Insights into the data dependency on anisotropy: an inversion prospective

Tariq Alkhalifah

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Running head: Anisotropy dependency

ABSTRACT

While velocity contrasts are responsible for most of the events recorded in our data, the long wavelength behavior of the velocity model is responsible for the geometrical shape of these events. For isotropic acoustic materials the wave dependency on the long (wave propagation) and short (scattering) wavelength velocity components is stationary with propagation angle. On the other hand, in representing a transversely isotropic with a vertical symmetry axis (VTI) medium with the NMO velocity, the anellepticity parameter $\eta$, the vertical scaling parameter $\delta$, the sensitivity of waves varies with polar angle for both the long and short wavelength features of the anisotropic dimensionless medium parameters ($\delta$ and $\eta$). For horizontal reflectors at reasonable depths, the long wavelength features of the $\eta$ model is reasonably constrained by the long offsets, while the short wavelength features, produce very week reflections at even reasonable offsets. Thus, for surface acquired seismic data, we could mainly invert for smooth $\eta$ responsible for the geometrical shape of reflections. On the other hand, while the $\delta$ long wavelength components mildly effects the recorded data, its short wavelength variations can produce reflections at even zero offset, with a behavior pattern synonymous to density. The lack of the long wavelength $\delta$ information
will mildly effect focusing, but will cause misplacement of events in depth. With low enough frequencies (very low), we may be able to recover the long wavelength $\delta$ using full waveform inversion (FWI). However, unlike velocity, the frequencies needed for that should be ultra low to produce long wavelength scattering based model information, as $\delta$ perturbations do not exert scattering at large offsets. For a combination given by the horizontal velocity, $\eta$, and $\epsilon$, the diving wave influence of $\eta$ is absorbed by the horizontal velocity, severely limiting the $\eta$ influence on FWI. As a result, with a good smooth $\eta$ estimation, for example, from tomography, we can focus FWI to invert for only the horizontal velocity, and maybe $\epsilon$ as a parameter to fit the amplitude. This is possibly the most practical inversion for surface seismic data in VTI media.
INTRODUCTION

The short wavelength components of the velocity model are responsible for most of the events recorded in our seismic data, referred to them as reflections. On the other hand, the long wavelength components control the shape of these events. In isotropic, constant density, acoustic materials, the wave sensitivity to both the long and the short wavelength components influence are angular independent. This feature does not change for velocity in the acoustic anisotropic representation, if such representation is based on a velocity and dimensionless parameters. In this case, the angular dependence is provided by the anisotropic parameters. This dependence also varies between the short and long wavelength components of these parameters.

Thus, multi-parameter inversion, needed in anisotropic materials (Burridge et al., 1998; Plessix and Cao, 2011; Prieux et al., 2011; Operto et al., 2013), benefits from an appropriate choice of parameters to represent the model. Finding a minimum set of parameters that can explain the data can lead to a better inversion. Alkhalifah and Plessix (2014) analytically analyzed the radial dependency (radiation pattern) of the anisotropic parameter perturbation in acoustic transversely isotropic media with vertical symmetry axis (VTI). They advocated using certain combinations of parameters for various FWI strategies, including those that start with a model obtained from MVA, and those obtained from inverting diving wave energy. Choosing the right inversion setup for resolving anisotropy can make the difference between interpretable high-resolution results and results that may not make a lot of sense (too smooth or too biased). Representing the VTI model using the NMO velocity, $v_n$ (Thomsen, 1986), and the anisotropy parameters, $\delta$ and $\eta$ offers the proper perturbation radiation pattern for an inversion that includes reflections and diving waves (Alkhalifah,
Since \( \delta \) mildly influences the geometrical aspects of the recorded wavefield, it can serve as a secondary parameter to fit the amplitude to compensate for the shortcomings of the acoustic model in representing the true elastic Earth. For an inversion with a hierarchical implementation in which diving waves are used first, a VTI model represented by the horizontal velocity, \( v_h \), and the anisotropy parameters, \( \eta \) \((v_h = v_n \sqrt{1 + 2\eta})\) and \( \epsilon \), offers a practical set necessary to reduce the tradeoff and provide reasonable resolution. In this case, \( \epsilon \) plays the role of amplitude fitting as it mildly effects the kinematics in the recorded wavefield (or in other words the horizontal projection of the propagator). In fact, I find this combination to be the most practical for FWI for reasons explained in this letter.

The angular influence of every point in the model can be constrained by the wave path angles of the recorded waves (it's plane wave component) that either went through it or scattered from it. The transmission provides the long wavelength information in the direction normal to the wave propagation, while the scattering defines the short wavelength in the direction normal to a potential reflector. If we are missing the needed propagation angles to constrain the anisotropic description at any wavelength scale, the description will be limited at that scale (Alkhalifah, 2015). Thus, missing the short wavelength of \( \eta \) or the long wavelength of \( \delta \), as is the case for a parameterization given by \( v_n, \eta, \) and \( \delta \), and a surface seismic P-wave experiment setup, means that we end up with a smooth \( \eta \) and a sharp \( \delta \), representing the \( \delta \) reflectivity only. Since most of the Earth is elastic and the difference in the wavefield using the acoustic representation is given by the reflectivity, the \( \delta \) reflectivity will carry in this case a lot of the amplitude residual. In this letter, I focus on the angular sensitivity of the wavefield to the short and long wavelength perturbations of the anisotropic parameter and explain what it means for FWI and parameter estimation in general using any wavefield-based method. The parameter analysis at the scattering point
is independent of the process used to constrain their influence. Inspired by the original
diagram of Claerbout (1985), shown in Figure 1, illustrating the influences of the different
scales of velocity, I develop such diagrams for anisotropic materials using the most feasible
parameter combination. The focus of the discussions and conclusions in this letter apply
mostly to surface seismic data acquisition for exploration or regional seismology. This
includes a focus on only $P$-waves, thus, justifying the acoustic assumption, with conclusions
that may apply more to the phase aspects of the inversion.

THE ANGULAR SENSITIVITY OF WAVES

Both the angular dependence of the kinematic and scattering components of waves have been
derived for purposes of traveltime calculation and reflection amplitude. Since we are working
in the local (with respect to wavelength) homogeneous and linear regime, the kinematics can
be represented by the moveout equations (Hake et al., 1984) and the scattering by linearized
approximations of the reflection coefficient (Ruger, 1997). I will, however, analyze those
with respect to the parametrization suggested and for the purpose of inversion.

The kinematic-transmission components

The long wavelength component of the velocity model, or the anisotropic parameters, typ-
ically control the majority of wave propagation characteristics, effecting the shape of the
waves, which is conveniently described, in the high frequency asymptotic limit, by trav-
eltimes. To analyze the behavior of this component for reflections, and since the angular
dependency is our focus, I utilize derived moveout approximations to describe the angular
dependency. These moveout equations are derived for sources and receivers on the Earth
surface for an effectively homogeneous medium, but we can always bring the sources and receivers down to the grid point of interest where the medium is locally homogenous.

Using the Taylor’s series expansion, we end up with the following 3-term approximation of the traveltime for VTI media as a function of offset, $X$, for the homogenous medium case (Hake et al., 1984; Alkhalifah and Tsvankin, 1995):

$$ t^2 = t_0^2 + \frac{X^2}{v_n^2} - \frac{2\eta X^4}{t_0^2 v_n^4}, $$

(1)

where $t_0$ is the zero-offset two-way time, or for horizontal events, the two-way vertical time ($t_0 = \frac{2Z\sqrt{1+2\delta}}{v_n}$, where the depth $Z$ is an unknown quantity in surface acquired seismic data).

Thus, the sensitivity of traveltime to velocity can be expressed approximately as

$$ \frac{\partial t}{\partial v_n} = \frac{X^2}{t^2 v_n^3} + \frac{8\eta X^4}{t^2 t_0^2 v_n^5} \approx -\frac{\tan^2 \frac{\theta}{2}}{v_n}, $$

(2)

stopping at the first term (leading-order influence), and

$$ \frac{\partial t}{\partial \eta} = -\frac{X^4}{t^2 t_0^2 v_n^4} \approx 4 \sin^2 \frac{\theta}{2} \tan^2 \frac{\theta}{2}, $$

(3)

where $\theta$ is the scattering angle (twice the reflection angle) between the source and receiver wavefields (or rays). Since the depth of the reflector is unknown, both parameters gain resolvability away from vertical ray path, in which the vertical path (zero offset) is used as a reference. In this case, the sensitivity of $v_n$ is proportional to $-\tan^2 \frac{\theta}{2}$, while the sensitivity of $\eta$ is proportional to $4 \sin^2 \frac{\theta}{2} \tan^2 \frac{\theta}{2}$. In the above approximation, the traveltime is insensitive to $\delta$. 
Finally, if we use $v_h$ and $\eta$ instead in equation 1, we obtain the following sensitivities:

\[
\frac{\partial t}{\partial v_h} = -\frac{(1 + 2\eta)X^2}{t^2 v_h^3} \approx -\frac{(1 + 2\eta) \tan^2 \frac{\theta}{2}}{v_h},
\] (4)

stopping at the first order influence, and

\[
\frac{\partial t}{\partial \eta} = \frac{2X^2}{t^2 v_h^2} \approx 2 \tan^2 \frac{\theta}{2}.
\] (5)

Note in this case, $v_h$ and $\eta$ have similar dependencies at the second-order term, which is a source of tradeoff.

**The scattering components**

In the scattering case, the angular dependency (radiation pattern) is extracted from the Born approximation. For acoustic VTI media, Alkhalifah and Plessix (2014) derived such patterns for two sets of anisotropic parameter combinations that they deem to be the most practical. Those are $v_n$, $\eta$, and $\delta$ in the case in which we have the opportunity to resolve long wavelength $v_n$ first, using for example travetimes, and $v_h$, $\eta$, and $\epsilon$ in the case in which we have the opportunity to invert diving waves first. We focus initially on the first combination since it is utilized often.

Considering the asymptotic Greens function (without multi-pathing) for a plane wave approaching location $x$ in the subsurface expressed in the frequency, $\omega$, domain

\[ G(x, k, \omega) = A(x, k) \exp (i k \cdot x), \]

where $k$ is the wavenumber vector for either the source or receiver wavefields, $i$ is equal
to $\sqrt{-1}$, and $A$ is the geometrical amplitude between location $x$ and a potential source or receiver. We can then write the single-scattered wavefield (Alkhalifah and Plessix, 2014)

$$u_s(k_s, k_r, \omega) = -\omega^2 s(\omega) \int d\mathbf{x} \frac{G(k_s, \mathbf{x}, \omega) G(k_r, \mathbf{x}, \omega)}{v_0^2(\mathbf{x}) \rho(\mathbf{x})} a_1(\mathbf{x}) \cdot \mathbf{r}_1(\mathbf{x})$$

(6)

with $s$ as the source function, $v_0$ is the background isotropic velocity, and

$$\mathbf{r}_1 = \begin{pmatrix} r_{vn} \\ r_\eta \\ r_\delta \end{pmatrix}; \quad a_1 = \begin{pmatrix} 2 \\ 2n_{sh}^2 n_{rh}^2 \\ -(n_{sz}^2 + n_{rz}^2) \end{pmatrix}.$$ 

(7)

The vector $\mathbf{r}_1$ includes the perturbations of the individual parameters, $v_n$, $\eta$, and $\delta$, from top to bottom. Thus, the coefficients of $a_1$ define the radiation patterns of each parameter for the given parameterization (Aki and Richards, 1980). The unit vectors $\mathbf{n}_s$ and $\mathbf{n}_r$ with the source incident angle, $\theta_s$, and the reflector dip angle $\phi$, are given by

$$\mathbf{n}_s = \begin{pmatrix} n_{sh} \\ n_{sz} \end{pmatrix} = \begin{pmatrix} \sin(\theta_s) \\ \cos(\theta_s) \end{pmatrix}; \quad \mathbf{n}_r = \begin{pmatrix} n_{rh} \\ n_{rz} \end{pmatrix} = \begin{pmatrix} -\sin(\theta_s + 2\phi) \\ \cos(\theta_s + 2\phi) \end{pmatrix}.$$ 

(8)

For a horizontal reflector in a VTI medium, the source and receiver wave angle are the same, and thus, the $\eta$ scattering potential is proportional to $\sin^4 \frac{\theta}{2}$, whereas, the velocity scattering is angle independent. The scattering potential of $\delta$ is proportional to $\cos^2 \frac{\theta}{2}$. Compared to the kinematics, which suffers from the depth ambiguity of the reflector near zero offset, minimizing the influence of $\delta$ in that case, the scattering has its highest resolution (as we see next) at zero offset, with $\delta$ perturbations playing a major role.

On the other hand, the radiation patterns of the parameterization $(v_n, \eta, \varepsilon)$ are given
by (Alkhalifah and Plessix, 2014)

\[
\begin{align*}
\mathbf{r}_2 &= \begin{pmatrix} r_{v_h} \\ r_{\eta} \\ r_{\epsilon} \end{pmatrix} ; \\
\mathbf{a}_2 &= \begin{pmatrix} 2 \\ -n_{sz}^2 n_{r_h}^2 - n_{r_z}^2 n_{sh}^2 \\ -(n_{sz}^2 + n_{r_z}^2) \end{pmatrix} 
\end{align*}
\] (9)

Thus, for a combination given by \(v_h, \eta,\) and \(\epsilon,\) the \(\eta\) influence is generally small and is predominantly located at about 90 degrees scattering angle, reasonably beyond the conventional offset of reflection seismic data and nowhere near the scattering angles associated with diving waves. Thus, the scattering potential of \(\eta\) in this case is weak for surface seismic data. As a result, we may ignore \(\eta\) in FWI, which allows us to focus on \(v_h.\)

**THE WAVENUMBER CONTENT FROM THE KINEMATICS AND SCATTERING**

The wavenumber content in the derived velocity model from the kinematics of the wavefield depends mainly on the density of events (reflections or diffractions) in which the kinematic influence can be measured (and constrained). Higher resolution models are obtained from data that have more events to constrain the kinematics of the wavefield. So the resolution of the model is dependent on the model and corresponding reflections. However, within the layer, the extracted model wavenumber is governed by the first Fresnel zone region of the gradient. In wavefield tomography and migration velocity analysis methods, our gradients are based on the adjoint state method, and thus, the model wavenumber is governed by diffraction tomography principles of transmission. In complex media, MVA methods are governed by the transmission components between our recording surface and the source of.
events in our model, usually under the single scattering assumption. These wavenumbers are generated by diffraction tomography principles, and thus, extracted from the plane wave decomposition of the Born approximation update kernel. In fact, in isotropic media, the update wavelength at a model point, is governed by the dip of a potential reflector and the scattering angle. Specifically, the local model wavenumber vector with respect to a potential scatterer in our model (Miller et al., 1987; Jin et al., 1992; Thierry et al., 1999) is given by,

$$k_m = k_s + k_r = 2\frac{\omega}{v_0} \cos \frac{\theta}{2} n, \quad (10)$$

which depends on, among other things, the angular frequency in a direction guided by a unit vector, n, normal to a potential reflector. Here, k_s and k_r are the source and receiver (or state and adjoint state) wavefield wavenumbers, respectively, at the model point. Both diving waves and image based reflections provide updates along the wave path with very low wavenumbers. It is usually limited to the first Fresnel zone, with a maximum wavenumber dependent on the frequency of the data, and resolution provided in the direction normal to the wave path. For diving waves that direction is mainly vertical, and for imaging wave path that direction is mainly horizontal. Therefore, the range of model wavenumbers extracted from transmitted waves start from zero, to theocratically the following limit in a homogenous isotropic background:

$$k_{\text{max}} = \frac{2\omega}{v_0} \sqrt{\frac{1 + 4l \frac{\omega}{v_0}}{1 + 2l \frac{\omega}{v_0}}}, \quad (11)$$

where l is the distance between the source and receiver. As l tends to zero (zero-offset) $k_{\text{max}} = \frac{2\omega}{v_0}$, and as l goes to infinity $k_{\text{max}} = 2\sqrt{\frac{\omega}{lv_0}}$, which is much smaller than for zero offset and tends to zero. This is expected as we get higher resolution if our wave path is
Conversely, the scattering part are described by the diffraction tomography components that are beyond the first Fresnel zone. For typical reflection events, equation 11 holds, but the scattering angle is low, resulting in high wavenumber components normal to a potential reflector and a maximum given by $k_{\text{max}} = \frac{2\omega}{v_0}$ at zero offset.

**MAPS OF RESOLVABILITY**

I will discuss the anisotropy parameter resolvability utilizing Claerbout’s epic diagram (Figure 1) that schematically explains the data sensitivity to various velocity scales. Similar graphs can be constructed for the anisotropic parameters based on the above discussed angular behavior of the sensitivity of the data. In classic imaging, we use the anisotropic parameters given by the long wavelength components of the model description to form the velocity needed for migration, or, in other words, to invert for the single scattering component exclusively. In inversion, we seek both components (short and long wavelengths), to produce data that include all events, including multi scattering. Since anisotropy induces angular variation, we add to Claerbout’s diagram an angle dimension.

Using the $v$, $\eta$, and $\delta$ combination, I show in Figure 2a the sensitivity of data to the scattering potential of velocity at various angles. Since the scattering angle is used, I assume that the depth of the horizontal reflector is unknown so knowledge of the velocity at low wavenumbers are gained through their dependency on angle. Also, since the velocity has an isotropic radiation pattern with this combination, it is similar to the isotropic case, and thus, as the scattering angle increases we obtain lower wavenumbers from the FWI process. Large scattering angles are usually available more up shallow, and thus, FWI performs better on
the shallow subsurface. For an $\eta$ perturbation, such dependency, as shown in Figure 4 occurs at high scattering angles, and more so for the low wavenumber components, as it is, as we saw earlier, proportional to $\tan^2 \frac{\theta}{2}$ as opposed to $\sin^2 \frac{\theta}{2}$ for scattering. Meanwhile, Figure 2c shows that the low wavenumbers of $\delta$ are not usually attainable from surface seismic data, and only the scattering components are available. These $\delta$ perturbations can be used to fit the amplitude at conventional reflection offsets as we try to fit reflections from an acoustic model to elastic data. However, in the rare case we have ultra low frequencies in the data (inducing model wavelengths that cover the whole model), we may be able to invert for low wavenumbers $\delta$, needed to place the reflections at their accurate depth (Figure 2d).

For a $v_h$, $\eta$, and $\epsilon$ combination, the main difference is in the $\eta$ influence. The scattering potential $v_h$ (Figure 3a) has the same radiation pattern as $v_\eta$, but the $\eta$ scattering potential is much smaller (Figure 3b). In fact, $\eta$ in this case does not effect diving waves as well. On the low wavenumber side, the $\eta$ influence is now shared with $v_h$ in the second order term, as evident from the moveout equations 4 and 5. This introduces a potential trade off in any MVA or MVA based tomography inversion, and thus, for this I recommend sticking with the first combination for the tomographic part of the velocity model building process. However, for the FWI part the $v_h$ based combination provides interesting features. The low influence of $\eta$ at low and high scattering angles, means that FWI can be performed for $v_h$ only, or with $\epsilon$ to absorb the potential amplitude misfit in reflections between the acoustic assumption and the elastic nature of our data.

**A SIMPLE EXAMPLE OF INFLUENCE**

Many of the angular dependency arguments mentioned above are drawn from a plane wave decomposition of the Born scattering approximation. Let us observe whether such argu-
ments hold using numerical calculations that goes beyond the Born approximation or the
plane wave representation. For a homogenous background NMO velocity, I consider a two-
layer anisotropic parameter model in which the first layer has three scatterers located at the
same depth (see Figure 4). For the first example, this anisotropic model corresponds to $\eta$,
while $\delta$ is set to zero. In this case, Figure 5a shows a shot gather for a shot located at 1280
m near the surface and wavefields computed using spectral methods (Wu and Alkhalifah,
2014). We first note that the $\eta$ scatterer below the shot has nearly no amplitude despite the
magnitude of the scatterer. A similar behavior occurred from the reflection corresponding
to the $\eta$ interface for small scattering angles. Only when the offset-to-depth ratio exceeds
one that we start to see some reasonable amplitude. For the other scatterers, the minimum
time wave path forms an angle with the vertical for either the incident or reflected wave,
and thus, we get scattering from $\eta$ perturbations since the scattering angle is not zero. On
the other hand, if we set $\eta$ constant equal 0.2 and make the model in Figure 4 represent
$\delta$, we obtain the shot gather in Figure 5b. Clearly, as expected, a $\delta$ perturbation induces
zero-offset scattering and reflections. However, the amplitude of these events decrease con-
siderably with offset. Also, with an $\eta = 0.2$ background, the reflection in Figure 5b arrives
at earlier times at far offset, compared to the case when $\eta = 0$ in Figure 5a. Thus, despite
that $\eta$ does not produce a lot of scattering, it clearly effects wave propagation. The opposite
conclusion can be made for $\delta$. If we set $\eta = 0$ and add 0.2 (long wavelength variation) to
the $\delta$ model in Figure 4, we obtain the shot gather in Figure 5c. Since the depth of the
reflector and scatterers are unknown in our acquired data and to maintain the same time at
zero offset, I adjust the depth of these events to compensate for the change in the vertical
velocity caused by adding 0.2 to the $\delta$ model. Thus, Figure 5c has the same time of events
at zero offset as Figures 5a and 5b, but the moveout is not effected by the added $\delta = 0.2$.
(compared with Figure 5a). As expected and known, $\delta$ may cause considerable scattering, but does not effect the wave propagation of the data we see at the Earth surface due to the tradeoff with depth.

**THE CASE FOR DIMENSIONLESS PARAMETERS**

Despite our desire to have the inverted parameters share the same units, and as a result have similar influence on the data, the strong angular influence of the parameters and the clear bias in our acquisition (only along the Earth surface) make such a desire unattainable, and thus, the utilization of the Hessian (specifically, pseudo Hessian, Shin et al. (2001)) is very important to FWI. It is also important in BFGS to start the Hessian update with a an initial pseudo Hessian, to speed up convergence.

Thus, the need for the scaling part of the Hessian in multi parameter inversion does not diminish when we use parameters with the same units. These parameters have angular dependence and our bias in the acquisition coverage will evidently favor certain parameters. However, the dimensionless nature of the anisotropic combinations used here allows for a better control of its behavior. Since the phenomena responsible for anisotropy tend to be similar within a layer or a region, the anisotropy is expected to change mildly, and such mild variation can be better constrained with the anisotropic parameters. This preference is also highlighted in Oh and Min (2014) as they suggested using the Poisson’s ratio instead of the $S$-wave velocity in inverting elastic data. The parameter dependence shown here holds for the elastic case if the Poisson’s ratio is used to represent the shear wave velocity.
DISCUSSIONS

The background model considered here, for simplicity, is isotropic, but many of the results, as they are related to the physics of scattering regardless of background, still holds. The \( \eta \) perturbation does not exert much back scattering in the two parameter combinations addressed here. As a result, it does not share back scattering (high resolution information of such perturbations) with the data. It also means that we should be incapable of inverting for high scattering resolution of \( \eta \) \((\sim \frac{\omega v_0}{v_H})\) using any method. A similar phenomena is encountered in inverting for shear wave velocity using converted waves. However, like converted waves, multi mode data may help obtain high resolution S-wave information and may also help in the elastic case to invert for \( \eta \).

Considering the above arguments, for surface seismic data of conventional acquisition geometries, the most practical combination for FWI is \( v_H, \eta \) and \( \epsilon \). This is because the data dependency on the scattering potential of \( \eta \) is very mild at the angles covered by such an acquisition. Specifically, the transmission influence of \( \eta \) on diving waves is now absorbed by \( v_H \). So, we can use tomography or MVA to invert for the best smooth \( v_H \) (or NMO velocity) and \( \eta \). Then we use both smooth models as starting parameters in FWI to update only \( v_H \), or \( v_H \) and \( \epsilon \). In this case, \( \epsilon \) serves to improve the amplitude fit to compensate for our acoustic assumption. Afterwards, we may use the extracted high resolution horizontal velocity to improve on the \( \eta \) from tomography. Follow that by additional iterations of FWI to finalize \( v_H \). This process can also be combined in a single objective function that inverts for both at different scales.

Of course, most of the conclusions drawn here are applicable mainly to surface seismic data. Other acquisition set ups like vertical seismic profiling data or cross well may require
different parameter combinations. Our analysis, however, demonstrates that any combination benefits from having one of the parameters with an isotropic radiation pattern and the others describing the deviation from that. In the anisotropic case, such parametrization is given by a velocity and dimensionless parameters. Another insight that we might propagate to other acquisition setups is the identification of the data sensitivity to the short and long wavelength model components, in which we establish a combination that makes the dependences unique to the parameters.

**CONCLUSIONS**

Through the analysis of the data dependency on the long and short wavelength components of the anisotropy parameters, we managed to gain knowledge on the role of the parameters in the inversion process, and understand our limitations. These limitations are driven by the physics of the wave sensitivity to the long wavelength and scattering components of the various parameters that describe the medium for a given pixel of our model space as a function of propagation (or scattering) angle. The location of this pixel with respect to the acquisition surfaces determines our ability to resolve the parameters. With a combination given by the NMO velocity, $\eta$, and $\delta$, the inverted $\eta$ will be generally smooth with information coming mainly from diving waves. We also, in this combination, can only invert for the scattering $\delta$. Ultra low frequencies are required to obtain background $\delta$ information. The scattering $\delta$ is useful in absorbing any amplitude misfit that may come from fitting an acoustic model to an elastic Earth. In a combination given by the horizontal velocity, $\eta$ and $\epsilon$, $\eta$ has minor influence on FWI for diving waves and reflections, and thus, will allow us to focus the inversion on the horizontal velocity when an accurate smooth $\eta$ field is obtained from, for example, tomography. The parameter $\epsilon$ in this case is used to help fit the am-
plitude. Regardless of parameterization, the limiting factors of the resolution holds for all parameters as the diffraction tomography principle applies to any parameter perturbation.

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LIST OF FIGURES

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direction, represented by frequencies. It shows that our recorded seismic data depends on
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2 Diagrams depicting the normalized data dependency on the model parameters (for
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Figure 4 added to that 0.2.
Figure 1: A diagram depicting the data dependency on the velocity wavelength in the vertical direction, represented by frequencies. It shows that our recorded seismic data depends on the low frequency velocity variation for its geometrical shape, and thus, we use Tomography or MVA to constrain it. Meanwhile, the high frequency variations in the velocity produces the reflectivity in our data.

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Figure 2: Diagrams depicting the normalized data dependency on the model parameters (for a combination given by $v_n$, $\eta$, and $\delta$) as a function of vertical model wavenumber and scattering angle, $\theta$, a) for perturbations in velocity, b) in $\eta$, c) in $\delta$, and d) in $\delta$ for very low frequency data.

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Figure 3: Diagrams depicting the normalized data dependency on the model parameters (for a combination given by $v_h$, $\eta$, and $\epsilon$) as a function of vertical model wavenumber and scattering angle, $\theta$, for perturbations in the a) horizontal velocity, and b) $\eta$. 

Alkhalifah –
Figure 4: The $\eta$ or $\delta$ models used to generate the shot gathers in Figures 5a, 5b, and 5c for a homogenous $v_n$ of 1000 m/s.

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Figure 5: Shot gathers obtained from a homogenous \( v_n = 1000 \text{ m/s} \), with a) \( \delta = 0 \) and \( \eta \) given by Figure 4, b) \( \eta = 0.2 \) and \( \delta \) given by Figure 4, and finally c) \( \eta = 0.0 \) and \( \delta \) given by Figure 4 added to that 0.2.

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