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Power Minimization of a Wireless Sensor Node under Different Rate Constraints

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Abstract

Future wireless networks are expected to handle a huge number of devices, including sensors, within a low energy consumption. In this scope, we present, in this paper, performance of wireless sensor networks (WSN). Specifically, we aim at finding the optimal transmit power of a node communicating with multiple receivers in a cognitive radio (CR) spectrum sharing framework, i.e., existence of an active primary user. We first present the optimal power with single secondary receiver, under instantaneous or average transmission rate constraints. Then, we propose a suboptimal solution for an easier, yet efficient, implementation and perform insightful asymptotical analysis for both schemes with Rayleigh fading. Afterwards, we extend our results to a multiple secondary receives CR scenario and present the corresponding optimal and suboptimal transmit power while satisfying independent peak/average and sum of peak/average transmission rate constraints. The corresponding numerical results are provided for Rayleigh and Nakagami-m fading channels. We characterize some transmission outage events depending

on system parameters.

I. Introduction

A. Motivation and Related Work

Wireless sensor networks (WSN) consist of the deployment of sensors over a broad area in order to acquire data. These networks have an increasing impact in a great variety of industrial, medical, and environmental applications. The energy consumption, transmission power, memory, and computational speed of the devices have a direct influence in the viability and cost of WSN. However, among other design criteria of sensor devices, the battery life-cycle is of crucial interest.

In fact, the energy consumption in WSN is directly related to the power control policy. As a result, power minimization has been considered in several network optimization problems from an architectural perspective together with the design of efficient algorithms for WSN. For instance, in [1], the authors provide an overview of the power control policies applied to wireless cellular networks. Furthermore, authors in [2] proposed a cross-layer algorithm that aims at minimizing the power consumption as well as dealing with scheduling and routing problems for WSN with data aggregation. In addition, an integer linear programming model has been proposed in order to address the problem of establishing energy-efficient state assignment to sensors under energy-

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efficient routing, clustering and coverage constraints [3] in addition to energy-efficient power control [4], [5]. In order to ensure reliable link quality between WSN nodes, a minimal rate should be met during transmission [6].

From another side, WSN can be deployed in an area where the frequency band is already used by other applications and systems labeled as primary users (PUs). In this case, the WSN nodes act as secondary users (SUs) in a cognitive radio (CR) context. The CR paradigm is based on the idea of sensing the environment to improve the communication without interfering with other users. In [7], a discussion of existing communication protocols and algorithms designed cognitive radio sensor networks (CRSN) is presented. When the SU is transmitting at the same time with the PU, this CR setting is called underlay mode [8]. In this case, the SU transmit power needs to respect a certain interference constraint [9]. In [10]–[13], the authors design an optimal and power allocation strategy to maximize the capacity of cognitive radio systems. In [14], the authors present power control and interference mitigation techniques based on interference alignment. Furthermore, in [15], a joint admission control and rate /power allocation schemes have been developed where the interference limits at primary receiving points are adapted depending on the traffic load of the primary network. In [16], [17], the authors presented derivation of the optimum power profile and the ergodic capacity for general fading channels with respect to average and peak transmit power along with interference outage constraints. In [18], [19], the instantaneous power was minimized vs. interference and power constraints. However, in such a problem the overall power is not limited and for some channel realization the channel could be high. In [20], a comparison between the instantaneous and average interference constraints in the CR framework is presented.

Within the activity of the WSN, transmitted signals can be sent to a single node, i.e., unicast,

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or multiple node simultaneously, i.e., broadcast. When commands are sent to the deployed nodes, broadcasting is the most used type of transmission since it is more efficient, simple and ensures data redundancy compared to unicasting. For instance, the authors in [21], [22] focus on WSN power allocation for multiple nodes without considering existence of a primary user.

B. Approach and Contributions

In all previous works and to the best of our knowledge, optimizing underlay WSN operating in a broadcast mode as a SU has not been considered. Hence, the focus of this work is on power control of underlay WSN performing a broadcast transmission. In our approach we first address the problem of minimizing the average power at each node while satisfying a rate constraint and an interference constraint dictated by the primary network. Then, we preset the results when the node is acting as a SU in a CR framework. Finally, we extend our result to multiple receivers case. More specifically, our contributions are as follows:

- Derive the optimal power in closed-form under instantaneous and average rate constraints in single-user mode.
- Present the asymptotic analysis at high and low power regime for Rayleigh and Nakagami-m channels.
- Derive the optimal power for a single node with single receiver in a CR framework.
- Derive the optimal power for broadcast mode with multiple receivers in a CR framework.

C. Outline of the Paper

The rest of the paper organized as follows. Section II presents the system model. In Section III, the power minimization problem of a sensor node in a non-cognitive radio environment is addressed under different rate constraints together with asymptotic analysis and numerical results.

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Section IV shows the optimal power allocation of a sensor node in a cognitive radio environment under different rate constraints. In Section V, the power minimization problem of a wireless sensor network with a single transmitter and multiple receivers in a cognitive radio environment while satisfying independent peak, independent average, sum of peak and sum of average transmission rate constraints, is addressed along with numerical simulations for each case. Section VI concludes the work.

II. System Model

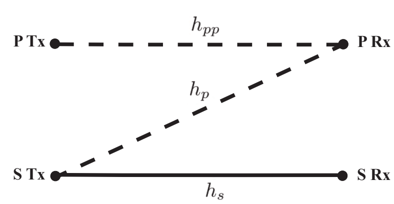

Figure 1: Spectrum sharing wireless sensor in unicast mode.

In this work, two scenarios are considered. The first one is depicted in Fig. 1 where two sensors are communicating through a fading channel (solid line) reflecting the unicast mode. The second scenario is about broadcasting to multiple receivers and is studied in Section V.

In Fig. 1, we consider a spectrum sharing communication system where a secondary sensor node (SN) is communicating with a secondary user receiver in the presence of a primary SN (dashed lines in Fig. 1) under certain interference constraints and the channel gain is denoted by h_s. For the sake of simplicity, interference from the primary user transmitter to the secondary SN receiver is neglected. The channel between the secondary SN transmitter and the primary SN receiver is denoted by h_p. We denote by γ_p and γ_s the modulus squared of the channel gains h_p and h_s, respectively. All fading channels are random variables assumed to be ergodic and

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stationary with continuous probability density functions (PDF) denoted by $f_{\gamma_p}(\gamma_p)$ and $f_{\gamma_s}(\gamma_s)$. For ease of the analysis, we assumed that the transmit SN is perfectly aware of the instantaneous channel gain modulus γ_s and is only aware of the statistics, i.e. PDF, of γ_p. In real life, these assumptions can be addressed by either applying traditional statistical approaches in the fading channel or performing an experiment where some data can be recorded and then apply machine learning or data mining algorithms to characterize the fading channel. Such analysis is out of the scope of this work. Our aim is to minimize the average power $P_{avg} = E_{\gamma_s}[P(\gamma_s)]$, where, $P(\gamma_s)$ is the allocated power when the channel gain is γ_s. We consider four different cases which are denoted as follows:

$$\begin{aligned} &\min_{P} \quad P_{avg} = E_{\gamma_s}[P(\gamma_s)]. \\ C1: &\quad \text{s. t} \quad R(\gamma_s) \geq R_{min}; \\ C2: &\quad \text{s. t} \quad E_{\gamma_s}[R(\gamma_s)] \geq R_{min}; \\ C3: &\quad \text{s. t} \quad \Pr\{P(\gamma_s)\cdot\gamma_p \geq Q_{peak}\} \leq \epsilon; \\ C4: &\quad \text{s. t} \quad P(\gamma_s) \leq P_{max} \end{aligned} \tag{1}$$

where $R(\gamma_s) = \log(1 + P(\gamma_s)\cdot\gamma_s)$, R_{min} is the minimum transmission rate required, Q_{peak} is the interference limit that the primary SN can tolerate from the secondary SN, and ϵ is the threshold for the level of interference allowed. $C1$ represents an instantaneous transmission rate constraint, while $C2$ is an average transmission rate constraint. The motivation of constraint $C3$ is justified by the fact the primary sensor network is communicating under a delay constraint and thus is interested to have the interference, in each (sufficiently long) coherence block, below a certain threshold. Since the secondary transmitter, SN is not aware of the perfect cross-link (CL) CSI, it cannot guarantee such constraint. Instead, it adopts a statistical protection of the primary communication, i.e., $C3$. The constraint $C4$ describes the maximum transmit power denoted P_{max} available at the node.

III. Optimal Power Allocation for a Single Sensor Node

A. Preliminary: System under Instantaneous Transmission Rate Constraint ($C1$)

Consider the minimization of the average power $P_{avg} = E_{\gamma_s}[P(\gamma_s)]$ subject to an instantaneous transmission rate constraint given by $R(\gamma_s) \geq R_{min}$. This optimization problem can be rewritten as

$$\begin{aligned} &\min_{P} P_{avg} = E_{\gamma_s}[P(\gamma_s)]. \\ &\text{s. t.} \;\; R(\gamma_s) \geq R_{min}. \end{aligned} \tag{2}$$

In order to solve this problem, we need to satisfy the constraint, i.e. $R(\gamma_s) = \log(1 + P(\gamma_s) \cdot \gamma_s) \geq R_{min}$. Solving for $P(\gamma_s)$ gives

$$P(\gamma_s) \geq \frac{e^{R_{min}} - 1}{\gamma_s}. \tag{3}$$

Therefore the minimum power required under an instantaneous transmission rate constraint is given by

$$P(\gamma_s) = \frac{e^{R_{min}} - 1}{\gamma_s} \quad \text{for } \gamma_s > 0; \tag{4}$$

Note that equation (4) is a function of the system parameter R_{min} as well as the channel gain γ_s.

B. System under Average Transmission Rate Constraint ($C2$)

Consider the minimization of the average power $P_{avg} = E_{\gamma_s}[P(\gamma_s)]$ subject to an average transmission rate constraint given by $E_{\gamma_s}[R(\gamma_s)] \geq R_{min}$. This optimization problem can be rewritten as

$$\begin{aligned} &\min_{P} P_{avg} = E_{\gamma_s}[P(\gamma_s)]. \\ &\text{s. t.} \;\; E_{\gamma_s}[R(\gamma_s)] \geq R_{min}. \end{aligned} \tag{5}$$

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1) Optimal Solution: To solve this problem, we can write the Lagrangian as follows,

$$\begin{aligned}\mathcal{L} = &\int_0^{\infty} P(\gamma_s) f_{\gamma_s}(\gamma_s)\, d\gamma_s \\ &- \lambda \left[\int_0^{\infty} \log\left(1 + P(\gamma_s) \cdot \gamma_s\right) f_{\gamma_s}(\gamma_s)\, d\gamma_s - R_{min} \right].\end{aligned} \tag{6}$$

Since both the objective function and the constraint (with the minus sign) are convex functions, then the solution that we obtain is a global minimum. With this argument, we can differentiate the Lagrangian with respect to $P(\gamma_s)$ using the method of variations to obtain the necessary and sufficient condition: $1 - \lambda \frac{\gamma_s}{1+P(\gamma_s)\cdot\gamma_s} = 0$ which means that

$$P(\gamma_s) = \left[\lambda - \frac{1}{\gamma_s}\right]^{+} = \begin{cases} \lambda - \frac{1}{\gamma_s} & \text{for } \lambda > \frac{1}{\gamma_s}, \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

where $[\cdot]^{+} = \max(0, \cdot)$. The power profile (7) is the well-known water-filling solution which arises in many information theoretical problems such as power allocation over n parallel channels [23]. From (7), we can see that a transmission occurs only when $\lambda > \frac{1}{\gamma_s}$. From the KKT complementary slackness condition [23], the constraint is achieved with equality therefore, substituting (7) into the constraint ($C2$), with equality, we have:

$$E_{\gamma_s \geq \frac{1}{\lambda}}\left[\log(\lambda \cdot \gamma_s)\right] = R_{min} \tag{8}$$

which gives the following implicit expression for λ by expanding the $\log$ in (8):

$$\lambda = \exp\left(\frac{R_{min} - \int_{\frac{1}{\lambda}}^{\infty} \log(\gamma_s) f_{\gamma_s}(\gamma_s)\, d\gamma_s}{\int_{\frac{1}{\lambda}}^{\infty} f_{\gamma_s}(\gamma_s)\, d\gamma_s} \right), \tag{9}$$

which can be solved numerically. Note that the power profile (7) involves solving the transcendental equation (9) only once for all channel realizations γ_s. Now our aim is to obtain a closed-form solution that only depends on the system parameters. Let us consider a variable μ such that $\mu = \frac{1}{\lambda}$. The power profile in (7) can be rewritten as

$$P(\gamma_s) = \left[\frac{1}{\mu} - \frac{1}{\gamma_s}\right]^{+}. \tag{10}$$

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Substituting (10) into the constraint $(C2)$ leads to $G(\mu) = R_{min}$ where the function $G(x)$ is defined by

$$G(x) = -\log(x)\left[1 - F_{\gamma_s}(x)\right] + \int_x^{\infty} \log(\gamma_s) f_{\gamma_s}(\gamma_s)\, d\gamma_s. \tag{11}$$

Provided that the integral exists, the function $G(x)$ can be shown to be monotonic decreasing function (see Appendix A), hence, invertible on $(0, \infty)$. Thus, the optimized power is given by

$$P(\gamma_s) = \left[\frac{1}{G^{-1}(R_{min})} - \frac{1}{\gamma_s}\right]^+, \tag{12}$$

where the presented optimal power does not, anymore, depend on λ.

2) On / Off Suboptimal Solution: The optimal solution presented in (12) would be difficult to implement because of all the numerical computations required in order to obtain the optimal power P. Consequently, we propose a simple suboptimal solution based on the On / Off power control. In this On / Off power control, there are two power levels either P_o when there is transmission or zero otherwise, i.e.,

$$P(\gamma_s) = P_o \quad \text{for } \gamma_s \geq 0. \tag{13}$$

where P_o is a constant power with regards to γ_s, obtained by solving

$$\int_0^{\infty} \log(1 + P_o \cdot \gamma_s) f_{\gamma_s}(\gamma_s)\, d\gamma_s = R_{min} \tag{14}$$

Interestingly, it is important to see that in this case, the CSIT is not needed. The objective function in this case is $P_{avg} = P_o$, whereas with the optimal power policy (12), the average power constraint is $P_{avg} = \frac{1-F_{\gamma_s}(\mu)}{\mu} - \frac{\int_\mu^\infty f(\gamma_s)\ d\gamma_s}{\gamma_s}$. In order to assess how suboptimal the proposed uniform power strategy is, we consider, next, Rayleigh fading channels.

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3) Application for Rayleigh Fading Channels:

- Optimal Scheme

Let us consider a Rayleigh fading channel, i.e. $f_{\gamma_s}(\gamma_s) = e^{-\gamma_s}$ for $\gamma_s \geq 0$. The optimal power is given by (12) with

$$G(x) = E_1(x) \text{ for } \gamma_s \geq 0 \tag{15}$$

where $E_1(\cdot)$ is the exponential integral function of first-order defined by $E_1(x) = \int_1^{\infty} \frac{e^{-xt}}{t} dt$, [24]. It is important to note that when $R_{min} \to 0$, (7) can be rewritten as (see Appendix B)

$$P(\gamma_s) \simeq \left[\frac{1}{W\left(\frac{1}{R_{min}}\right)} - \frac{1}{\gamma_s} \right]^+, \tag{16}$$

where $W(\cdot)$ is the principle branch of the Lambert W-function [24]. The asymptotic expressions (16) is obviously simpler than (12) in the sense that they do not require any inverse function computation. Note that, according to (16) and property of the Lambert W function, as $R_{min} \to \infty$, $P(\gamma_s) \to 0$. That is, the transmission power can be arbitrarily small without constraint on minimum rate. Furthermore, the Lambert W-function can be efficiently approximated by a sum of log, e.g., [25] and reference therein. This is appealing especially in regard to the limited computation capability of sensor networks.

- On / Off Suboptimal Scheme

In this case, P_o in (13) is obtained by solving the equation

$$e^{\frac{1}{P_o}} E_1\left(\frac{1}{P_o}\right) = R_{min}. \tag{17}$$

It is important to note that when $R_{min} \to 0$, (17) can be rewritten as (see Appendix C)

$$P_o \simeq R_{min}. \tag{18}$$

Note that (18) can be regarded as explicit expression of the on-off suboptimal power policy at low rate regimes. These expressions are, also, simple to evaluate as a function of R_{min}.

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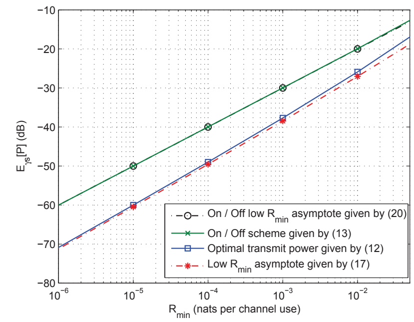

Figure 2: Performance comparison between optimal and On/Off suboptimal schemes for low R_{min} values under an average rate constraint.

4) Comparison between the Optimal and the On / Off Suboptimal Schemes : As shown in Fig. 2, is a performance comparison of the average transmit power of a sensor node as a function of R_{min} for a Rayleigh fading channel between the optimal scheme, the On/Off scheme and the asymptotical approximations for low values of R_{min}. Note also that in this regime of R_{min}, our approximation given by equation (16) matches perfectly with the optimal solution (red curve in Fig. 2). We also have a perfect match between the On/Off scheme and its low R_{min} approximation given by (18). As shown by Fig. 2, at low R_{min} CSI becomes valuable and our proposed scheme presents a noticeable gap to the optimal power profile. However, we expect this gap to decrease if the proposed power scheme adapts with respect to a threshold, say τ. That is, the transmitter uses a constant power if the channel gain is above a certain threshold τ and remains silent otherwise (e.g., in (13), τ was taken equal to 0). The threshold τ can be optimized for in order to get the best performance [26]. This approach has been widely studied in the context of channel capacity at low power regime that is basically the dual of the problem we are dealing with in this paper. For a discussion on the optimally of the on-off power scheme, please refer to the seminal work [27].

IV. Optimal Power Allocation for a Sensor Node in a Cognitive Radio Environment

A. Preliminary: System under Instantaneous Transmission Rate Constraint ($C1 \bigcup C3$)

Consider the minimization of the average power $P_{avg} = E_{\gamma_s}\left[P(\gamma_s)\right]$ subject to an instantaneous transmission rate constraint given by ($C1$), and an interference constraint given by ($C1 \bigcup C3$). This optimization problem can be rewritten as

$$\begin{aligned} \min_{P}\; & P_{avg} = E_{\gamma_s}\left[P(\gamma_s)\right] \\ \text{s. t.}\;\; & R(\gamma_s) \geq R_{min} \\ & \Pr\left\{P(\gamma_s)\gamma_p \geq Q_{peak}\right\} \leq \epsilon. \end{aligned} \tag{19}$$

For this case, the purpose of the interference constraint is to reduce the interference power detected at the primary receiver. Note that the interference constraint ($C1 \bigcup C3$) is equivalent to

$$P(\gamma_s) \leq P_{\gamma_p}(\epsilon)\,, \tag{20}$$

where $P_{\gamma_p}(\epsilon) = \frac{Q_{peak}}{F_{\gamma_p}^{-1}(1-\epsilon)}$ and $F_{\gamma_p}^{-1}$ is the inverse cumulative distribution function of γ_p. Now, we have an optimization problem with two opponent peak constraints: ($C1$) and (20). The optimal power corresponding to the subproblem, with the constraint ($C1$), is $\frac{e^{R_{min}}-1}{\gamma_s}$. Hence, by considering (20), the optimal power should satisfy $\frac{e^{R_{min}}-1}{\gamma_s} \leq P(\gamma_s) \leq P_{\gamma_p}(\epsilon)$. Consequently, the complete problem is given by

$$P(\gamma_s) = \begin{cases} \frac{e^{R_{min}}-1}{\gamma_s} & \gamma_s \geq \frac{e^{R_{min}}-1}{P_{\gamma_p}(\epsilon)} \\ \text{no solution} & \gamma_s < \frac{e^{R_{min}}-1}{P_{\gamma_p}(\epsilon)}. \end{cases} \tag{21}$$

From (21), we can see that when $\gamma_s < \frac{e^{R_{min}}-1}{P_{\gamma_p}(\epsilon)}$ there is no solution since it is not possible to satisfy both constraints at the same time. This situation is referred to *"outage"*. We can then

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control the *outage* occurrence by setting $\Pr\left\{\gamma_s < \frac{e^{R_{min}}-1}{P_{\gamma_p}(\epsilon)}\right\} < \eta$, where η is the threshold for the *outage* occurrence allowed. This new condition is achieved as long as

$$P_{\gamma_p}(\epsilon) > \frac{e^{R_{min}} - 1}{F_{\gamma_s}^{-1}(\eta)}. \tag{22}$$

B. System under Average Transmission Rate Constraint ($C4$)

Consider the minimization of the average power $P_{avg} = E_{\gamma_s}[P(\gamma_s)]$, subject to an average transmission rate constraint given by $E_{\gamma_s}[R(\gamma_s)] > R_{min}$, and an interference constraint given by $\Pr\{P(\gamma_s) \cdot \gamma_p \geq Q_{peak}\} \leq \epsilon$. This optimization problem can be rewritten as

$$\begin{aligned} &\min_P P_{avg} = E_{\gamma_s}[P(\gamma_s)] \\ &\text{s. t.} \;\; E_{\gamma_s}[R(\gamma_s)] > R_{min} \\ &\text{and } P(\gamma_s) \leq P_{\gamma_p}(\epsilon) \end{aligned} \tag{23}$$

1) Optimal Solution: To solve this problem, we can write the Lagrangian as

$$\begin{aligned} \mathcal{L} = &\int_0^\infty P(\gamma_s) f_{\gamma_s}(\gamma_s)\; d\gamma_s \\ &- \lambda\left[\int_0^\infty \log\left(1 + P(\gamma_s)\cdot\gamma_s\right)\; f_{\gamma_s}(\gamma_s)\, d\gamma_s - R_{min}\right] \\ &+ \mu\left[P - P_{\gamma_p}(\epsilon)\right] \end{aligned} \tag{24}$$

where λ and μ are the positive Lagrange multipliers. Since both the objective function and the constraints are convex functions, then the solution that we obtain is a global minimum. Differentiating the Lagrangian with respect to $P(\gamma_s)$, we get $1 + \mu - \lambda\frac{\gamma_s}{1+P(\gamma_s)\cdot\gamma_s} = 0$. By using, once again, the method of variations, we obtain

$$P(\gamma_s) = \left[\frac{1}{x} - \frac{1}{\gamma_s}\right]^+, \tag{25}$$

where $x = \frac{1+\mu}{\lambda}$. From the KKT conditions we know that:

- $\mu = 0$ if $\lambda - \frac{1}{\gamma_s} < P_{\gamma_p}(\epsilon)$

- $\mu > 0$ if $\frac{\lambda}{1+\mu} - \frac{1}{\gamma_s} = P_{\gamma_p}(\epsilon)$

Therefore, the optimum power profile can be derived as: (see Appendix D):

- if $E_{\gamma_s}\left[\log\left(1 + P_{\gamma_p}(\epsilon)\cdot\gamma_s\right)\right] - R_{min} < 0$, there is no solution (*outage* situation),
- if $R_{min} \le G\left(P_{\gamma_p}(\epsilon)^{-1}\right)$

$$P(\gamma_s) = \min\left\{\left[\frac{1}{G^{-1}(R_{min})} - \frac{1}{\gamma_s}\right]^+, P_{\gamma_p}(\epsilon)\right\} \tag{26}$$

where $G(x)$ is defined by (11).

- if $R_{min} > G\left(P_{\gamma_p}(\epsilon)^{-1}\right)$

$$P(\gamma_s) = \min\left\{\left[\frac{1}{H^{-1}(R_{min})} - \frac{1}{\gamma_s}\right]^+, P_{\gamma_p}(\epsilon)\right\} \tag{27}$$

where

$$\begin{aligned} H(x) = G(x) + \\ \int_{\frac{1}{\frac{1}{x} - P_{\gamma_p}(\epsilon)}}^{\infty} \log\left(\frac{1 + P_{\gamma_p}(\epsilon)\gamma_s}{\gamma_s}x\right) f_{\gamma_s}(\gamma_s)\, d\gamma_s \end{aligned} \tag{28}$$

2) On / Off Suboptimal Solution: The optimal solution presented in (25) would be difficult to implement due to the numerical computations required to obtain the optimal power P. Instead, we propose, a simple suboptimal solution based on the On / Off power control, i.e.

$$P(\gamma_s) = \begin{cases} P_o & \gamma_s \ge 0 \\ 0 & \text{otherwise.} \end{cases} \tag{29}$$

The procedure to solve this problem is the following: first we consider only the average rate constraint and we compute a power P_o by solving (14). We compare this power with the interference constraint, i.e. $P(\gamma_s) \le P_{\gamma_p}(\epsilon)$. Therefore, we can have one of two possible cases:

- if $P_o \le P_{\gamma_p}(\epsilon)$ then the power policy is given by (29),

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- if $P_o > P_{\gamma_p}(\epsilon)$ then, both constraints cannot be satisfied at the same time and there is no solution.

3) Application for Nakagami-m and Rayleigh Fading Channels: Let us consider a Nakagami-m fading channel, i.e. $f_\gamma(\gamma) = \frac{m^m}{\Gamma(m)}(\gamma)^{m-1}e^{-m\gamma}$ where $m \geq \frac{1}{2}$ and $\Gamma(\cdot)$ is the Gamma function. The optimal power profile is given by (26) and (27), where

$$G(x) = \frac{1}{\Gamma(m)} G_{2\,0}^{3\,0}\left(mx \middle| \begin{matrix} 1 & 1 & \\ 0 & 0 & m \end{matrix} \right), \tag{30}$$

where $G_{p\,q}^{i\,j}\left(z \middle| \begin{matrix} a_1 \ldots a_p \\ b_1 \ldots b_q \end{matrix} \right)$ is the Meijer G-function [24], which is a general function used to group most of the important special functions (polynomials, logarithms, Bessel functions, etc.) in mathematical analysis and physics. For the case of $m = 1$, Nakagami fading channel reduces to a Rayleigh fading channel where $G(x)$ is defined in (15). Table I summarizes the equations involved in a Nakagami-m and Rayleigh fading channel. In this table, the function $\Gamma(s,x)$ is the incomplete Gamma function and it is defined as $\int_x^\infty t^{s-1}e^{-t}dt$.

	$G(x)$	$F_{\gamma_p}^{-1}(x)$
Rayleigh	$E_1(x)$	$-\log(1-x)$
Nakagami-m	$\frac{1}{\Gamma(m)} G_{2\,0}^{3\,0}\left(mx \middle| \begin{matrix} 1\,1 \\ 0\,0\,m \end{matrix} \right)$	$\left[1 - \frac{\Gamma(m,mx)}{\Gamma(m)}\right]^{-1}$

Table I: Expressions of $G(x)$ and $F_{\gamma_p}^{-1}(x)$ for Nakagami-m and Rayleigh fading channel.

4) Comparison between the Theoretical and the Numerical Solutions: The derived results in (26) and (27) have been used in order to display Fig. 3 and Fig. 4. Figure 3 depicts the instantaneous transmit power of a sensor node as a function of γ_s for a Nakagami-m fading

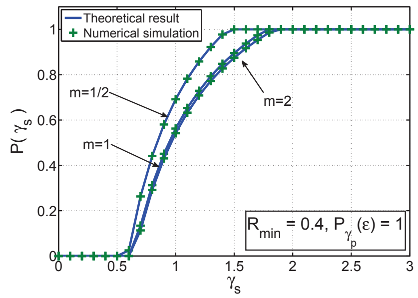

Figure 3: Power profile when $R_{min} > G\left(P_{\gamma_p}(\epsilon)^{-1}\right)$.

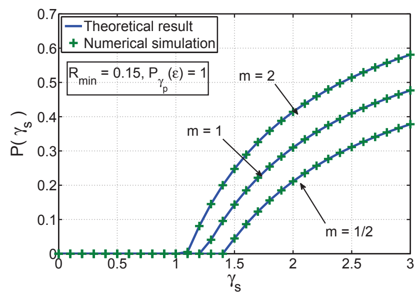

Figure 4: Power profile when $R_{min} \leq G\left(P_{\gamma_p}(\epsilon)^{-1}\right)$.

channel with a parameter $m = \frac{1}{2}$, 1 and 2. For this case, $R_{min} \leq G\left(P_{\gamma_p}(\epsilon)^{-1}\right)$ and our numerical simulations[1] match with our theoretical result given by (26) for the three different values of the parameter m.

Figure 4 depicts the transmit power of a sensor node as a function of γ_s for a Nakagami-m fading channel. Although we computed a power profile for a parameter $m = \frac{1}{2}$, 1 and 2, there is no significant difference between the shape of the profiles. For this case, $R_{min} > G\left(P_{\gamma_p}(\epsilon)^{-1}\right)$ and our numerical simulations match with our theoretical result given by (27) for the three different values of the parameter m. Note that the transmit power is always limited by P_{max}. Hence, when we consider the constraint ($C4$) in the previous cases, the optimal transmit power, P_{tx}, is given

[1]Numerical simulations solve the optimization problem without considering the explicit expressions using the *fmincon* tool in MATLAB.

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by

$$P_{tx} = \min\{P^*, P_{max}\}, \tag{31}$$

were P^* is the obtained power expressions in (3), (12), (13), (16), (18), (21), (26) and (27).

In Fig. 5, we plot the probability given by $\Pr\left\{P_{\gamma_p}(\epsilon)\,\gamma_p \geq Q_{peak}\right\}$ as function of ϵ where $P_{\gamma_p}(\epsilon) = \frac{Q_{peak}}{F_{\gamma_p}^{-1}(1-\epsilon)}$ and γ_p is randomly generated. The averaging is performed for N_{sim} realizations of γ_p. The probability is computed by averaging the realizations where the constraint is satisfied over all the generated realizations. As can be seen in Fig. 5, the constraint is perfectly respected for high a number of realization, i.e. $N_{sim} = 1.000.000$. However, the probability presents small fluctuations around ϵ for $N_{sim} = 10.000$.

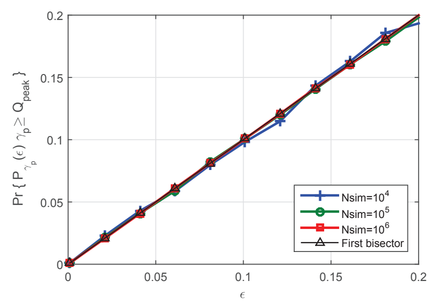

Figure 5: The probability that the received interference at the primary user is above a threshold $Q_{peak} = 7$dB vs. ϵ.

V. Power Allocation of a Wireless Sensor Network with Multiple Secondary Receivers

In this Section, we consider a spectrum sharing communication system where a secondary SN is communicating with multiple secondary user receivers. An example of multiple secondary receivers is when a cognitive node broadcasts a message to other cognitive nodes. The communication occurs in the presence of a primary SN under independent peak, independent average, sum

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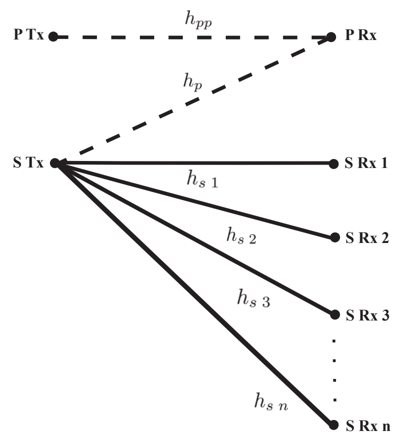

Figure 6: Wireless sensor link in a spectrum sharing environment in a broadcast mode.

of peak, sum of average, product of peak and product of average transmission rate constraints, as depicted in Fig. 6. These constraints are considered separately, one at the time. The channel between the secondary SN transmitter and the primary SN receiver is denoted by h_p while the channel between the secondary transmitter and each of the secondary receivers is denoted by $h_{s\,i}$ for every $i \in \{1, ..., n\}$. All fading channels are assumed to be ergodic and stationary with continuous probability density function. The signal received by each secondary user is given by:

$$r_{s\,i}(l) = h_{s\,i}(l)\, s(l) + w_{s\,i}(l) \tag{32}$$

where l is the discrete-time index, $s(l)$ is the channel input, $h_{s\,i}(l)$ is the channel gain and $w_{s\,i}(l)$ is a zero-mean circularly symmetric complex white Gaussian noise with spectral density N_o and is independent of $h_{s\,i}(l)$. We denote by $\gamma_{s\,i}$ the squared modulus of $h_{s\,i}$ $\forall i \in \{1, ..., n\}$. Our aim is to minimize the power $P(\mathbf{\Gamma_s}) = P \quad \forall i \in \{1, \ldots, n\}$, where $\mathbf{\Gamma_s} = \begin{bmatrix} \gamma_{s\,1}, & \gamma_{s\,2}, & \ldots, & \gamma_{s\,n} \end{bmatrix}$,

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considering four different cases, each corresponding to specific constraints, described as follows:

$$\begin{aligned} &\min_{P} \quad P_{avg} = E_{\mathbf{\Gamma_s}}\left[P(\mathbf{\Gamma_s})\right]. \\ C.1: \quad \text{s. t.} \quad & R_i\left(\mathbf{\Gamma_s}\right) \geq R_{min} \bigcup C.5 \quad \forall i \in \{1,\ldots,n\} \\ C.2: \quad \text{s. t.} \quad & E_{\mathbf{\Gamma_s}}\left[R_i\left(\mathbf{\Gamma_s}\right)\right] \geq R_{min} \bigcup C.5 \quad \forall i \in \{1,\ldots,n\} \\ C.3: \quad \text{s. t.} \quad & \sum_{i=1}^{n} R_i\left(\mathbf{\Gamma_s}\right) \geq R_{min} \bigcup C.5 \\ C.4: \quad \text{s. t.} \quad & E_{\mathbf{\Gamma_s}}\left[\sum_{i=1}^{n} R_i\left(\mathbf{\Gamma_s}\right)\right] \geq R_{min} \bigcup C.5 \\ C.5: \quad \text{s. t.} \quad & \Pr\left\{P\left(\mathbf{\Gamma_s}\right)\gamma_p \geq Q_{peak}\right\} \leq \epsilon, \end{aligned} \tag{33}$$

where R_{min}, Q_{peak}, ϵ, are defined in (1), and $R_i\left(\mathbf{\Gamma_s}\right) = \alpha_i \log\left(1 + P(\mathbf{\Gamma_s})\gamma_{s\,i}\right)$. The set $R_i\left(\mathbf{\Gamma_s}\right)$ $\forall i \in \{1,\ldots,n\}$ represents a set of achievable rates of the SISO Gaussian broadcast channel using a time sharing argument by the source SN that dedicates α_i percent of the available time resource to each sensor destination i in a time division duplex (TDD) manner. Setting $\sum_i \alpha_i = 1$ ensures that these rates fall inside the capacity region of the broadcast channel at hand. While this time sharing strategy is not optimal, it guarantees i) a non-zero rate to each sensor destination, ii) avoiding the usage of the optimal, yet complex, superposition coding at the source and successive interference cancellation at the destinations. Recall that the last property is of important interest especially in wireless sensor networks where it is desirable to maintain the complexity at a basic level. Without loss of generality, we set $\alpha_i = \frac{1}{n}$, for all i, and consider that n is absorbed by R_{min}. We then proceed similarly for $C.2$ and for the sum rate constraints $C.3$ and $C.4$. Recall that $C.1$ represents an independent peak transmission rate constraint, while $C.2$ represents an independent average transmission rate constraint. $C.3$ and $C.4$ are a sum of peak and a sum of average transmission rate constraints, respectively. All the constraints $C.1 - C.4$ have an additional outage constraint, $C.5$. The purpose of the constraint $C.5$ is to reduce the interference power detected at the primary SN receiver. It is easy to show that this constraint can be rewritten

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as $P(\mathbf{\Gamma_s}) \leq P_{\gamma_p}(\epsilon)$, where $P_{\gamma_p}(\epsilon)$ is defined after (20).

A. Independent Peak Transmission Rate Constraint (C.1)

Considering that all the channels are independent, the constraint $R_i(\mathbf{\Gamma_s}) \geq R_{min} \quad \forall i \in \{1,...,n\}$ is satisfied as long as $P(\mathbf{\Gamma_s}) = \max\limits_{i\in\{1,n\}} \left\{ \frac{e^{n \cdot R_{min}-1}}{\gamma_{s\ i}} \right\}$. Taking into account the interference constraint $C.5$, leads to the following 2 cases:

- if $\max\limits_{i\in\{1,...,n\}} \left\{ \frac{e^{n \cdot R_{min}-1}}{\gamma_{s\ i}} \right\} \leq P_{\gamma_p}(\epsilon)$ then, $P(\mathbf{\Gamma_s}) = \max\limits_{i\in\{1,...,n\}} \left\{ \frac{e^{n \cdot R_{min}-1}}{\gamma_{s\ i}} \right\}$,
- if $\max\limits_{i\in\{1,...,n\}} \left\{ \frac{e^{n \cdot R_{min}-1}}{\gamma_{s\ i}} \right\} > P_{\gamma_p}(\epsilon)$ then, there is no solution. We refer to this situation as an *outage*.

B. Independent Average Transmission Rate Constraint (C.2)

Recall that the optimum power profile for a single transmitter-single receiver channel in a cognitive radio environment with an average transmission rate constraint is given in Section IV.B.2 as follows:

- if $E_{\gamma_s}\left[\log\left(1 + P_{\gamma_p}(\epsilon) \cdot \gamma_s\right)\right] - R_{min} < 0$, then there is no solution (*outage* situation),
- if $R_{min} \leq G\left(P_{\gamma_p}(\epsilon)^{-1}\right)$, then $P(\gamma_s)$ is given by (26),
- if $R_{min} > G\left(P_{\gamma_p}(\epsilon)^{-1}\right)$, then $P(\gamma_s)$ is given by (27).

Assuming that the distribution of the fading channels is independent and identically distributed (i.i.d.), and using the solution given by (26) and (27) then, we have the following cases:

- if $E_{\mathbf{\Gamma_s}}\left[\log\left(1 + P_{\gamma_p}(\epsilon) \cdot \gamma_{s\ i}\right)\right] - n \cdot R_{min} < 0 \ \forall i \in \{1,...,n\}$, there is no solution (*outage* situation),
- if $n \cdot R_{min} \leq G\left(P_{\gamma_p}(\epsilon)^{-1}\right)$ for every channel $i \in \{1,...,n\}$ then,

$$P(\mathbf{\Gamma_s}) = \min\left\{ \max_{i\in\{1,...,n\}} \left[\frac{1}{G^{-1}(R_{min})} - \frac{1}{\gamma_{s\ i}}\right]^+, \ P_{\gamma_p}(\epsilon) \right\}, \tag{34}$$

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where $G(x)$ is defined in (11).

- if $n \cdot R_{min} > G\left(P_{\gamma_p}(\epsilon)^{-1}\right)$ for every channel $i \in \{1, ..., n\}$ then,

$$P(\mathbf{\Gamma_s}) = \min\left\{ \max_{i \in \{1,...,n\}} \left[\frac{1}{H^{-1}(n \cdot R_{min})} - \frac{1}{\gamma_{s\,i}} \right]^+ ,\ P_{\gamma_p}(\epsilon) \right\} \tag{35}$$

where $H(x)$ is defined by (28).

C. Sum of Peak Transmission Rate Constraint (C.3)

We can notice that the constraint $\sum_{i=1}^{n} R_i(\mathbf{\Gamma_s}) \geq R_{min}$ is satisfied as long as the polynomial

$$\prod_{i=1}^{n} \left(1 + P(\mathbf{\Gamma_s}) \cdot \gamma_{s\,i}\right) - e^{n \cdot R_{min}} \geq 0 \tag{36}$$

is valid. Expanding (36) gives a polynomial of the form $\sum_{i=0}^{n} a_i P^i (\gamma_{s\,1}, \gamma_{s\,2}, \ldots, \gamma_{s\,n})$ where all its coefficients a_i are larger than zero, except the independent coefficient $a_0 = 1 - e^{n \cdot R_{min}}$ which is lower than zero. It is important to mention that the rest of the coefficients a_i follow the following rule: each a_i is equivalent to the sum of the product of the $\binom{n}{i}$ combinations of the n channel gains taken i at a time. For example, if $n = 3$, the coefficients are: $a_0 = 1 - e^{n \cdot R_{min}}$, $a_1 = \gamma_{s\,1} + \gamma_{s\,2} + \gamma_{s\,3}$, $a_2 = \gamma_{s\,1}\gamma_{s\,2} + \gamma_{s\,1}\gamma_{s\,3} + \gamma_{s\,2}\gamma_{s\,3}$ and $a_3 = \gamma_{s\,1}\gamma_{s\,2}\gamma_{s\,3}$. According with Descartes' rule of signs, from the n roots of (36) only one is real and positive then, we can apply Newton's method to find it. The advantage of applying Newton's method is that we are able to always find the root P^* that we are interested in, independently of the initial guess for the algorithm. Taking into account the interference constraint $C.5$, leads to the following 2 cases:

- if $P^* \leq P_{\gamma_p}(\epsilon)$ then, $P(\mathbf{\Gamma_s}) = P^*$,
- if $P^* > P_{\gamma_p}(\epsilon)$ then, there is no solution.

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D. Sum of Average Transmission Rate Constraint (C.4)

Assuming that the fading channels are i.i.d. then, the transmission rate constraint can be rewritten as $E_{\gamma_s}\left[R\left(\gamma_s\right)\right] \geq R_{min}$, which results in a single integration, over one fading channel, multiplied by the number of receivers. Making use of the solution given by (26) and (27) then, we have the following cases:

- if $E_{\gamma_s}\left[\log\left(1 + P_{\gamma_p}\left(\epsilon\right) \cdot \gamma_s\right)\right] - R_{min} < 0$, there is no solution (*outage* situation),
- if $R_{min} \leq G\left(P_{\gamma_p}\left(\epsilon\right)^{-1}\right)$ for every channel $i \in \{1, ..., n\}$ then,

$$P\left(\mathbf{\Gamma_s}\right) = \min\left\{\max_{i\in\{1,\dots,n\}}\left[\frac{1}{G^{-1}\left(R_{min}\right)} - \frac{1}{\gamma_{s\,i}}\right]^{+},\ P_{\gamma_p}\left(\epsilon\right)\right\} \tag{37}$$

where $G\left(x\right)$ is defined by (11).

- if $R_{min} > G\left(P_{\gamma_p}\left(\epsilon\right)^{-1}\right)$ for every channel $i \in \{1, ..., n\}$ then,

$$P\left(\mathbf{\Gamma_s}\right) = \min\left\{\max_{i\in\{1,\dots,n\}}\left[\frac{1}{H^{-1}\left(R_{min}\right)} - \frac{1}{\gamma_{s\,i}}\right]^{+},\ P_{\gamma_p}\left(\epsilon\right)\right\} \tag{38}$$

where $H\left(x\right)$ is defined by (28). Note that the solution (37) and (38) is less restrictive than the one obtained for the independent average transmission rate constraint, i.e., (34) and (35), since the solution depends on the rate R_{min} instead of $n \cdot R_{min}$.

E. Numerical Results

Simulations for the single transmitter and multiple receivers case were performed considering Rayleigh fading channels and a value of $P_{\gamma_p}\left(\epsilon\right) = 2$ dB. Fig. 7 and 8 show the results for the independent peak transmission rate constraint. In addition, the outage zone corresponds to the rate-power region where the related optimization problem is not feasible, i.e., admits no solution. As shown in Fig. 7, for low values of R_{min}, the average power per sensor node is lower when

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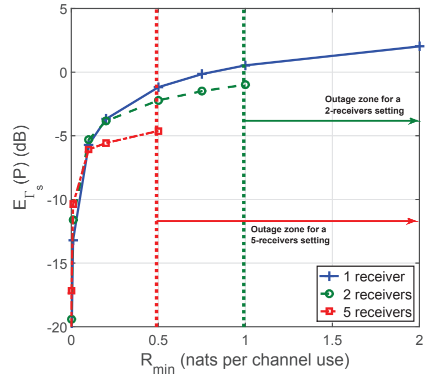

Figure 7: Average power consumed, vs. the rate constraint, for different number of receivers under an independent peak transmission rate constraint.

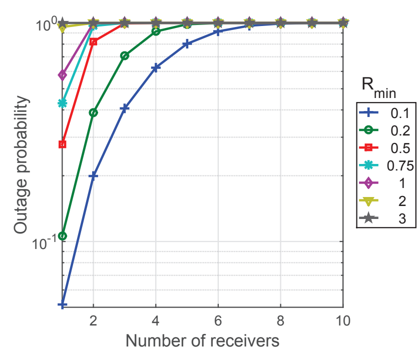

Figure 8: Outage probability as a function of the number of receivers under an independent peak transmission rate constraint.

there are a small number of receivers. Furthermore, for high values of R_{min}, as the number of receivers is increased, the average power per sensor node is reduced, but this also comes with the fact that it is not possible to achieve higher values of R_{min}. This observation makes sense since our power depends on the most demanding sensor, and as the number of receivers increases, the probability of an *outage* increases as well. This fact is shown in Fig. 8.

In Fig. 9, we plot the average rate given by $E_{\mathbf{\Gamma_s}}\left[R_i\left(\mathbf{\Gamma_s}\right)\right]$ as function of R_{min} where $P^*(\mathbf{\Gamma_s})$

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is given by (34) and (35) and the vector $\mathbf{\Gamma_s}$ is randomly generated. The averaging is performed for 100.000 realizations of $\mathbf{\Gamma_s}$. We show in Fig. 9 that the constraint is respected for all values of R_{min} given the different number of receivers. In addition, we notice that as the number of receivers increase, the corresponding rate becomes higher than R_{min} since the transmit power is adapted with worst channel.

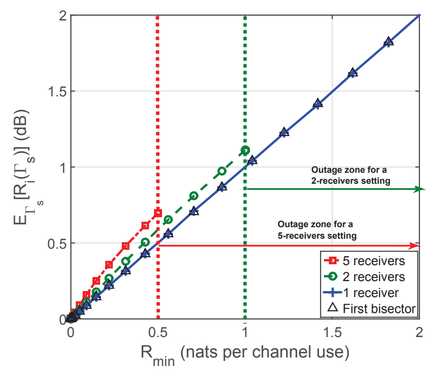

Figure 9: The independent average transmission rate vs. R_{min}, for different number of receivers under an independent peak transmission rate constraint.

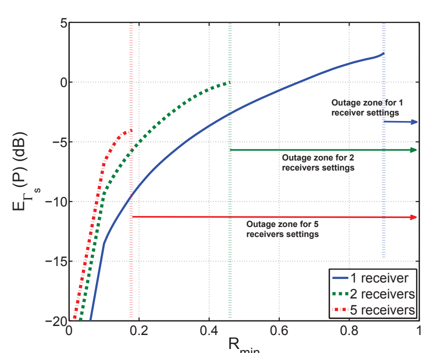

Figure 10: Average power consumed, vs. the rate constraint, for different number of receivers under an independent average transmission rate constraint.

Fig. 10 shows the average power per sensor node with respect to R_{min} when $P_{\gamma_p}(\epsilon) = 2$ for

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an independent average transmission rate constraint. As it is shown in the figure, higher values of R_{min} are achieved with a small number of receivers. Furthermore, with a large number of receivers, it is possible to achieve lower values of R_{min} for the same average power.

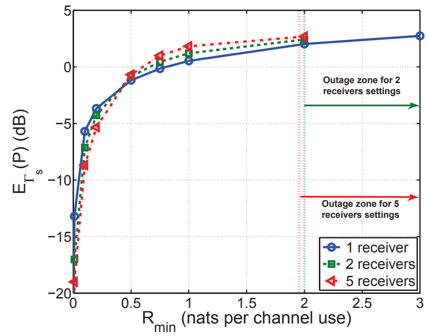

Figure 11: Average power consumed, vs. the rate constraint, for different number of receivers under a sum of peak transmission rate constraint.

- Fig. 11 and 12 show the results for the sum of peak transmission rate constraint. As it is shown in Fig. 11, for low values of R_{min}, the average power per sensor node is lower when there are a large number of receivers. Furthermore, for high values of R_{min}, as the number of receivers is increased, the average power per sensor node is increased as well, but this also comes with the fact that it is not possible to achieve higher values of R_{min}. In this scenario, the sensor nodes are working cooperatively in order to satisfy the sum of peak transmission rate constraint; therefore, as the number of receivers increases, the probability of an *outage* is reduced (as shown in Fig. 12).
- Fig. 13 shows the average power per sensor node with respect to R_{min} when $P_{\gamma_p}(\epsilon) = 2$ for a sum of average transmission rate constraint. As it is shown in the figure, as the number of receivers is increased, the average power per sensor node is reduced.

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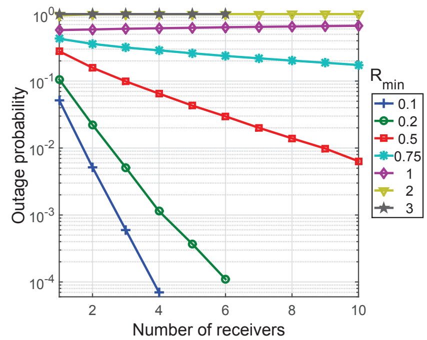

Figure 12: Outage probability as a function of the number of receivers under an independent peak transmission rate constraint.

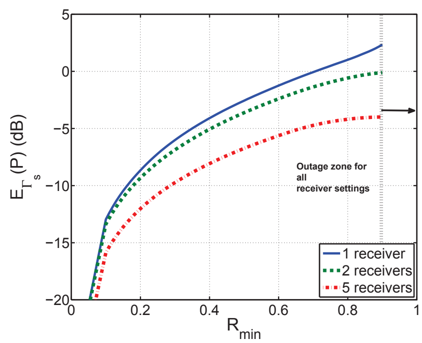

Figure 13: Average consumed power vs. the rate constraint, for different number of receivers under a sum of average transmission rate constraint.

F. Suboptimal Scheme for Independent Peak Transmission Rate Constraint (C.1)

Due to high outage probability for the independent peak transmission rate constraint, we propose a suboptimal solution. The idea is the following: if the power of the most demanding sensor is violating the outage constraint, then we ignore this sensor and look for the next most demanding one. We repeat this task until we find the sensor with the most demanding power that is satisfying all the constraints. If such sensor does not exist, we declare an *outage* situation and no information is sent. In order to implement this, a communication protocol should be established that can monitor the different channel gains for all the receivers in order to select the ones that are violating the constraints. A simulation for this suboptimal scheme was performed, and the results are shown in Fig. 14 and 15. In such figures, we can see a reduction in the average power per sensor node and also a reduction of the outage probability compared with Fig. 7 and 8. Furthermore, Fig. 15 shows that there is an optimal number of receivers to have a minimum outage probability depending on the value of R_{min}.

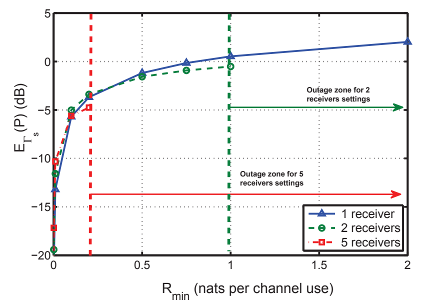

Figure 14: Average power consumed, vs. the rate constraint, for different number of receivers under an independent peak transmission rate constraint with suboptimal scheme.

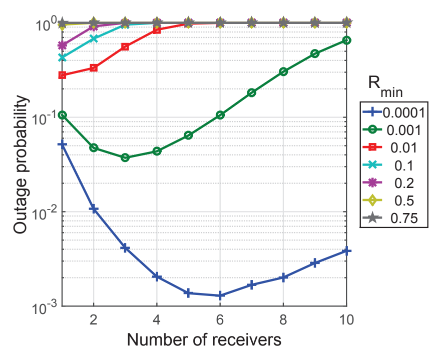

Figure 15: Outage probability as a function of the number of receivers under an independent peak transmission rate constraint with suboptimal scheme.

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VI. Conclusion

In this paper, a power minimization of sensor nodes in different scenarios is studied. First, the average transmit power of a point to point communication is minimized under instantaneous or average power constraint. Because of the numerical computations required to implement the optimal scheme, an on/off scheme is introduced and we show that it is optimal at high R_{min} and suboptimal for low R_{min}. Then, an interference constraint, related to the cognitive radio environment, is introduced in order to protect the primary communication. The corresponding general optimal power is derived and applied to Rayleigh and Nakagami-m fading channels. Our analysis reveals that there might be "*outage*" events, where it is not possible to satisfy all the constraints at the same time. These "*outage*" events have been identified and characterized. In addition, the problem of minimizing the transmit power of a wireless sensor network with a single transmitter and multiple receivers in a cognitive radio environment while satisfying different rate constraints is analyzed. Again, closed-form expressions of the optimal power are provided. A suboptimal scheme is proposed to overcome the excessive outage probability for the independent peak transmission rate constraint and simulations for Rayleigh fading channels show the improvement in performance.

APPENDIX A: Proof for Decremental Monotonicity of Equation (11)

Consider the function $G(x) = -\log(x)\left[1 - F_{|h|^2}(x)\right] + \int_x^{\infty} \log(|h|^2) f_{|h|^2}(|h|^2)\, d|h|^2$. Let us get the first derivative of $G(x)$. By applying Leibniz integral rule, we get

$$G'(x) = -\frac{1}{x}\int_x^{\infty} f_{|h|^2}\left(|h|^2\right) d|h|^2 = -\frac{1}{x}\left[1 - F_{|h|^2}(x)\right]$$

which means that $G'(x) < 0 \quad \forall\, x > 0$. Thus, $G(x)$ is a monotonic decreasing function.

APPENDIX B: Proof for Equation (16)

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We need to work with (9), considering that $f_{\gamma_s}(t) = e^{-t}$ for $t \geq 0$, i.e. From (9), we have

$$\lambda = \exp\left(\frac{R_{min} - \int_{\frac{1}{\lambda}}^{\infty} \log(\gamma_s) f_{|h|^2}(\gamma_s)\, d\gamma_s}{\int_{\frac{1}{\lambda}}^{\infty} f_{\gamma_s}(\gamma_s)\, d\gamma_s}\right)$$

and by assuming that $R_{min} \to 0$, the minimum power $P \to 0$ and from (7) we have $\lambda \to 0$ as well, therefore the equation above can be simplified by using the Taylor series expansion $\int_{\frac{1}{\lambda}}^{\infty} \log(\gamma_s) e^{-\gamma_s} d\gamma_s = \exp\left(-\frac{1}{\lambda}\right)\left[-\log(\lambda) + \lambda + O\left[\lambda^2\right]\right]$ which yields the following result,

$$\lambda = \exp\left(\frac{R_{min} - e^{-\frac{1}{\lambda}}(-\log\lambda + \lambda)}{e^{-\frac{1}{\lambda}}}\right)$$

Thus, $R_{min} = \lambda \exp\left(-\frac{1}{\lambda}\right)$ Solving this equation gives $\lambda = \frac{1}{W\left(\frac{1}{R_{min}}\right)}$ which yields (16).

APPENDIX C: Proof for Equation (18)

Let us consider (17) written as $e^{R_{min}} = \exp\left[e^{\frac{1}{P_o}} E_1\left(\frac{1}{P_o}\right)\right]$. It is known that as $R_{min} \to 0$, then $P_o \to 0$ thus, by taking a Taylor series expansion of the exponent in the RHS of the equation when $P_o \to 0$, we get that $e^{\frac{1}{P_o}} E_1\left(\frac{1}{P_o}\right) \simeq P_o + O\left[{P_o}^2\right]$ With this result, (17) can be written as

$$e^{R_{min}} \simeq \exp\left(P_o + O\left[{P_o}^2\right]\right)$$

Therefore, $P_o \simeq R_{min}$ which is the result given by (18).

APPENDIX D: Proof for Equations (26) and (27)

The derivative of (24) with respect to λ is given by

$$\frac{\partial \mathcal{L}}{\partial \lambda} = R_{min} - E_{\gamma_s}\left[\log\left(1 + P(\gamma_s) \cdot \gamma_s\right)\right]$$

Considering that $P(\gamma_s) = \min\left\{\left[\lambda - \frac{1}{\gamma_s}\right]^+,\ P_{\gamma_p}(\epsilon)\right\}$ then, $\frac{\partial \mathcal{L}}{\partial \lambda}$ evaluated at $\lambda = 0$ is equal to R_{min}, which means that the Lagrangian is increasing. In case that $\lim_{\lambda \to \infty} \frac{\partial \mathcal{L}}{\partial \lambda} = R_{min} - E_{\gamma_s}\left[\log\left(1 + P_{\gamma_p}(\epsilon) \cdot \gamma_s\right)\right] > 0$, then there is no solution (this is an "*outage*" situation). If a solution exists, we need to find the value of λ where $\frac{\partial \mathcal{L}}{\partial \lambda} = 0$, i.e. $E_{\gamma_s}\left[\log\left(1+ P(\gamma_s)\cdot\gamma_s\right)\right] = R_{min}$. To make the analysis simpler, we make the change $\lambda = \frac{1}{x}$, therefore

$$\int_x^{\infty} \min\left\{\log\left(\frac{\gamma_s}{x}\right),\ \log\left(1+P_{\gamma_p}(\epsilon)\cdot\gamma_s\right)\right\} \times f_{\gamma_s}(\gamma_s)\,d\gamma_s = R_{min}$$

From this point, we have two cases:

- if $x > P_{\gamma_p}(\epsilon)^{-1}$ then,
  $P(\gamma_s) = \min\left\{\left[\frac{1}{x} - \frac{1}{\gamma_s}\right]^{+},\ P_{\gamma_p}(\epsilon)\right\}$,
  where x is obtained from

$$G(x) = \int_x^{\infty} \log\left(\frac{\gamma_s}{x}\right) f_{\gamma_s}(\gamma_s)\,d\gamma_s = R_{min}$$

- if $x < P_{\gamma_p}(\epsilon)^{-1}$ then,
  $P(\gamma_s) = \min\left\{\left[\frac{1}{x} - \frac{1}{\gamma_s}\right]^{+},\ P_{\gamma_p}(\epsilon)\right\}$,
  where x is obtained from

$$H(x) = G(x) + \int_{\frac{1}{\frac{1}{x}-P_{\gamma_p}(\epsilon)}}^{\infty} \log\left(\frac{1+P_{\gamma_p}(\epsilon)\,\gamma_s}{\gamma_s}x\right) f_{\gamma_s}(\gamma_s)\,d\gamma_s = R_{min}$$

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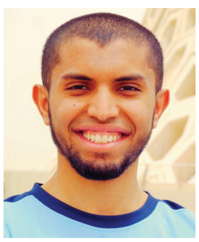

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