then, TX consumes $L \times p_i$ units of energy during this epoch from the energy stored in the battery in this epoch B_i. Just after this time slot, the energy harvester node scavenges an amount of energy of E_i. Hence, in the epoch $i + 1$ the stored energy in the battery is updated as follows:

$$B_{i+1} = min\{B_i - Lp_i + E_i, E_{max}\}, \quad \forall i \in [\![1, N-1]\!] \tag{4.6}$$

$$B_N = B_N - L_N p_N. \tag{4.7}$$

From (4.6), one can conclude that the energy stored in the battery follows a first-order Markov process that depends on the immediate past energy harvested and the transmitted power. The initial amount of the energy saved in the battery B_1 is equal to the amount of energy harvested E_0 by the node that engenders the beginning of the transmission of data, i.e., $B_1 = min\{E_0, E_{max}\}$.

4.2.5 Energy Harvesting Constraints

In this subsection, we will design the region of feasible power policies is constrained by EH constraints. In fact, the optimal scheduling is constrained by the energy harvested profile. To design the feasible region, we consider the causality constraint which is defined as follows:

$$\sum_{j=1}^{i} Lp_j \leq \sum_{j=0}^{i-1} E_j, \forall i \in [\![1, N]\!]. \tag{4.8}$$

Also, one can understand this constraint as if TX can transmit only energy which is already available in the battery, i.e.,

$$0 \leq L \times p_i \leq B_i, \quad \forall i \in [\![1, N]\!]. \tag{4.9}$$

In order to investigate the optimal power policy, we consider the energy storage constraint that can be expressed as:

$$\sum_{j=0}^{i} E_j - \sum_{j=1}^{i} Lp_j \leq E_{max}, \quad \forall i \in [\![1, N-1]\!]. \tag{4.10}$$

Also, this constraint can be reflected by (4.6) in the battery dynamics, which states that the energy stored in the battery during TS i must be less than E_{max}.

4.2.6 Overall Markov Model:

In this subsection, we investigate a model that can capture the energy and the battery model, jointly. Using the Markov chain appropriate to the EH dynamics (4.3) and the first-order Markov process that captures the battery dynamics (4.6), we can design a new first-order Markov process where the states of this process are defined as the joint state between the EH's state and the battery's state, i.e., the state at the initial TS: $S_1 \triangleq (B_1)$ which is assumed to be known to TX, the states during TSs are defined by: $\forall j \in [\![2, N]\!]$, $S_j \triangleq (E_{j-1}, B_j)$ and a last state $S_{N+1} \triangleq (B_{N+1})$ which describes the state of battery by the end of transmission. Hence, one can deduce from (4.3) and (4.6) that:

$$\mathbb{P}(S_i|S_1, S_2, ..., S_{i-1}) = \mathbb{P}(S_i|S_{i-1}), \forall i \in [\![2, N+1]\!]. \tag{4.11}$$

Consequently, the state transition probability can be rewritten as follows:

$$\mathbb{P}(S_i, S_{i-1}, ..., S_2|S_1) = \prod_{k=2}^{i} \mathbb{P}(S_k|S_{k-1}), \forall i \in [\![2, N+1]\!]. \tag{4.12}$$

Therefore, our objective is to formulate the throughput maximization problem given a deadline T subject to EH constraints under the assumption of imperfect CSI-T.

4.3 Optimal Power Policy

In this section, our objective is to formulate the throughput maximization problem given a deadline T subject to EH constraints under imperfect CSI-T. Then, we determine the online optimal power policy that maximizes the throughput of the EH communication system.

4.3.1 Problem Formulation

Our throughput maximization problem can be classified as a discrete-time Markov Decision Process (MDP). In fact, at the initiation of the communication TX has the information about the initial energy harvested. Then, TX decides to transmit with power p_i at the beginning of the TS i, taking into consideration the matrix of the transition probabilities. Our aim is to find an optimal power policy that maximizes the expected cumulative data rate during N TSs, i.e.,

$$(P_1): \begin{cases} \max_{\{p_i\}_{i=1}^{N}} \mathbb{E}_{S_2^N} \left\{ \mathbb{E}_{\underline{\gamma}|\underline{\hat{\gamma}}} \left[\sum_{i=1}^{N} \frac{L}{2} \log(1+\gamma p_i) \right] \mid \mathbf{M}, S_1 \right\} & \text{(4.13a)} \\ \text{s.t.} \begin{cases} 0 \leq Lp_i \leq B_i, \quad \forall i \in [\![1, N]\!] \\ B_{i+1} = min\{B_i - Lp_i + E_i, E_{max}\}, \forall i \in [\![1, N-1]\!] \\ B_{N+1} = B_N - Lp_N. \end{cases} & \text{(4.13b)} \end{cases}$$

where $\mathbb{E}_{S_2^N}$ designates the statistical expectation over all possible states during TSs $i = 2, ..., N$. $\mathbb{E}_{\underline{\gamma}|\underline{\hat{\gamma}}}$ designates the statistical expectation over the RV $\underline{\gamma}|\underline{\hat{\gamma}}$. Generally, the optimization problem in (4.13) cannot be solved independently for each time slot due to the dependence of constraints along TSs. For instance, the energy consumed currently affects the energy stored in the battery in the next TS, and therefore affects the future power allocation. Consequently, such sequential optimization problem with random EH amounts can be solved optimally using finite-horizon dynamic programming (DP) [31].

4.3.2 Online Power Policies with Fading Channel

In this section, we solve the problem (4.13) by using finite-horizon DP. In fact, the optimal power allocation is determined by backward induction method [32]. For instance, $\{p_1^*, p_2^*, ..., p_N^*\}$ is calculated in the time reversal order. The reward functions are calculated recursively as follows:

1) The last TS, $i = N$:

$$R_N(\hat{\gamma}_N, S_N, \alpha) = \max_{p_N} \mathbb{E}_{\gamma|\hat{\gamma}_N} \left[\frac{L}{2} \log(1+\gamma p_N) \mid \mathbf{M}, S_1 \right] \quad (4.14)$$
$$\text{s.t. } 0 \leq Lp_N \leq B_N.$$

Proposition 6. The optimal last state S_{N+1}^* of the first-order Markov process is

$S^*_{N+1} \triangleq (B_{N+1} = 0)$.

Proof. The proof is immediate and thus omitted. □

Proposition 6 states that TX transmits the last symbol with full energy available in the battery, this result is quite consistent with recent works where the causality constraint (4.10) is assumed to be satisfied with equality at the last time slot, e.g., [11]. Therefore, the reward function in the last time slot can be evaluated using $f_{\gamma|\hat{\gamma}}(\cdot)$ as follows:

$$R_N(\widehat{\gamma}_N, S_N, \alpha) = \frac{L}{2}\mathbb{E}_{\gamma|\widehat{\gamma}_N}\left[\log(1+\gamma\frac{B_N}{L})\right]. \tag{4.15}$$

As a consequence, by the end of the communication, we have:

$$\sum_{i=0}^{N-1} E_i = \sum_{i=1}^{N} Lp_i^*. \tag{4.16}$$

2) The TSs, $i = N-1, N-2, ..., 1$**:** During the TS i, the optimal power is determined by maximizing the reward function corresponding to this TS which is expressed as follows:

$$R_i(\hat{\gamma}_i, S_i, \alpha) = \max_{p_i \leq \frac{B_i}{L}} \left\{\mathbb{E}_{\gamma|\hat{\gamma}_i}\left(\frac{L}{2}\log(1+\gamma p_i)\right) + \overline{R}_{i+1}(\hat{\gamma}_{i+1}, S_{i+1}, \alpha)\right\}, \tag{4.17}$$

where $\overline{R}_{i+1}(\widehat{\gamma}_{i+1}, S_{i+1}, \alpha)$ designates the expected values of the future reward functions and it is defined as follows:

$$\overline{R}_{i+1}(\hat{\gamma}_{i+1}, S_{i+1}, \alpha) = \mathbb{E}_{\mathcal{S}^i}\left\{\mathbb{E}_{\gamma|\hat{\gamma}_{i+1}}\left[R_{i+1}(\hat{\gamma}_{i+1}, S_{i+1}, \alpha)\right] | \mathbf{M}, S_i\right\}, \tag{4.18}$$

where $\mathbb{E}_{\mathcal{S}^i}$ designates the statistical expectation over the set of possible states in the TS $i+1$ given the state S_i and the transition probabilities. The reward function in (4.18) can be understood as the maximal sum of the data rate in the current TS.

Taking into account the matrix $\mathbf{M}$ we maximize the expected cumulative data rate in the future TSs resulted from the current state and the current transmitted power. The reward functions $R_i(\widehat{\gamma}_i, S_i, \alpha)$ cannot be determined in a closed form.

Theorem 6. *Given a channel fading level estimation at different TSs denoted by the vector* $\hat{\gamma} = (\hat{\gamma}_1, \hat{\gamma}_2, ..., \hat{\gamma}_N)$*, the initial state* $S_1 \triangleq (B_1) \triangleq (E_0)$ *and the matrix of transition probabilities* $\mathbf{M}$ *related to Markov chain that models the EH process, the optimal power allocation for TSs* $i = 1, 2, ..., N$ *is given by the initial reward function* $R_1(\hat{\gamma}_1, S_1, \alpha)$*.*

Proof. The proof follows immediately by applying equations of Bellman [32] and the equations that resume the dynamic of the Markov process ((4.6) and (4.3)). □

Some properties of the optimal power policy

Proposition 7. Under the settings and the reward functions defined previously, one can deduce that $R_i(\widehat{\gamma}_i, S_i, \alpha)$ and $\overline{R}_i(\widehat{\gamma}_i, S_i, \alpha)$ are continuous, non-decreasing and concave in B_i.

Proof. We note that studying the continuity, the non-decreasing and the convexity properties of the reward functions in the belief state is quite well known in the MDP literature [33] [34]. The proof follows similar lines of proof of Theorem 1 and Theorem 2 in [15], thus the proof is omitted. □

The Proposition 7 states that the optimal cumulative data rate is concave and increasing in the initial state of the battery. This result is quite intuitive, the higher amount of energy harvested at the beginning of transmission, the higher is the maximum achievable expected data rate.

Remark 6. One can see easily that the optimal reward function $R_1(\widehat{\gamma}_1, S_1, \alpha)$ is non-increasing in α. This can be deduced immediately from the definition of the parameter α which measures the quality of the CSI-T.

We note that our framework can be used to capture some specific scenarios (the perfect CSI-T and the no CSI-T cases).

Special cases

Online optimal power policy with perfect CSI-T Using the procedure described above, one can determine the optimal power allocation when TX has a perfect knowledge of the channel during TSs. In order to capture this case, one can set $\alpha = 0$ and $f_{\gamma|\hat{\gamma}}(\gamma) = \delta(\gamma - \hat{\gamma})$.

Online optimal power policy without CSI-T In some situations, having an estimate of the channel is challenging due to the huge amount of noise in the environment. Then, the optimal power allocation in such situations can also be determined by setting $\alpha = 1$ and $f_{\gamma|\hat{\gamma}}(.) = f_{\gamma}(.)$ in the above procedure.

Online optimal power policy with static channel This work can be utilized also to retrieve the optimal reward function when the channel is equal to $\overline{\gamma}$ during TSs. For instance, one can perform the optimization problem under the setting $\alpha = 0$ and $\forall i \in [\![1, N]\!],\ \gamma_i = \overline{\gamma}$. This will provide a look up table where the optimal powers are stored.
Hence, our work is a generalization of maximizing the throughput considering the fading channel case.

4.4 Asymptotic Analysis

In this section, average throughput performance is studied when the CSI-T is partially available. Given an error variance α of the channel estimation, the average throughput can be written as follows: $\forall\ \alpha \in [0,1]$,

$$AT_{I.CSI-T}(\alpha) = \mathbb{E}_{S_1^N}\left\{\mathbb{E}_{\underline{\hat{\gamma}}}\left(\sum_{i=1}^{N}\frac{L}{2T}\mathbb{E}_{\gamma|\hat{\gamma}_i}\left[\log(1+\gamma p_i^*(\underline{\hat{\gamma}}))\right]\right) \mid \mathbf{M}\right\}. \tag{4.19}$$

Based on the developed optimal power policy, we present our asymptotic analysis in two extremes cases, the low ARR regime ($ARR \to 0$) and the high ARR regime ($ARR \to +\infty$), respectively.

4.4.1 Low ARR Regime

In this subsection, we evaluate the performance of the communication system when the energy harvested is scarce.

Proposition 8. Given that all energies are very low, given that the channel between TX and RX is a fading channel where the CSI-T is unavailable and a static channel equal to $\overline{\gamma}$. The AT of the system grows linearly with ARR, i.e.,

$$AT_{N.CSI-T}(ARR) \approx \frac{\mathbb{E}_{\tilde{\gamma}}(\tilde{\gamma})}{2}ARR \tag{4.20}$$

$$AT_{Stat}(ARR) \approx \frac{\overline{\gamma}}{2}ARR. \tag{4.21}$$

Proof. In the low ARR regime, we assume that all powers are very low. In the case where the CSI-T is unavailable, by setting $\alpha = 1$, we have:

$$AT_{N.CSI-T}(ARR) = \mathbb{E}_{S_1^N}\left\{\mathbb{E}_{\tilde{\gamma}}\left[\sum_{i=1}^{N}\frac{L}{2T}\log(1+\tilde{\gamma}p_i^*)\right] \mid \mathbf{M}\right\} \tag{4.22}$$

$$\approx \frac{1}{2}\mathbb{E}_{S_1^N}\left\{\mathbb{E}_{\tilde{\gamma}}\left[\sum_{i=1}^{N}\frac{L}{T}p_i^*\tilde{\gamma}\right] \mid \mathbf{M}\right\} \tag{4.23}$$

$$= \frac{1}{2}\mathbb{E}_{S_1^N}\left\{\sum_{i=1}^{N}\frac{L}{T}p_i^* \mid \mathbf{M}\right\}\mathbb{E}_{\tilde{\gamma}}(\tilde{\gamma}). \tag{4.24}$$

Using (4.16), then we have:

$$AT_{N.CSI-T}(ARR) = \frac{\mathbb{E}_{\tilde{\gamma}}(\tilde{\gamma})}{2}\mathbb{E}_{S_1^N}\left\{\sum_{i=0}^{N-1}\frac{E_i}{T} \mid \mathbf{M}\right\} \tag{4.25}$$

$$= \frac{\mathbb{E}_{\tilde{\gamma}}(\tilde{\gamma})}{2T}\sum_{i=0}^{N-1}\sum_{j=1}^{N_s}\mathbb{P}_{E_j^h}E_j^h \tag{4.26}$$

$$= \frac{\mathbb{E}_{\tilde{\gamma}}(\tilde{\gamma})}{2T}\mathbb{E}_{\underline{E}}\left\{\sum_{i=0}^{N-1}E_i\right\} = \frac{\mathbb{E}_{\tilde{\gamma}}(\tilde{\gamma})}{2}\ ARR. \tag{4.27}$$

The expression of the AT_{Stat} can be proved using similar lines of the proof for $AT_{N.CSI-T}$. □

Proposition 8 asserts that with no CSI-T, no gain is provided by fading in terms of AT given that the expected value of the fading level is equal to $\overline{\gamma}$. These results are quite consistent with those derived in the offline analysis. In fact, in the low ARR regime, the AT_{Stat} and $AT_{N.CSI-T}$ of the communication system studied in this work depend only on the ARR and the property of the channel. Hence, the model considered for the energy arrival process does not have an effect on the performance of the communication in terms of the data rate.

Conjecture 2. Given an error variance α, the AT in the low ARR regime is a linear combination of the AT when the fading level of the channel is known perfectly and the AT when is unavailable at TX, i.e., $\forall\ \alpha \in [0,1]$,

$$AT_{I.CSI-T,\alpha}(ARR) \approx (1-\alpha)\ AT_{P.CSI-T}(ARR) + \alpha\ AT_{N.CSI-T}(ARR). \tag{4.28}$$

This conjecture is based on numerical observations. Also, one can expect this result. Indeed, in Chapter 3, we have proved this result after determining a closed form expression of the optimal powers. In fact, we have remarked that, given the same channel realization, TX transmits the same powers for the cases where the CSI-T is assumed to be perfect or imperfect i.e., $\alpha \in [0,1[$. Also, we have proved that, in the low ARR regime, the AT of the system studied depends on the ARR and not on how energies are distributed across time. However, in the online analysis, we are not able to derive a closed form solution for the optimal powers as previously. Thus, a complete proof of this result is difficult or even impossible. But we noticed, from Proposition 8, that the AT_{Stat} and the $AT_{N.CSI-T}$ depend only on the ARR as proved in the offline analysis. Therefore, we expected to have the same results since we remarked that the performance of the communication system is just function of the ARR. Consequently, the AT at low ARR does not take into account the considered energy model and it depends only on the metric ARR. That is why, we surmise this conjecture which is proved by the numerical results in Section 4.5.

4.4.2 High ARR Regime

In this subsection, we evaluate the performance of the communication system when the energy harvested is abundant. In such a scenario, the state $E_1^h = 0$ is omitted because the node can harvest a very high amount of energy at each sensing instant.

Proposition 9. Provided that all powers are very high in the high ARR regime, then the AT increases as follows:

$$AT_{I.CSI-T,\alpha}(ARR) - AT_{Stat}(ARR) \approx \frac{\mathbb{E}\left[\log(\gamma)\right]}{2}, \quad \forall \alpha \in [0,1]. \tag{4.29}$$

Proof. See Appendix A.2. □

The result stated in Proposition 9, is similar to that derived in the offline analysis, in the high ARR regime. Considering a stochastic model for the energy arrival process, the gap between $AT_{I.CSI-T,\alpha}$ and AT_{Stat} is constant at high ARR, and the loss depends only on the fading distribution.

4.5 Numerical Results

In this section, we present some selected numerical results that illustrate our theoretical analysis. Indeed, we consider a point to point communication system in order to compare the average throughput of this system, for different scenarios of CSI-T availability.

4.5.1 Model for Simulation Results

In order to obtain the numerical results, the deadline of the transmission is fixed $T = 10$ sec, the maximum energy that can be stored by the battery is $E_{max} = 100$ J , also we consider a band-limited AWGN channel, with bandwidth W chosen $W = 1$ MHZ for simulations and the noise power spectral density is $N_0 = 10^{-19}$ W/Hz and $\sigma^2 = 1$. All the simulations are performed for 5×10^4 channel realizations. As mentioned in our considered model, the data rate sent to RX at TS i is calculated as $C(h_i, p_i) = \frac{W}{2} \log_2(1 + \frac{\gamma_i p_i}{N_0 W})$ bits/sec. We assume that the energy takes values in a finite set, i.e., $N_s = 3$, for simulations we fix $\mathcal{E} = \{E_1^h = 0, E_2^h = \beta, E_3^h = \eta\}$, such that $0 < \beta < \eta$. For the EH process, we assume that the matrix of the transition probabilities $\mathbf{M}$ is defined by:

$$\mathbf{M} = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}. \tag{4.30}$$

It is clear that the steady state probability in this case is given by $\pi^* = [\mathbb{P}_{E_1^h} \mathbb{P}_{E_2^h} \mathbb{P}_{E_3^h}] = \frac{1}{3}[1\ 1\ 1]$. Thus, in this case, the average energy that can be harvested at instant t_i is defined by:

$$\overline{E} = \mathbb{E}(E_i) = \sum_{j=1}^{N_s=3} \mathbb{P}_{E_j^h} E_j^h = \frac{\beta + \eta}{3}. \tag{4.31}$$

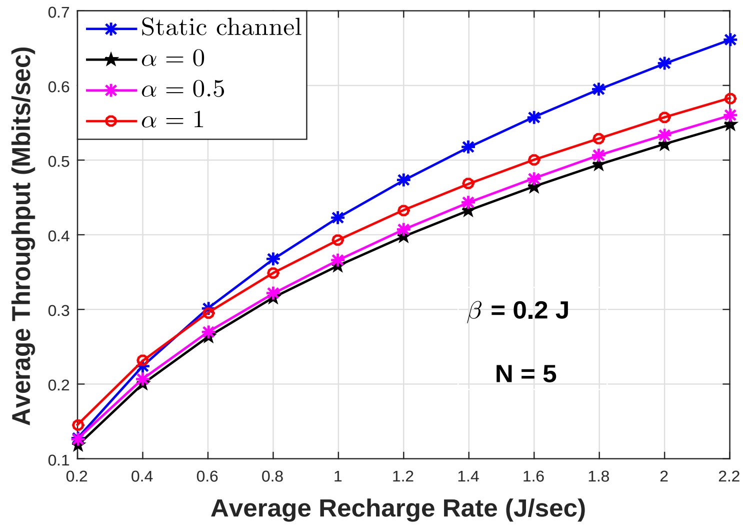

Figure 4.3: Performance of the online policies with exponential fading channel for various energy arrival, $E_{max} = 100$ J and $\sigma^2 = 1$.

4.5.2 Characteristics of the Optimal Power Policy

The optimal power policy is determined by induction. Fixing an error estimation variance α, the powers $p_1^*, p_2^*, ..., p_N^*$ are calculated recursively in the time reversal order. Starting by the evaluation of the reward function at the last TS N, using the closed form expression given in (4.15). Then, we calculate $\overline{R}_N(\widehat{\gamma}_N, S_N, \alpha)$ using (4.18), for different battery capacity B_N discretized in an adaptive step size depending on the regime where the system is running (low, medium and high ARR regime). We set the step size $\Delta B = 0,02$ J to plot figures 4.3, 4.4 and 4.6 and $\Delta B = 2 \times 10^{-7}$J to plot Fig.4.5. Also, $\overline{R}_N(\widehat{\gamma}_N, S_N, \alpha)$ is averaged over 5×10^4 channel realizations. Then, we calculate $R_{N-1}(\widehat{\gamma}_{N-1}, S_{N-1}, \alpha)$ solving (4.17) using a one line search method. The value of $R_{N-1}(\widehat{\gamma}_{N-1}, S_{N-1}, \alpha)$ is stored as a look-up table to be used for the next TSs. Then, the same procedure is applied for TSs $N-2, ..., 1$. At TS 1, TX determines the optimal reward function by evaluating $R_1(\underline{\hat{\gamma}}, S_1, \alpha)$. To evaluate the optimal AT, the reward function is averaged over 5×10^4 independent realizations of $\underline{\hat{\gamma}}$ and over all possible states for the initial state S_1.

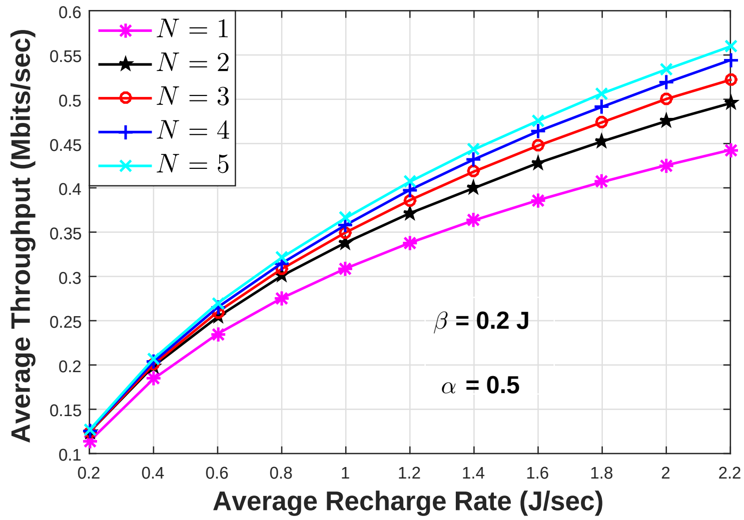

Figure 4.4: Performance of the online policies with exponential fading channel for various energy arrival, $E_{max} = 100$ J and $\sigma^2 = 1$.

We start by examining the AT with different values of α. In Fig.4.3, we can see that the AT is a non-increasing function in α as stated in our analytical claims. Also, we can observe that the EH system with a static channel equal to $\overline{\gamma} = 1$, performs better than the one experiencing fading. However, in the low ARR regime, we remark that the fading channel has a better performance (red curve). This points out the benefit provided by the fading to the EH system at low ARR. This highlights the gain provided by the available CSI-T. Indeed, this motivates us to investigate of utmost interest this gain at low ARR. Note that the performance depends on the states of the Markov chain associated to the EH process. In fact, having the same ARR does not mean having the same performance. In Fig.4.3, we set $\beta = 0, 2$J and the third state is fixed by the ARR. In Fig.4.4, we investigate the effect of varying the number of TSs N on the AT. As we remark, the EH communication system performs better as N increases. This is consistent with what we claim in previous parts of this chapter. Simulations in this figure are performed for a fading channel with error variance $\alpha = 0, 5$. As discussed previously, simulations in Fig.4.3 stir us to boost our analysis on the low ARR regime

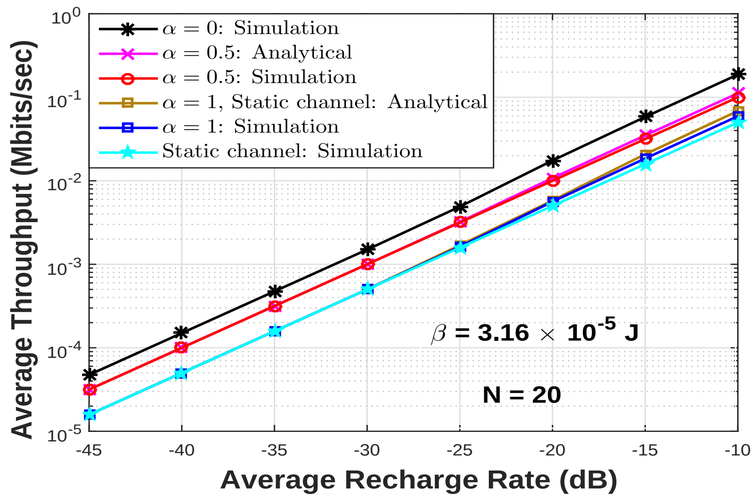

Figure 4.5: Performance of the online policies at low ARR, $E_{max} = 100$ J and $\sigma^2 = 1$.

presented in Fig.4.5. Such analysis is reasonable and very useful for EH applications that are running at low ARR. We observe in Fig.4.5 that results found are quite consistent with our theoretical claims. In fact, the EH system take advantage from the partial knowledge of the channel to enhance its performance. We can see that when the channel experiences fading without CSI-T, the communication system has almost the same performance as the case where the channel is static with $\overline{\gamma} = 1$, (the brown and the blue curves). We use (4.20) to plot the cyan curve, simulation results in the case of the static channel and the case where $\alpha = 1$ confirm our theoretical claims. Also, we plot the AT using expression (4.28) (magenta curve) for $\alpha = 0, 5$, we remark that this curve is close to the simulated AT for this settings.

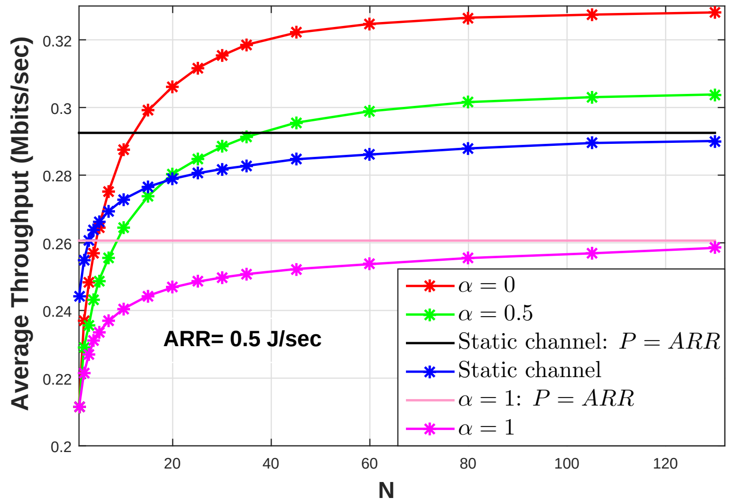

Figure 4.6: Performance of the online policies for N very large, $E_{max} = 100$ J and $\sigma^2 = 1$.

Next, we investigate the performance of the EH for large sensing instants. We observe in Fig.4.6 that the EH system performs better when the channel experiences fading. This is reasonable because, the ARR assumed for this simulation is considered low. Furthermore, we remark that beyond a certain value of N the AT saturates.

4.6 Conclusion

Differently from the Chapter 3, we have proposed a stochastic model for the energy arrival process. Indeed, a first-order Markov has been defined as the joint process of the energy and the battery state. Furthermore, we have investigated the optimal power policy that maximizes the throughput using the DP framework. Afterwards, an asymptotic analysis of the AT has been provided when ARR is either very low or very high, similarly as the analysis provided for the offline model. In fact, considering the low ARR regime, we have deduced that the online system, considered in Chapter 4, can reach the same performance as the offline system, considered in Chapter 3, in terms of data transmitted during the deadline T. So, the knowledge of the energy amounts does not provide an additional benefit for our system, in the low ARR regime. We have also addressed the issue of very large number of sensing instants in order to have an insight into the performance of the communication system where the energy harvester node is designed so that the sensing frequency is very high.

Chapter 5

Summary

The motivation for this project was to study the fundamental communication limits of EH-WSNs.

In this thesis, we have focused on optimal power scheduling of point-to-point communication system where TX is powered by the EH technique and has access only to an estimate version for the fading level of the channel. On the other hand, the CSI-R is assumed to be perfectly known. In Chapter 3, we considered the offline case where the energy arrival is known before the beginning of data transmission. The optimization was for the extreme assumptions of having either perfect CSI-T or no CSI-T available. This work also focused on the practical wireless system where the CSI-T is imperfect, by introducing a model to estimate the channels conditions. In Chapter 4, we have extended our work for the online case where the energy arrival was modeled by a random Markov process. Using dynamic programming, we obtain the optimal power allocation for different cases taking into account the availability of the CSI-T. Besides, we showed also the performances of communication under EH constraints. In fact, we analyzed the behavior of communication system powered by EH nodes where the average recharge rate asymptotically goes to zero and when it is very high.

Thus, this work provides an interesting study of the point-to-point communication system under EH constraints. Indeed, the analysis has been done from a throughput maximization point of view assuming imperfect CSI-T. It has been shown that this scenario can capture all particular cases analyzed previously.

To sum up, considering the low ARR regime, TX transmits data opportunistically by choosing the best channel conditions during the deadline T to send data symbols. In addition to that, in this regime, it can be deduced from this work that the CSI-T provides a linear gain with respect to ARR in terms of data rate. Also, we have deduced that the online system, considered in Chapter 4, can reach the same performance as the offline system, considered in Chapter 3, in terms of data transmitted during the deadline T. However, the CSI-T does not provide an additional gain with respect to no CSI-T case in the high ARR regime.

Finally, I have to say that conducting this project was a great opportunity for me to apply my academic knowledge, also to develop new skills by working with professionals during this valuable experience.

Chapter 6

Future work

In the light of this project, the conducted work can be guidance for future research. In fact, many possible extensions can be done. First, we will extend our work by analyzing the communication limits of energy harvesting wireless sensor networks in the context of MIMO communication in place of point-to-point communication. There is also the possibility of changing the type of communication by considering EH in the context of cognitive communications. Another possible extension is also to model the channel fading as a stochastic process and try to design the optimal power policies and analyzing the performance of EH systems. In addition to that, we can improve the model used by considering that data, to be transmitted, is not available at TX before communication. In fact, we can consider also that data packets arrives as well as the arrivals of the harvested energy during the course of communication taking into account arrival times and sizes.

REFERENCES

[1] M. Pickavet, W. Vereecken, P. Audenaert, B. Vermeulen, C. Develder, D. Colle, B. Dhoedt, and P. Demeester, “Worldwide energy needs for ict: The rise of power-aware networking,” *ANTS’08. 2nd International Symposium on Advanced Networks and Telecommunication Systems, Systems and Computers, 2008.*, pp. 1 – 3, Dec. 2008.

[2] A. Kansal, J. Hsu, S. Zahedi, and M. B. Srivastava, “Power management in energy harvesting sensor networks,” *ACM Trans. Embed. Comput. Syst.*, vol. 6, no. 4, Sep. 2007. [Online]. Available: http://doi.acm.org/10.1145/1274858.1274870

[3] B. Gurakan, O. Ozel, J. Yang, and S. Ulukus, “Energy cooperation in energy harvesting communications,” *IEEE Transactions on Communications.*, vol. 61, no. 12, pp. 4884–4898, December 2013.

[4] D. Gündüz and B. Devillers, “Two-hop communication with energy harvesting,” in *2011 4th IEEE International Workshop on Computational Advances in Multi-Sensor Adaptive Processing (CAMSAP).*, Dec 2011, pp. 201–204.

[5] O. Orhan and E. Erkip, “Energy harvesting two-hop networks: Optimal policies for the multi-energy arrival case,” in *2012 35th IEEE Sarnoff Symposium (SARNOFF).*, May 2012, pp. 1–6.

[6] C. Huang, R. Zhang, and S. Cui, “Throughput maximization for the gaussian relay channel with energy harvesting constraints,” *IEEE Journal on Selected Areas in Communications.*, vol. 31, no. 8, pp. 1469–1479, August 2013.

[7] M. K. Watfa, H. AlHassanieh, and S. Selman, “Multi-hop wireless energy transfer in wsns,” *IEEE Communications Letters.*, vol. 15, no. 12, pp.

1275–1277, 2011.

[8] P. Grover and A. Sahai, “Shannon meets tesla: Wireless information and power transfer.” in *ISIT*, 2010, pp. 2363–2367.

[9] R. Zhang and C. K. Ho, “Mimo broadcasting for simultaneous wireless information and power transfer,” *IEEE Transactions on Wireless Communications.*, vol. 12, no. 5, pp. 1989–2001, 2013.

[10] S. Park, H. Kim, and D. Hong, “Cognitive radio networks with energy harvesting,” *IEEE Transactions on Wireless Communications.*, vol. 12, no. 3, pp. 1386–1397, 2013.

[11] O. Ozel, K. Tutuncuoglu, J. Yang, S. Ulukus, and A. Yener, “Transmission with energy harvesting nodes in fading wireless channels: Optimal policies,” *IEEE Journal on Selected Areas in Communications.*, vol. 29, no. 8, pp. 1732–1743, September 2011.

[12] B. Devillers and D. Gündüz, “A general framework for the optimization of energy harvesting communication systems with battery imperfections,” *Journal of Communications and Networks.*, vol. 14, no. 2, pp. 130–139, 2012.

[13] K. Tutuncuoglu and A. Yener, “Optimum transmission policies for battery limited energy harvesting nodes,” *IEEE Transactions on Wireless Communications.*, vol. 11, no. 3, pp. 1180–1189, 2012.

[14] J. Yang and S. Ulukus, “Optimal packet scheduling in an energy harvesting communication system,” *IEEE Transactions on Communications.*, vol. 60, no. 1, pp. 220–230, January 2012.

[15] C. K. Ho and R. Zhang, “Optimal energy allocation for wireless communications with energy harvesting constraints,” *IEEE Transactions on Signal Processing.*, vol. 60, no. 9, pp. 4808–4818, Sept 2012.

[16] V. Sharma, U. Mukherji, V. Joseph, and S. Gupta, “Optimal energy management policies for energy harvesting sensor nodes,” *IEEE Transactions on Wireless Communications.*, vol. 9, no. 4, pp. 1326–1336, April 2010.

[17] J. Xu and R. Zhang, “Throughput optimal policies for energy harvesting wireless transmitters with non-ideal circuit power,” *IEEE Journal on Selected Areas in Communications.*, vol. 32, no. 2, pp. 322–332, February 2014.

[18] Z. Wang, V. Aggarwal, and X. Wang, “Power allocation for energy harvesting transmitter with causal information,” *IEEE Transactions on Communications.*, vol. 62, no. 11, pp. 4080–4093, 2014.

[19] R. Ma and W. Zhang, “Optimal power allocation for energy harvesting communications with limited channel feedback,” in *2014 IEEE Global Conference on Signal and Information Processing (GlobalSIP).* IEEE, 2014, pp. 193–197.

[20] M.-L. Ku, Y. Chen, and K. Liu, “Data-driven stochastic models and policies for energy harvesting sensor communications,” *IEEE Journal on Selected Areas in Communications.*, vol. 33, no. 8, pp. 1505–1520, 2015.

[21] E. Biglieri, R. Calderbank, A. Constantinides, A. Goldsmith, A. Paulraj, and H. V. Poor, *MIMO wireless communications.* Cambridge university press, 2007.

[22] M. Medard, “The effect upon channel capacity in wireless communications of perfect and imperfect knowledge of the channel,” *IEEE Transactions on Information Theory.*, vol. 46, no. 3, pp. 933–946, 2000.

[23] Z. Rezki and M.-S. Alouini, “Ergodic capacity of cognitive radio under imperfect channel-state information,” *IEEE Transactions on Vehicular Technology.*, vol. 61, no. 5, pp. 2108–2119, Jun 2012.

[24] J. Zhang, C. Yuen, C.-K. Wen, S. Jin, K.-K. Wong, and H. Zhu, “Large system secrecy rate analysis for swipt mimo wiretap channels,” *IEEE Transactions on Information Forensics and Security.*, vol. 11, no. 1, pp. 74–85, 2016.

[25] X. Chen, C. Yuen, and Z. Zhang, “Wireless energy and information transfer tradeoff for limited-feedback multiantenna systems with energy beamforming,” *IEEE Transactions on Vehicular Technology.*, vol. 63, no. 1, pp. 407–412, 2014.

[26] D. Gündüz, K. Stamatiou, N. Michelusi, and M. Zorzi, "Designing intelligent energy harvesting communication systems," *IEEE Communications Magazine.*, vol. 52, no. 1, pp. 210–216, 2014.

[27] V. Raghunathan, A. Kansal, J. Hsu, J. Friedman, and M. Srivastava, "Design considerations for solar energy harvesting wireless embedded systems," in *International Conference on Information Processing in Sensor Networks (IPSN)*, 2005, pp. 457–462.

[28] K. Tutuncuoglu and A. Yener, "Short-term throughput maximization for battery limited energy harvesting nodes," in *2011 IEEE International Conference on Communications (ICC).* IEEE, 2011, pp. 1–5.

[29] ——, "Optimum transmission policies for battery limited energy harvesting nodes," *IEEE Transactions on Wireless Communications.*, vol. 11, no. 3, pp. 1180–1189, 2012.

[30] C. K. Ho, P. D. Khoa, and P. C. Ming, "Markovian models for harvested energy in wireless communications," in *2010 IEEE International Conference on Communication Systems (ICCS).*, Nov 2010, pp. 311–315.

[31] M. L. Puterman, *Markov decision processes: discrete stochastic dynamic programming.* John Wiley & Sons, 2014.

[32] D. P. Bertsekas, D. P. Bertsekas, D. P. Bertsekas, and D. P. Bertsekas, *Dynamic programming and optimal control.* Athena Scientific Belmont, MA, 1995, vol. 1, no. 2.

[33] S. C. Albright, "Structural results for partially observable markov decision processes," *Operations Research*, vol. 27, no. 5, pp. 1041–1053, 1979.

[34] W. S. Lovejoy, "Some monotonicity results for partially observed markov decision processes," *Operations Research*, vol. 35, no. 5, pp. 736–743, 1987.

[35] A. Goldsmith and P. Varaiya, "Capacity of fading channels with channel side information," *IEEE Transactions on Information Theory.*, vol. 43, no. 6, pp. 1986–1992, Nov 1997.

[36] S. Boyd and L. Vandenberghe, *Convex Optimization*. New York, NY, USA: Cambridge University Press, 2004.

[37] M. K. Simon and M.-S. Alouini, *Digital Communication over Fading Channels*. Wiley-Interscience, 2005, vol. 95.

APPENDICES

A Appendix A:

A.1 Proof of Theorem 1

From the optimization problem (P_1), we construct a new problem (P_1') where energy storage constraint is removed and the optimization time of interest is i_1.

$$(P_1'): \begin{cases} \max\limits_{p_i \geq 0} \sum\limits_{i=i_0+1}^{i_1} \frac{L_i}{2} \log(1+\overline{\gamma} p_i) & \text{(A.1a)} \\ \text{s.t.} \sum\limits_{j=i_0+1}^{i} L_j p_j \leq \sum\limits_{j=i_0}^{i-1} E_j, \forall i \in [\![i_0+1, i_1]\!]. & \text{(A.1b)} \end{cases}$$

Now, we formulate an auxiliary optimization problem $(\tilde{P}_1)$, from (P_1'), where only the last constraint is considered:

$$(\tilde{P}_1): \begin{cases} \max\limits_{p_i \geq 0} \sum\limits_{i=i_0+1}^{i_1} \frac{L_i}{2} \log(1+\overline{\gamma} p_i) & \text{(A.2a)} \\ \text{s.t.} \sum\limits_{j=i_0+1}^{i_1} L_j p_j \leq \sum\limits_{j=i_0}^{i_1-1} E_j. & \text{(A.2b)} \end{cases}$$

It is clear that the optimal powers for $(\tilde{P}_1)$ is an upper bound on that of (P_1'), since any feasible power policy for (P_1') is feasible for $(\tilde{P}_1)$. First we solve $(\tilde{P}_1)$, denoting

$T_1 = \sum_{j=i_0+1}^{i_1} L_j$, we have

$$\sum_{i=i_0+1}^{i_1} \frac{L_i}{2}\log(1+\overline{\gamma}p_i) = T_1 \sum_{i=i_0+1}^{i_1} \frac{L_i}{2T_1}\log(1+\overline{\gamma}p_i). \qquad (A.3)$$

Using the Jensen inequality,

$$\sum_{i=i_0+1}^{i_1} \frac{L_i}{2}\log(1+\overline{\gamma}p_i) \leq \frac{T_1}{2}\log(1+\overline{\gamma}\sum_{i=i_0+1}^{i_1} \frac{L_i p_i}{T_1}). \qquad (A.4)$$

Based on our constraint, $\sum_{j=i_0+1}^{i_1} L_j p_j \leq \sum_{j=i_0}^{i_1-1} E_j$, we have

$$\sum_{i=i_0+1}^{i_1} \frac{L_i}{2}\log(1+\overline{\gamma}p_i) \leq \frac{T_1}{2}\log(1+\overline{\gamma}\sum_{j=i_0}^{i_1-1} \frac{E_j}{T_1}). \qquad (A.5)$$

Let us consider the powers given by Theorem 1 for $k = 1$, during epochs i_0+1,i_0+2,...,i_1. Then, we attain an objective value $\frac{T_1}{2}\log(1+\gamma\sum_{j=i_0}^{i_1-1}\frac{E_j}{T_1})$. Thus the optimal value of $(\tilde{P}_1)$ is the upper bound in (A.5). Hence, the power scheme suggested is optimal for the problem (P_1'). Let us now consider this optimal power policy for $(\tilde{P}_1)$ and try to verify if this solution satisfies the constraint in (P_1'). Let $i \in [\![i_0+1, i_1]\!]$, based on the definition of i_1 in Theorem 1 we have:

$$\sum_{j=i_0+1}^{i} L_j p_j = \sum_{j=i_0+1}^{i} L_j \frac{\sum_{k=i_0}^{i_1-1} E_k}{T_1} \leq \sum_{j=i_0+1}^{i} L_j \frac{\sum_{k=i_0}^{i-1} E_k}{\sum_{k=i_0+1}^{i} L_k} \leq \sum_{k=i_0}^{i-1} E_j. \qquad (A.6)$$

So p_i^* is feasible also for (P_1'), and we achieve an objective value $\frac{T_1}{2}\log(1+\overline{\gamma}\sum_{j=i_0}^{i_1-1}\frac{E_j}{T_1})$. We have the gap between P_1 and P_1' is zero. Thus the solution p_i^* is the optimal power policy for P_1. It is clear that the optimal power scheme requires that the energy harvested during the epochs $i_0+1, ..., i_1$ be consumed totally during these time slots. So, the constraint $\sum_{j=1}^{i} L_j p_j \leq \sum_{j=0}^{i-1} E_j, \forall i \in [\![1, N]\!]$ can be decoupled before and after i_1, therefore solving the optimization problem is equivalent to optimize the powers over epochs $i_0+1, ..., i_1$ and $i_1+1, ..., N$ separately. Without loss of generality, we can do the same steps to the optimization problem P'_k defined as follows:

$$(P'_k) : \begin{cases} \max\limits_{p_i \geq 0} \sum\limits_{i=i_{k-1}+1}^{i_k} \frac{L_i}{2} \log(1+\overline{\gamma} p_i) & \text{(A.7a)} \\ \text{s.t.} \sum\limits_{j=i_{k-1}+1}^{i} L_j p_j \leq \sum\limits_{j=i_{k-1}}^{i-1} E_j, i \in [\![i_{k-1}+1, i_k]\!]. & \text{(A.7b)} \end{cases}$$

As mentioned before the energy harvested amounts during epochs $i_{k-1}+1$,$i_{k-1}+2$,...,i_k are consumed totally by the end of the epoch i_k. Therefore, the causality constraint can be decoupled before and after i_k, so we optimize p_i over $i \in [\![i_{k-1}+1, i_k]\!]$ separately. Thus the optimal solutions for $(P'_1), (P'_2), ...$ are also optimal for our main maximization throughput problem.

A.2 Proof of Theorem 4

From the optimization problem (P_6), we construct a new problem $(\hat{P}_1)$ where energy storage constraint is removed and the optimization time of interest is i_1.

$$(\hat{P}_1) : \begin{cases} \max\limits_{p_i \geq 0} \sum\limits_{i=i_0+1}^{i_1} \frac{L_i}{2} \mathbb{E}_{\gamma|\hat{\gamma}} \left(\log(1+\gamma p_i)\right) & \text{(A.8a)} \\ \text{s.t.} \sum\limits_{j=i_0+1}^{i} L_j p_j \leq \sum\limits_{j=i_0}^{i-1} E_j, \forall i \in [\![i_0+1, i_1]\!]. & \text{(A.8b)} \end{cases}$$

First we solve $(\hat{P}_1)$, denoting $T_1 = \sum\limits_{j=i_0+1}^{i_1} L_j$, since the log function is concave and the $\hat{\gamma}_i$ are i.i.d, so

$$\sum_{i=i_0+1}^{i_1} \frac{L_i}{2} \mathbb{E}_{\gamma|\hat{\gamma}} \left(\log(1+\gamma p_i)\right) \leq \sum_{i=i_0+1}^{i_1} \frac{L_i}{2} \log\left(1+\mathbb{E}_{\gamma|\hat{\gamma}_i}(\gamma) p_i\right). \tag{A.9}$$

We have $\mathbb{E}_{\gamma|\hat{\gamma}_i}(\gamma) = (1-\alpha)\hat{\gamma}_i + \alpha \mathbb{E}_{\tilde{\gamma}}(\tilde{\gamma})$, then $\forall i \in [\![i_0+1, i_1]\!]$

$$\mathbb{E}_{\gamma|\hat{\gamma}} \left(\log(1+\gamma p_i)\right) \leq \log\left(1 + \left((1-\alpha)\hat{\gamma}_i + \alpha \mathbb{E}_{\tilde{\gamma}}(\tilde{\gamma})\right) p_i\right). \tag{A.10}$$

Based on the definition of i_1, we have $\forall i \in [\![i_0+1, i_1]\!]$

$$\mathbb{E}_{\gamma|\hat{\gamma}} \left(\log(1+\gamma p_i)\right) \leq \log\left(1+\mathbb{E}_{\gamma|\hat{\gamma}_{i_1}}(\gamma) p_i\right). \tag{A.11}$$

So,

$$\sum_{i=i_0+1}^{i_1} \frac{L_i}{2} \mathbb{E}_{\gamma|\hat{\gamma}} \left(\log(1+\gamma p_i)\right) \leq \sum_{i=i_0+1}^{i_1} \frac{L_i T_1}{2T_1} \log\left(1+\mathbb{E}_{\gamma|\hat{\gamma}_{i_1}}(\gamma) p_i\right). \tag{A.12}$$

Using the Jensen inequality,

$$\sum_{i=i_0+1}^{i_1} \frac{L_i}{2} \mathbb{E}_{\gamma|\hat{\gamma}} \left(\log(1+\gamma p_i)\right) \leq \frac{T_1}{2} \log(1+\mathbb{E}_{\gamma|\hat{\gamma}_{i_1}}(\gamma) \sum_{i=i_0+1}^{i_1} \frac{L_i p_i}{T_1}). \tag{A.13}$$

Based on our constraint, $\sum\limits_{j=i_0+1}^{i_1} L_j p_j \leq \sum\limits_{j=i_0}^{i_1-1} E_j$, we have

$$\sum_{i=i_0+1}^{i_1} \frac{L_i}{2} \log(1+\gamma_i p_i) \leq \frac{T_1}{2} \log(1+\mathbb{E}_{\gamma|\hat{\gamma}_{i_1}}(\gamma) \sum_{j=i_0}^{i_1-1} \frac{E_j}{T_1}) \tag{A.14}$$

$$\leq \frac{\mathbb{E}_{\gamma|\hat{\gamma}_{i_1}}(\gamma)}{2} \sum_{j=i_0}^{i_1-1} E_j. \tag{A.15}$$

Let us consider the powers given by the above Theorem 3 for $k = 1$, during epochs $i_0 + 1$,$i_0 + 2$,...,i_1. Using this power policy, the throughput of the communication system is:

$$T^* = \frac{L_{i_1}}{2}\mathbb{E}_{\gamma|\hat{\gamma}_{i_1}}\left\{\log\left(1+\gamma\sum_{j=i_0}^{i_1-1}\frac{E_j}{L_{i_1}}\right)\right\}. \tag{A.16}$$

Since the amounts of the energy harvested are very small, the throughput can be approximated as follows:

$$T^* \approx \frac{L_{i_1}}{2}\mathbb{E}_{\gamma|\hat{\gamma}_{i_1}}(\gamma)\sum_{j=i_0}^{i_1-1}\frac{E_j}{L_{i_1}} = \frac{(1-\alpha)\hat{\gamma_{i_1}} + \alpha\mathbb{E}_{\tilde{\gamma}}(\tilde{\gamma})}{2}\sum_{j=i_0}^{i_1-1}E_j. \tag{A.17}$$

Hence, by adopting the power scheme suggested the upper bound can be achieved with equality. Thus, the optimal power allocation during epochs $i_0 + 1, ..., i_1$ is derived. The optimal power scheme requires that the energy harvested during the these epochs should be consumed totally during the time slot i_1. So, the constraint $\sum_{j=1}^{i} L_j p_j \leq \sum_{j=0}^{i-1} E_j, \forall i \in [\![1, N]\!]$ can be decoupled before and after i_1, therefore solving the optimization problem is equivalent to optimize the powers over epochs $i_0 + 1, ..., i_1$ and $i_1 + 1, ..., N$ separately. Without loss of generality, we can do the same steps to the optimization problem $(\hat{P}_k)$ defined as follows:

$$(\hat{P}_k): \begin{cases} \max\limits_{p_i\geq 0} \sum\limits_{i=i_{k-1}+1}^{i_k} \frac{L_i}{2}\mathbb{E}_{\gamma|\hat{\gamma}}\left(\log(1+\gamma p_i)\right) & \text{(A.18a)} \\ \text{s.t.} \sum\limits_{j=i_{k-1}+1}^{i} L_j p_j \leq \sum\limits_{j=i_{k-1}}^{i-1} E_j, \forall i \in [\![i_{k-1}+1, i_k]\!]. & \text{(A.18b)} \end{cases}$$

As mentioned before the energy harvested amounts during epochs $i_{k-1} + 1$,$i_{k-1} + 2$,...,i_k are consumed totally by during the epoch i_k. Therefore, the causality constraint can be decoupled before and after i_k, so we optimize p_i over $i \in [\![i_{k-1} + 1, i_k]\!]$ separately. Thus the optimal solutions for $(\hat{P}_1), (\hat{P}_2), ...$ are also optimal for our main maximization throughput problem.

A.3 Proof of Theorem 5

In the low ARR regime, the powers are very low according to Theorem 4. We denote $\underline{\hat{\gamma}} = (\hat{\gamma}_1, \hat{\gamma}_2, ..., \hat{\gamma}_N)$ So we have :

$$
\begin{aligned}
AT_{I.CSI,\alpha}(ARR) =& \mathbb{E}_{\underline{E}}\left\{\mathbb{E}_{\underline{\hat{\gamma}}}\left(\sum_{i=1}^{N}\frac{L_i}{2T}\mathbb{E}_{\gamma|\hat{\gamma}_i}\left\{\log(1+\gamma p_i^*(\underline{\hat{\gamma}}))\right\}\right)\right\} && \text{(A.19)}\\
\approx& \mathbb{E}_{\underline{E}}\left\{\mathbb{E}_{\underline{\hat{\gamma}}}\left(\sum_{i=1}^{N}\frac{L_i}{2\,T}\mathbb{E}_{\gamma|\hat{\gamma}_i}\left(\gamma p_i^*(\underline{\hat{\gamma}})\right)\right)\right\} && \text{(A.20)}\\
=& \mathbb{E}_{\underline{\hat{\gamma}}}\left(\sum_{i=1}^{N}\mathbb{E}_{\underline{E}}\left\{\frac{L_i p_i^*(\underline{\hat{\gamma}})}{2T}\right\}\mathbb{E}_{\gamma|\hat{\gamma}_i}(\gamma)\right) && \text{(A.21)}\\
=& (1-\alpha)\;\mathbb{E}_{\underline{\hat{\gamma}}}\left(\mathbb{E}_{\underline{E}}\left\{\sum_{i=1}^{N}\frac{L_i p_i^*(\underline{\hat{\gamma}})\hat{\gamma}_i}{2T}\right\}\right) + \alpha\;\mathbb{E}_{\underline{\hat{\gamma}}}\left(\mathbb{E}_{\tilde{\gamma}}(\tilde{\gamma})\;\mathbb{E}_{\underline{E}}\left\{\sum_{i=1}^{N}\frac{L_i p_i^*(\underline{\hat{\gamma}})}{2T}\right\}\right). && \text{(A.22)}
\end{aligned}
$$

Assuming that the causality constraint must be satisfied in the last epoch (3.13), so:

$$
\begin{aligned}
\mathbb{E}_{\underline{\hat{\gamma}}}\left(\mathbb{E}_{\tilde{\gamma}}(\tilde{\gamma})\;\mathbb{E}_{\underline{E}}\left\{\sum_{i=1}^{N}\frac{L_i p_i^*(\underline{\hat{\gamma}})}{2T}\right\}\right) &= \frac{1}{2}\mathbb{E}_{\tilde{\gamma}}(\tilde{\gamma})\;\mathbb{E}_{\underline{E}}\left\{\sum_{i=0}^{N-1}\frac{E_i}{T}\right\} && \text{(A.23)}\\
&= \frac{\mathbb{E}_{\tilde{\gamma}}(\tilde{\gamma})}{2}ARR \approx AT_{N.CSI-T}(ARR). && \text{(A.24)}
\end{aligned}
$$

Using the approximation $\log(1+x) \underset{0}{\approx} x$, we have:

$$
\begin{aligned}
AT_{P.CSI-T}(ARR) =& AT_{I.CSI-T,0}(ARR) && \text{(A.25)}\\
=& \mathbb{E}_{\underline{E}}\left(\mathbb{E}_{\underline{\hat{\gamma}}}\left\{\sum_{i=1}^{N}\frac{L_i}{2T}\log(1+\hat{\gamma}_i p_i^*(\underline{\hat{\gamma}}))\right\}\right) && \text{(A.26)}\\
\approx& \mathbb{E}_{\underline{\hat{\gamma}}}\left(\mathbb{E}_{\underline{E}}\left\{\sum_{i=1}^{N}\frac{L_i p_i^*(\underline{\hat{\gamma}})\hat{\gamma}_i}{2T}\right\}\right). && \text{(A.27)}
\end{aligned}
$$

Using the power profile derived in Theorem 3 and Theorem 4, we have:

$$\mathbb{E}_{\underline{\hat{\gamma}}}\left(\mathbb{E}_{\underline{E}}\left\{\sum_{i=1}^{N}\frac{L_i p_i^*(\underline{\hat{\gamma}})\hat{\gamma}_i}{2T}\right\}\right) \approx \mathbb{E}_{\underline{\hat{\gamma}}}\left(\mathbb{E}_{\underline{E}}\left\{\sum_{k}\frac{L_i p_{i_k}^*(\underline{\hat{\gamma}})\hat{\gamma}_{i_k}}{2T}\right\}\right). \tag{A.28}$$

Then (3.58) follows immediately.

A.4 Proof of Proposition 5

Let $AT_{I.CSI,\alpha}$ the AT of our system when the error variance is equal to α. It is obvious that $AT_{I.CSI,\alpha}$ is decreasing in α, hence:

$$0 \leq AT_{I.CSI,1}(ARR) \leq AT_{I.CSI,\alpha}(ARR) \leq AT_{I.CSI,0}(ARR), \qquad \forall \alpha \in [0,1]. \tag{A.29}$$

Lemma 1. Given the same energy profile, the optimal power policy when the channel is perfectly known is related to the optimal power policy when the channel is static. In fact, the optimal powers in (3.38) satisfy this property:

$$\mathbb{E}_{\underline{\gamma}}\left\{p_i^*(\underline{\gamma})\right\} = p_{static,i}^*, \tag{A.30}$$

where, $\underline{\gamma} = (\gamma_n)_n$ and $p_{static,i}^*$ is the optimal power delivered during the epoch i in the case where the channel is static.

Proof. Let us evaluate $\mathbb{E}_{\underline{\gamma}}\left\{p_i^*(\underline{\gamma})\right\}$ between two successive energy arrivals $E_{i-1}^{(h)}$ and $E_i^{(h)}$ where the channel fading level changes $K-1$ times between these energy. Denoting L_j^i the duration of epoch j corresponding to $(j-1)^{th}$ change in the fading level and $L^{(i)}$ the duration between the arrival instants of the two energies. We have all γ_n are i.d.d., so $\mathbb{E}_{\underline{\gamma}}\left\{p_i^*(\underline{\gamma})\right\}$ is similar for all epochs corresponding to a change in the fading level. Thus, $\mathbb{E}_{\underline{\gamma}}\left\{p_i^*(\underline{\gamma})\right\}$ depends only on the energy arrivals. Therefore, to maximize the throughput, the average energy used during all epochs between the two energy arrivals should be equal to the energy used when the channel is static between the two instants of energy arrivals. So, we have:

$$\mathbb{E}_{\underline{\gamma}}\left\{\sum_{j=1}^{K} L_j^{(i)} p_{ij}^*(\underline{\gamma})\right\} = L^{(i)} p_{static,i}^*. \tag{A.31}$$

Then, $\quad \mathbb{E}_{\underline{\gamma}}\left\{p_i^*(\underline{\gamma})\right\} \sum_{j=1}^{K} L_j^{(i)} = L^{(i)} p_{static,i}^*.$

By definition $\sum_{j=1}^{K} L_j^{(i)} = L^{(i)}$. Hence, Lemma 1 follows immediately. □

Let us evaluate the AT when $\alpha = 0$, i.e., we have no error in estimation. Let a fading channel with $\underline{\hat{\gamma}} = (\hat{\gamma}_1, \hat{\gamma}_2, ..., \hat{\gamma}_N)$ be the fading level during transmission.

$$AT_{I.CSI,0}(ARR) = \mathbb{E}_{\underline{E}}\left\{\mathbb{E}_{\underline{\hat{\gamma}}}\left(\sum_{i=1}^{N} \frac{L_i}{2T} \log(1 + \gamma_i p_i^*(\underline{\hat{\gamma}}))\right)\right\} \tag{A.32}$$

$$\approx \mathbb{E}_{\underline{E}}\left\{\sum_{i=1}^{N} \frac{L_i}{2T} \mathbb{E}_{\underline{\hat{\gamma}}}\left(\log \hat{\gamma}_i\right)\right\} + \mathbb{E}_{\underline{E}}\left\{\mathbb{E}_{\underline{\hat{\gamma}}}\left(\sum_{i=1}^{N} \frac{L_i}{2T}\left(\log\ p_i^*(\underline{\hat{\gamma}})\right)\right)\right\}. \tag{A.33}$$

We have $(\hat{\gamma}_i)_{i \in [\![1,N]\!]}$ are i.i.d., so:

$$\mathbb{E}_{\underline{E}}\left\{\sum_{i=1}^{N} \frac{L_i}{2T} \mathbb{E}_{\underline{\hat{\gamma}}}\left(\log \hat{\gamma}_i\right)\right\} = \mathbb{E}_{\underline{E}}\left\{\sum_{i=1}^{N} \frac{L_i}{2T} \mathbb{E}_{\hat{\gamma}_i}\left(\log \hat{\gamma}_i\right)\right\} \tag{A.34}$$

$$= \mathbb{E}_{\underline{E}}\left\{\sum_{i=1}^{N} \frac{L_i}{2T} \mathbb{E}_{\hat{\gamma}}\left(\log \hat{\gamma}\right)\right\} \tag{A.35}$$

$$= \frac{\mathbb{E}_{\hat{\gamma}}\left(\log \hat{\gamma}\right)}{2}. \tag{A.36}$$

Because the log function is concave, so:

$$\mathbb{E}_{\underline{E}}\left\{\mathbb{E}_{\underline{\hat{\gamma}}}\left(\sum_{i=1}^{N} \frac{L_i}{2T} \log\ p_i^*(\underline{\hat{\gamma}})\right)\right\} \leq \mathbb{E}_{\underline{E}}\left\{\sum_{i=1}^{N} \frac{L_i}{2T} \log\left(\mathbb{E}_{\underline{\hat{\gamma}}}(p_i^*(\underline{\hat{\gamma}}))\right)\right\}. \tag{A.37}$$

Then, using Lemma 1, we have:

$$AT_{I.CSI,0}(ARR) \leq \mathbb{E}_{\underline{E}}\left\{\sum_{i=1}^{N}\frac{L_i}{2T}\log\left(\mathbb{E}_{\underline{\hat{\gamma}}}(p_i^*(\underline{\hat{\gamma}}))\right)\right\} + \frac{\mathbb{E}_{\hat{\gamma}}\left(\log\hat{\gamma}\right)}{2} \tag{A.38}$$

$$= \mathbb{E}_{\underline{E}}\left\{\sum_{i=1}^{N}\frac{L_i}{2T}\log(p_{i,Static}^*)\right\} + \frac{\mathbb{E}_{\hat{\gamma}}\left(\log\hat{\gamma}\right)}{2} \tag{A.39}$$

$$= AT_{Stat} + \frac{\mathbb{E}_{\hat{\gamma}}\left(\log\hat{\gamma}\right)}{2}. \tag{A.40}$$

Now, let us evaluate AT when $\alpha = 1$, i.e., TX have not any information about the channel conditions.

$$AT_{I.CSI,1}(ARR) = \mathbb{E}_{\underline{E}}\left\{\mathbb{E}_{\tilde{\gamma}}\left(\sum_{i=1}^{N}\frac{L_i}{2}\log(1+\tilde{\gamma}p_i^*)\right)\right\} \tag{A.41}$$

$$\approx \frac{1}{2}\mathbb{E}_{\underline{E}}\left\{\sum_{i=1}^{N}\frac{L_i}{T}(\log(p_i^*) + \mathbb{E}_{\tilde{\gamma}}(\log(\tilde{\gamma})))\right\} \tag{A.42}$$

$$\approx AT_{Stat} + \frac{\mathbb{E}_{\tilde{\gamma}}\left[\log(\tilde{\gamma})\right]}{2}. \tag{A.43}$$

The channel model considered, in this work, $\tilde{\gamma}$ and $\hat{\gamma}$ are assumed i.i.d. Then (3.59) follows immediately.

B Appendix B:

B.1 Proof of Proposition 9

Lemma 2. Given a random EH process and the matrix of transition probabilities $\mathbf{M}$, the optimal policy when the CSI is unavailable at TX is the same as the power policy in the case when the channel is static during communication.

Proof. Let us denote $(p_i^*)_i$ the optimal powers for the throughput maximization problem in the case where the channel is static, i.e., the powers maximize the following objective function:

$$\max_{\{p_i\}_{i=1}^N} \mathbb{E}_{S_2^N} \left[\sum_{i=1}^{N} \frac{L}{2} \log(1 + \overline{\gamma} p_i) \mid \mathbf{M}, S_1 \right]. \tag{B.1}$$

The problem in the no CSI-T setting is:

$$\max_{\{p_i\}_{i=1}^N} \mathbb{E}_{S_2^N} \left\{ \mathbb{E}_\gamma \left[\sum_{i=1}^{N} \frac{L}{2} \log(1 + \gamma p_i) \right] \mid \mathbf{M}, S_1 \right\}. \tag{B.2}$$

Since $(p_i^*)_i$ is the optimal power policy when the channel is static, and it is clear form the objective function that the optimal powers do not depend on the channel gain $\overline{\gamma}$. The optimal solution depends only on the energies harvested along time. Thus, $(p_i^*)_i$ does not depend on the static channel gain, then we have :

$$\sum_{i=1}^{N} \frac{L}{2} \log(1 + \gamma p_i) \leq \sum_{i=1}^{N} \frac{L}{2} \log(1 + \gamma p_i^*), \ \forall \gamma \tag{B.3}$$

$$\mathbb{E}_{S_2^N}\left[\sum_{i=1}^{N}\frac{L}{2}\log(1+\gamma p_i)\mid \mathbf{M},S_1\right]\leq \mathbb{E}_{S_2^N}\left[\sum_{i=1}^{N}\frac{L}{2}\log(1+\gamma p_i^*)\mid \mathbf{M},S_1\right] \tag{B.4}$$

$$\max_{\{p_i\}_{i=1}^N}\ \mathbb{E}_{\gamma}\left\{\mathbb{E}_{S_2^N}\left[\sum_{i=1}^{N}\frac{L}{2}\log(1+\gamma p_i)\mid \mathbf{M},S_1\right]\right\}\leq \mathbb{E}_{\gamma}\left\{\mathbb{E}_{S_2^N}\left[\sum_{i=1}^{N}\frac{L}{2}\log(1+\gamma p_i^*)\mid \mathbf{M},S_1\right]\right\}. \tag{B.5}$$

So, $\mathbb{E}_{\gamma}\left\{\mathbb{E}_{S_2^N}\left[\sum_{i=1}^{N}\frac{L}{2}\log(1+\gamma p_i^*)\mid \mathbf{M},S_1\right]\right\}$ is an upper bound for the problem (B.2) that can be achieved by the power policy $(p_i^*)_i$, also this policy is feasible because it satisfies the EH constraints. Consequently, the optimal power policy is $(p_i^*)_i$. □

Let $\alpha = 1$, the TX have not information about the channel. we have just proved in Lemma 2 that the optimal power allocation is the same as the case where the channel is static. Then, we have:

$$\begin{aligned} AT_{N.CSI-T} &= \mathbb{E}_{S_1^N}\left\{\mathbb{E}_{\tilde{\gamma}}\left(\sum_{i=1}^{N}\frac{L}{2}\log(1+\tilde{\gamma}p_i^*)\right)\mid \mathbf{M}\right\} && \text{(B.6)}\\ &\approx \frac{1}{2}\mathbb{E}_{S_1^N}\left\{\sum_{i=1}^{N}\frac{L}{T}(\log(p_i^*)+\mathbb{E}_{\tilde{\gamma}}(\log(\tilde{\gamma})))\mid \mathbf{M}\right\} && \text{(B.7)}\\ &\approx AT_{Stat}+\frac{\mathbb{E}_{\tilde{\gamma}}\left[\log(\tilde{\gamma})\right]}{2}. && \text{(B.8)} \end{aligned}$$

Let a fading channel with $\underline{\hat{\gamma}} = (\hat{\gamma}_1, \hat{\gamma}_2, ..., \hat{\gamma}_N)$ be an estimates of the fading level during transmission.

Lemma 3. Given the same initial state, the optimal power policy when the channel is perfectly known is related to the optimal power policy when the channel is static. In fact, the optimal powers satisfy the property:

$$\mathbb{E}_{\underline{\hat{\gamma}}}\left\{p_i^*(\underline{\hat{\gamma}})\right\} = p_{stat,i}^*, \tag{B.9}$$

where, $\underline{\hat{\gamma}}$ is a channel realization and $p_{stat,i}^*$ is the optimal power delivered during the TS i in the case where the channel is static.

Proof. Let us evaluate $\mathbb{E}_{\underline{\hat{\gamma}}}\left\{p_i^*(\underline{\hat{\gamma}})\right\}$ between two successive energy arrivals E_{i-1} and E_i. We have the average is over possible values of the channel, so $\mathbb{E}_{\underline{\hat{\gamma}}}\left\{p_i^*(\underline{\hat{\gamma}})\right\}$ depends only on the energy arrivals. Therefore, to maximize the throughput, the average energy (averaging with respect to the channel) consumed during the TS i between the two energy arrivals should be allocated optimally. Since, the average energy used does not depend on the channel, an optimal energy profile should be similar to the optimal energy profile when the CSI-T is unavailable. Using result stated in Lemma 2, the average energy consumed during this TS should be equal to the energy used when the channel is static between the two instants of energy arrivals. So, we have:

$$\mathbb{E}_{\underline{\hat{\gamma}}}\left\{Lp_i^*(\underline{\hat{\gamma}})\right\} = Lp_{stat,i}^*.$$

Hence, Lemma 3 follows immediately. □

Now, let us evaluate the AT when $\alpha = 0$:

$$AT_{P.CSI-T} = \mathbb{E}_{S_1^N}\left\{\mathbb{E}_{\underline{\hat{\gamma}}}\left[\sum_{i=1}^{N}\frac{L}{2T}\log(1+\hat{\gamma}_i p_i^*(\underline{\hat{\gamma}}))\right] \mid \mathbf{M}\right\} \tag{B.10}$$

$$\approx \mathbb{E}_{S_1^N}\left\{\sum_{i=1}^{N}\frac{L}{2T}\mathbb{E}_{\underline{\hat{\gamma}}}\left(\log\hat{\gamma}_i\right)\right\} + \mathbb{E}_{S_1^N}\left\{\mathbb{E}_{\underline{\hat{\gamma}}}\left[\sum_{i=1}^{N}\frac{L}{2T}\log(p_i^*(\underline{\hat{\gamma}}))\right] \mid \mathbf{M}\right\}. \tag{B.11}$$

We have $(\hat{\gamma}_i)_{i\in[\![1,N]\!]}$ are i.i.d., so:

$$\mathbb{E}_{S_1^N}\left\{\sum_{i=1}^{N}\frac{L}{2T}\mathbb{E}_{\underline{\hat{\gamma}}}\left(\log\hat{\gamma}_i\right)\right\} = \sum_{i=1}^{N}\frac{L}{2T}\mathbb{E}_{\hat{\gamma}_i}\left(\log\hat{\gamma}_i\right) \tag{B.12}$$

$$= \sum_{i=1}^{N}\frac{L}{2T}\mathbb{E}_{\hat{\gamma}}\left(\log\hat{\gamma}\right) \tag{B.13}$$

$$= \frac{\mathbb{E}_{\hat{\gamma}}\left(\log\hat{\gamma}\right)}{2}. \tag{B.14}$$

Because the log function is concave, so:

$$\mathbb{E}_{S_1^N}\left\{\mathbb{E}_{\underline{\hat{\gamma}}}\left[\sum_{i=1}^{N}\frac{L}{2T}\log(p_i^*(\underline{\hat{\gamma}}))\right] \mid \mathbf{M}\right\} \leq \mathbb{E}_{S_1^N}\left\{\sum_{i=1}^{N}\frac{L}{2T}\log(\mathbb{E}_{\underline{\hat{\gamma}}}\left[p_i^*(\underline{\hat{\gamma}})\right]) \mid \mathbf{M}\right\}. \quad \text{(B.15)}$$

Then, we have:

$$AT_{P.CSI-T} \leq \mathbb{E}_{S_1^N}\left\{\sum_{i=1}^{N}\frac{L}{2T}\log(\mathbb{E}_{\underline{\hat{\gamma}}}\left[p_i^*(\underline{\hat{\gamma}})\right]) \mid \mathbf{M}\right\} + \frac{\mathbb{E}_{\hat{\gamma}}\left(\log\hat{\gamma}\right)}{2} \quad \text{(B.16)}$$

Using Lemma 3

$$AT_{P.CSI-T} \leq \mathbb{E}_{S_1^N}\left\{\sum_{i=1}^{N}\frac{L}{2T}\log(p_{stat,i}^*) \mid \mathbf{M}\right\} + \frac{\mathbb{E}_{\hat{\gamma}}\left(\log\hat{\gamma}\right)}{2} \quad \text{(B.17)}$$

$$= AT_{Stat} + \frac{\mathbb{E}_{\hat{\gamma}}\left(\log\hat{\gamma}\right)}{2}. \quad \text{(B.18)}$$

Then (4.29) follows immediately.

One can remark that the proof of Proposition 5, in the offline analysis, is a special case of this proof.

C Papers Submitted and Under Preparation

• M.-R. Znaidi, Z. Rezki, H. Tembine, and M.-S. Alouini, "Performance Limits of Energy Harvesting Communications under Imperfect Channel State Information", *Accepted for ICC 2016*, Kuala Lumpur, Malaysia.

• M.-R. Znaidi, Z. Rezki, and M.-S. Alouini, "On Communications under Stochastic Energy Harvesting with Noisy Channel State Information", Submitted to GLOBECOM 2016.

• M.-R. Znaidi, Z. Rezki, H. Tembine, and M.-S. Alouini, "Performance Limits of Energy Harvesting Communications with Noisy Channel State Information at the Transmitter-Part I: Offline Analysis", *To be submitted to IEEE Transactions on Communications.*

• M.-R. Znaidi, Z. Rezki, and M.-S. Alouini, "Performance Limits of Energy Harvesting Communications with Noisy Channel State Information at the Transmitter-Part II: Online Analysis", *To be submitted to IEEE Transactions on Communications.*