# Low Complexity Precoder and Receiver Design for Massive MIMO Systems: A Large System Analysis using Random Matrix Theory

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# ABSTRACT


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Massive MIMO systems are shown to be a promising technology for next generations of wireless communication networks. The realization of the attractive merits promised by massive MIMO systems requires advanced linear precoding and receiving techniques in order to mitigate the interference in downlink and uplink transmissions. This work considers the precoder and receiver design in massive MIMO systems.

We first consider the design of the linear precoder and receiver that maximize the minimum signal-to-interference-plus-noise ratio (SINR) subject to a given power constraint. The analysis is carried out under the asymptotic regime in which the number of the BS antennas and that of the users grow large with a bounded ratio. This allows us to leverage tools from random matrix theory in order to approximate the parameters of the optimal linear precoder and receiver by their deterministic approximations. Such a result is of valuable practical interest, as it provides a handier way to implement the optimal precoder and receiver. To reduce further the complexity, we propose to apply the truncated polynomial expansion (TPE) concept on a per-user basis to approximate the inverse of large matrices that appear on the expressions of

the optimal linear transceivers. Using tools from random matrix theory, we determine deterministic approximations of the SINR and the transmit power in the asymptotic regime. Then, the optimal per-user weight coefficients that solve the max-min SINR problem are derived. The simulation results show that the proposed precoder and receiver provide very close to optimal performance while reducing significantly the computational complexity.

As a second part of this work, the TPE technique in a per-user basis is applied to the optimal linear precoding that minimizes the transmit power while satisfying a set of target SINR constraints. Due to the emerging research field of green cellular networks, such a problem is receiving increasing interest nowadays. Closed form expressions of the optimal parameters of the proposed low complexity precoding for power minimization are derived. Numerical results show that the proposed power minimization precoding approximates well the performance of the optimal linear precoding while being more practical for implementation.

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# TABLE OF CONTENTS

# LIST OF ABBREVIATIONS

| | |
|---|---|
| OLP | Optimal Linear Precoding |
| A-OLP | Asymptotically Optimal Linear Precoding |
| OLR | Optimal Linear Receiver |
| A-OLR | Asymptotically Optimal Linear Receiver |
| TPE | Truncated Polynomial Expansion |
| US-TPE | User Specific TPE |
| SNR | Signal to Noise Ratio |
| SINR | Signal to Interference-plus-Noise Ratio |
| TDD | Time Division Duplex |
| MMSE | Minimum Mean Square Error |
| BS | Base Station |
| CSI | Channel State Information |
| MIMO | Multiple Input Multiple Output |
| MU-MIMO | Multi-User Multiple Input Multiple Output |
| ZF | Zero Forcing |
| RZF | Regularized Zero Forcing |
| LTE | Long-Term Evolution (standard for wireless communication) |
| LTE-A | LTE-Advanced |
| UE | User Equipement |

# LIST OF SYMBOLS

| | |
|---|---|
| $M$ | Number of BS antennas |
| $K$ | Number of users |
| $\eta$ | Channel state information parameter |
| $\rho$ | Effective SNR |
| $\gamma_k$ | Target SINR of user $k$ |
| $P$ | Transmit power |
| $P_{\max}$ | Maximal transmit power |
| $r$ | Average per user rate |
| $r_{\max}$ | Maximal allowed rate |
| $\overline{P}$ | Asymptotic equivalent of the transmit power |
| $\sigma^2$ | Noise variance |
| $\beta_k$ | Channel attenuation coefficient of user $k$ |
| $\mathbf{h}_k$ | Channel vector of user $k$ |
| $\mathbf{H}_k$ | Channel matrix |
| $\widehat{\mathbf{H}}_k$ | Estimate of the channel matrix |
| $\tau$ | Achievable weighted SINR |

# LIST OF FIGURES

# Chapter 1

# Introduction

## 1.1 Context and Motivation

MIMO technology for wireless communications is now incorporated into emerging wireless broadband standards like LTE. The basic idea behind MIMO technology is that the more antennas the transmitter and the receiver are equipped with, the more the available signal paths, the better the performance in terms of data rate and energy efficiency. Massive MIMO is an emerging technology that aims to profit further from the benefits of MIMO systems by increasing significantly the number of antennas at the transceivers. In massive MIMO, the base station (BS) is equipped with a few hundreds of antennas, simultaneously serving many tens of users in the same time-frequency resource.

Due to their finer spatial resolution achieved by the use of a huge number of antennas at the base station, these systems can focus energy into small regions, thereby reducing the inter-user interference. It is thus no surprise that at equal performances, massive MIMO systems require less energy than usual MIMO systems [1–4]. This however comes at the cost of an increasing computational complexity, since as the number of users increases, the mitigation of the inter-user interference becomes more challenging. In fact, the realization of the attractive merits promised by massive MIMO systems requires the BS to employ advanced linear precoding and receiv-

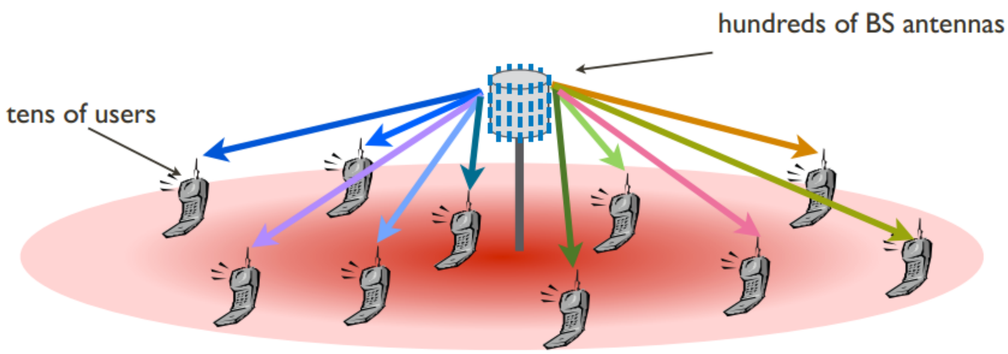


Figure 1.1: Single cell massive MIMO

ing techniques in order to mitigate the induced interference in downlink and uplink transmissions respectively. The classical precoding and receiving techniques are very computationally demanding in large scale systems since they involve computation of very large-dimension matrices inversion. Thus, more efficient precoding and receiving schemes in terms of computational complexity are needed for massive MIMO systems.

## 1.2 Contribution and Literature Overview

The design of these transmit and receive techniques can be based on several performance metrics of interest that reflect different design properties. Among them, we distinguish the minimization of the transmit power, [5–7] and the maximization of the minimum SINR [8,9]. In this work, we focus on the problem of designing the optimal linear precoding (OLP) and the optimal linear receiver (OLR) that maximize the minimum SINR while satisfying a total power constraint.

In particular, we consider the case of a single-cell multi-user MIMO system in which the BS makes use of $M$ antennas in order to communicate with $K$ single-antenna UEs. Within this setting, and assuming perfect channel state information (CSI) at the BS, the OLP is known to be tightly related to the OLR by the uplink-downlink duality principle, which allows to convert the downlink optimization problem into its equivalent counterpart in the dual uplink variables. In particular, by uplink-downlink duality, it was shown that the optimal OLP involves solving a fixed-point problem, the solutions of which correspond to the powers allocated to the UEs of the dual uplink network. Although computationally feasible, the OLP and OLR might be difficult to implement in practice for two major reasons: ($i$) First, they both involve the computation of a matrix inverse at each coherence period. Second, ($ii$) they require the solving the fixed point system of equations yielding the powers of the uplink dual network. In order to solve the latter issue, we follow the same approach as in recent works [10, 11]. Particularly, we consider the asymptotic regime wherein $M$ and $K$ grow large which allows us to leverage recent results from random matrix theory. The analysis is performed under the assumption of imperfect CSI for the UEs. Under this setting, we show that the powers associated with the uplink dual networks converge to deterministic values that depend only on the channel statistics. The obtained expressions can be thus plugged into the OLP and OLR as a replacement of the fixed point solutions, without causing a large loss in performances for sufficiently large $M$ and $K$. Additionally, using these results, we derive the asymptotic SINR that is achieved in the downlink and uplink networks.

Although the asymptotic uplink powers are computed only when the channel statistics change, the complexity of the OLP and OLR are still high, since they require insersion of a large matrices. To overcome this issue, we propose to make use of the truncated polynomial expansion (TPE) technique. This method has already been applied to the minimum mean squared error (MMSE) detector in [12–15] and more recently to the regularized-zero forcing (RZF) precoder in [16, 17]. However, a close look to these works reveal that they all assumed the same TPE coefficients for all user equipments (UEs). This has induced a loss on the available degrees of freedom, and more importantly restricted the application of the TPE to only some particular cases [16, 17]. In light of this observation, we propose in this work to apply the TPE technique on a per-UE basis. More explicitly, the TPE concept is applied to the precoding and/or receiving vectors associated with each UE rather than to the whole precoding or receiving matrix. The resulting transceivers will be thus coined user specific TPE (US-TPE) receiver and precoder. In order to approximate their corresponding SINRs, we assume again that $M$ and $K$ grow large and resort to tools from random matrix theory. The obtained expressions are then optimized with the purpose of maximizing the minimum SINR in both downlink and uplink. In doing so, we prove that the design of the TPE transceivers can be formulated into a standard max-min optimization problems, as such met within the framework of max-min SINR optimization problem previously studied in [6, 8, 9]. This leads to a novel TPE precoder and receiver schemes which are shown by simulations to achieve the same performance as the OLP and OLR under different working scenarios.

An other contribution of this thesis is applying the TPE technique to the optimal linear precoding that minimizes the transmit power while satisfying a set of SINR constraints. Such a problem is receiving an increasing interest nowadays due to the emerging research area of green cellular networks [18]. We consider the same settings as in the first problem with the same channel model and assuming perfect channel state information. Within this setting, the structure of OLP is well-known and can be derived using different approaches [10, 19, 20]. As most linear precoding schemes (a notable exception is the maximum ratio transmission scheme), a major difficulty towards the implementation of OLP is that it requires to compute matrix inversions in every channel coherence period with a computational complexity proportionally to $MK^2$. This makes it unsuited for scenarios with highly varying channels or with large values of $M$ and $K$. To overcome this drawback, in this work the TPE concept is applied to OLP for power minimization on a per-UE basis. In other words, the polynomial truncation artifice is applied separately to each precoding vector rather than to the all precoding matrix. This allows to formulate the optimization problem of the weighting coefficients in a convex form. A further reduction in complexity is obtained by assuming that $M$ and $K$ grow large with a bounded ratio. Such an assumption allows us to approximate the signal-to-noise-plus-interference ratio (SINR) and the transmit power by deterministic quantities that depend only on the channel statistics rather than on its instantaneous realizations [16, 17, 21]. All this leads to a novel TPE precoder scheme, which is shown to achieve the same performance of OLP but with much lower complexity.

## 1.3 Outline of the thesis

The remainder of this thesis is organized as follows. In the next chapter, some results from random matrix theory that will be used in this work are presented. In chapter 3, the system model is introduced and the max-min SINR optimization problem is formulated. In chapter 4, asymptotic analysis of the OLP and the OLR that solve the max-min SINR problem is conducted using tools from random matrix theory while the proposed low complexity precoder and receiver are presented in chapter 5. In chapter 6, we propose a low complexity precoding that solves the power minimization problem. Finally, some conclusions and implications are drawn in chapter 7.

## 1.4 Notations

Boldface upper case is used for denoting matrices, $\mathbf{X}$, and lower case for column vectors, $\mathbf{x}$. The transpose, conjugate and hermitian of $\mathbf{X}$ are denoted $\mathbf{X}^T$ , $\mathbf{X}^*$ and $\mathbf{X}^H$ respectively. $\mathrm{tr}(\mathbf{X})$ is used for denoting the trace of $\mathbf{X}$. $\mathbf{x} \sim \mathcal{CN}(\overline{\mathbf{x}}, \mathbf{Q})$ denotes a circularly symmetric complex Gaussian random vector with mean $\mathbf{x}$ and covariance matrix $\mathbf{Q}$. Moreover, $\mathbf{I}_M$ denotes the $M \times M$ identity matrix. Given an infinitely differentiable function $f(t)$, the $n$-th derivative at $t_0$ is denoted $f^{(n)}(t_0)$ and it is denoted by $f^{(n)}$ when $t = 0$ for notational simplicity. The operator $\mathrm{diag}\left(\{v_k\}_{k=1}^{K}\right)$

is the diagonal matrix having $v_1, \cdots v_K$ as diagonal elements. The notation $f(t) = \mathcal{O}(g(t))$ means $|\frac{f(t)}{g(t)}|$ is bounded while $f(t) = o(g(t))$ means $|\frac{f(t)}{g(t)}|$ tends to zero when $x \to \infty$.

# Chapter 2

# Random Matrix Theory Tools

This chapter presents the main results from Random Matrix Theory that are used throughout this work. These results are very useful in chapter 4 and 5 where asymptotic approximations, also known as deterministic equivalents, of the SINR, the transmit power and several other random quantities are computed. Numerical results are given in this chapter to analyze the accuracy of the asymptotic approximations .

## 2.1 Random matrix theory development

Random Matrix Theory has been a very active research area of mathematics with several applications such as theoretical physics, number theory, financial mathematics, and recently in wireless communications. The main objective of Random Matrix Theory is to understand some properties ( such as statistics of eigenvalues) of matrices with random entries drawn from different probability distributions. This field was introduced by Wishart in 1928 in his results on matrices with Gaussian entries and fixed dimensions. Since then, many asymptotic results have been established. For instance, Wigner provided in 1967 the first results on the asymptotic eigenvalue distributions of symmetric matrices which had been applied in nuclear physics.

Recently, Random Matrix theory has received an increasing interest in the field of wireless communications providing asymptotic analysis of some important metrics such as channel capacity, SINR and transmit power. In this section, we present some results about the asymptotic equivalents of some relevant quantities to our work and we discuss their accuracy.

### 2.1.1 Resolvent and Gram matrices

We first define the Resolvent and Gram matrices which are very useful in random matrix theory. Let $\mathbf{H} = (H_{m,k})$ be a $M \times K$ random matrix. $\mathbf{H}\mathbf{H}^H$ and $\mathbf{Q}(t) = (t\mathbf{H}\mathbf{H}^H + \mathbf{I}_M)^{-1}$ are said to be the Gram and resolvent matrices corresponding to $\mathbf{H}$.

Since the dimension of the considered matrices can grow to infinity, the entries of $\mathbf{H}$ need to be normalized by $\frac{1}{K}$ so as to ensure a finite energy per row and column of $\mathbf{H}$. More explicitly, $\mathbf{H}$ is defined as:

$$H_{m,k} = \frac{\sigma_{m,k} W_{m,k}}{\sqrt{K}}$$

where $(W_{m,k})$ is a sequence of i.i.d random variables with zero mean and variance 1, and $(\sigma_{m,k})$ is a bounded sequence of positive real numbers referred to as variance profile.

According to the structure of $(\sigma_{m,k})$ three kinds of variance profiles can be distinguished:

- Separable profile: The variance profile is said to be separable if there exist positive sequences $d_m$ and $\tilde{d}_k$ such that:

$$\sigma_{m,k}^2 = d_m \tilde{d}_k$$

- Limit profile: The variance profile is said to be limit if there exists a continuous function $f(x, y)$ defined on $[0, 1]^2$ such that:

$$\sigma_{m,k}^2 = f\left(\frac{m}{M}, \frac{k}{K}\right)$$

- General profile: If the variance profile is not separable, it is said to be general.

### 2.1.2 Deterministic equivalents

Since the asymptotic spectral measure of Gram matrices does not always exist, [22], asymptotic equivalents that dictate the asymptotic behavior of the spectral measure have been ivestigated. Such asymptotic equivalents are shown to be unique and to always exist for any variance profile, [22].

Hereafter, we present these asymptotic equivalents for separable variance profiles (which is the case in our work) as well as some of their properties. But first, we need to define the following quantities:

**Definition 1.** Let $(\sigma_{m,k}) = d_m \tilde{d}_k$ be a separable variance profile. Based on this variance profile, we define the following diagonal matrices:

$$\mathbf{D} = \mathrm{diag}\left(d_1, ..., d_M\right) \qquad \text{and} \qquad \tilde{\mathbf{D}} = \mathrm{diag}(\tilde{d}_1, ..., \tilde{d}_K)$$

The following theorem, adapted from [22], states the main result that will be next exploited in the asymptotic analysis. For notational convenience, we define first the matrix $\tilde{\mathbf{Q}}(t) = (t\mathbf{H}^H\mathbf{H} + \mathbf{I}_K)^{-1}$.

**Theorem 1.** [22] *The system of two functional equations:*

$$\begin{cases} \delta(t) &= \frac{1}{K}\mathrm{Tr}\left(\mathbf{D}\left(\mathbf{I}_M + t\tilde{\delta}(t)\mathbf{D}\right)^{-1}\right) \\ \tilde{\delta}(t) &= \frac{1}{K}\mathrm{Tr}\left(\tilde{\mathbf{D}}\left(\mathbf{I}_K + t\delta(t)\tilde{\mathbf{D}}\right)^{-1}\right) \end{cases}$$

*admits a unique solution* $(\delta, \tilde{\delta})$*. Moreover, letting* $t_0 \in [0, +\infty[$*, the system admits a unique point-wise solution* $(\delta(t_0), \tilde{\delta}(t_0))$ *such that* $\delta(t_0) > 0$ *and* $\tilde{\delta}(t_0) > 0$*. Let* $\mathbf{T}(t)$ *and* $\tilde{\mathbf{T}}(t)$ *be the matrix functions defined as:*

$$\mathbf{T}(t) = \left(\mathbf{I}_M + t\tilde{\delta}(t)\mathbf{D}\right)^{-1} \qquad \textit{and} \qquad \tilde{\mathbf{T}}(t) = \left(\mathbf{I}_K + t\delta(t)\tilde{\mathbf{D}}\right)^{-1}$$

*Assuming that* $0 < \lim\,\inf \frac{K}{M} \leq \lim\,\sup \frac{K}{M} < \infty$*, we have*

$$\frac{1}{K}\mathrm{Tr}\left(\mathbf{A}\left(\mathbf{Q}(t) - \mathbf{T}(t)\right)\right) \xrightarrow[M,K\to+\infty]{\text{a.s.}} 0 \quad \textit{and} \quad \frac{1}{K}\mathrm{Tr}\left(\tilde{\mathbf{A}}\left(\tilde{\mathbf{Q}}(t) - \tilde{\mathbf{T}}(t)\right)\right) \xrightarrow[M,K\to+\infty]{\text{a.s.}} 0.$$

*where* $\tilde{\mathbf{A}}$ *and* $\mathbf{A}$ *are test matrices with bounded spectral norm.*

$\mathbf{T}(t)$, $\tilde{\mathbf{T}}(t)$, $\delta(t)$ and $\tilde{\delta}(t)$ can be computed iteratively as the fixed point of a 2 dimensional system of equations.

**Algorithm 1** Iterative algorithm for asymptotic equivalents computation

$\delta \leftarrow 0$
$\tilde{\delta} \leftarrow 0$
$\epsilon \leftarrow 10^{-3}$
**repeat**
  $\delta^n \leftarrow \delta$
  $\tilde{\delta}^n \leftarrow \tilde{\delta}$
  $\mathbf{T} = \left(\mathbf{I}_N + t\tilde{\delta}^n(t)\mathbf{D}\right)^{-1}$
  $\tilde{\mathbf{T}} = \left(\mathbf{I}_K + t\delta^n(t)\tilde{\mathbf{D}}\right)^{-1}$
  $\delta \leftarrow \frac{1}{K}\mathrm{Tr}\left(\mathbf{DT}\right)$
  $\tilde{\delta} \leftarrow \frac{1}{K}\mathrm{Tr}\left(\tilde{\mathbf{D}}\tilde{\mathbf{T}}\right)$
**until** $\left(\frac{\delta^n - \delta}{\delta^n} < \epsilon\right) \& \left(\frac{\tilde{\delta}^n - \tilde{\delta}}{\tilde{\delta}^n} < \epsilon\right)$

## 2.2 Numerical results

In this section, numerical results are used to study computational complexity of the deterministic equivalents, as well as their accuracy for finite size dimensions. We consider a separable variance profile with the diagonal matrices $\tilde{\mathbf{D}}$ and $\mathbf{D}$ defined as:

$$\tilde{\mathbf{D}} = \mathrm{diag}(\beta_1, ..., \beta_K),$$

where $(\beta_k)_{1 \leq k \leq K}$ are positive scalars that will be considered as the channel attenuation coefficients in the next chapters. And

$$\mathbf{D} = \mathrm{eig}(\boldsymbol{\Phi}),$$

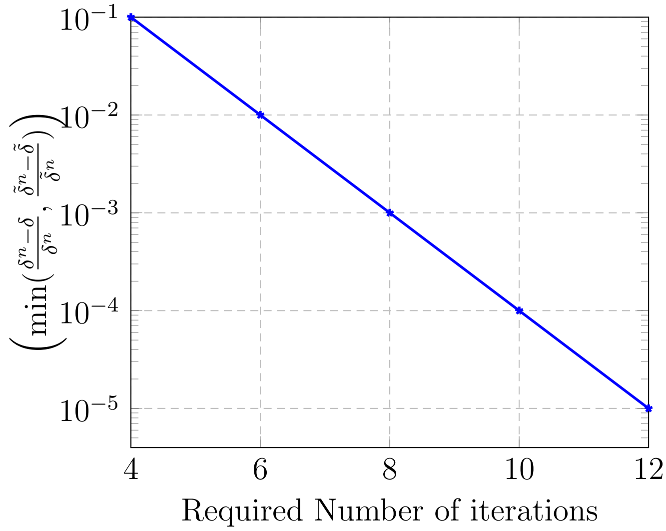


Figure 2.1: Required number of iterations vs. accuracy

where the $M \times M$ matrix $\mathbf{\Phi}$ is defined as:

$$[\mathbf{\Phi}]_{i,j} = \begin{cases} a^{j-i}, & i \leq j, \\ (a^{i-j})^*, & i > j, \end{cases}$$

where $a \in (0, 1)$.

### 2.2.1 Number of iterations

The computation of the deterministic equivalents is in general performed using algorithms similar to algorithm 1 with the exception of some particular cases where closed-form expressions for the deterministic equivalents exist. In figure 2.1, we evaluate the number of required iterations in algorithm 1 for different values of accuracy $\left(\min(\frac{\delta^n-\delta}{\delta^n}, \frac{\tilde{\delta}^n-\tilde{\delta}}{\tilde{\delta}^n})\right)$ and for $M = 100$, $K = 50$ and $t = 1$ . We note that only few iterations are required to reach the convergence.

### 2.2.2 Accuracy of the deterministic equivalents

As long as the matrix dimensions $M$ and $K$ are large enough, the deterministic equivalent $\frac{1}{K}\mathrm{Tr}\left(\mathbf{T}(t)\right)$ aproximates accurately the asymptotic behavior of the random quantity $\frac{1}{K}\mathrm{Tr}\left(\mathbf{Q}(t)\right)$. This is shown in figure 1.2 where the quantity $\mathbb{E}\frac{1}{K}|\mathrm{Tr}\left(\mathbf{Q}(t)\right) - \mathrm{Tr}\left(\mathbf{T}(t)\right)|$ is presented for different values of $M$ and different ratios $\frac{M}{K}$. We observe that, for both cases $\frac{M}{K} = 2$ and $\frac{M}{K} = 10$, the error decreases as long as the matrix dimensions $M$ and $K$ become larger. It is important to note that the error is small even for finite values of $K$

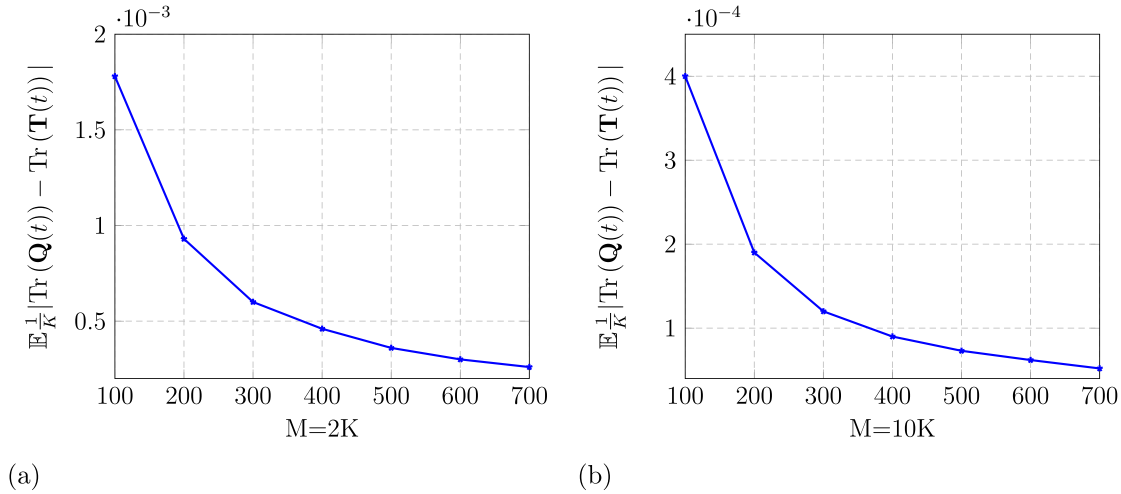


Figure 2.2: Accuracy of the asymptotic equivalents

## 2.3 Conclusion

In this chapter, we have presented a general overview of the important results of random matrix theory that will be used later in this work. In particular, results about a class of asymptotic equivalents approximating the resolvent matrix have been stated, and numerical algorithm that allows their computation for the separable variance profile has been described. It has been shown that only few number of iterations are required to get convergence, which is to be compared with the other alternative based on highly massive Monte Carlo iterations. Moreover, simulations show that the accuracy of the deterministic equivalents is guaranteed even for finite sizes.

# Chapter 3

# Max-Min SINR precoder and receiver

In this chapter, the system and the transmission models are introduced. Then, the problem of designing the optimal linear precoder and receiver that maximize the minimum SINR under a certain power constraint is formally stated. This problem is usually solved by exploiting the duality between uplink and downlink. The expressions of the optimal linear precoding and receiver that solve this problem are presented. The latter are the starting point of this work since they will be compared with our proposed lower complexity precoding and receiver.

## 3.1 System model

We consider a single-cell massive MIMO system in which the BS makes use of $M$ antennas to communicate with $K$ single antenna UEs. The $K$ UEs are randomly chosen from a large set of UEs within the coverage area. We characterize the location of the UE $k$ by the distance $d_k$ that separates it from the base station. The large-scale channel fading at distance $d_k$, which is assumed to be dominated by the path-loss, is denoted by $\beta_k$ and is assumed to be the same for all the BS antennas. Such an assumption is reasonable since the distances between the BS antennas are negligible compared to the distance between the BS and the UE. The channel vector $\mathbf{h}_k$ for UE $k$ is thus modeled as:

$$\mathbf{h}_k = \sqrt{\beta_k}\mathbf{z}_k, \tag{3.1}$$

where $\mathbf{z}_k \sim \mathcal{CN}(\mathbf{0}_{M\times 1}, \mathbf{I}_M)$. For notational convenience, we introduce $\tilde{\mathbf{D}} = \mathrm{diag}(\beta_1, \cdots, \beta_K)$ the diagonal matrix collecting the users channel attenuation coefficients.

### 3.1.1 Downlink

We assume that the BS employs Gaussian codebook and linear precoding. The precoding matrix is denoted by $\mathbf{G} = [\mathbf{g}_1, \cdots, \mathbf{g}_K]$ while the data symbol vector is denoted by $\mathbf{s} = [s_1, \cdots, s_K]^T$ with $s_k \sim \mathcal{CN}(0, 1)$. Then, the transmit signal vector $\mathbf{x}$ is :

$$\mathbf{x} = \mathbf{G}\mathbf{s}, \tag{3.2}$$

whereas the received signal at UE $k$ is:

$$y_k = \sqrt{\rho}\mathbf{h}_k^H \mathbf{g}_k s_k + \sqrt{\rho} \sum_{n=1, n\neq k}^{K} \mathbf{h}_k^H \mathbf{g}_n s_n + n_k, \tag{3.3}$$

where $n_k \sim \mathcal{CN}(0, 1)$ stands for the additive Gaussian noise and $\rho$ is the effective SNR. The SINR at the $k$-th UE is:

$$\mathrm{SINR}_k^{dl} = \frac{\rho \mathbf{h}_k^H \mathbf{g}_k \mathbf{g}_k^H \mathbf{h}_k}{\sum_{i\neq k} \rho \mathbf{h}_k^H \mathbf{g}_i \mathbf{g}_i^H \mathbf{h}_k + 1}. \tag{3.4}$$

The average per UE transmit power at the BS can be expressed as:

$$P = \frac{1}{K}\,\mathrm{tr}(\mathbf{G}\mathbf{G}^H). \tag{3.5}$$

### 3.1.2 Uplink

We assume that the BS employs linear receiver. Denoting $\mathbf{G}_{ul} = [\mathbf{g}_{1,ul}, \cdots, \mathbf{g}_{K,ul}]$ as the receiver matrix, the SINR at the BS of the UE $k$ is expressed as:

$$\mathrm{SINR}_k^{ul} = \frac{\rho \mathbf{h}_k^H \mathbf{g}_{k,ul} \mathbf{g}_{k,ul}^H \mathbf{h}_k}{\sum_{i \neq k} \rho \mathbf{h}_i^H \mathbf{g}_{k,ul} \mathbf{g}_{k,ul}^H \mathbf{h}_i + \|\mathbf{g}_{k,ul}\|^2}, \tag{3.6}$$

where $\rho$ is the effective SNR and $\{q_k\}$ are the transmit powers of the UEs.

The average per UE transmit power in the uplink can be expressed as:

$$P = \frac{1}{K} \sum_{k=1}^{K} q_k. \tag{3.7}$$

## 3.2 Problem formulation

### 3.2.1 Downlink

We propose to determine the optimal linear precoder (OLP) that solves the following the max-min SINR problem:

$$\mathcal{P}_{\mathrm{dl}} : \begin{cases} \max_{\mathbf{g}_1, \cdots, \mathbf{g}_K} & \min \frac{\mathrm{SINR}_k^{dl}}{\gamma_k} \\ \text{subject to} & \frac{1}{K} \operatorname{tr}(\mathbf{G}\mathbf{G}^H) \leq P_{\max} \end{cases} \tag{3.8}$$

where $\{\gamma_k\}$ are fixed coefficients defining the priority of the UEs and $\overline{P}$ is a fixed power constraint. To this end, we write $\mathbf{g}_k$ as:

$$\mathbf{g}_k = \sqrt{\frac{p_k}{K}} \mathbf{u}_k, \tag{3.9}$$

with $p_k > 0$ and $\|\mathbf{u}_k\| = 1$. Vector $\mathbf{u}_k$ represents the normalized beamforming vector and $\frac{p_k}{K}$ is the transmit power allocated to UE $k$. With these notations, the SINR in the downlink corresponding to UE $k$ writes as:

$$\mathrm{SINR}_k^{dl} = \frac{\rho \frac{p_k}{K} |\mathbf{h}_k^H \mathbf{u}_k|^2}{\sum_{i\neq k} \rho \frac{p_i}{K} |\mathbf{h}_k^H \mathbf{u}_i|^2 + 1}, \tag{3.10}$$

whereas the constraint in $\mathcal{P}_{\mathrm{dl}}$ is equivalent to:

$$\frac{1}{K}\sum_{k=1}^{K} p_k \leq P_{\max}. \tag{3.11}$$

The optimization of $P_{\mathrm{dl}}$ over the set of unit-norm vectors $\mathbf{u}_k$ and $\{p_k\}$ has recently been resolved in [23], where it has been shown that the optimal vectors $\{\mathbf{u}_\ell^\star\}$ are given by:

$$\mathbf{u}_\ell^\star = \frac{\mathbf{v}_\ell^\star}{\|\mathbf{v}_\ell^\star\|}, \tag{3.12}$$

with $\mathbf{v}_\ell^\star = \left(\frac{\rho}{K}\sum_{k\neq\ell} q_k^\star \mathbf{h}_k \mathbf{h}_k^H + \mathbf{I}_M\right)^{-1} \mathbf{h}_\ell$ and $q_1^\star, \cdots, q_K^\star$ being the unique positive solutions to the following fixed-point equations:

$$q_\ell^\star = \frac{\gamma_\ell K P_{\max}}{\sum_{n=1}^{K} \frac{\gamma_n \mathbf{h}_\ell^H \left(\frac{1}{K}\sum_{k\neq\ell} q_k^\star \mathbf{h}_k \mathbf{h}_k^H + \frac{1}{\rho}\mathbf{I}_M\right)^{-1} \mathbf{h}_\ell}{\mathbf{h}_n^H \left(\frac{1}{K}\sum_{k\neq n} q_k^\star \mathbf{h}_k \mathbf{h}_k^H + \frac{1}{\rho}\mathbf{I}_M\right)^{-1} \mathbf{h}_n}}. \tag{3.13}$$

It is also well-known that, at optimality, the powers $\{p_k\}$ are the unique solution to the set of equations such that the SINR are such that

$$\frac{\mathrm{SINR}_1^{dl}}{\gamma_1} = \frac{\mathrm{SINR}_2^{dl}}{\gamma_2} = \cdots = \frac{\mathrm{SINR}_K^{dl}}{\gamma_K}. \tag{3.14}$$

Let $\tau^\star = \frac{\mathrm{SINR}_1^{dl}}{\gamma_1}$ denote the achievable weighted SINR. It has been shown in [23] that $\tau^\star$ satisfies:

$$\tau^\star = \frac{K P_{\max}}{\sum_{n=1}^{K} \gamma_n \left(\frac{1}{K}\mathbf{h}_n^H \left(\frac{1}{K}\sum_{k\neq\ell} q_\ell^\star \mathbf{h}_k \mathbf{h}_k^H + \frac{\mathbf{I}_M}{\rho}\right)^{-1} \mathbf{h}_n\right)^{-1}}, \tag{3.15}$$

which also implies that

$$q_\ell^\star = \frac{\gamma_\ell \tau^\star}{\frac{1}{K}\mathbf{h}_\ell^H \left(\frac{1}{K}\sum_{k\neq\ell} q_k^\star \mathbf{h}_k \mathbf{h}_k^H + \frac{\mathbf{I}_M}{\rho}\right)^{-1} \mathbf{h}_\ell}. \tag{3.16}$$

Denote by $\mathbf{p}^\star = [p_1^\star, \cdots, p_K^\star]^T$ the vector of the optimal transmit powers. Solving the set of equations in (3.14), $\mathbf{p}^\star$ can be shown to be given by:

$$\mathbf{p}^\star = \frac{\tau^\star}{\rho}\left(\mathbf{I}_K - \tau^\star \boldsymbol{\Gamma}\mathbf{F}\right)^{-1} \boldsymbol{\Gamma}\mathbf{1}_K, \tag{3.17}$$

where $\boldsymbol{\Gamma} = \text{diag}\left\{\frac{K\gamma_k}{\left|\mathbf{h}_k^H \mathbf{u}_k^\star\right|^2}\right\}_{k=1}^K$ and $\mathbf{F}$ is a $K \times K$ matrix defined as:

$$[\mathbf{F}]_{k,i} = \begin{cases} 0 & \text{if} \quad k = i \\ \frac{1}{K}|\mathbf{h}_k^H \mathbf{u}_i^\star|^2 & \text{if} \quad k \neq i. \end{cases} \tag{3.18}$$

Instead of working with these random quantities that depends on the channel realization, we will determine later, in the asymptotic regime when M and K go large with a fixed ratio, deterministic equivalents of $\tau^*$, $\mathbf{p}^*$ and $\mathbf{q}^*$. These deterministic equivalents depends only on the statistics of the channel and they are not computed at every channel realization.

### 3.2.2 Dual Uplink

From uplink-downlink duality [23], it is known that vectors $\{\mathbf{u}_k\}$ represent the optimal receive beamforming vectors and $\{q_k^\star\}$ the optimal powers allocated to the UEs that would be used by the dual network in order to solve the following max-min uplink SINR problem:

$$\mathcal{P}_{\text{ul}} : \begin{cases} \max_{\substack{\mathbf{u}_1,\cdots,\mathbf{u}_K \\ q_1,\cdots,q_K}} & \min \frac{\text{SINR}_k^{ul}}{\gamma_k} \\ \text{subject to} & \frac{1}{K}\sum_{k=1}^K q_k \leq P_{\max} \end{cases} \tag{3.19}$$

where

$$\mathrm{SINR}_k^{ul} = \frac{\rho \frac{q_k}{K} |\mathbf{h}_k^H \mathbf{u}_k|^2}{\sum_{i \neq k} \rho \frac{q_i}{K} |\mathbf{h}_i^H \mathbf{u}_k|^2 + \|\mathbf{u}_k\|^2}. \tag{3.20}$$

The OLR reduces thus to $\{\mathbf{u}_\ell^\star\}$ given by

$$\mathbf{u}_\ell^\star = \frac{\mathbf{v}_\ell^\star}{\|\mathbf{v}_\ell^\star\|} \tag{3.21}$$

with $\mathbf{v}_\ell^\star = \left( \frac{\rho}{K} \sum_{k \neq \ell} q_k^\star \mathbf{h}_k \mathbf{h}_k^H + \mathbf{I}_M \right)^{-1} \mathbf{h}_\ell$. It follows thus that the optimal receive beamforming vectors $\{\mathbf{u}_k\}$ that solves $P_{\mathrm{ul}}$ coincides with the MMSE receiver that maximizes the $\left\{\mathrm{SINR}_k^{ul}\right\}$.

As it is seen in (3.13), the computation of the uplink transmit powers $\{q_k^\star\}$, which can be also interpreted as Lagrange multipliers for the downlink optimization problem $\mathcal{P}_{\mathrm{ul}}$, is done by solving a system of fixed point equations. Moreover, at every iteration, inverses of large-dimension matrices are needed. This makes the implementation of the OLP and the OLR very computationally demanding in massive MIMO systems. This motivates us to compute deterministic approximations of $\{q_k^\star\}$ using results from random matrix theory.

## 3.3 Conclusion

In this chapter, we have introduced the system model and we have formulated the first main problem considered in this work and the motivation behind solving this problem. Particularly, we have presented the expressions of the recently proposed OLP and OLR that solve this problem and we have explained the necessity of more practical techniques in terms of computational complexity.

# Chapter 4

# Large system analysis of the OLP and the OLR

In this chapter, we give an asymptotic analysis of the OLP and the OLR that solve the max-min SINR problem. Particularly, asymptotic equivalents of the parameters of these precoding and receiving techniques are computed. Based on this asymptotic analysis, deterministic equivalents of the SINRs of the UEs are determined which allow us to solve the max-min SINR problem in the asymptotic regime and obtain new asymptotically optimal precoder and receiver respectively named A-OLP and A-OLR.

## 4.1 Asymptotic regime

As shown in the previous section, the OLP and OLR are parametrized by the set of $\{q_k^\star\}$ and $\{p_k^\star\}$, the computation of which requires solving a set of fixed-point equations. This not only induces a high computational complexity especially when $M$ and $K$ are large, but it also renders the expressions of the OLP and OLR less instrumental to get insights on how these parameters depend on the channel statistics. To overcome this issue, we exploit the fact that $M$ and $K$ are large and make use of results from random matrix theory in order to get accurate approximations for $\{q_k^\star\}$

and $\{p_k^\star\}$. These results are then leveraged to get accurate approximations of the asymptotic uplink and downlink SINRs.

For technical purposes, we shall consider the following asymptotic regime:

**Assumption 1.** *We assume that both $M$ and $K$ grow large, their ratio being bounded below and above as follows:*

$$1 < \liminf \frac{M}{K} \leq \limsup \frac{M}{K} < \infty.$$

We shall also assume that the channel attenuation coefficients $\{\beta_k\}$ satisfy:

**Assumption 2.** *The channel attenuation coefficients $\{\beta_k\}$ satisfy:*

$$0 < \liminf \{\beta_k\} \leq \limsup \{\beta_k\} < \infty.$$

We assume the BS has imperfect knowledge of the instantaneous channel realizations which can be modeled as [24], [21], [25]:

$$\widehat{\mathbf{h}}_k = \sqrt{\beta_k}(\sqrt{1-\eta^2}\mathbf{w}_k + \eta\mathbf{v}_k) = \sqrt{1-\eta^2}\mathbf{h}_k + \sqrt{\beta_k}\eta\mathbf{v}_k, \tag{4.1}$$

where $\mathbf{h}_k$ is the real channel, $\mathbf{v}_k \sim \mathcal{CN}(\mathbf{0}_{M\times 1}, \mathbf{I}_M)$ stands for additive Gaussian noise. The scalar parameter $\eta \in [0,1]$ indicates the quality of the instantaneous CSI, where $\eta = 0$ corresponds to perfect instantaneous CSI and $\eta = 1$ corresponds to having only statistical channel knowledge. The matrix collecting the channel vectors will be denoted $\widehat{\mathbf{H}} = [\widehat{\mathbf{h}}_1, \cdots, \widehat{\mathbf{h}}_K]$.

## 4.2 Asymptotic performance analysis of OLP and OLR

When only imperfect CSI is available, the optimal linear precoding is not known. As a solution, we assume that the true channels are simply replaced by their estimates. Replacing $\mathbf{h}_k$ by $\widehat{\mathbf{h}}_k$ in (3.13) and (3.12) yields the following OLP precoder:

$$\widehat{\mathbf{g}}_\ell = \sqrt{\frac{\widehat{p}_\ell}{K}}\frac{\widehat{\mathbf{v}}_\ell}{\|\widehat{\mathbf{v}}_\ell\|}, \tag{4.2}$$

with $\widehat{\mathbf{v}}_\ell = \left(\frac{\rho}{K}\sum_{k\neq\ell}\widehat{q}_k\widehat{\mathbf{h}}_k\widehat{\mathbf{h}}_k^H + \mathbf{I}_M\right)^{-1}\widehat{\mathbf{h}}_\ell$. Powers $\{\widehat{p}_k\}$ are obtained by solving (3.17) after replacing $\mathbf{h}_k$ by $\widehat{\mathbf{h}}_k$ and $\{\widehat{q}_\ell\}$ being the fixed point solutions to the following system of equations:

$$\widehat{q}_\ell = \frac{\gamma_\ell K P_{\max}}{\sum_{n=1}^K \frac{\gamma_n \widehat{\mathbf{h}}_\ell^H\left(\frac{1}{K}\sum_{k\neq\ell}\widehat{q}_k\widehat{\mathbf{h}}_k\widehat{\mathbf{h}}_k^H + \frac{1}{\rho}\mathbf{I}_M\right)^{-1}\widehat{\mathbf{h}}_\ell}{\widehat{\mathbf{h}}_n^H\left(\frac{1}{K}\sum_{k\neq n}\widehat{q}_k\widehat{\mathbf{h}}_k\widehat{\mathbf{h}}_k^H + \frac{1}{\rho}\mathbf{I}_M\right)^{-1}\widehat{\mathbf{h}}_n}}. \tag{4.3}$$

We further define $\hat{\tau}$ as the weighted achievable SINR.

$$\hat{\tau} = \frac{K P_{\max}}{\sum_{n=1}^K \gamma_n\left(\frac{1}{K}\widehat{\mathbf{h}}_n^H\left(\frac{1}{K}\sum_{k\neq\ell}\widehat{q}_\ell\widehat{\mathbf{h}}_\ell\widehat{\mathbf{h}}_\ell^H + \frac{\mathbf{I}_M}{\rho}\right)^{-1}\widehat{\mathbf{h}}_n\right)^{-1}}. \tag{4.4}$$

Our first main result derives asymptotic approximations of $\hat{\tau}$ and $\widehat{q}_k$:

**Theorem 2.** *Under the settings of Assumptions 1 and 2, we have*

$$\widehat{\tau} - \overline{\tau} \xrightarrow[M,K\to+\infty]{\text{a.s.}} 0,$$

$$\max_{1\leq k\leq K}|\widehat{q}_k - \overline{q}_k| \xrightarrow[M,K\to+\infty]{\text{a.s.}} 0,$$

*where*

$$\bar{q}_k = \frac{\gamma_k \bar{\tau}}{\rho \beta_k \left[ \frac{M}{K} - \frac{1}{K} \sum_{i=1}^{K} \frac{\gamma_i \bar{\tau}}{1+\gamma_i \bar{\tau}} \right]}, \tag{4.5}$$

*and $\bar{\tau}$ is the unique positive solution to the following fixed point equation:*

$$\bar{\tau} = \frac{\rho P_{\max} \left[ \frac{M}{K} - \frac{1}{K} \sum_{i=1}^{K} \frac{\gamma_i \bar{\tau}}{1+\gamma_i \bar{\tau}} \right]}{\frac{1}{K} \sum_{i=1}^{K} \frac{\gamma_i}{\beta_i}}. \tag{4.6}$$

*Proof.* See Appendix B. □

As seen from (4.5)and (4.6), to compute $\{q_k\}$ only the UEs priority coefficients $\{\gamma_k\}$ and the channel attenuation coefficients $\{\beta_k\}$ are needed. The latter can be easily estimated since they change slowly with time. The asymptotic value $\bar{q}_k$ of the Lagrange multiplier $q_k$, which play also the role of the transmit power of the UE $k$ in the dual uplink problem, is found to be proportional to $\gamma_k$ such that the UE with higher priority is allocated more power, and inversely proportional to $\beta_k$ such that UEs with bad channels are assigned more transmit power.

We now proceed to computing the asymptotic SINR and the asymptotic transmit powers in the downlink.

**Theorem 3.** *Under the settings of Assumptions 1 and 2, we have*

$$\max_{1 \leq k \leq K} |\widehat{p}_k - \bar{p}_k| \xrightarrow[M,K \to +\infty]{\text{a.s.}} 0,$$
$$\max_{1 \leq k \leq K} \left| \text{SINR}_k^{dl} - \overline{\text{SINR}}_k^{dl} \right| \xrightarrow[M,K \to +\infty]{\text{a.s.}} 0,$$

*where*

$$\bar{p}_k = \frac{\gamma_k \bar{\tau}}{\rho \beta_k \xi} \left[ \frac{\rho \beta_k P_{\max}}{(1+\gamma_k \bar{\tau})^2} + 1 \right], \tag{4.7}$$

*and*

$$\overline{\mathrm{SINR}}_k^{dl} = \frac{\rho \bar{p}_k (1-\eta^2)\beta_k \xi}{\left[\rho \frac{1+2\eta^2\gamma_k\bar{\tau}+\eta^2\gamma_k^2\bar{\tau}^2}{(1+\gamma_k\bar{\tau})^2}\beta_k P_{\max}\right]+1}, \tag{4.8}$$

*and the positive quantity* $\xi$ *is given by*

$$\xi = \left(\frac{M}{K} - \frac{1}{K}\sum_{i=1}^{K}\frac{[\gamma_i\bar{\tau}]^2}{[1+\gamma_i\bar{\tau}]^2}\right). \tag{4.9}$$

*Proof.* See Appendix C. □

First, from (4.8), it is clear that the asymptotic SINR is a decreasing function of $\eta$. When $\eta = 0$, the SINR at the UE $k$ is maximal and it is equal to $\gamma_k\bar{\tau}$ and when $\eta$ tends to zero the SINR tends to zero. Also, the asymptotic UEs power allocations $\{\bar{p}_k\}$ are independent of $\eta$. This result was expected in fact, we assumed the quality of the CSI is the same for all the users. Let us now proceed to computing the asymptotic SINR in the uplink.

**Theorem 4.** *Under the settings of Assumptions 1 and 2, we have*

$$\max_{1\leq k\leq K}\left|\mathrm{SINR}_k^{ul} - \overline{\mathrm{SINR}}_k^{ul}\right| \xrightarrow[M,K\to+\infty]{\text{a.s.}} 0,$$

*where*

$$\overline{\mathrm{SINR}}_k^{ul} = \frac{\rho \bar{q}_k (1-\eta^2)\beta_k \xi}{\frac{\rho}{K}\sum_{i=1}^{K}\beta_i\bar{q}_i\frac{1+2\eta^2\gamma_i\bar{\tau}+\eta^2\gamma_i^2\bar{\tau}^2}{(1+\gamma_i\bar{\tau})^2}+1}. \tag{4.10}$$

*Proof.* The proof is similar to the downlink case and is thus omitted. □

As mentioned before, at optimality, there is uplink-downlink duality in the perfect CSI case, and the achievable weighted SINR $\tau^\star$ is the same in both uplink and downlink. This duality holds true also in the asymptotic regime as proven in the following corollary.

**Corollary 1.** *If* $\eta = 0$*, then*

$$\frac{\overline{\mathrm{SINR}}_k^{ul}}{\gamma_k} = \frac{\overline{\mathrm{SINR}}_k^{dl}}{\gamma_k} = \bar{\tau}. \tag{4.11}$$

*Proof.* See Appendix D . □

An important consequence of Theorems 2 and 3 is that the performances would remain asymptotically the same if $\{\overline{q}_k\}$ and $\{\overline{p}_k\}$ had been used instead of $\{\widehat{q}_k\}$ and $\{\widehat{p}_k\}$, in which case the precoding vector and receiver associated with the $k$-th UE are given by:

$$\mathbf{g}_k = \sqrt{\frac{\overline{p}_k}{K}} \frac{\mathbf{v}_k}{\|\mathbf{v}_k\|}, \tag{4.12}$$

with $\mathbf{v}_k = \left(\frac{\rho}{K}\sum_{i=1}^{K}\overline{q}_i\widehat{\mathbf{h}}_i\widehat{\mathbf{h}}_i^H + \mathbf{I}\right)^{-1}\widehat{\mathbf{h}}_k$. It is worth mentioning that, unlike parameters $\{\widehat{q}_k\}$ and $\{\widehat{p}_k\}$ which depend on the instantaneous realization of all channel vectors $\left\{\widehat{\mathbf{h}}_k\right\}$, parameters $\{\overline{p}_k\}$ and $\{\overline{q}_k\}$ depend only on the large-scale channel statistics. As a consequence, when plugged into the OLP and/or the OLR, these parameters are not required to be computed at every channel realization but only once per coherence period.

## 4.3 Asymptotically optimal linear precoder and receiver

In the previous section, we started from the expressions of the OLP and the OLR to determine asymptotic equivalents of their parameters $\{\widehat{q}_k\}$ and $\{\widehat{p}_k\}$. In this section, we follow a different approach; we consider the same asymptotic optimal beamforming directions determined in the previous section and given by (4.12). By considering the transmit powers in both uplink and downlink as fixed constants, we determine asymptotic equivalents of the SINRs. Then, the transmit powers are optimized in the asymptotic regime.

Exploiting the results of the previous section concerning the deterministic equivalents of the SINR, our optimization problem in the downlink can be written as

$$\mathcal{P}_{\text{dl},1}:\begin{cases}\max_{p_1,\cdots,p_K} & \min \frac{\rho p_k(1-\eta^2)\beta_k\xi}{\gamma_k\left[\left(\rho\frac{1+2\eta^2\gamma_k\bar{\tau}+\eta^2\gamma_k^2\bar{\tau}^2}{(1+\gamma_k\bar{\tau})^2}\frac{\beta_k}{K}\sum_{i=1}^K p_i\right)+1\right]}\\ \text{subject to} & \frac{1}{K}\sum_{k=1}^K p_k \leq P_{\max}\end{cases} \tag{4.13}$$

And its dual problem in the uplink is

$$\mathcal{P}_{\text{ul},1}:\begin{cases}\max_{q_1,\cdots,q_K} & \min \frac{\rho q_k(1-\eta^2)\beta_k\xi}{\gamma_k\left[\frac{\rho}{K}\sum_{i=1}^K q_i\beta_i\frac{1+2\eta^2\gamma_i\bar{\tau}+\eta^2\gamma_i^2\bar{\tau}^2}{(1+\gamma_i\bar{\tau})^2}+1\right]}\\ \text{subject to} & \frac{1}{K}\sum_{k=1}^K q_k \leq P_{\max}\end{cases} \tag{4.14}$$

For notational convenience, we define the diagonal matrix $\boldsymbol{\beta}$ and the vector $\mathbf{f}$ as:

$$\boldsymbol{\beta} = \text{diag}(\frac{\gamma_1}{\rho\xi\beta_1(1-\eta^2)},\cdots,\frac{\gamma_K}{\rho\xi\beta_K(1-\eta^2)}), \tag{4.15}$$

and

$$[\mathbf{f}]_i = \frac{\rho\beta_i}{K}\frac{1+2\eta^2\gamma_i\bar{\tau}+\eta^2\gamma_i^2\bar{\tau}^2}{(1+\gamma_i\bar{\tau})^2}, \quad \forall i=1,\cdots,K. \tag{4.16}$$

Using these notations, $\mathcal{P}_{\text{dl},1}$ and $\mathcal{P}_{\text{ul},1}$ can be rewritten as

$$\mathcal{P}_{\text{dl},1}:\begin{cases}\max_{p_1,\cdots,p_K} & \min \frac{p_k}{\left[\boldsymbol{\beta}(\mathbf{f}\mathbf{1}^T\mathbf{p}+\mathbf{1})\right]_k}\\ \text{subject to} & \frac{1}{K}\sum_{k=1}^K p_k \leq P_{\max}\end{cases} \tag{4.17}$$

$$\mathcal{P}_{\text{ul},1}:\begin{cases}\max_{q_1,\cdots,q_K} & \min \frac{q_k}{[\boldsymbol{\beta}(\mathbf{1}\mathbf{f}^T\mathbf{q}+\mathbf{1})]_k}\\ \text{subject to} & \frac{1}{K}\sum_{k=1}^K q_k \leq P_{\max}\end{cases} \tag{4.18}$$

It is known that at optimality, the weighted SINR for all UEs are the same, and the weighted power constraint becomes tight. Thus, the optimal power vectors $\tilde{\mathbf{p}}$ and $\tilde{\mathbf{q}}$

satisfy [8, 9]:

$$\frac{1}{\tilde{\tau}}\tilde{\mathbf{p}} = \boldsymbol{\beta}\left(\mathbf{f}\mathbf{1}^T + \frac{1}{KP_{\max}}\mathbf{1}\mathbf{1}^T\right)\tilde{\mathbf{p}}, \tag{4.19}$$

$$\frac{1}{\tilde{\tau}}\tilde{\mathbf{q}} = \boldsymbol{\beta}\left(\mathbf{1}\mathbf{f}^T + \frac{1}{KP_{\max}}\mathbf{1}\mathbf{1}^T\right)\tilde{\mathbf{q}}, \tag{4.20}$$

From the above equations, it can be shown that $\tilde{\mathbf{p}}$ and $\tilde{\mathbf{q}}$ are proportional to the Perron eigenvectors of the non negative matrices $\boldsymbol{\beta}\left(\mathbf{f}\mathbf{1}^T + \frac{1}{KP_{\max}}\mathbf{1}\mathbf{1}^T\right)$ and $\boldsymbol{\beta}\left(\mathbf{1}\mathbf{f}^T + \frac{1}{KP_{\max}}\mathbf{1}\mathbf{1}^T\right)$ respectively. Thus, it follows that

$$\tilde{\mathbf{p}} = \frac{KP_{\max}}{\mathbf{1}^T\boldsymbol{\beta}(\mathbf{f} + \frac{1}{KP_{\max}}\mathbf{1})}\boldsymbol{\beta}(\mathbf{f} + \frac{1}{KP_{\max}}\mathbf{1}), \tag{4.21}$$

$$\tilde{\mathbf{q}} = \frac{KP_{\max}}{\mathbf{1}^T\boldsymbol{\beta}\mathbf{1}}\boldsymbol{\beta}\mathbf{1}, \tag{4.22}$$

which can be expressed more explicitly as

$$\tilde{p}_k = \frac{P_{\max}\gamma_k\frac{1+2\eta^2\gamma_k\bar{\tau}+\eta^2\gamma_k^2\bar{\tau}^2}{(1+\gamma_k\bar{\tau})^2} + \frac{\gamma_k}{\beta_k}}{\frac{1}{K}\sum_{i=1}^K \gamma_i\frac{1+2\eta^2\gamma_i\bar{\tau}+\eta^2\gamma_k^2\bar{\tau}^2}{(1+\gamma_i\bar{\tau})^2} + \frac{\gamma_i}{\beta_i P_{\max}}}, \tag{4.23}$$

$$\tilde{q}_k = \frac{P_{\max}\gamma_k}{\frac{\beta_k}{K}\sum_{i=1}^K \frac{\gamma_i}{\beta_i}}, \tag{4.24}$$

Note that $\tilde{\mathbf{q}}$ is exactly equal to $\bar{\mathbf{q}} = [\bar{q}_1, \cdots, \bar{q}_K]^T$ obtained in the previous section by computing deterministic equivalents of $\{\widehat{q}_k\}$ and thus the asymptotic performance of the OLR is the same as the asymptotically optimal linear receiver (A-OLR) parametrized by $\tilde{\mathbf{q}}$. However, this is not the case for $\tilde{\mathbf{p}}$. The new precoding parametrized by $\tilde{\mathbf{p}}$ will be named asymptotically optimal linear precoding (A-OLP). The performance of the OLP and the A-OLP will be compared later. In fact, while computing $\tilde{\mathbf{p}}$ we are taking into account the quality of CSI, we expect that the proposed A-OLP will provide better performance than the OLP when $\eta \neq 0$. This result will be seen next by simulation.

## 4.4 Numerical results

In this section, numerical simulations are used to compare the OLP and the A-OLP. Moreover, Monte-Carlo (MC) simulations are used to validate the analysis in the asymptotic regime. The UEs are assumed to be uniformly distributed in a cell with radius $D = 250 \ \ m$. The pathloss $\beta_k$ between the BS and a UE $k$ with distance $x_k$ from the BS is modeled as

$$\beta_k = \frac{1}{1 + \left(\frac{x_k}{d_0}\right)^{\delta}} \tag{4.25}$$

where $\delta = 3.8$, $d_0 = 30 \ \ m$. The performance is measured in terms of the average achievable rate of the UEs:

$$r = \frac{1}{K} \sum_{k=1}^{K} \mathbb{E}\left[\log_2(1 + \mathrm{SINR_k})\right] \tag{4.26}$$

where the expectation is computed with respect to the different channel realization. Without loss of generality, we assume that the working signal-to-noise ratio (SNR) is $\rho = 20dB$. The UEs priorities $\{\gamma_k\}$ are randomly chosen from the interval [1,2].

In figure 4.1,we plot the average per UE rate vs. CSI parameter $\eta$ in the downlink when $K = 32$, $P_{\max} = 5$ Watt and $\rho = 20dB$. Markers represents the asymptotic values whereas the error bars indicate the standard deviation of the MC results. We observe that the A-OLP outperforms the OLP when imperfect CSI is available ($\eta \neq 0$).

## 4.5 Conclusion

In this chapter, we have analyzed the asymptotic performance of the OLP and OLR that solve the max-min SINR problem. This asymptotic analysis allows us to propose

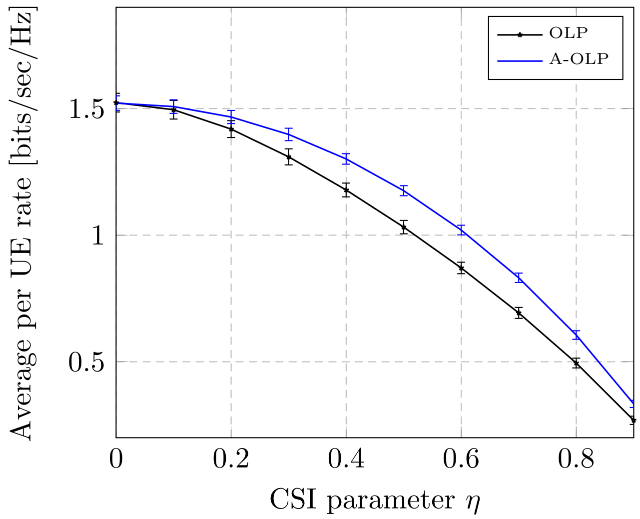


Figure 4.1: Average per UE rate vs. $\eta$ when K=32, M=128, $P_{\max}$=5 Watt and $\rho = 20dB$

new asymptotically optimal precoder and receiver respectively named A-OLP and A-OLR. The A-OLR is expected to provide the same performance of the OLR while reducing the computational complexity. The A-OLP is expected to provide better performance compared to the OLP when only imperfect CSI is available at the BS. This result will be shown by simulation in the next chapter.

# Chapter 5

# Low complexity TPE-based precoder and receiver

In this chapter, alternatives of the OLP and OLR that solve the Max-Min SINR problem are proposed. Applying the TPE technique, new precoder and receiver are proposed and optimized in the asymptotic regime. A comparison in terms of computational complexity between the proposed transceivers and the optimal ones is presented where it is shown that our proposed transceivers are more efficient to implement although their performance in terms of data rate is very close to that of the optimal ones.

## 5.1 User specific TPE precoding and receiver

A major difficulty towards implementing the optimal max-min SINR precoding and receiver is that they require to compute matrix inversions at every channel coherence period. This makes them unsuited for scenarios with large $M$ and $K$. A typical solution to overcome this issue is to resort to the TPE concept. Basically, the TPE consists in replacing the matrix inverse by a matrix polynomial involving $J$ terms, where $J$ is fixed and stands for the truncation order. The traditional way to apply the TPE has been to consider the same truncation coefficients for all users, [16, 17, 26].

In this work, we propose instead to apply the TPE concept on a user basis. The TPE coefficients are then optimized in order to solve a max-min SINR problem. The proposed TPE based technique, which we refer to as User Specific TPE (US-TPE), has two major advantages. First, it increases the degrees of freedom, which shall result in improved performances, and second, as will be shown in the course of this work, it allows to write the optimization problem in a convex form.

### 5.1.1 User Specific TPE precoder

Applying the TPE on a per-UE basis, the precoding vector associated with the $k$-th UE writes as:

$$\mathbf{g}_{k,\mathrm{TPE}}^{dl} = \sqrt{p_{k,\mathrm{TPE}}}\frac{1}{\sqrt{K}}\frac{\mathbf{v}_{k,\mathrm{TPE}}}{\|\mathbf{v}_{k,\mathrm{TPE}}\|}, \tag{5.1}$$

with

$$\mathbf{v}_{k,\mathrm{TPE}} = \sum_{\ell=0}^{J-1} w_{\ell,k}^{dl}\left(\frac{\widehat{\mathbf{H}}\mathbf{R}\widehat{\mathbf{H}}^H}{K}\right)^{\ell}\frac{\widehat{\mathbf{h}}_k}{\sqrt{K}} \tag{5.2}$$

where $\frac{1}{K}p_{k,\mathrm{TPE}}$ is the transmit power allocated to UE $k$, $J$ is the truncation order and $\mathbf{R} = \mathrm{diag}(\overline{q}_1, \cdots, \overline{q}_K)$. Plugging $\mathbf{g}_{k,\mathrm{TPE}}^{dl}$ into (6.2) and denoting by $\mathbf{w}_{k,dl} = \left[w_{0,k}^{dl}, \cdots, w_{J-1,k}^{dl}\right]^T$, the SINR corresponding to the $k$-th UE can be written as:

$$\mathrm{SINR}_{k,\mathrm{TPE}}^{\mathrm{dl}} = \frac{\rho p_{k,\mathrm{TPE}}\frac{\mathbf{w}_k^H\mathbf{a}_k\mathbf{a}_k^H\mathbf{w}_k}{\mathbf{w}_k^H\mathbf{E}_k\mathbf{w}_k}}{\left(\frac{\rho}{K}\sum_{i\neq k} p_{i,\mathrm{TPE}}\frac{\mathbf{w}_i^H\mathbf{B}_{k,i}\mathbf{w}_i}{\mathbf{w}_i^H\mathbf{E}_i\mathbf{w}_i} + 1\right)} \tag{5.3}$$

where $\mathbf{a}_k \in \mathbb{C}^{J\times 1}$, $\mathbf{E}_k \in \mathbb{C}^{J\times J}$, and $\mathbf{B}_{i,k} \in \mathbb{C}^{J\times J}$ are given by:

$$[\mathbf{a}_k]_\ell = \frac{1}{K}\mathbf{h}_k^H \left(\frac{\widehat{\mathbf{H}}\mathbf{R}\widehat{\mathbf{H}}^H}{K}\right)^\ell \widehat{\mathbf{h}}_k \tag{5.4}$$

$$[\mathbf{B}_{k,i}]_{\ell,m} = \frac{1}{K}\mathbf{h}_k^H \left(\frac{\widehat{\mathbf{H}}\mathbf{R}\widehat{\mathbf{H}}^H}{K}\right)^\ell \widehat{\mathbf{h}}_i\widehat{\mathbf{h}}_i^H \left(\frac{\widehat{\mathbf{H}}\mathbf{R}\widehat{\mathbf{H}}^H}{K}\right)^m \mathbf{h}_k \tag{5.5}$$

$$[\mathbf{E}_k]_{\ell,m} = \frac{1}{K}\widehat{\mathbf{h}}_k^H \left(\frac{\widehat{\mathbf{H}}\mathbf{R}\widehat{\mathbf{H}}^H}{K}\right)^{\ell+m} \widehat{\mathbf{h}}_k \tag{5.6}$$

$$\tag{5.7}$$

The transmit power at the BS can be expressed as:

$$P = \sum_{k=1}^{K} \frac{p_{k,\mathrm{TPE}}}{K} \tag{5.8}$$

### 5.1.2 User Specific TPE receiver

The TPE concept is now applied to the OLR. Let $\left\{\frac{q_{k,\mathrm{TPE}}}{K}\right\}$ be the set of UL transmit powers. The receive beamforming vector associated with the $k$-th UE is thus given by:

$$\mathbf{g}_{k,\mathrm{TPE}}^{ul} = \sum_{\ell=0}^{J-1} w_{\ell,k}^{ul} \left(\frac{\widehat{\mathbf{H}}\mathbf{R}\widehat{\mathbf{H}}^H}{K}\right) \frac{\widehat{\mathbf{h}}_k}{\sqrt{K}} \tag{5.9}$$

Plugging $\mathbf{g}_{k,\mathrm{TPE}}^{ul}$ into (3.20) and denoting $\mathbf{w}_k^{ul} = \left[w_{0,k}^{ul}, \cdots, w_{J-1,k}^{ul}\right]^T$, the SINR associated with the $k$th UE is given by:

$$\mathrm{SINR}_{k,\mathrm{TPE}}^{ul} = \frac{\rho q_{k,\mathrm{TPE}}\mathbf{w}_k^H\mathbf{a}_k\mathbf{a}_k^H\mathbf{w}_k}{\sum_{i\neq k}\frac{\rho}{K}q_{i,\mathrm{TPE}}\mathbf{w}_k^H\mathbf{B}_{i,k}\mathbf{w}_k + \mathbf{w}_k^H\mathbf{E}_k\mathbf{w}_k} \tag{5.10}$$

where $\mathbf{a}_k$, $\mathbf{B}_{i,k}$ and $\mathbf{E}_k$ are given by (5.4), (5.5) and (5.6).

### 5.1.3 Complexity analysis

There are two main aspects to take into account when comparing the computational complexities of TPE transceivers and the optimal ones. The first one is the number of arithmetic operations (additions, multiplications and divisions) while the second aspect is the implementation complexity.

Let us begin with the number of operations. It has been shown in [16] that for the RZF precoding, the total number of arithmetic operations per coherence period is:

$$C_O = 3K^2M + 2KM + \frac{K^3}{2} - \frac{K^2}{2} + \frac{K}{2} + T_{data}(2KM - M), \tag{5.11}$$

where $T_{data}$ stands for the number of downlink data symbols transmitted per coherence period. Since the expressions of the optimal transceivers is very similar to that of the RZF precoding, without taking into account the complexity of the fixed point equations in (3.13), it can be shown that the OLP and the OLR require at least the same number of operations as RZF precoding.

On the other hand, the total number of arithmetic operations per coherence period for the TPE precoding proposed in [16] is:

$$C_{TPE} = T_{data}((4J - 2)MK - (J - 2)K), \tag{5.12}$$

It can be shown that our proposed TPE transceivers require asymptotically the same number of operations. More clearly, the asymptotic complexity in terms of number of iterations when $M$ and $K$ are large, is $\mathcal{O}(K^2M)$ for the optimal transceivers and $\mathcal{O}(KM)$ for our proposed transceivers. This is a huge difference in large-scale systems where $M$ and $K$ are large.

Let us focus now on the second aspect. In practice, the implementation costs in terms of hardware complexity, time delays, and energy consumption are very important. The main benefit of the TPE transceivers is that they enable multistage hardware implementation where the computations, especially additions and multiplications, are parallelized over multiple processing cores. In comparison, the optimal transceivers involve many divisions, which are less efficient and accurate than additions and multiplications, and are not easily parallelized. Therefore, the $C_O$ arithmetic operations per coherence period are handled by a single processing core, which needs a higher clock-frequency and energy consumption than the TPE precoding implementation.

## 5.2 Analysis and Optimization of the TPE-based precoder and receiver

In this section, we consider the asymptotic regime defined in section 4.1. We show that the SINR at the TPE precoding and receiver converges to a deterministic limit, that depends only on the weight coefficients $\left\{\mathbf{w}_k^{dl}\right\}_{k=1}^{K}$ or $\left\{\mathbf{w}_k^{ul}\right\}_{k=1}^{K}$, and the large-scale channel statistics. These deterministic equivalents are then used in order to find the optimal weights that maximise the minimum asymptotic SINR in the downlink and the uplink.

### 5.2.1 Large-scale Analysis

From the expressions of the SINR in (5.3) and (5.10), it follows that it suffices to determine the deterministic equivalents for the quantities $\{\mathbf{w}_k^{\mathrm{H}}\mathbf{a}_k\mathbf{a}_k^{\mathrm{H}}\mathbf{w}_k\}$, $\left\{\sum_{i\neq k}\frac{p_{i,\mathrm{TPE}}}{K}\mathbf{w}_i^{\mathrm{H}}\mathbf{B}_{k,i}\mathbf{w}_i\right\}$, $\left\{\sum_{i\neq k}\frac{q_{i,\mathrm{TPE}}}{K}\mathbf{w}_k^{\mathrm{H}}\mathbf{B}_{i,k}\mathbf{w}_k\right\}$ and $\{\mathbf{w}_k^{\mathrm{H}}\mathbf{E}_k\mathbf{w}_k\}$ in order to find that of the transmitted power and the SINRs at the uplink and the downlink. To this end, we will make use of tools of random matrix theory.

**Lemma 5.** *Let Assumption hold true. For $t > 0$, let $\delta(t)$ be the unique positive solution of the following equation:*

$$\delta(t) = \frac{M}{K\left(1 + \frac{t}{K}\sum_{i=1}^{K}\frac{\beta_i \bar{q}_i}{1+t\delta(t)\bar{q}_i\beta_i}\right)}, \tag{5.13}$$

*and $\tilde{\mathbf{T}}(t)$ and $\mathbf{T}(t)$ the matrices defined as*

$$\tilde{\mathbf{T}}(t) = (\mathbf{I}_k + t\delta(t)\tilde{\mathbf{D}})^{-1}, \tag{5.14}$$

$$\mathbf{T}(t) = \frac{1}{1 + \frac{t}{K}\operatorname{tr}\left(\tilde{\mathbf{D}}\tilde{\mathbf{T}}(t)\right)}\mathbf{I}_N, \tag{5.15}$$

*Define $\overline{X}_k(t)$ and $\overline{Z}_{k,i}(t)$ as:*

$$\overline{X}_k(t) = \frac{\beta_k\delta(t)}{1 + t\bar{q}_k\beta_k\delta(t)}, \tag{5.16}$$

$$\overline{Z}_{k,i}(t,u) = \frac{\beta_i\overline{f}_k(t,u)\bar{\alpha}(t,u)}{(1 + t\delta(t)\beta_i\bar{q}_i)(1 + u\delta(u)\beta_i\bar{q}_i)}, \tag{5.17}$$

*where*

$$\overline{f}_k(t,u) = \beta_k\rho\left(\eta^2 + \frac{1-\eta^2}{(1+\bar{q}_k\beta_k t\delta(t))(1+\bar{q}_k\beta_k u\delta(u))}\right), \tag{5.18}$$

*and*

$$\bar{\alpha}(t,u) = \frac{\frac{1}{K}\operatorname{tr}\left(\mathbf{T}(t)\mathbf{T}(u)\right)}{1 - \frac{tu}{K^2}\operatorname{tr}\left(\mathbf{T}(t)\mathbf{T}(u)\right)\operatorname{tr}\left(\tilde{\mathbf{D}}^2\mathbf{R}^2\tilde{\mathbf{T}}(t)\tilde{\mathbf{T}}(u)\right)}. \tag{5.19}$$

*Let $\overline{\mathbf{a}}_k$ be the $J \times 1$ vector defined as:*

$$[\overline{\mathbf{a}}_k]_\ell = \frac{(-1)^\ell}{\ell!}\sqrt{1-\tau^2}\,\overline{X}_k^{(\ell)} \tag{5.20}$$

$$. \tag{5.21}$$

*Call $\overline{\mathbf{B}}_{i,k}$ and $\overline{\mathbf{E}}_k$ the $J \times J$ matrices whose elements are given by:*

$$\left[\overline{\mathbf{B}}_{k,i}\right]_{\ell,m} = \frac{(-1)^{\ell+m}}{\ell! m!} \overline{Z}_{k,i}^{(\ell+m)} \tag{5.22}$$

$$\left[\overline{\mathbf{E}}_{k}\right]_{\ell,m} = \frac{(-1)^{\ell+m}}{(\ell+m)!} \overline{X}_{k}^{(\ell+m)} \tag{5.23}$$

*Then, under the settings of Assumptions 1 and 2, we have*

$$\max_{1 \le k \le K} \left| \mathrm{SINR}_{k,\mathrm{TPE}}^{dl} - \overline{\mathrm{SINR}}_{k,\mathrm{TPE}}^{dl} \right| \xrightarrow[M,K \to +\infty]{\text{a.s.}} 0,$$

$$\max_{1 \le k \le K} \left| \mathrm{SINR}_{k,\mathrm{TPE}}^{ul} - \overline{\mathrm{SINR}}_{k,\mathrm{TPE}}^{ul} \right| \xrightarrow[M,K \to +\infty]{\text{a.s.}} 0,$$

*and*

$$P - \overline{P} \xrightarrow[M,K \to +\infty]{\text{a.s.}} 0,$$

*where*

$$\overline{P} = \frac{1}{K} \sum_{k=1}^{K} \mathbf{w}_k^H \overline{\mathbf{E}}_k \mathbf{w}_k, \tag{5.24}$$

*and*

$$\overline{\mathrm{SINR}}_{k,\mathrm{TPE}}^{dl} = \frac{\rho p_{k,\mathrm{TPE}} \frac{\mathbf{w}_{k,dl}^H \overline{\mathbf{a}}_k \overline{\mathbf{a}}_k^H \mathbf{w}_{k,dl}}{\mathbf{w}_{k,dl}^H \overline{\mathbf{E}}_k \mathbf{w}_{k,dl}}}{\sum_{i \neq k} \frac{\rho}{K} p_{i,\mathrm{TPE}} \frac{\mathbf{w}_{i,dl}^H \overline{\mathbf{B}}_{k,i} \mathbf{w}_{i,dl}}{\mathbf{w}_{i,dl}^H \overline{\mathbf{E}}_k \mathbf{w}_{i,dl}} + 1}, \tag{5.25}$$

$$\overline{\mathrm{SINR}}_{k,\mathrm{TPE}}^{ul} = \frac{\rho q_{k,\mathrm{TPE}} \mathbf{w}_{k,ul}^H \overline{\mathbf{a}}_k \overline{\mathbf{a}}_k^H \mathbf{w}_{k,ul}}{\sum_{i \neq k} \frac{\rho}{K} q_{i,\mathrm{TPE}} \mathbf{w}_{k,ul}^H \overline{\mathbf{B}}_{i,k} \mathbf{w}_{k,ul} + \mathbf{w}_{k,ul}^H \overline{\mathbf{E}}_k \mathbf{w}_{k,ul}}. \tag{5.26}$$

*Proof.* See Appendix E. □

With the asymptotic equivalents of the SINR and the transmit power on hand, we are ready now to determine the optimal parameters of the TPE based receiver and precoder.

### 5.2.2 Optimization of the US-TPE precoding and receiver

In the sequel, we exploit the asymptotic analysis developed in the previous section in order to compute the weighting vectors $\mathbf{w}_{k,dl}$ and $\mathbf{w}_{k,ul}$ that maximizes the minimum SINRs over all users. To begin with, we operate the following change of variables $\mathbf{c}_{k,dl} = \frac{\overline{\mathbf{E}}_k^{\frac{1}{2}}\mathbf{w}_{k,dl}}{\|\overline{\mathbf{E}}_k^{\frac{1}{2}}\mathbf{w}_{k,dl}\|}$, $\mathbf{c}_{k,ul} = \frac{\overline{\mathbf{E}}_k^{\frac{1}{2}}\mathbf{w}_{k,ul}}{\|\overline{\mathbf{E}}_k^{\frac{1}{2}}\mathbf{w}_{k,ul}\|}$. With this change of variable at hand, the asymptotic SINRs in the downlink and uplink write as:

$$\overline{\mathrm{SINR}}_{k,\mathrm{TPE}}^{dl} = \frac{\rho p_{k,\mathrm{TPE}}\ \mathbf{c}_{k,dl}^H \overline{\mathbf{E}}_k^{-\frac{1}{2}} \overline{\mathbf{a}}_k \overline{\mathbf{a}}_k^H \overline{\mathbf{E}}_k^{-\frac{1}{2}} \mathbf{c}_{k,dl}}{\sum_{i\neq k} \frac{\rho}{K} p_{i,\mathrm{TPE}}\ \mathbf{c}_{i,dl}^H \overline{\mathbf{E}}_i^{-\frac{1}{2}} \overline{\mathbf{B}}_{k,i} \overline{\mathbf{E}}_i^{-\frac{1}{2}} \mathbf{c}_{i,dl} + 1} \tag{5.27}$$

$$\overline{\mathrm{SINR}}_{k,\mathrm{TPE}}^{ul} = \frac{\rho q_{k,\mathrm{TPE}} \mathbf{c}_{k,ul}^H \overline{\mathbf{E}}_k^{-\frac{1}{2}} \overline{\mathbf{a}}_k \overline{\mathbf{a}}_k^H \overline{\mathbf{E}}_k^{-\frac{1}{2}} \mathbf{c}_{k,ul}}{\sum_{i\neq k} \frac{\rho}{K} q_{i,\mathrm{TPE}} \mathbf{c}_{k,ul}^H \overline{\mathbf{E}}_k^{-\frac{1}{2}} \overline{\mathbf{B}}_{i,k} \overline{\mathbf{E}}_k^{-\frac{1}{2}} \mathbf{c}_{k,ul} + 1} \tag{5.28}$$

where the parameters $\{\mathbf{c}_{k,dl}\}$, $\{\mathbf{c}_{k,ul}\}$, $\{p_{k,\mathrm{TPE}}\}$ and $\{q_{k,\mathrm{TPE}}\}$ should be set to the values that solve the following uplink and downlink optimization problems:

$$\begin{aligned} P_{ul}: \quad & \max_{\{\mathbf{c}_{k,ul}\},\{q_{k,TPE}\}} \ \min_{1\leq k\leq K} \frac{\overline{\mathrm{SINR}}_{k,\mathrm{TPE}}^{ul}}{\gamma_k} \\ & \text{subject to} \quad \sum_{k=1}^{K} q_{k,\mathrm{TPE}} \leq K P_{\max} \\ & \qquad q_{k,\mathrm{TPE}} \geq 0,\ k = 1,\cdots,K. \end{aligned}$$

$$\begin{aligned} P_{dl}: \quad & \max_{\{\mathbf{c}_{k,dl}\},\{p_{k,TPE}\}} \ \min_{1\leq k\leq K} \frac{\overline{\mathrm{SINR}}_{k,\mathrm{TPE}}^{dl}}{\gamma_k} \\ & \text{subject to} \quad \sum_{k=1}^{K} p_{k,\mathrm{TPE}} \leq K P_{\max} \\ & \qquad p_{k,\mathrm{TPE}} \geq 0,\ k = 1,\cdots,K. \\ & \qquad \|\mathbf{c}_{k,dl}\| = 1. \end{aligned}$$

Problems $P_{ul}$ and $P_{dl}$ have the same structure as the traditional max-min SINR problem encountered in the design of the OLP and OLR. This problem has been addressed in the literature using different approaches relying either on standard convex optimization tools [6] or on tools from nonlinear Perron-Frobenius theory [8, 9]. As a direct consequence of the existing results from these works, we can deduce that problems $P_{ul}$ and $P_{dl}$ are connected by the duality principle, implying that at optimality, all the weighted asymptotic SINRs are equal to the same value $\tau^*_{\mathrm{TPE}}$:

$$\tau^*_{\mathrm{TPE}} = \frac{\overline{\mathrm{SINR}}^{ul}_{k,\mathrm{TPE}}}{\gamma_k} = \frac{\overline{\mathrm{SINR}}^{dl}_{k,\mathrm{TPE}}}{\gamma_k}, \qquad k = 1, \cdots, K. \tag{5.29}$$

Moreover, the optimal uplink powers $q^*_{k,\mathrm{TPE}}$ are solutions of the following fixed-point system of equations:

$$q^*_{k,\mathrm{TPE}} = \frac{\gamma_k K P_{\max}}{\sum_{\ell=1}^{K} \frac{\gamma_\ell \overline{\mathbf{a}}_k^T \overline{\mathbf{E}}_k^{-\frac{1}{2}} \left(\sum_{i\neq k} \frac{\rho}{K} q^*_{i,\mathrm{TPE}} \overline{\mathbf{E}}_k^{-\frac{1}{2}} \overline{\mathbf{B}}_{i,k} \overline{\mathbf{E}}_k^{-\frac{1}{2}} + \mathbf{I}_J\right)^{-1} \overline{\mathbf{E}}_k^{-\frac{1}{2}} \overline{\mathbf{a}}_k}{\overline{\mathbf{a}}_\ell^T \overline{\mathbf{E}}_\ell^{-\frac{1}{2}} \left(\sum_{j\neq \ell} \frac{\rho}{K} q^*_{j,\mathrm{TPE}} \overline{\mathbf{E}}_\ell^{-\frac{1}{2}} \overline{\mathbf{B}}_{j,\ell} \overline{\mathbf{E}}_\ell^{-\frac{1}{2}} + \mathbf{I}_J\right)^{-1} \overline{\mathbf{E}}_\ell^{-\frac{1}{2}} \overline{\mathbf{a}}_\ell}} \tag{5.30}$$

whereas the optimal weighting vector are given by:

$$\begin{aligned} &\mathbf{c}^*_{k,ul} = \mathbf{c}^*_{k,dl} \\ &= \frac{\left(\sum_{i\neq k} \frac{\rho}{K} q^*_{i,\mathrm{TPE}} \overline{\mathbf{E}}_k^{-\frac{1}{2}} \overline{\mathbf{B}}_{i,k} \overline{\mathbf{E}}_k^{-\frac{1}{2}} + \mathbf{I}_J\right)^{-1} \overline{\mathbf{E}}_k^{-\frac{1}{2}} \overline{\mathbf{a}}_k}{\left\|\left(\sum_{i\neq k} \frac{\rho}{K} q^*_{i,\mathrm{TPE}} \overline{\mathbf{E}}_k^{-\frac{1}{2}} \overline{\mathbf{B}}_{i,k} \overline{\mathbf{E}}_k^{-\frac{1}{2}} + \mathbf{I}_J\right)^{-1} \overline{\mathbf{E}}_k^{-\frac{1}{2}} \overline{\mathbf{a}}_k\right\|} \end{aligned} \tag{5.31}$$

The power allocation vector $\mathbf{p}^*_{\mathrm{TPE}} = [p_{1,\mathrm{TPE}}, \cdots, p_{K,\mathrm{TPE}}]^T$ is obtained so that the weighted SINRs in the uplink are all equal to $\tau^*_{,\mathrm{TPE}}$. These constraints produce a linear system whose solution is:

$$\mathbf{p}_{\mathrm{TPE}} = \frac{\tau^*_{\mathrm{TPE}}}{\rho} \left(\mathbf{I}_K - \tau^*_{\mathrm{TPE}} \mathbf{\Gamma}_{\mathrm{TPE}} \mathbf{F}_{\mathrm{TPE}}\right)^{-1} \mathbf{\Gamma}_{\mathrm{TPE}} \mathbf{1}_K, \tag{5.32}$$

where:

$$\mathbf{\Gamma}_{\rm TPE} = {\rm diag}\left\{\left(\mathbf{c}_{k,dl}^H \overline{\mathbf{E}}_k^{-\frac{1}{2}} \overline{\mathbf{a}}_k \overline{\mathbf{a}}_k^H \overline{\mathbf{E}}_k^{-\frac{1}{2}} \mathbf{c}_{k,dl}\right)^{-1}\right\}_{k=1}^{K}, \quad (5.33)$$

and

$$[\mathbf{F}_{\rm TPE}]_{k,i} = \begin{cases} 0 & \text{if } k = i \\ \frac{1}{K}\mathbf{c}_{i,dl}^H \overline{\mathbf{E}}_k^{-\frac{1}{2}} \overline{\mathbf{B}}_{k,i} \overline{\mathbf{E}}_k^{-\frac{1}{2}} \mathbf{c}_{i,dl} & \text{if } k \neq i \end{cases} \quad (5.34)$$

It appears thus that the optimal design parameters intervening in the user specific TPE precoding and receiver have the same structure as those obtained when dealing with the design of the OLP and OLR. However, the TPE presents three main advantages: (i) First, unlike the OLP and the OLR, the design of the TPE account for the statistics on the channel estimation error and thus is expected to achieve higher robustness in case of a high errors on the channel estimates. (ii) Second, the optimal parameters depend only on the large scale channel statistics and can be thus computed beforehand, or at least be updated at the rate of the change of the channel statistics. (iii) Third, it allows a considerable reduction in the complexity since it does not involve the computation of a matrix inverse, requiring only about $\mathcal{O}(KM)$ arithmetic operations. This has to be compared with the OLP and OLR that involve $\mathcal{O}(K^2M)$ arithmetic operations. For more details about complexity analysis, we refer the reader to [16] where the complexity of the RZF precoding is computed and compared with the complexity of the TPE precoding.

## 5.3 Numerical results

In this section, numerical simulations are used to compare the optimal linear precoding and the proposed US-TPE precoding in the downlink and the MMSE receiver and the proposed US-TPE receiver in the uplink. Moreover, Monte-Carlo (MC) simulations are used to validate the analysis in the asymptotic regime. The UEs are assumed to be uniformly distributed in a cell with radius $D = 250\ \ m$. The pathloss

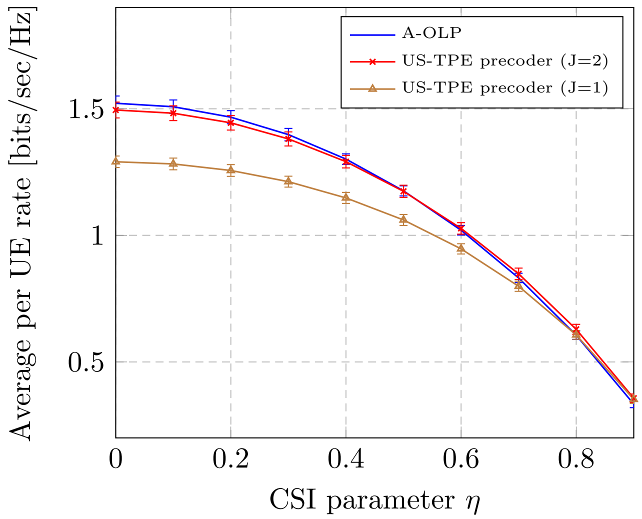


Figure 5.1: Average per UE rate vs. $\eta$ when K=32, M=128, $P_{\max}$=5 Watt and $\rho = 20dB$

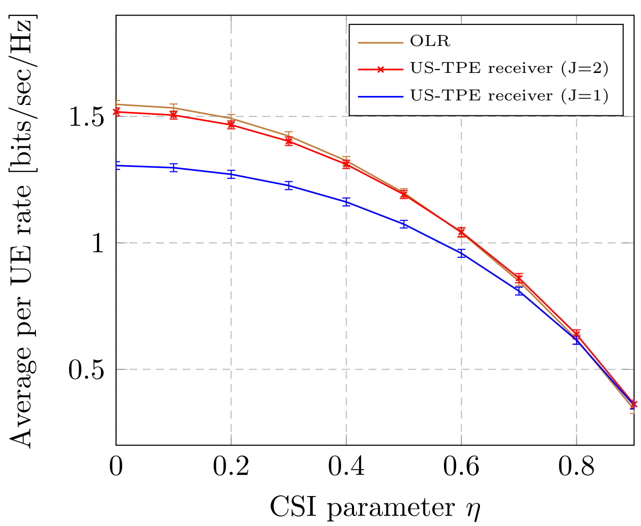


Figure 5.2: Average per UE rate vs. $\eta$ when K=32, M=128, $P_{\max}$=5 Watt and $\rho = 20dB$

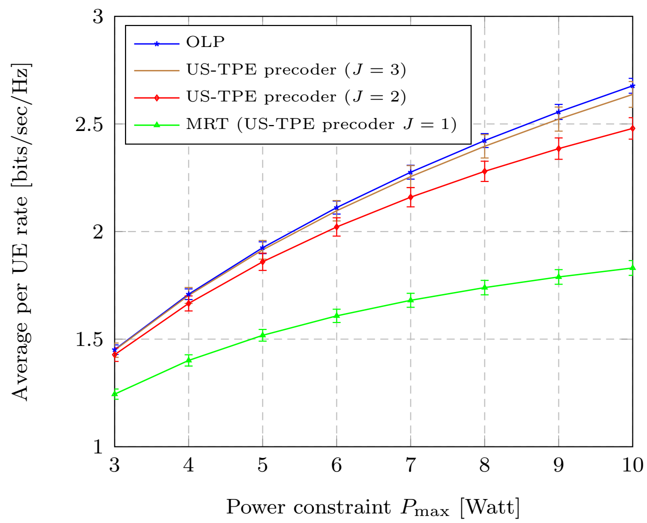


Figure 5.3: Average per UE rate vs. power constraint $P_{\max}$ when $K = 32$, $M = 128$, $\rho = 20$ dB and $\eta = 0$.

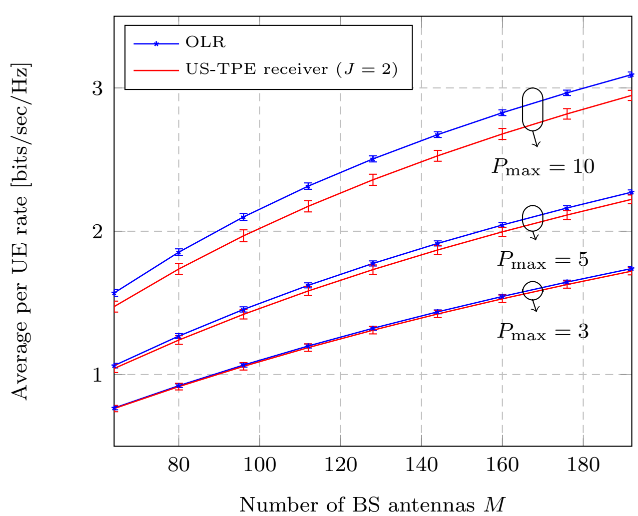


Figure 5.4: Average per UE rate vs. $M$ when $K = 32$, $\rho = 20$ dB and $\eta = 0$.

$\beta_k$ between the BS and a UE $k$ with distance $x_k$ from the BS is modeled as

$$\beta_k = \frac{1}{1 + \left(\frac{x_k}{d_0}\right)^{\delta}} \tag{5.35}$$

where $\delta = 3.8$, $d_0 = 30 \quad m$. The performance is measured in terms of the average achievable rate of the UEs:

$$r = \frac{1}{K}\sum_{k=1}^{K} \mathbb{E}\left[\log_2(1 + \mathrm{SINR_k})\right] \tag{5.36}$$

where the expectation is computed with respect to the different channel realization. Without loss of generality, we assume that the working signal-to-noise ratio (SNR) is $\rho = 20dB$. The UEs priorities $\{\gamma_k\}$ are randomly chosen from the interval [1,2].

Figure 5.1 plots the average per UE rate vs. CSI parameter $\eta$ in the downlink when $K = 32$, $P_{\max} = 5$ Watt and $\rho = 20dB$. Markers represents the asymptotic values whereas the error bars indicate the standard deviation of the MC results. The curves presented in this figure correspond to the A-OLP and the US-TPE precoding for different values of the truncation order $J$. The performance of the proposed TPE precoding is very close to that of the A-OLP when near to perfect CSI is available ($\eta$ small). And when $\eta$ is getting larger, the proposed US-TPE precoder outperforms the A-OLP even for small values of $J$.

In figure 5.2, we plot the average per UE rate vs. CSI parameter $\eta$ in the uplink when $K = 32$, $P_{\max} = 5$ Watt and $\rho = 20dB$. The curves presented in this figure correspond to the OLR and the US-TPE receiver. As in the case of the downlink, we conclude from this figure that even for small values of the truncation order $J$ ($J = 2$) the US-TPE receiver exhibits interesting performance compared to the OLR.

Figure 5.3 plots the average per UE rate vs. the power constraint $P_{\max}$ in the downlink when $K = 32$, $M = 128$, $\eta = 0$ and $\rho = 20dB$. First, it is important to note that the performance of the US-TPE precoding is very good compared to the OLP for different values of the power constraint. Besides, we can conclude from this figure that for small values of the power constraint it is better to choose small value of $J$ ($J = 2$ for instance). However, for larger values of $P_{\max}$, bigger values of $J$ should be chosen since they approximate better the OLP.

Figure 5.4 illustrates the uplink average rate per UE vs. $M$ for different values of the power constraint $P_{\max}$. As seen, with $J = 2$ the TPE-based receiver provides interesting performance compared OLR for all the values of $M$. Besides, as in downlink, the gap is getting larger when $P_{\max}$ is getting higher. However, we can increase the truncation order $J$ to enhance the performance of the proposed receiver and get closer to the OLR. Also, it is important to notice from this figure that the gap between the proposed receiver and the OLR becomes constant for large values of $M$.

## 5.4 Conclusion

In this chapter, low complexity precoder and receiver has been proposed for the Max-Min SINR problem. Then, these proposed transceivers have been compared with the optimal linear transceivers that solve the same problem. It has been shown that the performance of the proposed schemes is very close to that of the optimal ones while requiring less computational complexity.

# Chapter 6

# Power Minimization Precoding

In this chapter, we propose a low complexity precoding for the power minimization problem. Starting from the expression of the optimal linear precoding and applying the TPE technique in a per-user basis, a new precoding scheme is obtained. Based on this scheme, asymptotic equivalents of the SINR and the transmit power are computed. Then, the optimal parameters of the proposed precoding are optimized in the asymptotic regime.

## 6.1 Problem statement

We consider the same setting of the system model defined in section 3.1 with perfect CSI available at the BS. We assume that the BS employs Gaussian codebook and linear precoding. The precoding matrix is denoted by $\mathbf{G} = [\mathbf{g}_1, \cdots, \mathbf{g}_K]$ while the data symbol vector for all UEs is called $\mathbf{s} = [s_1, \cdots, s_K]^{\mathrm{T}}$ with $s_k \sim \mathcal{CN}(0, 1)$. The transmit signal vector $\mathbf{x}$ is $\mathbf{x} = \mathbf{G}\mathbf{s}$ whereas the received signal at UE $k$ can be written as:

$$y_k = \mathbf{h}_k^H \mathbf{g}_k s_k + \sum_{i=1, i\neq k}^{K} \mathbf{h}_k^H \mathbf{g}_i s_i + n_k \tag{6.1}$$

where $n_k \sim \mathcal{CN}(0, \sigma^2)$ accounts for thermal noise. The SINR at UE $k$ is thus given by:

$$\mathrm{SINR}_k = \frac{|\mathbf{h}_k^{\mathrm{H}} \mathbf{g}_k|^2}{\sum_{i=1, i \neq k}^{K} |\mathbf{h}_k^{\mathrm{H}} \mathbf{g}_i|^2 + \sigma^2} \tag{6.2}$$

whereas the total transmit power at the BS is computed as $P = \mathrm{tr}(\mathbf{G}\mathbf{G}^{\mathrm{H}})$.

The objective of this work is to design a low-complexity linear precoding scheme that minimizes the transmit power at the BS while maintaining target user rates $\{r_k\}$. More formally, the power minimization problem is formulated as:

$$\begin{aligned} \mathcal{P}: \ \underset{\mathbf{G}}{\text{minimize}} \quad & \mathrm{tr}(\mathbf{G}\mathbf{G}^{\mathrm{H}}) \\ \text{subject to} \quad & \mathrm{SINR}_k \geq \gamma_k \quad k = 1, \cdots, K. \end{aligned}$$

where $\gamma = [\gamma_1, \ldots, \gamma_K]^T$ is the vector of target SINRs given by $\gamma_k = 2^{r_k} - 1$ (under the assumption of Gaussian codebooks). The solution of ($\mathcal{P}$) is well-known and given by [6, 27, 28]:

$$\mathbf{G}_{\mathrm{OLP}} = \left( \sum_{i=1}^{K} q_i^* \mathbf{h}_i \mathbf{h}_i^{\mathrm{H}} + M \mathbf{I}_M \right)^{-1} \mathbf{H} \sqrt{\mathrm{diag}\left(\mathbf{p}^*\right)} \tag{6.3}$$

where $\{q_k^*\}$ are obtained as the unique positive solution to the following set of fixed point equations:

$$\left(1 + \frac{1}{\gamma_k}\right) q_k^* = \frac{1}{\mathbf{h}_k^{\mathrm{H}} \left( \sum_{i=1}^{K} q_i^* \mathbf{h}_i \mathbf{h}_i^{\mathrm{H}} + M \mathbf{I}_M \right)^{-1} \mathbf{h}_k} \ \forall k \tag{6.4}$$

while $\mathbf{p}^* = [p_1^*, \ldots, p_K^*]^T$ is such that the SINR constraints are all satisfied with equality. This yields $\mathbf{p}^* = \sigma^2 \mathbf{A}^{-1} \mathbf{1}_K$ where the $(k, i)$-th element of $\mathbf{A}$ is

$$[\mathbf{A}]_{k,i} = \begin{cases} \frac{1}{\gamma_k} |\mathbf{h}_k^{\mathrm{H}} \mathbf{v}_k^*|^2 & \text{if} \quad k = i \\ -|\mathbf{h}_k^{\mathrm{H}} \mathbf{v}_i^*|^2 & \text{if} \quad k \neq i \end{cases} \tag{6.5}$$

with $\mathbf{v}_k^*$ being the $k$-th column of $\mathbf{V}^* = (\sum_{i=1}^{K} q_i^* \mathbf{h}_i \mathbf{h}_i^{\mathrm{H}} + M\mathbf{I})^{-1}\mathbf{H}$. The major difficulties towards the implementation of $\mathbf{G}_{\mathrm{OLP}}$ are as follows. Firstly, $\mathbf{G}_{\mathrm{OLP}}$ is parameterized by $\mathbf{q}^*$ and $\mathbf{p}^*$. Both require matrix inversions every coherence period. As for $\mathbf{q}^*$, it needs also to be evaluated by an iterative procedure due to the fixed-point equations in (6.4). Once $\mathbf{q}^*$ and $\mathbf{p}^*$ are computed, the application of $\mathbf{G}_{\mathrm{OLP}}$ to the data vector requires a matrix inversion. All this is a computationally demanding task when $M$ and $K$ grow large as envisioned in large-scale MIMO systems.

A possible way to reduce the complexity in the computation of $\mathbf{q}^*$ and $\mathbf{p}^*$ is to resort to the asymptotic analysis. Indeed, when $M$ and $K$ grow large with a bounded ratio we have that [7, 10, 20]

$$\max_{1 \leq k \leq K} |q_k^* - \bar{q}_k| \xrightarrow[M,K\rightarrow+\infty]{\text{a.s.}} 0 \tag{6.6}$$

$$\max_{1 \leq k \leq K} |p_k^* - \bar{p}_k| \xrightarrow[M,K\rightarrow+\infty]{\text{a.s.}} 0 \tag{6.7}$$

with

$$\bar{q}_k = \frac{\gamma_k}{\beta_k \xi} \tag{6.8}$$

$$\bar{p}_k = \frac{\gamma_k}{\beta_k \xi^2}\left(\bar{P} + \frac{\sigma^2}{\beta_k}(1+\gamma_k)^2\right) \tag{6.9}$$

where $\bar{P} = \frac{\sigma^2 \sum_{i=1}^{K} \frac{\gamma_i}{\beta_i}}{M\xi}$ and $\xi = 1 - \frac{1}{M}\sum_{i=1}^{K} \frac{\gamma_i}{1+\gamma_i}$. From the above results, it follows that both $\mathbf{q}^*$ and $\mathbf{p}^*$ admit simple closed-form deterministic approximations that depend solely on the large-scale channel statistics. This information can be easily observed and estimated accurately at the BS because it changes slowly with time (relative to the small-scale fading).

Replacing $\{q_k^*\}$ and $\{p_k^*\}$ with $\{\bar{q}_k\}$ and $\{\bar{p}_k\}$ yields the so-called asymptotically

OLP (A-OLP) given by:

$$\mathbf{G}_{\text{A-OLP}} = \left( \sum_{i=1}^{K} \bar{q}_i \mathbf{h}_i \mathbf{h}_i^{\text{H}} + M \mathbf{I}_M \right)^{-1} \mathbf{H} \sqrt{\text{diag}\,(\bar{\mathbf{p}})} \tag{6.10}$$

where $\bar{\mathbf{p}} = [\bar{p}_1, \ldots, \bar{p}_K]^T$. Although the parameters $\{\bar{q}_k\}$ and $\{\bar{p}_k\}$ are no longer computed at every channel realization, the computation of $\mathbf{G}_{\text{A-OLP}}$ can be a task of a prohibitively high complexity when $M$ and $K$ are large. To address this issue, a TPE approach will be adopted in the next section.

## 6.2 TPE based Precoding

### 6.2.1 User Specific TPE precoding

As defined in the previous chapter, the TPE of the precoding vector associated with the $k$-th UE is computed as:

$$\mathbf{g}_k = \sum_{\ell=0}^{J-1} w_{\ell,k} \left( \frac{\mathbf{H}\mathbf{R}\mathbf{H}^H}{K} \right)^{\ell} \frac{\mathbf{h}_k}{K} \quad \forall k \tag{6.11}$$

where $J$ is the truncation order and $\mathbf{R} = \text{diag}(\bar{q}_1, \cdots, \bar{q}_K)$. Plugging $\mathbf{g}_k$ in (6.11) into (6.2) and denoting by $\mathbf{w}_k = [w_{0,k}, \cdots, w_{J-1,k}]^T$, the SINR associated with the $k$-th UE can be written as:

$$\text{SINR}_k = \frac{\mathbf{w}_k^H \mathbf{a}_k \mathbf{a}_k^H \mathbf{w}_k}{\sum_{i \neq k} \frac{1}{K} \mathbf{w}_i^H \mathbf{B}_{k,i} \mathbf{w}_i + \sigma^2} \quad \forall k \tag{6.12}$$

where $\mathbf{a}_k \in \mathbb{C}^J$ and $\mathbf{B}_{k,i} \in \mathbb{C}^{J \times J}$ are random quantities given by:

$$[\mathbf{a}_k]_\ell = \frac{1}{K} \mathbf{h}_k^H \left( \frac{\mathbf{H}\mathbf{R}\mathbf{H}^H}{K} \right)^{\ell} \mathbf{h}_k \tag{6.13}$$

$$[\mathbf{B}_{k,i}]_{\ell,m} = \frac{1}{K} \mathbf{h}_k^H \left( \frac{\mathbf{H}\mathbf{R}\mathbf{H}^H}{K} \right)^{\ell} \mathbf{h}_i \mathbf{h}_i^H \left( \frac{\mathbf{H}\mathbf{R}\mathbf{H}^H}{K} \right)^{m} \mathbf{h}_k. \tag{6.14}$$

The transmit power at the BS can be expressed as:

$$P = \frac{1}{K}\sum_{k=1}^{K}\mathbf{w}_k^H\mathbf{E}_k\mathbf{w}_k \tag{6.15}$$

where $\mathbf{E}_k \in \mathbb{C}^{J\times J}$ is defined as:

$$[\mathbf{E}_k]_{\ell,m} = \frac{1}{K}\mathbf{h}_k^H\left(\frac{\mathbf{H}\mathbf{R}\mathbf{H}^H}{K}\right)^{(\ell+m)}\mathbf{h}_k. \tag{6.16}$$

Note that the matrices $\mathbf{a}_k$, $\mathbf{B}_{k,i}$ and $\mathbf{E}_k$ are the same as in the previous chapter when perfect CSI is available. We recall their expression for completeness. As $M$ and $K$ grow simultaneously large, we have

$$\mathrm{SINR}_k - \overline{\mathrm{SINR}}_k \xrightarrow[M,K\to+\infty]{\text{a.s.}} 0, \quad P - \overline{P} \xrightarrow[M,K\to+\infty]{\text{a.s.}} 0 \tag{6.17}$$

with

$$\overline{\mathrm{SINR}}_k = \frac{\mathbf{w}_k^H\overline{\mathbf{a}}_k\overline{\mathbf{a}}_k^H\mathbf{w}_k}{\sum_{i\neq k}\frac{1}{K}\mathbf{w}_i^H\overline{\mathbf{B}}_{k,i}\mathbf{w}_i + \sigma^2} \tag{6.18}$$

and

$$\overline{P} = \frac{1}{K}\sum_{k=1}^{K}\mathbf{w}_k^H\overline{\mathbf{E}}_k\mathbf{w}_k \tag{6.19}$$

where the entries of $\overline{\mathbf{a}}_k \in \mathbb{C}^J$, $\overline{\mathbf{B}}_{k,i} \in \mathbb{C}^{J\times J}$ and $\overline{\mathbf{E}}_k \in \mathbb{C}^{J\times J}$ as well as the proof are given in Appendix E. These asymptotic equivalents are used in the sequel to determine the optimal weight vectors $\{\mathbf{w}_k\}$ that minimize the transmit power while satisfying a set of target SINRs.

### 6.2.2 Optimization of the US-TPE precoding

We propose to solve the following optimization problem:

$$\begin{aligned} \mathcal{P}_1 : \underset{\mathbf{w}_1,\cdots,\mathbf{w}_K}{\text{minimize}} \quad & \frac{1}{K}\sum_{k=1}^{K} \mathbf{w}_k^T \overline{\mathbf{E}}_k \mathbf{w}_k \\ \text{subject to} \quad & \frac{1}{\gamma_k}\mathbf{w}_k^T \overline{\mathbf{a}}_k \overline{\mathbf{a}}_k^T \mathbf{w}_k \geq \sum_{i\neq k} \frac{1}{K}\mathbf{w}_i^T \overline{\mathbf{B}}_{k,i}\mathbf{w}_i + \sigma^2 \end{aligned} \tag{6.20}$$

which can be easily shown to be non-convex. To reformulate it in a convex form, we use the same trick of [27] (see also [28]). In particular, we note that the power and the SINRs keep the same values if $\forall k$ $\mathbf{w}_k$ is replaced by $-\mathbf{w}_k$. Therefore, we can assume that $\mathbf{w}_k^T\overline{\mathbf{a}}_k$ is positive $\forall k$. It is worth mentioning that this artifice cannot be used if the same weight vector is used for all UEs, i.e., $\forall k$ $\mathbf{w}_k = \mathbf{w}$. This easily follows from observing that if $\forall k$ $\mathbf{w}_k = \mathbf{w}$ then we cannot ensure that $\forall k$ $\mathbf{w}^T\overline{\mathbf{a}}_k > 0$. This explains why the traditional TPE can be applied only to some particular cases [16, 26]. By rewriting the SINR constraints as [28]:

$$\begin{aligned} &\frac{1}{\gamma_k}\mathbf{w}_k^T \overline{\mathbf{a}}_k \overline{\mathbf{a}}_k^T\ \mathbf{w}_k \geq \gamma_k\Big(\sum_{i\neq k}\frac{1}{K}\mathbf{w}_i^T\overline{\mathbf{B}}_{k,i}\mathbf{w}_i + \sigma^2\Big) \Leftrightarrow \\ &\frac{1}{\sqrt{\gamma_k\sigma^2}}\overline{\mathbf{a}}_k^T\ \mathbf{w}_k \geq \sqrt{\sum_{i\neq k}\frac{1}{\sigma^2 K}\mathbf{w}_i^T\overline{\mathbf{B}}_{k,i}\mathbf{w}_i + 1} \end{aligned} \tag{6.21}$$

the problem $\mathcal{P}_1$ becomes convex. Moreover, it can be easily shown that the Slater condition is fulfilled [28]. Therefore, the Karush-Kuhn-Tucker (KKT) conditions are necessary and sufficient for the reformulated problem and thus for the original one [28]. The Lagrangian of $\mathcal{P}_1$ is defined as:

$$\begin{aligned} &\mathcal{L}(\mathbf{w}_1,\cdots,\mathbf{w}_K,\lambda_1,\cdots,\lambda_K) = \frac{1}{K}\sum_{k=1}^{K}\mathbf{w}_k^T\overline{\mathbf{E}}_k\mathbf{w}_k + \\ &\sum_{k=1}^{K}\lambda_k\Big(\sum_{i\neq k}\frac{1}{\sigma^2 K}\mathbf{w}_i^T\overline{\mathbf{B}}_{k,i}\mathbf{w}_i + 1 - \frac{1}{\gamma_k\sigma^2}\mathbf{w}_k^T\overline{\mathbf{a}}_k\overline{\mathbf{a}}_k^T\mathbf{w}_k\Big) \end{aligned} \tag{6.22}$$

It is known that the optimal $\{\mathbf{w}_k\}$ satisfy $\forall k \ \frac{\partial \mathcal{L}}{\partial \mathbf{w}_k} = 0$. After some algebraic manipulations, we get

$$\mathbf{w}_k = \frac{K\lambda_k}{\sigma^2\gamma_k}\Big(\overline{\mathbf{E}}_k + \sum_{i\neq k}\frac{\lambda_i}{\sigma^2}\overline{\mathbf{B}}_{i,k}\Big)^{-1}\overline{\mathbf{a}}_k(\overline{\mathbf{a}}_k^T\mathbf{w}_k). \tag{6.23}$$

Since $\overline{\mathbf{a}}_k^T\mathbf{w}_k$ is a scalar, the optimal $\mathbf{w}_k$ corresponding to UE $k$ is proportional to the vector $\left(\overline{\mathbf{E}}_k + \sum_{i\neq k}\frac{\lambda_i}{\sigma^2}\overline{\mathbf{B}}_{i,k}\right)^{-1}\mathbf{a}_k$. The positive scalars $\lambda_k$ are obtained as the unique solution of the following set of equations:

$$\lambda_k = \frac{\gamma_k\sigma^2}{K\overline{\mathbf{a}}_k^T\left(\overline{\mathbf{E}}_k + \sum_{i\neq k}\frac{\lambda_i}{\sigma^2}\overline{\mathbf{B}}_{i,k}\right)^{-1}\overline{\mathbf{a}}_k}. \tag{6.24}$$

Putting all the above results together, the optimal vector $\mathbf{w}_k^*$ is found to be:

$$\mathbf{w}_k^* = \sqrt{p_k^*}\frac{\left(\overline{\mathbf{E}}_k + \sum_{i\neq k}\frac{\lambda_i}{\sigma^2}\overline{\mathbf{B}}_{i,k}\right)^{-1}\overline{\mathbf{a}}_k}{\left\|\left(\overline{\mathbf{E}}_k + \sum_{i\neq k}\frac{\lambda_i}{\sigma^2}\overline{\mathbf{B}}_{i,k}\right)^{-1}\overline{\mathbf{a}}_k\right\|} = \sqrt{p_k^*}\bar{\mathbf{w}}_k \tag{6.25}$$

where $\bar{\mathbf{w}}_k$ are the beamforming directions and $p_k^*$ are such that the SINR constraints in $\mathcal{P}_1$ are all satisfied with equality:

$$p_k^*\bar{\mathbf{w}}_k^T\overline{\mathbf{a}}_k\overline{\mathbf{a}}_k^T\bar{\mathbf{w}}_k = \gamma_k\sum_{i\neq k}\frac{p_i^*}{K}\bar{\mathbf{w}}_i^T\overline{\mathbf{B}}_{k,i}\bar{\mathbf{w}}^T + \gamma_k\sigma^2 \quad \forall k. \tag{6.26}$$

This yields:

$$\mathbf{p}^* = \sigma^2\mathbf{F}^{-1}\mathbf{1}_K \tag{6.27}$$

where the $(k,i)$-th element of the matrix $\mathbf{F}$ is

$$[\mathbf{F}]_{k,i} = \begin{cases} \frac{1}{\gamma_k}\bar{\mathbf{w}}_k^T\bar{\mathbf{a}}_k\bar{\mathbf{a}}_k^T\bar{\mathbf{w}}_k & \text{if} \quad k=i \\ -\frac{1}{K}\bar{\mathbf{w}}_i^T\overline{\mathbf{B}}_{k,i}\bar{\mathbf{w}}_i^T & \text{if} \quad k \neq i. \end{cases} \tag{6.28}$$

The precoding vectors obtained as in (6.25) are referred to as US-TPE in the sequel.

## 6.3 Simulation results

In this section, we consider the same simulation settings as in section 5.3.The UEs target rates $\{r_k\}$ are randomly chosen from the interval [0.1, 3] bits/sec/Hz.

Figure 6.1 plots the total transmit power vs. the number of BS antennas $M$ of all the investigated precoders. Comparisons are also made with the PA-RZF precoding proposed in [29]. As it is seen, US-TPE with $J=3$ requires almost the same amount of power of OLP and provides a marginal saving with respect to both PA-RZF in [29] and TPE in [26]. As far as computational complexity is concerned, it requires, like the traditional TPE, $\mathcal{O}(KM)$ arithmetic operations. This has to be compared with the PA-RZF and the OLP, which involve $\mathcal{O}(K^2M)$ arithmetic operations per coherence period.

Figure 6.2 illustrates the total transmit power vs. $M$ for different values of the truncation order $J$. Obviously, as $J$ increases, the average transmitted power of the US-TPE decreases, but with a slower rate. From the results of figure 6.2, choosing $J=3$ seems to be a good option since it leads to sufficiently close performance as the OLP.

The impact of UE target rates is investigated in Figure 6.3, where we report the transmit power vs. the maximum allowed rate $r_{\max}$. The UEs rates are chosen randomly in the interval [0.1, $r_{\max}$]. As seen, for values of $r_{\max}$ smaller than 4 [bit/s/Hz], $J=3$ is enough to achieve the same performance of OLP. On the other hand, higher

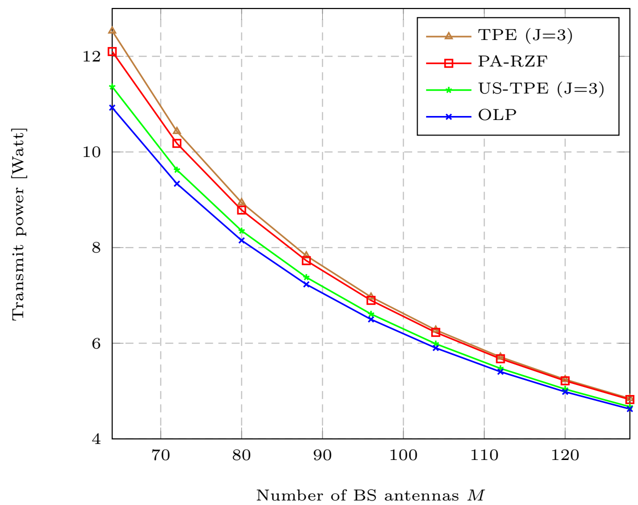


Figure 6.1: Average per UE transmit power in Watt vs. $M$ when $K = 32$.

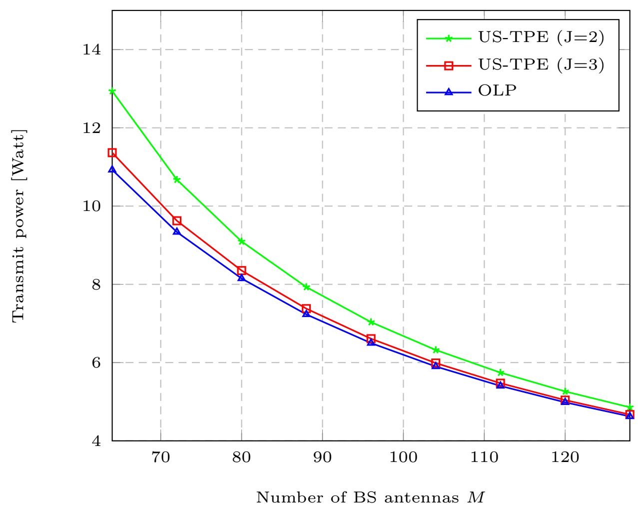


Figure 6.2: Transmit power in Watt vs. $M$ when $K = 32$.

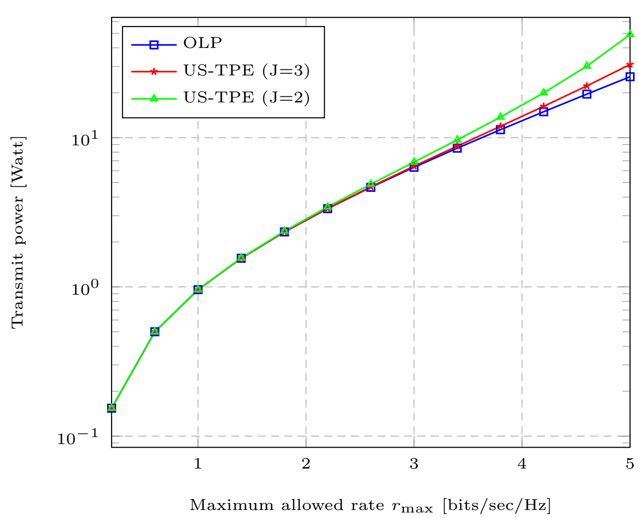


Figure 6.3: Transmit power in Watt vs. maximal allowed rate $r_{\max}$ when $M = 128$ and $K = 32$.

values of the truncation order $J$ must be used to further enhance performance when $r_{\max} \geq 4$.

## 6.4 Conclusion

In this chapter, based on the TPE technique, a low complexity precoding that minimizes the transmit power while satisfying a set of target SINR constraints has been proposed and optimized in the asymptotic regime. Considering the asymptotic regime, the optimal parameters of this precoding scheme are computed only when the channel statistics change. Numerical results showed that the proposed precoding provided the same power consumption of OLP under different operating conditions while reducing significantly the computational complexity.

# Chapter 7

# Conclusion

## 7.1 Summary

In this work, we have considered a single-cell massive MIMO system in wich the BS equipped $M$ antennas communicates with $K$ UEs. We have studied the problem of designing linear transceivers that maximize the SINR while satisfying a certain power constraint. We have considered the asymptotic regime, where the number of the BS antennas $M$ and the number of the UEs $K$ go large with the same pace, to determine asymptotic equivalents of the parameters of the OLP and the OLR. We have proposed low complexity precoding and receiving techniques based on truncated polynomial expansion respectively named user specific TPE precoding and receiver. Based on this new precoding and decoding techniques, asymptotic equivalents of the SINR and the transmit power, that depends only on the channel statistics, have been determined. Then, the parameters of the proposed transceivers has been jointly optimized. The numerical results show that the proposed low complexity precoder and receiver can achieve a-close-to-optimal performance.

We also focused on the design of a linear TPE precoder to achieve the same performance of the OLP minimizing the total power consumption while ensuring target UE rates. Differently from traditional TPE schemes, the proposed one applied the truncated polynomial expansion concept on a per-UE basis. This allowed to determine in closed-form the optimal TPE coefficients that approximate with the best accuracy the performance of OLP. In order to further facilitate the design of the proposed precoder, we considered the asymptotic regime in which the number of UEs and BS antennas grow simultaneously large with a bounded ratio. Such an assumption allows to approximate the transmit power and the SINRs by deterministic quantities that depend only on the channel statistics. Numerical results showed that the proposed TPE provided the same power consumption of OLP under different operating conditions.

## 7.2 Perspectives

Several aspects may be considered to improve this work. For example, studying the performance of the proposed low complexity precoder and receiver in a multi-cell scenario will be interesting. In fact, the effect of pilot contamination and inter-cell interference on the performance of our proposed precoding and receiving techniques should be analyzed. Moreover, a more practical system model taking into account the mobility of the UEs can be considered where the channel attenuation coefficients are function of time. Finally, the TPE technique could be applied in other communication and signal processing problems in order to reduce computational complexity.

# APPENDICES

## A Useful Lemmas

This appendix gathers some technical results from random matrix theory concerning the asymptotic behaviour of large random matrices. Throughout this section, we denote by $\mathbf{X} = [\mathbf{x}_1, \cdots, \mathbf{x}_K]$ a $M \times K$ standard complex Gaussian matrix. Let $t > 0$ and $\mathbf{R} = \mathrm{diag}\,(\alpha_1, \cdots, \alpha_K)$. We define the resolvent matrix of $\mathbf{X}\mathbf{R}\mathbf{X}^{\mathrm{H}}$ as:

$$\mathbf{Q}(t) = \left(\frac{t}{K}\sum_{i=1}^{K}\alpha_i \mathbf{x}_i \mathbf{x}_i^{\mathrm{H}} + \mathbf{I}_M\right)^{-1} = \left(\frac{t}{K}\mathbf{X}\mathbf{R}\mathbf{X}^{\mathrm{H}} + \mathbf{I}_M\right)^{-1}. \tag{A.1}$$

Define also $\mathbf{Q}_k(t)$ as:

$$\mathbf{Q}_k(t) = \left(\frac{t}{K}\sum_{i\neq k}\alpha_i \mathbf{x}_i \mathbf{x}_i^{\mathrm{H}} + \mathbf{I}_M\right)^{-1} \tag{A.2}$$

which is obtained from $\mathbf{Q}(t)$ by removing the contribution of vector $\mathbf{x}_k$. The following Lemma recalls some classical identities involving the resolvent matrix, which will be extensively used in our derivations:

**Lemma 6.** *The following identities hold true:*

1. *Inverse of resolvents:*

$$\mathbf{Q}(t) = \mathbf{Q}_k(t) - \frac{t\alpha_k \mathbf{Q}_k(t)\mathbf{x}_k\mathbf{x}_k^H \mathbf{Q}_k(t)}{1 + \frac{t\alpha_k}{K}\mathbf{x}_k^H \mathbf{Q}_k(t)\mathbf{x}_k}. \tag{A.3}$$

*2. Rank-one perturbation result: For any matrix* $\mathbf{A}$*, we have:*

$$\operatorname{tr}\mathbf{A}\left(\mathbf{Q}(t)-\mathbf{Q}_k(t)\right) \leq \|\mathbf{A}\|_2 .$$

**Lemma 7** (Convergence of quadratic forms)**.** *Let* $\mathbf{x} = [X_1, \ldots, X_M]^T \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_M)$*. Let* $\mathbf{A}$ *be an* $M \times M$ *random matrix independent of* $\mathbf{x}_M$*, with a bounded spectral norm. Then, we have*

$$\frac{1}{M}\mathbf{x}^H\mathbf{A}\mathbf{x} - \frac{1}{M}\operatorname{tr}(\mathbf{A}) \xrightarrow[M\to+\infty]{\text{a.s.}} 0.$$

The following Lemma provides results allowing to approximate random quantities involving the resolvent matrix when their dimensions grow simultaneously large:

**Lemma 8.** *Let* $\delta(t)$ *be the unique positive solution to the following equation:*

$$\delta(t) = \frac{M}{K\left(1 + \frac{t}{K}\sum_{i=1}^{K}\frac{\alpha_i}{1+t\delta(t)\alpha_i}\right)}. \tag{A.4}$$

*Consider the asymptotic regime in which* $M$ *and* $K$ *grow to infinity with:*

$$0 < \liminf \frac{M}{K} < \limsup \frac{M}{K} < \infty.$$

*Let* $[a, b]$ *be a closed bounded interval in* $[0, \infty)$*. the following convergences holds true:*

$$\sup_{t\in[a,b]}\left|\frac{1}{K}\operatorname{tr}\mathbf{Q}(t) - \delta(t)\right| \xrightarrow[M,K\to+\infty]{\text{a.s.}} 0.$$

*Moreover, if* $\mathbf{y}_1, \cdots, \mathbf{y}_K$ *denotes standard complex Gaussian vectors independent from* $\mathbf{x}_1, \cdots, \mathbf{x}_K$*, we have:*

$$\max_{1\leq j\leq K}\sup_{t\in[a,b]}\left|\mathbf{y}_j^H\mathbf{Q}(t)\mathbf{y}_j - \delta(t)\right| \xrightarrow[M,K\to+\infty]{\text{a.s.}} 0.$$

Note that, as a consequence of the rank-one perturbation Lemma, the above convergences can be transferred to the resolvent matrix $\mathbf{Q}_k(t)$. As a matter of fact, we also have:

$$\sup_{t\in[a,b]} \left| \frac{1}{K} \operatorname{tr} \mathbf{Q}_k(t) - \delta(t) \right| \xrightarrow[M,K\to+\infty]{\text{a.s.}} 0.$$

and

$$\max_{1\leq j\leq K} \sup_{t\in[a,b]} \left| \mathbf{y}_j^{\mathrm{H}} \mathbf{Q}(t) \mathbf{y}_j - \delta(t) \right| \xrightarrow[M,K\to+\infty]{\text{a.s.}} 0.$$

# B Proof of Theorem 2

The objective of this section is to determine deterministic equivalents for $\{\widehat{q}_k\}$. To this end, define $\hat{\tau}$ as:

$$\hat{\tau} = \frac{KP_{\max}}{\sum_{n=1}^{K} \gamma_n \left( \frac{1}{K} \widehat{\mathbf{h}}_n^{\mathrm{H}} \left( \frac{1}{K} \sum_{k \neq n} \widehat{q}_k \widehat{\mathbf{h}}_k \widehat{\mathbf{h}}_k^{\mathrm{H}} + \frac{1}{\rho} \mathbf{I}_M \right)^{-1} \widehat{\mathbf{h}}_n \right)^{-1}}. \tag{B.1}$$

Then, $\{\widehat{q}_\ell\}$ can be expressed as:

$$\widehat{q}_\ell = \frac{\gamma_\ell \hat{\tau}}{\frac{1}{K} \widehat{\mathbf{h}}_\ell^{\mathrm{H}} \left( \frac{1}{K} \sum_{k \neq \ell} \widehat{q}_k \widehat{\mathbf{h}}_k \widehat{\mathbf{h}}_k^{\mathrm{H}} + \frac{1}{\rho} \mathbf{I}_M \right)^{-1} \widehat{\mathbf{h}}_\ell}. \tag{B.2}$$

For the sake of the proof, it is more convenient to study the asymptotic convergence of $\left\{ \widehat{d}_k \right\}$ defined as

$$\widehat{d}_k = \frac{\gamma_k \hat{\tau}}{\widehat{q}_k \beta_k}, \tag{B.3}$$

since, as will be seen next unlike $\{q_k\}$, all $\{d_k\}$ converge to the same quantity. Let $\widehat{\mathbf{z}}_k = \beta_k^{-1/2} \widehat{\mathbf{h}}_k$. It can be thus easily shown that $\{d_k\}$ are the positive solutions to the following fixed-point equations:

$$\widehat{d}_k = \frac{1}{K} \widehat{\mathbf{z}}_k^{\mathrm{H}} \left( \sum_{m \neq k} \frac{\hat{\tau} \gamma_m}{K \widehat{d}_m} \widehat{\mathbf{z}}_m \widehat{\mathbf{z}}_m^{\mathrm{H}} + \frac{1}{\rho} \mathbf{I}_M \right)^{-1} \widehat{\mathbf{z}}_k. \tag{B.4}$$

Note that direct application of standard random matrix theory tools to the quadratic form arising in the expressions of $\left\{ \widehat{d}_k \right\}$ is not analytically correct, since coefficients $\{d_k\}$ and $\hat{\tau}$ are function of the channel vectors $\{\mathbf{z}_k\}$. However, one would expect coefficients $\left\{\widehat{d}_m\right\}_{m\neq k}$ to be weakly dependent of $\widehat{\mathbf{z}}_k$, and thus considering $\left\{\widehat{d}_k\right\}$ as deterministic, although not properly correct, would lead to infer about their asymptotic behaviour. Based on these intuitive expectations and using the results of Appendix A, we can claim that $\{d_k\}$ satisfy the following convergence:

$$\max_{1\leq k\leq K}\left|\frac{\widehat{d}_k}{\widetilde{d}}-1\right| \xrightarrow[M,K\to+\infty]{\text{a.s.}} 0,$$

where $\widetilde{d}$ is the unique positive solution to the following equation:

$$1=\frac{\frac{M}{K}}{\frac{\widetilde{d}}{\rho}+\frac{1}{K}\sum_{m=1}^{K}\frac{\gamma_m\widehat{\tau}}{1+\gamma_m\widehat{\tau}}}. \tag{B.5}$$

Note that a unique solution to the above equation exists, since function

$$f:x\mapsto\frac{\frac{M}{K}}{\frac{x}{\rho}+\frac{1}{K}\sum_{m=1}^{K}\frac{\gamma_m\widehat{\tau}}{1+\gamma_m\widehat{\tau}}},$$

is decreasing with $f(0)>1$ and $\lim_{x\to+\infty}f(x)=0$. We will now provide a rigourous proof to this claim. To this end, we will make use of the approach developed in [30]. Building on the same ideas of [30], we define $e_k=\frac{\widehat{d}_k}{\widetilde{d}}$, and assume without loss of generality that $e_1<\cdots,e_K$. We can thus write $\left\{\widehat{d}_k\right\}$ as:

$$\widehat{d}_k=\frac{1}{K}\widehat{\mathbf{z}}_k^{\mathrm{H}}\left(\sum_{m\neq k}\frac{\widehat{\tau}\gamma_m\widehat{\mathbf{z}}_m\widehat{\mathbf{z}}_m^{\mathrm{H}}}{Ke_m\widetilde{d}}+\frac{1}{\rho}\mathbf{I}_M\right)^{-1}\widehat{\mathbf{z}}_k. \tag{B.6}$$

Dividing the above equation by $\widetilde{d}$, we get:

$$e_k=\frac{1}{K}\widehat{\mathbf{z}}_k^{\mathrm{H}}\left(\sum_{m\neq k}\frac{\widehat{\tau}\gamma_m\widehat{\mathbf{z}}_m\widehat{\mathbf{z}}_m^{\mathrm{H}}}{Ke_m}+\frac{\widetilde{d}}{\rho}\mathbf{I}_M\right)^{-1}\widehat{\mathbf{z}}_k. \tag{B.7}$$

From monotonicity arguments, it follows that:

$$e_K \leq \frac{1}{K}\widehat{\mathbf{z}}_K^{\mathrm{H}}\left(\sum_{m\neq K}\frac{\widehat{\tau}\gamma_m\widehat{\mathbf{z}}_m\widehat{\mathbf{z}}_m^{\mathrm{H}}}{Ke_K}+\frac{\widetilde{d}}{\rho}\mathbf{I}_M\right)^{-1}\widehat{\mathbf{z}}_K, \tag{B.8}$$

or equivalently:

$$1 \leq \frac{1}{K}\widehat{\mathbf{z}}_K^{\mathrm{H}}\left(\sum_{m\neq k}\frac{\widehat{\tau}\gamma_m\widehat{\mathbf{z}}_m\widehat{\mathbf{z}}_m^{\mathrm{H}}}{K}+\frac{\widetilde{d}e_K}{\rho}\mathbf{I}_M\right)^{-1}\widehat{\mathbf{z}}_K. \tag{B.9}$$

To prove that $\max_{1\leq k\leq K}|e_k-1| \xrightarrow[M,K\to+\infty]{\text{a.s.}} 0$, we proceed by contradiction. Assume that there exists $\ell>0$ such that $\limsup e_K > 1+\ell$. Then, $e_K$ is infinitely often larger than $1+\ell$. Let us restrict ourselves to such a subsequence. Therefore, we have:

$$1 \leq \frac{\rho}{\widetilde{d}(1+\ell)}\frac{1}{K}\widehat{\mathbf{z}}_K^{\mathrm{H}}\left(\frac{\rho\hat{\tau}}{\widetilde{d}(1+\ell)}\sum_{m\neq k}\frac{\gamma_m\widehat{\mathbf{z}}_m\widehat{\mathbf{z}}_m^{\mathrm{H}}}{K}+\mathbf{I}_M\right)^{-1}\widehat{\mathbf{z}}_K. \tag{B.10}$$

We shall now invoke the convergence results of Lemma 2 in Appendix A. But before that, we need to check that $\frac{\rho\hat{\tau}}{\widetilde{d}}$ stay almost surely in a bounded interval. This can be shown by noticing that function $f$ can be bounded above and below by:

$$\frac{\frac{M}{K}}{\frac{x}{\rho}+1} \leq f(x) \leq \frac{M\rho}{Kx},$$

and thus:

$$\rho\left(\frac{M}{K}-1\right) \leq \widetilde{d} \leq \frac{M\rho}{K}.$$

Now, we are ready to apply the results of Lemma 8. Using this Lemma, we have:

$$\left|\frac{\rho}{\widetilde{d}(1+\ell)}\frac{1}{K}\widehat{\mathbf{z}}_K^{\mathrm{H}}\left(\frac{\rho\hat{\tau}}{\widetilde{d}(1+\ell)}\sum_{m\neq k}\frac{\gamma_m\widehat{\mathbf{z}}_m\widehat{\mathbf{z}}_m^{\mathrm{H}}}{K}+\mathbf{I}_M\right)^{-1}\widehat{\mathbf{z}}_K-\widehat{\mu}\right| \xrightarrow[M,K\to+\infty]{\text{a.s.}} 0,$$

where $\widehat{\mu}$ is the unique solution to the following equation:

$$\widehat{\mu} = \frac{M}{K}\left(\frac{\widetilde{d}(1+\ell)}{\rho} + \frac{1}{K}\sum_{m=1}^{K}\frac{\gamma_m\widehat{\tau}}{1+\frac{\gamma_m}{1+\gamma_m\widehat{\mu}\widehat{\tau}}}\right)^{-1}.$$

The above convergence along with (B.10) implies that:

$$\begin{aligned}1 &\leq \frac{\rho}{\widetilde{d}(1+\ell)}\frac{1}{K}\widehat{\mathbf{z}}_K^{\mathrm{H}}\left(\frac{\rho\hat{\tau}}{\widetilde{d}(1+\ell)}\sum_{m\neq k}\frac{\gamma_m\widehat{\mathbf{z}}_m\widehat{\mathbf{z}}_m^{\mathrm{H}}}{K}+\mathbf{I}_M\right)^{-1}\widehat{\mathbf{z}}_K \\ &\leq \widehat{\mu}+\epsilon_M,\end{aligned}$$

for $\epsilon_M \xrightarrow[M,K\to+\infty]{\text{a.s.}} 0$. Now note that $\widehat{\mu} = f(\frac{\widetilde{d}(1+\ell)}{\rho})$. Since $f(\widetilde{d}) = 1$ and $f$ is decreasing, $1-\epsilon_M \leq \widehat{\mu} = f(\frac{\widetilde{d}(1+\ell)}{\rho}) < 1$. Tending $n$ to infinity raises a contradiction. This proves that $\limsup e_K \leq 1$ for all large $K$. Using similar arguments, we can prove that $\liminf e_1 \geq 1$. Gathering both results, we finally obtain:

$$\max_{1\leq k\leq K}\left|\frac{\widehat{d}_k}{\widetilde{d}}-1\right| \xrightarrow[M,K\to+\infty]{\text{a.s.}} 0.$$

Note that $\widetilde{d}$ is still random because of its dependence on $\hat{\tau}$. Further work is needed to find a deterministic equivalent for $\hat{\tau}$. Recalling that $\widehat{q}_k = \frac{\gamma_k\widehat{\tau}}{\widehat{d}_k\beta_k}$ and $\frac{1}{K}\sum_{k=1}^{K}\widehat{q}_k = P_{\max}$, we obtain:

$$\hat{\tau} = \frac{P_{\max}}{\frac{1}{K}\sum_{k=1}^{K}\frac{\gamma_k}{\widehat{d}_k\beta_k}}.$$

Using the convergence $\max_{1\leq k\leq K}\left|\widehat{d}_k-\widetilde{d}\right| \xrightarrow[M,K\to+\infty]{\text{a.s.}} 0$, we thus have:

$$\hat{\tau} = \frac{P_{\max}}{\frac{1}{K}\sum_{k=1}^{K}\frac{\gamma_k}{\widetilde{d}\beta_k}} + o(1).$$

where $o(1)$ denotes a sequence converging to zero almost surely. Replacing $\widetilde{d}$ by $\frac{\hat{\tau}}{K}\sum_{k=1}^{K}\frac{\gamma_k}{\beta_k P_{\max}}$, we finally get that:

$$\hat{\tau} = \frac{M}{K}\left(\frac{1}{K}\sum_{\ell=1}^{K}\frac{\gamma_\ell}{\rho\beta_\ell P_{\max}} + \frac{1}{K}\sum_{m=1}^{K}\frac{\gamma_m}{1+\widehat{\tau}\gamma_m}\right)^{-1} + o(1).$$

Using the above equation, we are tempted to discard the vanishing terms and to state that a deterministic equivalent by $\hat{\tau}$ is given by $\overline{\tau}$, the unique solution to the following equation:

$$\overline{\tau} = \frac{M}{K}\left(\frac{1}{K}\sum_{\ell=1}^{K}\frac{\gamma_\ell}{\rho\beta_\ell P_{\max}} + \frac{1}{K}\sum_{m=1}^{K}\frac{\gamma_m}{1+\overline{\tau}\gamma_m}\right)^{-1}. \tag{B.11}$$

This is indeed true, since straightforward calculations lead to the following identity:

$$\widehat{\tau} - \overline{\tau} = o(1) + \frac{\frac{M}{K}\left[\frac{1}{K}\sum_{m=1}^{K}\frac{\gamma_m^2}{(1+\overline{\tau}\gamma_m)(1+\widehat{\tau}\gamma_m)}\right]}{\left(\alpha + \frac{1}{K}\sum_{j=1}^{K}\frac{\gamma_j}{1+\overline{\tau}\gamma_j}\right)\left(\alpha + \frac{1}{K}\sum_{j=1}^{K}\frac{\gamma_j}{1+\widehat{\tau}\gamma_j}\right)},$$

with $\alpha = \frac{1}{K}\sum_{\ell=1}^{K}\frac{\gamma_\ell}{\rho\beta_\ell P_{\max}}$. Using again the expressions of $\hat{\tau}$ and $\overline{\tau}$, we have:

$$\widehat{\tau} - \overline{\tau} = o(1) + \hat{\tau}\overline{\tau}\frac{K}{M}\left[\frac{1}{K}\sum_{m=1}^{K}\frac{\gamma_m^2\left(\widehat{\tau}-\overline{\tau}\right)}{(1+\overline{\tau}\gamma_m)(1+\widehat{\tau}\gamma_m)}\right].$$

Hence,

$$|\widehat{\tau} - \overline{\tau}| \leq \frac{K}{M}\,|\widehat{\tau} - \overline{\tau}| + o(1),$$

and thus:

$$\widehat{\tau} - \overline{\tau} \xrightarrow[M,K\to+\infty]{\text{a.s.}} 0.$$

Let $\overline{d} = \frac{\overline{\tau}\frac{1}{K}\sum_{k=1}^{K}\frac{\gamma_k}{\beta_k}}{\overline{P}}$, we thus have:

$$\max_{1\leq k\leq K}\left|\widehat{d}_k - \overline{d}\right| \xrightarrow[M,K\to+\infty]{\text{a.s.}} 0.$$

Putting the convergences of $\widehat{\tau}$ and $\left\{\widehat{d}_k\right\}$ together, the convergence of $\widehat{q}_k$ directly follows.

# C Proof of Theorem 3

The aim of this section is to prove the convergence of $\left\{\mathrm{SINR}_k^{dl}\right\}$ and $\{\hat{p}_k\}$ to $\left\{\overline{\mathrm{SINR}}_k^{dl}\right\}$ and $\{\overline{p}_k\}$. The proof of the convergence of $\{\hat{p}_k\}$ follows along the same arguments as those used for $\{\hat{q}_k\}$, and will be thus omitted. Based on these convergences, we can prove using standard calculations from random matrix theory, that the asymptotic behaviour of $\left\{\mathrm{SINR}_k^{dl}\right\}$ remains almost surely the same if we replace $\hat{p}_k$ by $\overline{p}_k$ and $\hat{q}_k$ by $\overline{q}_k$. This brings us to study the following $\widetilde{\mathrm{SINR}}_k^{dl}$ given by:

$$\widetilde{\mathrm{SINR}}_k^{dl} = \frac{\frac{\rho\overline{p}_k}{K}\left|\mathbf{h}_k^{\mathrm{H}}\overline{\mathbf{u}}_k\right|^2}{\sum_{i\neq k}\frac{\rho\overline{p}_i}{K}\left|\mathbf{h}_k^{\mathrm{H}}\overline{\mathbf{u}}_i\right|^2+1},$$

with:

$$\overline{\mathbf{u}}_i = \frac{\overline{\mathbf{v}}_i}{\|\overline{\mathbf{v}}_i\|},$$

vector $\overline{\mathbf{v}}_i$ being given by:

$$\overline{\mathbf{v}}_i = \left(\frac{1}{K}\sum_{j=1}^{K}\rho\overline{q}_j\widehat{\mathbf{h}}_j\widehat{\mathbf{h}}_j^H+\mathbf{I}_M\right)^{-1}\widehat{\mathbf{h}}_i.$$

Denote by $S_k$ and $I_k$ the signal and interference terms:

$$S_k = \frac{\rho\overline{p}_k}{K}\left|\mathbf{h}_k^{\mathrm{H}}\overline{\mathbf{u}}_k\right|^2,$$
$$I_k = \sum_{i\neq k}\frac{\rho\overline{p}_i}{K}\left|\mathbf{h}_k^{\mathrm{H}}\overline{\mathbf{u}}_i\right|^2,$$

Let $\mathbf{Q}(\rho) = \left(\frac{1}{K}\sum_{j=1}^{K}\rho\overline{q}_j\widehat{\mathbf{h}}_j\widehat{\mathbf{h}}_j^H+\mathbf{I}_M\right)^{-1}$ denote the resolvent matrix associated with $\frac{1}{K}\sum_{j=1}^{K}\overline{q}_j\widehat{\mathbf{h}}_j\widehat{\mathbf{h}}_j^H$. For $i\in\{1,\cdots,K\}$, denote by $\mathbf{Q}_i(\rho)$ the resolvent matrix obtained by removing the contribution of $\mathbf{h}_i$:

$$\mathbf{Q}_i(\rho) = \left( \frac{1}{K} \sum_{j \neq i} \rho \overline{q}_j \widehat{\mathbf{h}}_j \widehat{\mathbf{h}}_j^H + \mathbf{I}_M \right)^{-1}$$

Then, since $\frac{\mathbf{Q}(\rho)\widehat{\mathbf{h}}_i}{\left\|\mathbf{Q}(\rho)\widehat{\mathbf{h}}_i\right\|} = \frac{\mathbf{Q}_i(\rho)\widehat{\mathbf{h}}_i}{\left\|\mathbf{Q}_i(\rho)\widehat{\mathbf{h}}_i\right\|}$, $S_k$ can be written as:

$$S_k = \rho \overline{p}_k \frac{\left| \frac{1}{K} \mathbf{h}_k^{\mathrm{H}} \mathbf{Q}_k(\rho) \widehat{\mathbf{h}}_k \right|^2}{\frac{1}{K} \widehat{\mathbf{h}}_k^{\mathrm{H}} \mathbf{Q}_k(\rho)^2 \widehat{\mathbf{h}}_k}$$

Applying the result of Lemma 2 in Appendix A, we prove that:

$$\max_{1 \leq k \leq K} \left| \frac{\rho}{K} \mathbf{h}_k^{\mathrm{H}} \mathbf{Q}_k(\rho) \widehat{\mathbf{h}}_k - \beta_k \sqrt{1-\eta^2} \mu \right| \xrightarrow[M,K \to +\infty]{\text{a.s.}} 0$$

where $\mu$ is the unique positive solution to the following equation:

$$\mu = \frac{M}{K} \rho \left( 1 + \frac{\rho}{K} \sum_{i=1}^{K} \frac{\overline{q}_i \beta_i}{1 + \overline{q}_i \beta_i \mu} \right)^{-1}$$

To handle the denominator in the expression of $S_k$, we use the fact that:

$$\max_{1 \leq k \leq K} \left| \frac{1}{K} \widehat{\mathbf{h}}_k^H \mathbf{Q}_k(\rho)^2 \widehat{\mathbf{h}}_k - \frac{\beta_k}{K} \operatorname{tr} \mathbf{Q}(\rho)^2 \right| \xrightarrow[M,K \to +\infty]{\text{a.s.}} 0,$$

which is a consequence of the asymptotic properties of quadratic forms and the rank-one perturbation in Lemma 1. Now, applying the results of [31, Proposition 3 item 2], we obtain:

$$\frac{1}{K} \operatorname{tr} \mathbf{Q}(\rho)^2 - \frac{\mu^2}{\frac{M}{K}\rho^2 - \mu^2 \rho^2 \frac{M}{K} \frac{1}{K} \sum_{i=1}^{K} \frac{\beta_i^2 \overline{q}_i^2}{(1+\mu \beta_i \overline{q}_i)^2}} \xrightarrow[M,K \to +\infty]{\text{a.s.}} 0.$$

From the results of Appendix A, we can prove that $\mu = \frac{\gamma_k \overline{\tau}}{\overline{q}_k \beta_k}$. Using this relation into the above equation, we obtain:

$$\max_{1\leq k\leq K}\left|\frac{1}{K}\widehat{\mathbf{h}}_k^H\mathbf{Q}_k(\rho)^2\widehat{\mathbf{h}}_k-\beta_k\tilde{\mu}\right|\xrightarrow[M,K\to+\infty]{\text{a.s.}}0,$$

where:

$$\widetilde{\mu}=\frac{\mu^2}{\frac{M}{K}\rho^2-\frac{M}{K}\rho^2\frac{1}{K}\sum_{i=1}^K\frac{(\gamma_i\overline{\tau})^2}{(1+\gamma_i\overline{\tau})^2}}.$$

Putting all these results togother yields the following convergence:

$$\max_{1\leq k\leq K}\left|S_k-\frac{\frac{1}{\rho}\overline{p}_k\beta_k(1-\eta^2)\mu^2}{\tilde{\mu}}\right|\xrightarrow[M,K\to+\infty]{\text{a.s.}}0.$$

We now proceed to computing the interference term. This term can be written as:

$$I_k=\frac{\rho}{K^2}\mathbf{h}_k^H\mathbf{Q}(\rho)\widehat{\mathbf{H}}_k\mathbf{D}_k\widehat{\mathbf{H}}_k^H\mathbf{Q}(\rho)\mathbf{h}_k,$$

where $\mathbf{D}_k$ is a $K-1\times K-1$ diagonal matrix given by:

$$\mathbf{D}_k=\text{diag}\left\{\frac{\overline{p}_1}{\frac{1}{K}\widehat{\mathbf{h}}_1^H\mathbf{Q}(\rho)^2\widehat{\mathbf{h}}_1},\cdots,\frac{\overline{p}_{k-1}}{\frac{1}{K}\widehat{\mathbf{h}}_{k-1}^H\mathbf{Q}(\rho)^2\widehat{\mathbf{h}}_{k-1}},\frac{\overline{p}_{k+1}}{\frac{1}{K}\widehat{\mathbf{h}}_{k+1}^H\mathbf{Q}(\rho)^2\widehat{\mathbf{h}}_{k+1}}\right.$$
$$\left.,\cdots,\frac{\overline{p}_K}{\frac{1}{K}\widehat{\mathbf{h}}_K^H\mathbf{Q}(\rho)^2\widehat{\mathbf{h}}_K}\right\}$$

Using the fact that:

$$\widehat{\mathbf{h}}_k^H\mathbf{Q}(\rho)^2\widehat{\mathbf{h}}_k=\frac{\widehat{\mathbf{h}}_k^H\mathbf{Q}_k(\rho)^2\widehat{\mathbf{h}}_k}{\left(1+\rho\overline{q}_k\frac{1}{K}\widehat{\mathbf{h}}_k^H\mathbf{Q}_k(\rho)\widehat{\mathbf{h}}_k\right)^2}$$

and exploiting the already established convergences, we can prove that:

$$\max_{1\leq k\leq K}\left|\frac{\overline{p}_k}{\frac{1}{K}\widehat{\mathbf{h}}_k^H\mathbf{Q}(\rho)^2\widehat{\mathbf{h}}_k}-\frac{\overline{p}_k(1+\overline{q}_k\beta_k\mu)^2}{\beta_k\tilde{\mu}}\right|\xrightarrow[M,K\to+\infty]{\text{a.s.}}0.$$

It entails from the above convergence that matrix $\mathbf{D}_k$ converges in operator norm to

$\overline{\mathbf{D}}_k$ obtained by replacing the random elements of $\mathbf{D}_k$ by their asymptotic equivalents. Studying the asymptotic behaviour of $I_k$ amounts thus to considering $\tilde{I}_k$ given by:

$$\tilde{I}_k = \frac{\rho}{K^2}\mathbf{h}_k^H\mathbf{Q}(\rho)\widehat{\mathbf{H}}_k\overline{\mathbf{D}}_k\widehat{\mathbf{H}}_k^H\mathbf{Q}(\rho)\mathbf{h}_k.$$

Using the decomposition of $\mathbf{Q}$ as:

$$\mathbf{Q}(\rho) = \mathbf{Q}_k(\rho) - \frac{\frac{\rho}{K}\overline{q}_k\mathbf{Q}_k(\rho)\widehat{\mathbf{h}}_k\widehat{\mathbf{h}}_k^H\mathbf{Q}_k(\rho)}{1+\frac{\rho}{K}\overline{q}_k\widehat{\mathbf{h}}_k^H\mathbf{Q}_k(\rho)\widehat{\mathbf{h}}_k},$$

we can expand $\tilde{I}_k$ as:

$$\begin{aligned}\tilde{I}_k &= \frac{\rho}{K^2}\mathbf{h}_k^H\mathbf{Q}_k(\rho)\widehat{\mathbf{H}}_k\overline{\mathbf{D}}_k\widehat{\mathbf{H}}_k^H\mathbf{Q}_k(\rho)\mathbf{h}_k\\
&- \frac{\rho^2\overline{q}_k}{K^3}\frac{\mathbf{h}_k^H\mathbf{Q}_k(\rho)\widehat{\mathbf{h}}_k\widehat{\mathbf{h}}_k^H\mathbf{Q}_k(\rho)\widehat{\mathbf{H}}_k\overline{\mathbf{D}}_k\widehat{\mathbf{H}}_k\mathbf{Q}_k(\rho)\mathbf{h}_k}{(1+\rho\frac{\overline{q}_k}{K}\widehat{\mathbf{h}}_k^H\mathbf{Q}_k\widehat{\mathbf{h}}_k)}\\
&- \frac{\rho^2\overline{q}_k}{K^3}\frac{\mathbf{h}_k^H\mathbf{Q}_k(\rho)\widehat{\mathbf{H}}_k\overline{\mathbf{D}}_k\widehat{\mathbf{H}}_k^H\mathbf{Q}_k(\rho)\widehat{\mathbf{h}}_k\widehat{\mathbf{h}}_k^H\mathbf{Q}_k(\rho)\mathbf{h}_k}{(1+\frac{\rho\overline{q}_k}{K}\widehat{\mathbf{h}}_k^{\mathrm{H}}\mathbf{Q}_k\widehat{\mathbf{h}})}\\
&+ \frac{\rho^3\overline{q}_k^2}{K^4}\frac{\left|\mathbf{h}_k^H\mathbf{Q}_k(\rho)\widehat{\mathbf{h}}_k\right|^2\widehat{\mathbf{h}}_k^H\widehat{\mathbf{Q}}_k(\rho)\widehat{\mathbf{H}}_k\overline{\mathbf{D}}_k\widehat{\mathbf{H}}_k^H\widehat{\mathbf{h}}_k}{\left(1+\frac{\rho\overline{q}_k}{K}\widehat{\mathbf{h}}_k^H\mathbf{Q}_k(\rho)\widehat{\mathbf{h}}_k\right)^2}\end{aligned}$$

This expression make arise classical quadratic forms that can be studied using the trace Lemma. We thus have:

$$\begin{aligned}\tilde{I}_k &= \frac{\rho}{K^2}\beta_k\operatorname{tr}\mathbf{Q}_k(\rho)\widehat{\mathbf{H}}_k\overline{\mathbf{D}}_k\widehat{\mathbf{H}}_k^H\mathbf{Q}_k(\rho)\\
&- \frac{\overline{q}_k\beta_k^2(1-\eta^2)\mu}{1+\overline{q}_k\beta_k\mu}\frac{\rho}{K^2}\operatorname{tr}\mathbf{Q}_k(\rho)\widehat{\mathbf{H}}_k\overline{\mathbf{D}}_k\widehat{\mathbf{H}}_k^H\mathbf{Q}_k(\rho)\\
&- \frac{\overline{q}_k\beta_k^2(1-\eta^2)\mu}{1+\overline{q}_k\beta_k\mu}\frac{\rho}{K^2}\operatorname{tr}\mathbf{Q}_k(\rho)\widehat{\mathbf{H}}_k\overline{\mathbf{D}}_k\widehat{\mathbf{H}}_k^H\mathbf{Q}_k(\rho)\\
&+ \frac{\overline{q}_k^2(1-\eta^2)\mu^2\beta_k^3}{(1+\overline{q}_k\beta_k\mu)^2}\frac{\rho}{K^2}\operatorname{tr}\mathbf{Q}_k(\rho)\widehat{\mathbf{H}}_k\overline{\mathbf{D}}_k\widehat{\mathbf{H}}_k^H\mathbf{Q}_k(\rho)+\epsilon_k\\
&= \frac{(1+2\eta^2\beta_k\overline{q}_k\mu+\eta^2\mu^2\overline{q}_k^2\beta_k^2)}{(1+\overline{q}_k\beta_k\mu)^2}\frac{\rho\beta_k}{K^2}\operatorname{tr}\mathbf{Q}_k(\rho)\widehat{\mathbf{H}}_k\overline{\mathbf{D}}_k\widehat{\mathbf{H}}_k^H\mathbf{Q}_k(\rho)+\epsilon_k\end{aligned}$$

where $\epsilon_k$ is a random sequence converging to zero almost surely uniformly in $k$, that is $\max_k |\epsilon_k| \xrightarrow[M,K\to+\infty]{\text{a.s.}} 0$. Using the fact that $\mu\overline{q}_k\beta_k = \gamma_k\overline{\tau}$, we obtain:

$$\tilde{I}_k = \frac{1+2\eta^2\gamma_k\overline{\tau}+\eta^2\gamma_k^2\overline{\tau}^2}{(1+\gamma_k\overline{\tau})^2}\frac{\rho\beta_k}{K^2}\operatorname{tr}\mathbf{Q}_k(\rho)\widehat{\mathbf{H}}_k\overline{\mathbf{D}}\widehat{\mathbf{H}}_k^H\mathbf{Q}_k(\rho)+\epsilon_k$$

We will now handle the term $\frac{1}{K^2}\operatorname{tr}\mathbf{Q}_k(\rho)\widehat{\mathbf{H}}_k\overline{\mathbf{D}}\widehat{\mathbf{H}}_k^H\mathbf{Q}_k(\rho)$. Note that due to the rank-one perturbation Lemma and the convergence in operator norm of $\mathbf{D}_k$ to $\overline{\mathbf{D}}_k$, matrices $\mathbf{Q}_k(\rho)$ and $\overline{\mathbf{D}}_k$ can be replaced by $\mathbf{Q}(\rho)$ and $\mathbf{D}_k$. In doing so, we prove that $\tilde{I}_k$ is almost surely equivalent to:

$$\begin{aligned}
&\frac{(1+2\eta^2\beta_k\overline{q}_k\mu+\eta^2\mu^2\overline{q}_k^2\beta_k^2)}{(1+\overline{q}_k\beta_k\mu)^2}\frac{\rho\beta_k}{K^2}\operatorname{tr}\mathbf{Q}(\rho)\widehat{\mathbf{H}}_k\mathbf{D}\widehat{\mathbf{H}}_k^H\mathbf{Q}(\rho)\\
&=\frac{(1+2\eta^2\beta_k\overline{q}_k\mu+\eta^2\mu^2\overline{q}_k^2\beta_k^2)}{(1+\overline{q}_k\beta_k\mu)^2}\frac{\rho\beta_k}{K^2}\sum_{i=1,i\neq k}^{K}\frac{\widehat{\mathbf{h}}_i^H\mathbf{Q}(\rho)^2\widehat{\mathbf{h}}_i\overline{p_i}}{\frac{1}{K}\widehat{\mathbf{h}}_i^H\mathbf{Q}(\rho)^2\widehat{\mathbf{h}}_i}\\
&=\frac{(1+2\eta^2\beta_k\overline{q}_k\mu+\eta^2\mu^2\overline{q}_k^2\beta_k^2)}{(1+\overline{q}_k\beta_k\mu)^2}\frac{\rho\beta_k}{K}\sum_{i=1,i\neq k}^{K}\overline{p}_i.
\end{aligned}$$

Since $\frac{1}{K}\sum_{i=1,i\neq k}^{K}\overline{p}_i = P_{\max}+O(1/K)$, we thus have:

$$\max_{1\leq k\leq K}\left|I_k-\frac{(1+2\eta^2\beta_k\overline{q}_k\mu+\eta^2\mu^2\overline{q}_k^2\beta_k^2)}{(1+\overline{q}_k\beta_k\mu)^2}\rho\beta_kP_{\max}\right|\xrightarrow[M,K\to+\infty]{\text{a.s.}}0.$$

And since $\beta_k\overline{q}_k\mu=\gamma_k\bar{\tau}$,we have

$$\max_{1\leq k\leq K}\left|I_k-\frac{(1+2\eta^2\gamma_k\bar{\tau}+\eta^2\gamma_k^2\bar{\tau}^2)}{(1+\gamma_k\bar{\tau})^2}\rho\beta_kP_{\max}\right|\xrightarrow[M,K\to+\infty]{\text{a.s.}}0.$$

Putting the above results together yields the results of Theorem 3.

# D Proof of Corollary 1

Hereafter, we will prove that when $\eta = 0$, $\overline{\mathrm{SINR}}_k^{dl} = \overline{\mathrm{SINR}}_k^{dl} = \gamma_k \bar{\tau}$. Starting from (4.8), setting $\eta = 0$ and replacing $\bar{p}_k$ by their expressions, we get easily

$$\overline{\mathrm{SINR}}_k^{dl} = \gamma_k \bar{\tau}.$$

On the other hand, we have

$$\overline{\mathrm{SINR}}_k^{ul} = \frac{\rho \bar{q}_k \beta_k \xi}{\frac{\rho}{K} \sum_{i=1}^{K} \beta_i \bar{q}_i \frac{1}{(1+\gamma_i \bar{\tau})^2} + 1}.$$

Dividing the numerator and denominator by $\rho$ and multiplying by $\mu$, we get

$$\overline{\mathrm{SINR}}_k^{ul} = \frac{\bar{q}_k \beta_k \xi \mu}{\frac{1}{K} \sum_{i=1}^{K} \mu \beta_i \bar{q}_i \frac{1}{(1+\gamma_i \bar{\tau})^2} + \frac{\mu}{\rho}}.$$

Since $\gamma_k \bar{\tau} = \beta_k \bar{q}_k \mu$, we get

$$\overline{\mathrm{SINR}}_k^{ul} = \frac{\gamma_k \bar{\tau} \xi}{\frac{1}{K} \sum_{i=1}^{K} \bar{\tau} \gamma_i \frac{1}{(1+\gamma_i \bar{\tau})^2} + \frac{\mu}{\rho}}.$$

Since $\mu$ can be expressed as,

$$\mu = \rho \left[ \frac{M}{K} - \frac{1}{K} \sum_{i=1}^{K} \frac{\gamma_i \bar{\tau}}{1 + \gamma_i \bar{\tau}} \right], \tag{D.1}$$

then

$$\begin{aligned}\overline{\mathrm{SINR}}_k^{ul} &= \frac{\gamma_k \bar{\tau} \xi}{\frac{1}{K}\sum_{i=1}^{K} \frac{\gamma_i \bar{\tau}}{(1+\gamma_i \bar{\tau})^2} + \frac{M}{K} - \frac{1}{K}\sum_{i=1}^{K} \frac{\gamma_i \bar{\tau}}{1+\gamma_i \bar{\tau}}} \\ &= \frac{\gamma_k \bar{\tau} \xi}{\frac{M}{K} - \frac{1}{K}\sum_{i=1}^{K} \frac{(\gamma_i \bar{\tau})^2}{(1+\gamma_i \bar{\tau})^2}} \\ &= \gamma_k \bar{\tau}.\end{aligned}$$

# E Proof of Lemma 5

In this proof, we compute deterministic equivalents of the entries of the matrices $\mathbf{a}_k$, $\mathbf{E}_k$ and $\mathbf{B}_{k,i}$. First, we introduce the following functionals

$$X_k(t) = \frac{1}{K}\widehat{\mathbf{h}}_k^{\mathrm{H}}\mathbf{Q}(t)\widehat{\mathbf{h}}_k,$$
$$Y_k(t) = \frac{1}{K}\mathbf{h}_k^{H}\mathbf{Q}(t)\widehat{\mathbf{h}}_k,$$

The coefficients of the matrices $\mathbf{a}_k$ and $\mathbf{E}_k$ can be written as a function of the higher derivatives of these functionals taken at $t = 0$ as follows

$$[\mathbf{a}_k]_\ell = \frac{(-1)^\ell}{\ell!}Y_k^{(\ell)},$$
$$[\mathbf{E}_k]_{\ell,m} = \frac{(-1)^{\ell+m}}{\ell!m!}X_k^{(\ell+m)}.$$

Thus, we need to compute deterministic equivalents of $Y_k^{(\ell)}$ and $X_k^{(\ell)}$. In [16], it has been shown that it suffices to determine deterministic equivalents of $X_k(t)$ and $Y_k(t)$ and then take their derivatives at $t = 0$.

We begin first by treating $X_k^{(\ell)}$. Using Lemma 6, we can write

$$\frac{1}{K}\widehat{\mathbf{h}}_k^{\mathrm{H}}\mathbf{Q}(t)\widehat{\mathbf{h}}_k = \frac{\frac{1}{K}\widehat{\mathbf{h}}_k^{\mathrm{H}}\mathbf{Q}_k(t)\widehat{\mathbf{h}}_k}{1 + \frac{t\bar{q}_k}{K}\widehat{\mathbf{h}}_k^{\mathrm{H}}\mathbf{Q}_k(t)\widehat{\mathbf{h}}_k}.$$

Lemma 7 along with the rank-one perturbation property in Lemma 6, imply that

$$\frac{1}{K}\widehat{\mathbf{h}}_k^{\mathrm{H}}\mathbf{Q}_k(t)\widehat{\mathbf{h}}_k - \frac{1}{K}\beta_k \operatorname{tr}\left(\mathbf{Q}(t)\right) \xrightarrow[M,K\to+\infty]{\text{a.s.}} 0.$$

Using Lemma 8, we can conclude that

$$\frac{1}{K}\widehat{\mathbf{h}}_k^{\mathrm{H}}\mathbf{Q}_k(t)\widehat{\mathbf{h}}_k - \beta_k\delta(t) \xrightarrow[M,K\to+\infty]{\text{a.s.}} 0. \tag{E.1}$$

Then,

$$X_k(t) - \overline{X}_k(t) \xrightarrow[M,K\to+\infty]{\text{a.s.}} 0. \tag{E.2}$$

where

$$\overline{X}_k(t) = \frac{\beta_k\delta(t)}{1 + t\bar{q}_k\beta_k\delta(t)}.$$

Using Corollary 6 in [16], we have

$$X_k^{(\ell)} - \overline{X}_k^{(\ell)} \xrightarrow[M,K\to+\infty]{\text{a.s.}} 0. \tag{E.3}$$

Then,

$$\mathbf{w}_k^{\mathrm{H}}\mathbf{a}_k\mathbf{a}_k^{\mathrm{H}}\mathbf{w}_k - \mathbf{w}_k^{\mathrm{H}}\overline{\mathbf{a}}_k\overline{\mathbf{a}}_k^{\mathrm{H}}\mathbf{w}_k \xrightarrow[M,K\to\infty]{a.s.} 0.$$

Again using Lemma 6, we can write

$$\frac{1}{K}\mathbf{h}_k^{\mathrm{H}}\mathbf{Q}(t)\widehat{\mathbf{h}}_k = \frac{\frac{1}{K}\mathbf{h}_k^{\mathrm{H}}\mathbf{Q}_k(t)\widehat{\mathbf{h}}_k}{1 + \frac{t\bar{q}_k}{K}\widehat{\mathbf{h}}_k^{\mathrm{H}}\mathbf{Q}_k(t)\widehat{\mathbf{h}}_k}$$

The asymptotic equivalent of the quadratic form $\frac{1}{K}\mathbf{h}_k^{\mathrm{H}}\mathbf{Q}_k(t)\widehat{\mathbf{h}}_k$, is the same as $\frac{\sqrt{1-\eta^2}}{K}\widehat{\mathbf{h}}_k^{\mathrm{H}}\mathbf{Q}_k(t)\widehat{\mathbf{h}}_k$. Thus,

$$Y_k(t) - \overline{Y}_k(t) \xrightarrow[M,K\to+\infty]{\text{a.s.}} 0. \tag{E.4}$$

where

$$\overline{Y}_k(t) = \frac{\sqrt{1-\eta^2}\beta_k\delta(t)}{1 + t\bar{q}_k\beta_k\delta(t)}.$$

Using again Corollary 6 in [16], we have

$$Y_k^{(\ell)} - \overline{Y}_k^{(\ell)} \xrightarrow[M,K\to+\infty]{\text{a.s.}} 0. \tag{E.5}$$

Then,

$$\mathbf{w}_k^{\mathrm{H}}\mathbf{E}_k\mathbf{w}_k - \mathbf{w}_k^{\mathrm{H}}\overline{\mathbf{E}}_k\mathbf{w}_k \xrightarrow[M,K\to\infty]{a.s.} 0.$$

It remains to study the convergence of the interference term,

$$\sum_{i\neq k}\frac{\rho p_i}{K}\mathbf{w}_i^{\mathrm{H}}\mathbf{B}_{k,i}\mathbf{w}_i = \sum_{\ell=0}^{J-1}\sum_{m=0}^{J-1}\sum_{i\neq k}\frac{\rho p_i}{K}w_{\ell,i}w_{m,i}[\mathbf{B}_{k,i}]_{\ell,m}$$

Let

$$\mathbf{D}_k = \mathrm{diag}(p_1 w_{\ell,1}w_{m,1},\cdots,p_{k-1}w_{\ell,k-1}w_{m,k-1},p_{k+1}w_{\ell,k+1}w_{m,k+1},\cdots,p_K w_{\ell,K}w_{m,K}).$$

For a fixed $(\ell,m)$, the term $\sum_{i\neq k}\frac{\rho p_i}{K}w_{\ell,i}w_{m,i}[\mathbf{B}_{k,i}]_{\ell,m}$ is treated separately,

$$\begin{aligned}
&\sum_{i\neq k}\frac{\rho p_i}{K}w_{\ell,i}w_{m,i}[\mathbf{B}_{k,i}]_{\ell,m}\\
&=\sum_{i\neq k}\frac{\rho p_i}{K^2}w_{\ell,i}w_{m,i}\mathbf{h}_k^H\left(\frac{\widehat{\mathbf{H}}\mathbf{R}\widehat{\mathbf{H}}^H}{K}\right)^{\ell}\widehat{\mathbf{h}}_i\widehat{\mathbf{h}}_i^H\left(\frac{\widehat{\mathbf{H}}\mathbf{R}\widehat{\mathbf{H}}^H}{K}\right)^{m}\mathbf{h}_k\\
&=\frac{\rho}{K^2}\mathbf{h}_k^H\left(\frac{\widehat{\mathbf{H}}\mathbf{R}\widehat{\mathbf{H}}^H}{K}\right)^{\ell}\widehat{\mathbf{H}}_k\mathbf{D}_k\widehat{\mathbf{H}}_k^H\left(\frac{\widehat{\mathbf{H}}\mathbf{R}\widehat{\mathbf{H}}^H}{K}\right)^{m}\mathbf{h}_k\\
&=\frac{(-1)^{\ell+m}}{m!\ell!}Z_k^{(\ell,m)}
\end{aligned}$$

where $Z_k^{(\ell,m)}$ is the $(\ell,m)$ derivative taken on $t=0$ and $u=0$ of the functional $Z_k(t,u)$ defined as

$$Z_k(t,u)=\frac{\rho}{K^2}\mathbf{h}_k^H\mathbf{Q}(t)\widehat{\mathbf{H}}_k\mathbf{D}_k\widehat{\mathbf{H}}_k^H\mathbf{Q}(u)\mathbf{h}_k.$$

Using same techniques as in Appendix C, one can prove that

$$Z_k(t,u)=\overline{f}_k(t,u)\frac{1}{K^2}\operatorname{tr}\mathbf{Q}_k(t)\widehat{\mathbf{H}}_k\overline{\mathbf{D}}_k\widehat{\mathbf{H}}_k^H\mathbf{Q}_k(u)+\epsilon_k,$$

where

$$\overline{f}_k(t,u) = \beta_k \rho \left( \eta^2 + \frac{1-\eta^2}{(1+\bar{q}_k \beta_k t \delta(t))(1+\bar{q}_k \beta_k u \delta(u))} \right)$$

and $\epsilon_k$ is a random sequence converging to zero almost surely uniformly in $k$, that is $\max_k |\epsilon_k| \xrightarrow[M,K\to+\infty]{\text{a.s.}} 0$. It remains to deal with the quantity $\frac{1}{K^2} \operatorname{tr} \mathbf{Q}_k(t) \widehat{\mathbf{H}}_k \overline{\mathbf{D}}_k \widehat{\mathbf{H}}_k^H \mathbf{Q}_k(u)$. Using the same techniques as in [16, Lemma 15], one can prove that

$$\frac{1}{K^2} \operatorname{tr} \mathbf{Q}_k(t) \widehat{\mathbf{H}}_k \overline{\mathbf{D}}_k \widehat{\mathbf{H}}_k^H \mathbf{Q}_k(u) - \bar{\alpha}(t,u) \frac{1}{K} \sum_{i \neq k} \frac{p_i w_{\ell,i} w_{m,i} \beta_i}{(1+t\delta(t)\beta_i \bar{q}_i)(1+u\delta(u)\beta_i \bar{q}_i)} \xrightarrow[M,K\to+\infty]{\text{a.s.}} 0,$$

where

$$\bar{\alpha}(t,u) = \frac{\frac{1}{K} \operatorname{tr}(\mathbf{T}(t)\mathbf{T}(u))}{1 - \frac{tu}{K^2} \operatorname{tr}(\mathbf{T}(t)\mathbf{T}(u)) \operatorname{tr}\left(\tilde{\mathbf{D}}^2 \mathbf{R}^2 \tilde{\mathbf{T}}(t) \tilde{\mathbf{T}}(u)\right)}$$

And finally

$$Z_k(t,u) - \frac{1}{K} \sum_{i \neq k} \overline{Z}_{k,i}(t,u) \xrightarrow[M,K\to+\infty]{\text{a.s.}} 0,$$

where

$$\overline{Z}_{k,i}(t,u) = \frac{\beta_i \overline{f}_k(t,u) \bar{\alpha}(t,u)}{(1+t\delta(t)\beta_i \bar{q}_i)(1+u\delta(u)\beta_i \bar{q}_i)}.$$

And then

$$\sum_{i \neq k} \frac{\rho p_i}{K} \mathbf{w}_i^{\mathrm{H}} \mathbf{B}_{k,i} \mathbf{w}_i - \sum_{i \neq k} \frac{\rho p_i}{K} \mathbf{w}_i^{\mathrm{H}} \overline{\mathbf{B}}_{k,i} \mathbf{w}_i \xrightarrow[M,K\to+\infty]{\text{a.s.}} 0,$$

where

$$[\overline{\mathbf{B}}_{k,i}]_{\ell,m} = \frac{(-1)^{\ell+m}}{m!\ell!} \overline{Z}_k^{(\ell,m)}.$$

Similarly, it can be shown that

$$\sum_{i \neq k} \frac{\rho q_i}{K} \mathbf{w}_k^{\mathrm{H}} \mathbf{B}_{i,k} \mathbf{w}_k - \sum_{i \neq k} \frac{\rho q_i}{K} \mathbf{w}_k^{\mathrm{H}} \overline{\mathbf{B}}_{i,k} \mathbf{w}_k \xrightarrow[M,K\to+\infty]{\text{a.s.}} 0,$$

Plugging all these results together yields the convergence results of Lemma 5.

# Papers Submitted and Under Preparation

• H. Sifaou, A. Kammoun, L. Sanguinetti, M. Debbah, M. S. Alouini, ‘”Polynomial Expansion of the Precoder for Power Minimization in Large-Scale MIMO Systems”, *in Proceedings of ICC 2016*, Kuala Lumpur, Malysia, 2016.

• H. Sifaou, A. Kammoun, L. Sanguinetti, M. Debbah, M. S. Alouini, ‘”Max-Min SINR Low Complexity Transceiver Design for Single Cell Massive MIMO”, *in Proceedings of SPAWC 2016*, Edinburgh, UK, 2016.

• H. Sifaou, A. Kammoun, L. Sanguinetti, M. Debbah, M. S. Alouini, ‘”Low complexity precoding and receiver for the Max-Min SINR in multiuser massive MIMO systems”, *Under Preparation*, 2016.

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