Sub-Patch Roughness in Earthquake Rupture Investigations

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Abstract

Fault geometric complexities exhibit fractal characteristics over a wide range of spatial scales (μm to km) and strongly affect the rupture process at corresponding scales. Numerical rupture simulations provide a framework to quantitatively investigate the relationship between a fault’s roughness and its seismic characteristics. Fault discretization however introduces an artificial lower limit to roughness. Individual fault patches are planar and sub-patch roughness –roughness at spatial scales below fault-patch size– is not incorporated. Does negligence of sub-patch roughness measurably affect the outcome of earthquake rupture simulations? We approach this question with a numerical parameter space investigation and demonstrate that sub-patch roughness significantly modifies the slip-strain relationship –a fundamental aspect of dislocation theory. Faults with sub-patch roughness induce less strain than their planar-fault equivalents at distances beyond the length of a slipping fault. We further provide regression functions that characterize the stochastic effect sub-patch roughness.

Index Terms

7209 Earthquake dynamics and mechanics
7230 Seismicity and seismotectonics
7260 Theory and modeling

Key Points:

- Discretization introduces an artificial lower limit on fault roughness in rupture simulations
- We show that neglecting sub-patch roughness significantly modifies the slip-strain relationship
- We quantify the statistical effect of sub-patch roughness and provide means to implement it
1. Introduction

A fault’s geometric deviation from an idealized planar surface is often referred to as fault roughness or fault geometric complexity. Over the past 30 years, numerous studies have established the fractal character of fault roughness, spanning a wide range of spatial scales [e.g., Brown and Scholz, 1985; Power et al., 1987; Sagy et al., 2007; Candela et al., 2012]. Kinks, steps, and bends that constitute fault roughness may be found at $<\mu m$ length-scale, as well as $>10s$-of-$km$ length-scale. While large-scale geometric fault complexity is often referred to as fault segmentation, and fault roughness may be interpreted as being due to small-scale effects, there is no clear separation of scales. We therefore refer simply to fault roughness. These fault geometric complexities were shown to strongly affect a fault’s seismic behavior at the corresponding spatial scales. For example, $km$-scale step-overs or fault bends frequently serve as rupture initiation and termination locations. The probability of multi-segment rupture is directly related to amplitude and wavelength of those roughness expressions [e.g., Harris and Day, 1999; Oglesby, 2005; Wesnousky, 2006], affecting the dynamic rupture process and resulting ground-shaking distribution [Oglesby and Mai, 2012; Aochi and Ulrich, 2015]. Roughness at smaller scales affects the fault’s overall resistance to slip, its aseismic vs. seismic behavior, the break-down of shear resistance during sliding, the magnitude of stress concentration and heterogeneity, and the deformation and damage of the fault-hosting rock [e.g., Candela et al., 2012; Shi and Day, 2013; and references therein]. Characterizing fault roughness and quantifying its effect on the rupture process is therefore important to improve the overall understanding of earthquake faulting and the associated seismic radiation and deformation field.

Numerical rupture simulations provide a framework to investigate the relation between a fault’s geometric complexity and its seismic characteristics, including multi-
segment rupture probability, source-scaling properties, coseismic deformation, and seismic radiation pattern. Corresponding modeling approaches incorporate roughness via a discretization step in which a fault is subdivided into smaller fault patches that are shifted and tilted to off-planar positions to generate fractal or otherwise complex fault geometries [e.g. Zielke and Arrowsmith, 2008; Dieterich and Smith, 2009; Dunham et al., 2011; Fang and Dunham, 2013; Shi and Day, 2013]. However, only roughness at length-scales larger than the chosen fault-patch size can be incorporated—the patches themselves are still planar and roughness at smaller spatial scales is not accounted for. Currently, it is not known if neglecting sub-patch roughness—that is, fault roughness at spatial scales smaller than model discretization—significantly modifies the outcome of the aforementioned rupture simulations. Similarly, despite the generally acknowledged importance of geometric complexity on a fault’s rupture characteristics, it is not accounted for in related studies like kinematic source inversions [Mai and Thingbaijam, 2014]. In those studies, a fault is approximated as a single plane or by a small number of fault segments, each again subdivided into a number of planar fault patches. This simplification is made because fault geometries at depth are generally not well known, but also for computational reasons. Disregarding the geometrically complex nature of faults in source inversion studies is therefore not only a matter of limited geological knowledge but also of feasibility. However, the effects of such simplifications on the corresponding earthquake-source inversion results are not well understood [Mai et al., 2016].

In the present study, we show how sub-patch fault roughness affects the slip-strain relationship—a fundamental aspect of dislocation theory—and thus influences for example numerical earthquake rupture simulations. We compute the slip-induced principal strain vectors for a slipping fault patch with sub-patch roughness and compare them with the corresponding planar-patch strain vectors. We do this for many rough-patch realizations to statistically constrain the roughness-related strain field change. After a description of the
model formulation, we identify the level of sub-patch discretization that sufficiently captures the patches’ fractal character. Then, we present the dependence of principal strain vector components (their length and orientation) on fault patch roughness with respect to their planar-patch counterparts. We provide regression functions that statistically constrain the effect of sub-patch roughness on the slip-induced deformation field.

2. Model Formulation

We define a circular fault patch with unit-length diameter and divide it into a large number of approximately equal-sized triangular sub-patches, using the “DistMesh” algorithm by Persson and Strang [2004]. The circular fault was chosen to minimize potential shape-related strain field contributions. For brevity and to avoid confusion, we refer to this circular fault patch as “fault” for the remainder of this publication. The actual number of sub-patches (~72k) that sufficiently well approximates the fault geometry’s fractal character was found in a convergence test (see section 3.1 below).

We parameterize complex fault geometries using 2-D random fields (Fig. 1a) that follow the von Karman auto correlation function (ACF) [e.g., Mai and Beroza, 2002]. In the Fourier domain, its power spectral density $P(k)$ is given by

$$P(k) = \frac{a_x a_z}{(1 + k^2)^{\frac{H+1}{2}}}, \quad (1)$$

where $k$ is wave number, $H$ is Hurst exponent, and $a_x$ and $a_z$ are correlation length in along-strike and along-dip direction (Eq. 1). Hurst exponent $H$, along with $a_x$ and $a_z$, constrain the spectral decay as a function of wave number (and hence determine the spatial roughness pattern) but not the roughness amplitude. A second parameter is needed to fully characterize fault roughness. Following previous studies [e.g., Mandelbrot, 1983; Power and Tullis, 1991; Candela et al., 2012; Fang and Dunham, 2013], we implement a pre-factor $C$ that defines the
Roughness amplitude at reference wave number (e.g., \( k = 1 \)).

We model fault roughness to be self-similar \((H = 1)\) as opposed to self-affine \((H < 1)\). While this approach may represent a simplification, Shi and Day [2013] noted that a power law with \( H = 1 \) connects map-scale and outcrop-scale roughness data better than any single power law curve with, for example, \( H = 0.8 \) [see Candela et al., 2012]. Further, adopting a self-similar roughness model enables us to provide scale-independent roughness quantification and fault representations: our fault may equally well represent a circular patch with 1 m diameter as well as 10 km diameter. In our simulations, we generate 6000 random fault geometry realizations, using a constant Hurst-exponent \((H = 1)\), while independently varying pre-factor \( C \) (0.0 – 0.03) and correlation lengths \( a_x \) and \( a_z \) (10-30% of fault diameter). Roughness is then quantified as a single-value metric via the dimensionless quantity \( \sigma_{RMS} \) (Eq. 2),

\[
\sigma_{RMS} = \sqrt{\frac{1}{A} \sum_{i=1}^{N} h_i^2 da},
\]

where \( L \) is 1-D fault length, \( A \) is fault area, \( N \) is the number of sub-patches, \( h \) is sub-patch height above the planar reference, and \( da \) is sub-patch area. Equation (2) is a 2-D normalized representation of the classical 1-D \( \sigma_{RMS} \) roughness metric [e.g., Power et al., 1988]. With the adopted parameters, we generate faults with \( \sigma_{RMS} \) between 0 and 0.03, covering the range of roughness observations from real faults which roughly center on \( \sigma_{RMS} = 0.01 \) [e.g., Sagy et al., 2007; Candela et al., 2012].

We apply 0.1 m of slip in 90° rake direction on all triangular sub-patches that constitute our circular fault and compute the resulting deformation field for the homogenous elastic full-space [Nikhoo and Walter, 2015]. Observation locations form a 9x9x9 cube, with the fault at the center of this cube (Fig. 1c). The spacing between observation locations equals the diameter of the slipping circular fault.
The slip-induced strain tensors are rotated to define principal strain vectors $\varepsilon_i (i = 1, 2, 3)$ (Fig. 1d). We use sub-index rough to indicate strain vectors from rough-fault realizations and sub-index planar for their planar-fault equivalents (e.g., $\varepsilon_{r,\text{rough}}$ and $\varepsilon_{\text{p,planar}}$). Length and orientation of rough-fault strain vectors are documented with respect to the corresponding planar-fault counterparts. Thus, for each observation location and for each roughness realization we calculate three angles $\Delta^i(\varepsilon_{r,\text{rough}}, \varepsilon_{\text{p,planar}})$ and three normalized lengths $|\varepsilon_{r,\text{rough}}|/|\varepsilon_{\text{p,planar}}|$ to constrain the roughness-induced variations in the slip-strain relationship. Note that strain component $|\varepsilon_{2,\text{planar}}|$ may take zero and non-zero values, depending on observation location relative to patch orientation. For observation locations where $|\varepsilon_{2,\text{planar}}| = 0$, we normalize $|\varepsilon_{2,\text{rough}}|$ not with $|\varepsilon_{2,\text{planar}}|$ as we do otherwise, but instead use the square root of the strain tensors’ second invariant $J_2$ (Eq. 3, 4) to define $\varepsilon_2$’s normalized length and indicate it with sub-index zero.

$$|\varepsilon_{2,\text{zero}}| = \frac{|\varepsilon_{2,\text{rough}}|}{\sqrt{J_2}}, \text{ with}$$

$$J_2 = \left(\varepsilon_{1,\text{planar}}^2 + \varepsilon_{3,\text{planar}}^2\right)/2, \text{ if } |\varepsilon_{2,\text{planar}}| = 0.$$  

3. Results

3.1. Identifying Patch Discretization Level

As exemplified by the coastline paradox [e.g., Mandelbrot, 1983], fractals do not have a well-defined, deterministic length or shape. Instead, their total length varies inversely with the length of the ruler that is used to measure it. This observation also applies to fractal fault surfaces, where total length and ruler length correspond to fault- and patch area respectively. However, for our investigation on sub-patch roughness, we need to identify the discretization level that sufficiently well approximates the fault’s fractal character. We address this issue
with an optimization approach, assuming that this condition is met when the change in fault area relative to the change in fault discretization becomes sufficiently small. For example, if increasing the number of sub-patches by an order of magnitude further increases fault area only by a fraction of a percent, we may consider the corresponding spatial resolution to be sufficient to deterministically approximate this fractal. To identify this model resolution, we generate 1000 random rough-fault realizations for 14 different discretization levels ranging from 150 to ~2.8 million sub-patches. We keep \( H \) and \( C \) constant for all realizations (\( H = 1; C = 0.01 \)) while allowing correlation length \( a_x \) and \( a_z \) to vary independently between 10-30\% of the fault diameter. For each discretization level, we determine the mean area of the 1000 rough faults and normalize it by the area of a planar fault at same discretization level (Fig. 2a). As expected, increasing the spatial sampling causes the area of the rough faults to grow, however at a decreasing rate. Finer and finer discretization will have lesser and lesser effect on fault area increase. We further compute the slope of the area vs. sub-patch number relationship (Fig. 2a). The resulting L-curve behavior indicates that the area increase rate saturates for discretization levels above \( \geq 72k \) sub-patches. While discretization beyond this level will continue to increase area, the additional contribution will be small, and is assumed to be negligible. Therefore, we conclude that in the context of the present study, \( \sim 72k \) sub-patches sufficiently well approximate the roughness characteristics of our circular fault. This conclusion is further supported by an additional convergence test that relates the change of \( |\varepsilon_{i,\text{rough}}|/|\varepsilon_{i,\text{planar}}| \) (\( i = 1, 3 \)) to discretization level (Fig. 2b), using the same roughness parameters that we adopted in the area convergence test. The resulting plots corroborate our observations for fault-area convergence (Fig. 2a): increasing the spatial sampling causes \( |\varepsilon_{i,\text{rough}}|/|\varepsilon_{i,\text{planar}}| \) of the rough faults to decrease, however, at lower and lower rate. The corresponding L-curve behavior shows that the rate of strain decrease saturates for discretization levels above \( \geq 72k \) sub-patches, indicating that this discretization level is
sufficient to provide a deterministic approximation of a fractal fault surface. Hence, we adopt this value for the remainder of our study. Assuming that the patch has a 1 km diameter, the average triangular sub-patch area is therefore ~11 m².

3.2. Roughness Induced Changes to \( \varepsilon_i \)

Fig. 3a presents \(|\varepsilon_{i,\text{rough}}|/|\varepsilon_{i,\text{planar}}|\) as a function of \( \sigma_{\text{RMS}} \), combining the results from all roughness realizations at all observation locations (with the exception of \(|\varepsilon_{2,\text{zero}}|\), depicted in Fig. S2). Fig. 3b presents a subset of Fig. 3a, exemplifying the effect of sub-patch roughness at a single observation location. While the single-location point cloud is less dense than the combined-location point cloud, they both share the same overall pattern. That is, the relative length of the principal strain vectors \(|\varepsilon_{i,\text{rough}}|/|\varepsilon_{i,\text{planar}}|\) decreases on average as function of \( \sigma_{\text{RMS}} \) while its variability increases.

Faults with sub-patch roughness induce less strain than their planar-fault equivalents at distances beyond the length of a slipping fault.

At locations where \(|\varepsilon_{2,\text{planar}}| = 0\), we find that the average of \(|\varepsilon_{2,\text{zero}}|\) exhibits no dependency on \( \sigma_{\text{RMS}} \) while its variability increases with \( \sigma_{\text{RMS}} \) (Fig. S2). Combined- and single-location point clouds of \( \Delta \gamma(\varepsilon_{i,\text{rough}}, \varepsilon_{i,\text{planar}}) \) also share the same overall pattern, with increasing angle and variation thereof as \( \sigma_{\text{RMS}} \) increases (Fig. 3c, d). For faults with sub-patch roughness, the orientation of \( \varepsilon_{i,\text{rough}} \) increasingly deviates from the corresponding \( \varepsilon_{i,\text{planar}} \) orientation as a function of \( \sigma_{\text{RMS}} \). The resemblance of combined- and single-location point clouds indicates that the identified relations of \(|\varepsilon_{i,\text{rough}}|/|\varepsilon_{i,\text{planar}}|\) and \( \Delta \gamma(\varepsilon_{i,\text{rough}}, \varepsilon_{i,\text{planar}}) \) with respect to \( \sigma_{\text{RMS}} \) are not a result of spatial averaging (Fig. S3). Since the strain vector components of \( \varepsilon_{i,\text{rough}} \) are reported relative to \( \varepsilon_{i,\text{planar}} \), neither \(|\varepsilon_{i,\text{rough}}|/|\varepsilon_{i,\text{planar}}|\) nor \( \Delta \gamma(\varepsilon_{i,\text{rough}}, \varepsilon_{i,\text{planar}}) \) exhibit distance dependency (Fig. S4).
Next, we examine the statistical properties of the relative length and relative orientation changes, \( |\varepsilon_{\text{rough}}|/|\varepsilon_{\text{planar}}| \) and \( \Delta^\circ(\varepsilon_{\text{rough}}, \varepsilon_{\text{planar}}) \). Their probability density functions PDFs within individual roughness bins (Fig. 4 and S1, insets) are very well characterized by a Weibull distribution for \( |\varepsilon_{\text{rough}}|/|\varepsilon_{\text{planar}}| \) (with \( i = 1, 3 \)) and \( \Delta^\circ(\varepsilon_{\text{rough}}, \varepsilon_{\text{planar}}) \) (with \( i = 1, 2, 3 \)), and a Normal distribution (for \( |\varepsilon_{2,\text{rough}}|/|\varepsilon_{2,\text{planar}}| \) and \( |\varepsilon_{2,\text{zero}}| \)), given as

\[
\begin{align*}
  f(x | \lambda, k) &= \begin{cases} 
  \frac{k}{\lambda} \left(\frac{x}{\lambda}\right)^{k-1} e^{-\left(\frac{x}{\lambda}\right)^k} & x > 0 \\
  0 & x \leq 0 
  \end{cases}, \text{ and} \\
  f(x | \mu, \sigma) &= \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}},
\end{align*}
\]

In Eq. 5, \( \lambda \) is the scale parameter, \( k \) is the shape parameter, in Eq. 6, \( \mu \) is mean, and \( \sigma \) is standard deviation. The goodness of these fits may be somewhat expected, considering the well-known qualities of the Weibull distribution in characterizing mechanical damage processes. The apparent difference between observed and modeled probabilities for the single-location probability distributions may be due to the relatively low number of data points that contributes to those histograms (e.g., \( \sim 120 \) for the exemplary location and roughness bin). This number is generally insufficient to unambiguously constrain \( \lambda \) and \( k \) (or \( \mu \) and \( \sigma \)), so that some variability in the PDF shapes for single-location probability fits can be expected (Fig. S3).

For the exemplary roughness bin \( \sigma_{\text{RMS}} = 0.01 \) (Fig. 3), fitting the Weibull distribution to \( |\varepsilon_{\text{rough}}|/|\varepsilon_{\text{planar}}| \) (\( i = 1, 3 \)) returns \( \lambda \approx 0.97 \) and \( k \approx 60 \) for scale- and shape parameter respectively. This translates to a \( 3.5 \pm 4\% \) (\( 2\sigma \)) decrease in strain vector length for \( \varepsilon_{1,\text{rough}} \) and \( \varepsilon_{3,\text{rough}} \) with respect to the corresponding planar-fault values. Similarly, fitting the Weibull distribution to \( \Delta^\circ(\varepsilon_{2,\text{rough}}, \varepsilon_{2,\text{planar}}) \) for \( \varepsilon_{1,\text{rough}} \) and \( \varepsilon_{3,\text{rough}} \) returns \( \lambda \approx 0.54 \) and \( k \approx 1.66 \).
translating to a $0.5 \pm 0.6^\circ$ ($2\sigma$) deviation in principal strain vector orientation relative to the planar-fault reference case.

3.3 Regression of Weibull Parameters

In the previous section, we showed that Weibull and Normal PDF provide the means to statistically describe rough-fault principal strain vectors $\mathbf{\varepsilon}_{\text{rough}}$ with respect to their planar-fault equivalents $\mathbf{\varepsilon}_{\text{planar}}$. It is helpful to determine how the corresponding fitting parameters evolve as a function of $\sigma_{\text{RMS}}$. To examine this question, we bin data by roughness value (Fig. 3 and S1; bin step size = $10^{-3}$; bin width = $\pm 10^{-3}$). For each bin, normalized histograms of $|\mathbf{\varepsilon}_{\text{rough}}|/|\mathbf{\varepsilon}_{\text{planar}}|$ and $\Delta^\circ(\mathbf{\varepsilon}_{\text{rough}}, \mathbf{\varepsilon}_{\text{planar}})$ are modeled with the corresponding probability distribution. Fig. 4 presents the fitting parameters for Weibull and Normal distribution as a function of $\sigma_{\text{RMS}}$ along with regression functions that capture the parameters’ dependence on $\sigma_{\text{RMS}}$. The scale parameter $\lambda$ for $|\mathbf{\varepsilon}_{\text{rough}}|/|\mathbf{\varepsilon}_{\text{planar}}|$ ($i = 1, 3$) is well described by a $3^{\text{rd}}$ order polynomial of $\sigma_{\text{RMS}}$, while the corresponding shape parameter $k$ follows a rational function (Fig. 4a). For $\Delta^\circ(\mathbf{\varepsilon}_{\text{rough}}, \mathbf{\varepsilon}_{\text{planar}})$ ($i = 1, 3$), we find that $k$ is well described by $1^{\text{st}}$ order polynomial, while $\lambda$ exhibits no dependence on $\sigma_{\text{RMS}}$ (Fig. 4b). Corresponding regression functions are also provided for the relative length and orientation change of $\mathbf{\varepsilon}_{2,\text{rough}}$ and $\mathbf{\varepsilon}_{2,\text{zero}}$ (Fig. 4c, d). Combined, they provide the means to sample from the inverse Weibull and Normal cumulative density functions (Eq. 7 and 8 respectively) and hence define the stochastic, roughness-related changes in principal strain vector $\mathbf{\varepsilon}_i$, relative to the planar-fault equivalent

\[
f(\Pi \mid \lambda,k) = -\lambda \left[ \ln(1-\Pi) \right]^{1/k}, \quad \text{and} \quad F^{-1}(\Pi \mid \mu,\sigma) = \left\{ x : F(x \mid \mu,\sigma) = \Pi \right\}, \quad \text{with} \quad \Pi = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{t} e^{-\frac{(t-\mu)^2}{2\sigma^2}} \, dt \quad (7)
\]

\[
\]
where $\mathcal{P}$ is sampled probability [0, 1]. For example, assume a fault with sub-patch roughness $\sigma_{\text{RMS}} = 0.01$ for which we want to determine $|\varepsilon_{1,\text{rough}}|$. Employing the regression functions provided in Fig. 4, together with this roughness value, we determine $\lambda \approx 0.97$ and $k \approx 60$ for $|\varepsilon_{1,\text{rough}}|$. If the randomly sampled value for $\mathcal{P}$ is 0.3, then Eq. 7 provides $f(\mathcal{P} | \lambda, k) = |\varepsilon_{1,\text{rough}}|/|\varepsilon_{1,\text{planar}}| = 0.954$. That is, for this stochastic realization (with $\mathcal{P} = 0.3$), $|\varepsilon_{1,\text{rough}}|/|\varepsilon_{1,\text{planar}}|$ is $\approx 4.5\%$ below the expected value for the planar-fault reference case. For $\mathcal{P} = 0.9$ and $0.1$, $|\varepsilon_{1,\text{rough}}|/|\varepsilon_{1,\text{planar}}|$ decreases by $\approx 1.6\%$ and $\approx 6.5\%$ respectively. Multiplication of $|\varepsilon_{1,\text{rough}}|/|\varepsilon_{1,\text{planar}}|$ with $|\varepsilon_{1,\text{planar}}|$ –the expected value from the planar-fault– provides a strain vector component $|\varepsilon_{1,\text{rough}}|$ in absolute terms, stochastically incorporating sub-patch roughness. This procedure may be applied to all strain vector components (lengths and orientations of $\varepsilon_i$) utilizing the regression functions in Fig. 4 and the outlined procedure.

4. Conclusion

The results that we present here provide a comprehensive, statistical representation of the stochastic effect of sub-patch fault roughness on the slip-strain relationship relative to the planar-fault reference case. Investigators may incorporate sub-patch roughness by a) computing Green’s functions for planar fault; b) selecting a sub-patch roughness level to implement; c) using the regression equations developed in this study to determine the stochastic change in $\varepsilon_i$’s length and orientation in correspondence to the implemented roughness level, and d) applying these changes to the planar-fault Green’s functions (i.e., corresponding principal strain vectors). For verification purposes, we adopt the outlined approach and determine the slip-induced principal strain vectors for 6000 roughness realizations (using a planar fault and applying the provided regression functions). As is indicated in Fig. S5, our sub-patch roughness approximation is able to stochastically
reproduce the slip-induced deformations of a rough, highly discretized fault model.

Does sub-patch roughness significantly affect earthquake source inversions and dynamic rupture modeling? Assuming the previous example of a fault patch with sub-patch roughness $\sigma_{RMS} = 0.01$, the length of principal strain vectors decreases by $\sim 3.5 \pm 4\% \ (2\sigma)$ while their orientation changes by $\sim 0.5 \pm 0.6^\circ \ (2\sigma)$. That is, a fault with this level of sub-patch roughness requires $\sim 3.5 \pm 4\% \ (2\sigma)$ more slip than its planar-fault equivalent to statistically generate the same amount of deformation at given observation locations. Kinematic source inversions that utilize planar fault patches (to invert an earthquake’s deformation field) therefore are expected to underestimate the amount of fault slip that caused the observed deformation. However, the uncertainties that are currently associated with source inversions are much larger than this exemplary value [Mai and Thingbaijam, 2014; Mai et al., 2016], suggesting that the effect of sub-patch roughness is now (and in the foreseeable future) not resolvable in source inversion studies.

While sub-patch roughness may affect the outcome of source inversions only marginally, we expect a more pronounced effect on numerical rupture simulations. Negligence of sub-patch roughness –i.e., the simplifying assumption that fault roughness is limited to wavelengths above the utilized fault patch size– artificially increases the strain-transfer efficiency. Slip-induced amounts of strain are consistently too high (with respect to the expected values when sub-patch roughness is implemented), implying that current earthquake rupture simulations generally have a built-in overshoot. That is, the elastic medium and other fault patches embedded in it, receive consistently too much energy (in form of elastic deformation). As a result, the overall resistivity to slip is consistently too low, which affects for example rupture propagation potential and multi-segment rupture probability. We suggest that implementation of sub-patch roughness will help to remove the current built-in overshoot, making earthquake rupture simulations more realistic. Here, we
provide an approach to implement sub-patch roughness in quasi-static rupture simulations. While computationally expensive and potentially more difficult to implement, a desirable future step will be to perform a similar parameter space investigation for dynamic rupture simulations.

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Figure 1. a) Exemplary fault roughness realizations, using the same random field but different pre-factors. Colors indicate height above planar reference while contrast represents the corresponding curvature. The circular patches are divided into ~72k triangular sub-patches. b) 0.1 m of slip in 90° rake direction is applied on all triangular sub-patches. c) The slipping circular fault is surrounded by 9x9x9 observation locations. Presented here is a portion of a observation plane to visualize the spatial relation between observation points and the slipping fault. Minimum distance between observation location and the center of the slipping fault is the fault’s diameter. For each observation location (except the center location itself) the slip-induced strain tensor is computed and rotated to constrain principal strain vectors $\varepsilon_i$ ($i = 1, 2, 3$). d) The length and orientation of $\varepsilon_i$rough varies as a function of fault roughness, and are documented relative to the corresponding values from a planar fault $\varepsilon_i$planar.
Figure 2. a) The normalized fault area of the fractal fault surface increases with the number of sub-patches used to deterministically approximate it. The rate at which fault area grows decreases substantially once a certain discretization level is reached. The slope of the area vs. sub-patch number relationship exhibits a well-expressed L-curve behavior. A fault discretization with ~72k sub-patches is sufficient to capture the fractal character of a self-similar fault surface. b) Strain decrease (expressed as length change of principal strain components relative to the planar reference case) for different fault discretization levels using a given roughness scenario ($\sigma_{RMS} = 0.01$). The resulting relationship and its slope trace the observations in (a) indicating that discretization with ~72k sub-patches is sufficient to approximate the fault’s fractal character and its effect on the slip-strain relationship.
Figure 3. a) Normalized length of $\varepsilon_i$ ($i = 1, 2, 3$) as function of roughness metric $\sigma_{RMS}$ for all observation locations (except for $\varepsilon_{i,zero}$ which is presented in Fig. S1). b) Subset of (a) for an exemplary single observation location. Note the strong relationship between the amount of strain induced at the observation locations and the roughness of the slipping fault: a rougher fault yields smaller amounts of induced strain. c) Orientation change of $\varepsilon_i$ ($i = 1, 2, 3$) with respect to the planar-fault reference case as function of $\sigma_{RMS}$. d) Subset of (c) for the exemplary single observation location. The deviation from the expected orientations of $\varepsilon_i$ increases with $\sigma_{RMS}$ (see main text for detailed discussion). The insets show the observed and modeled probability distributions for the indicated roughness bin.
Figure 4. Regression of Weibull parameters $\lambda$ and $k$, and Gaussian parameters $\mu$ and $\sigma$ for relative length change, $|\varepsilon_{i,\text{rough}}|/|\varepsilon_{i,\text{planar}}|$ (a, c), and relative orientation change, $\Delta^\circ(\varepsilon_{i,\text{rough}}, \varepsilon_{i,\text{planar}})$ (b, d) as a function of $\sigma_{\text{RMS}}$. Regression coefficients are provided for all applicable components in the embedded empirical equations. The inserted histograms refer to the probability distribution of scale parameter $k$ for $\varepsilon_i$'s orientation, which exhibits no dependency on $\sigma_{\text{RMS}}$ but still shows some variability.