Skeletonized Least Squares Wave Equation Migration

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SUMMARY

The theory for skeletonized least squares wave equation migration (LSM) is presented. The key idea is, for an assumed velocity model, the source-side Green's function and the geophone-side Green's function are computed by a numerical solution of the wave equation. Only the early-arrivals of these Green's functions are saved and skeletonized to form the migration Green's function (MGF) by convolution. Then the migration image is obtained by a dot product between the recorded shot gathers and the MGF for every trial image point. The key to an efficient implementation of iterative LSM is that at each conjugate gradient iteration, the MGF is reused and no new finite-difference (FD) simulations are needed to get the updated migration image. It is believed that this procedure combined with phase-encoded multi-source technology will allow for the efficient computation of wave equation LSM images in less time than that of conventional reverse time migration (RTM).

INTRODUCTION

Wave equation migration methods can be very expensive compared to diffraction stack migration methods. The conventional RTM approach requires a numerical solution to the wave equation for every source position. Hence, much research effort has been spent in reducing the costs of RTM.

We propose to reduce the costs of both standard RTM and least squares RTM by skeletonizing the migration Green's function (Schuster and Hu, 2000) $\Gamma(\mathbf{g},\mathbf{x},\mathbf{s},t) = G(\mathbf{x},t|\mathbf{s}) * G(\mathbf{x},t|\mathbf{g})$ into a skeletonized one $\tilde{\Gamma}(\mathbf{g},\mathbf{x},\mathbf{s},t) = \tilde{G}(\mathbf{x},t|\mathbf{s}) * \tilde{G}(\mathbf{x},t|\mathbf{g})$ with only a few non-zero samples; each sample is at the arrival time of an important early-arrival event (e.g., a primary reflection or a multiple arrival reflection). Thus the onerous storage costs (Zhou and Schuster, 2002; Cao, 2007) of a MGF trace with 1001 samples is reduced to a sparse trace with just 10 or so samples. The sparsity of this MGF can also reduce migration artifacts by eliminating unnecessary events for high-quality migration images.

Once the skeletonized MGF is saved, it does not need to be recomputed at each LSM iteration so this can result in almost two orders of magnitude reduction in cost for iterative least squares migration (Nemeth et al., 1999; Aoki and Schuster, 2009) or waveform inversion. If it is combined with phase-encoded multi-source technology (Dai and Schuster, 2009; Zhan et al., 2009; Krebs et al., 2009), the cost savings can be even greater.

This paper is divided into three parts: theory, numerical results, and conclusions. We briefly introduce the theory first, followed by the synthetic tests on the 2D SEG/EAGE salt model that demonstrate the effectiveness of this method. At the end, we draw some conclusions.

THEORY

The theory for generalized diffraction stack migration (GDM) was described by Schuster (2002), who reformulated the equations of RTM so that they can be reinterpreted as a GDM algorithm,

$$m_{mig}(\mathbf{x}) = \int \Gamma(\mathbf{g},\mathbf{x},\mathbf{s},t) \otimes d(\mathbf{g},\mathbf{s},t)|_{t=0}\, d\mathbf{g}\, d\mathbf{s}\, dt, \qquad (1)$$

with the MGF defined as

$$\Gamma(\mathbf{g},\mathbf{x},\mathbf{s},t) = G(\mathbf{x},t|\mathbf{s}) * G(\mathbf{x},t|\mathbf{g}), \qquad (2)$$

where $*$ denotes temporal convolution and $\otimes$, together with $t = 0$, represents the correlation at zero-lag time (which is equivalent to a dot product in the data coordinates). The $d(\mathbf{g},\mathbf{s},t)$ term represents the second time derivative of the trace at the geophone point $\mathbf{g}$ with the source point at $\mathbf{s}$, while the $G(\mathbf{x},t|\mathbf{s})$ and $G(\mathbf{x},t|\mathbf{g})$ terms represent the scattered Green's functions which, respectively, propagate the energy from the surface source point at $\mathbf{s}$ and the geophone point $\mathbf{g}$ to the subsurface trial image point $\mathbf{x}$. The term $\Gamma(\mathbf{g},\mathbf{x},\mathbf{s},t)$ acts as the migration operator or focusing kernel (Schuster, 2002) which migrates the reflection in $d(\mathbf{g},\mathbf{s},t)$ to the trial image point $\mathbf{x}$. It is obtained by a FD solution to the wave equation with a point source $\mathbf{s}$ on the surface and a scattering point at the image point $\mathbf{x}$, and convolving this source-side Green's function $G(\mathbf{x},t|\mathbf{s})$ with the geophone-side Green's function $G(\mathbf{x},t|\mathbf{g})$.

A simple diagram shown in Figure 1 illustrates the difference between the diffraction stack migration and GDM operators plotted in data-space coordinates. Figure 1a shows the traditional migration curve for diffraction stack migration which is also known as Kirchhoff migration. The usual interpretation is that the migration image at $\mathbf{x}$ is given by summing the trace amplitudes along the hyperbola, i.e., the migration image is the dot product of the recorded shot gathers with the migration Green's function (a single hyperbolic curve shown in Figure 1a). Figure 1b illustrates the idea behind GDM: take the dot product of the recorded shot gathers with the generalized migration Green's function $\Gamma(\mathbf{g},\mathbf{x},\mathbf{s},t)$. Note that all events are included in the generalized MGF operator such as direct waves, multiples, reflections and diffractions as well.

But the major problem with the above approach is that the migration Green's function $\Gamma(\mathbf{g},\mathbf{x},\mathbf{s},t)$ is a five-dimensional matrix with the dimension size determined by the model size, the number of sources and geophones, and the number of samples within a trace. It means that $\Gamma(\mathbf{g},\mathbf{x},\mathbf{s},t)$ is too expensive to be stored. To reduce the the cost of I/O and storage of the $\Gamma(\mathbf{g},\mathbf{x},\mathbf{s},t)$, we only store the early-arrivals of $G(\mathbf{x},t|\mathbf{s})$ followed by skeletonization to reduce the size of the MGF by at least two orders of magnitude.

The skeletonized least squares GDM algorithm is as follows.

1. Compute the Green's functions $G(\mathbf{x},t|\mathbf{s})$ by a numerical solution to the wave equation for a point source at

source position $\mathbf{s}$ and trial image point at $\mathbf{x}$. Since $\mathbf{s}$ occupies the same positions as $\mathbf{g}$, then $G(\mathbf{x},t|\mathbf{g})$ is considered the same as $G(\mathbf{x},t|\mathbf{s})$.

2. Skeletonize $G(\mathbf{x},t|\mathbf{s})$ to $\tilde{G}(\mathbf{x},t|\mathbf{s})$. Early-arrivals are used and each important early-arrival event is replaced by a single time sample after skeletonization. In this way, a calculated Green's function trace with 501 samples is reduced to a sparse trace with about 25 samples.
3. The skeletonized Green's function $\tilde{G}(\mathbf{x},t|\mathbf{s})$ is associated with the source at $\mathbf{s}$ and $\tilde{G}(\mathbf{x},t|\mathbf{g})$ is that for the geophone at $\mathbf{g}$. Those two Green's functions are convolved to generate the MGF:

$$\tilde{\Gamma}(\mathbf{g},\mathbf{x},\mathbf{s},t) = \tilde{G}(\mathbf{x},t|\mathbf{s}) * \tilde{G}(\mathbf{x},t|\mathbf{g}). \tag{3}$$

4. The MGF $\tilde{\Gamma}(\mathbf{g},\mathbf{x},\mathbf{s},t)$ is a Kirchhoff-like kernel that describes, for a single shot gather, pseudo-hyperbolas of multi-arrivals in $\mathbf{g}-t$ space. The reflection energy in a recorded shot gather $d(\mathbf{g},\mathbf{s},t)$ is then summed along such pseudo-hyperbolas to give the migration image,

$$m_{mig}(\mathbf{x}) = \sum_{\mathbf{s}}\sum_{\mathbf{g}}\sum_{t}\tilde{\Gamma}(\mathbf{g},\mathbf{x},\mathbf{s},t)d(\mathbf{g},\mathbf{s},t). \tag{4}$$

5. The above $\tilde{\Gamma}(\mathbf{g},\mathbf{x},\mathbf{s},t)$ is used for iterative LSM by a conjugate gradient method.

The advantages of skeletonized least squares GDM are that 1). the high-frequency approximation of Kirchhoff migration is largely not needed; 2). the storage requirement for MGF is reduced by more than two orders of magnitude compared to storing every sample in a calculated MGF; 3). these MGFs do not need to be recalculated for each LSM iteration; 4). inclusion of just a few important early-arrival events can significantly reduce artifacts seen in conventional RTM images.

In contrast, the main drawback of skeletonized least squares GDM is that the choice of important events is somewhat arbitrary and so can lead to missing important information in the migration operator. However, the migration operator is only as accurate as our knowledge of the subsurface velocity model so important events might be just the early-arrivals.

NUMERICAL RESULTS

The skeletonized least squares GDM method is tested on synthetic data associated with the 2D SEG/EAGE salt model (see Figure 2). These traces were computed by FD solutions to the 2D acoustic wave equation. The data include 162 shot gathers with a peak-frequency of 15 Hz, with 176 traces in each shot gather, the shot and geophone intervals are 97.6 m and 24.4 m, respectively. To save computation time, we down-sampled the traces from 4001 time samples with a time interval of 0.001 to 1001 time samples with a time interval of 0.004 s in each trace.

A simple filtering plus thresholding scheme is used to skeletonize the Green's function. Convolution of the skeletonized traces (early-arrivals windowed with a window length of 5 periods) of both the source-side and the geophone-side Green's functions is computed and the MGFs are saved to disk.

Figure 3a shows the migration of the 2D SEG/EAGE data set using the GDM scheme. The shallow structures are contaminated by the large amplitude artifacts due to the strong reflection at the ocean bottom. Artifacts are also clearly noticeable around the up boundary of the salt at 1.5 km. Figure 3b shows the result after applying a low-cut filter. The major part of the artifacts are removed.

A preconditioned conjugate gradient LSM method is then implemented with Figure 3a as the starting model. All of the migration Green's functions are available, so no new simulations are needed to get the updated migration image at each iteration. This is the key point for an efficient implementation of iterative LSM, and the result after 10 iterations is shown in Figure 3c. These strong artifacts are successfully eliminated by the least squares iterations. Compared to Figure 3b, the structure below the salt dome and faults are more clearly resolved in Figure 3c. This is demonstrated in Figure 4 which shows zoom views of Figure 3b and Figure 3c.

CONCLUSIONS

The theory for skeletonized least squares wave equation migration is presented. The key idea is to solve the wave equation only once to get the Green's functions. These Green's functions are skeletonized into a few samples and saved to disk. The skeletonized MGF is obtained through convolution of source-side and geophone-side Green's functions followed by a filtering plus thresholding scheme. Both storage and I/O costs are greatly reduced compared to saving the entire MGF, and the GDM image is computed by a dot product of the MGF with the recorded shot gathers. The outstanding feature of skeletonized least squares GDM is that in the least squares mode, the MGF is reused and does not require new solutions to the wave equation. Hence, least squares GDM might not be significantly more costly than standard RTM. If this procedure is combined with multi-source technology, we believe that the cost of it is much less than that of conventional RTM. This method can also be used with one-way wave equation methods such as phase-shift migration and is applicable for rapid migration velocity analysis as well.

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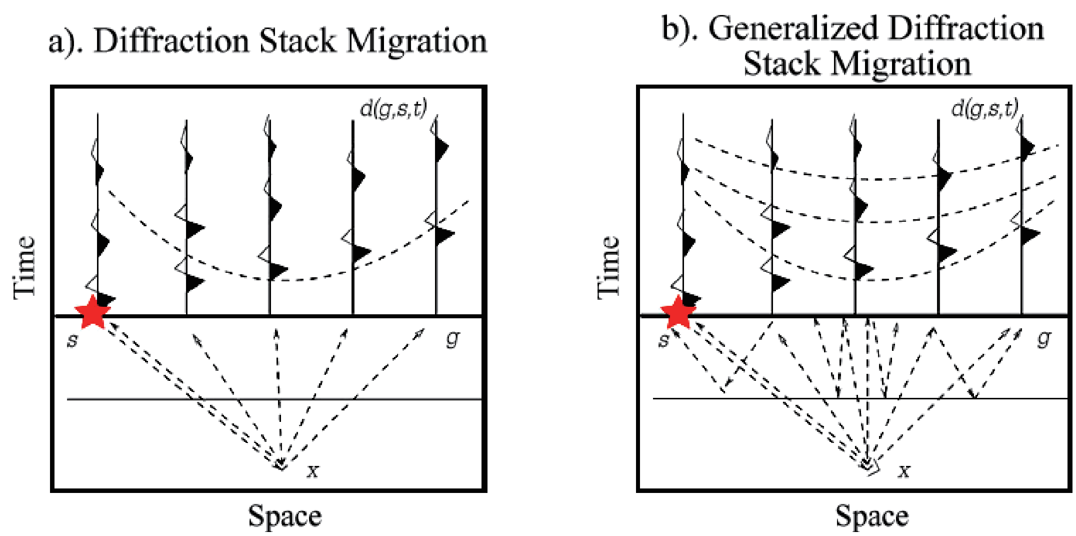

Figure 1: a) Simple diffraction stack operator (dashed curve) and data, which only contains first arrival scattering information. b) Generalized diffraction stack operator (dashed curves) which contains all events in the migration model, including multiples, diffractions and reflections (Schuster, 2002).

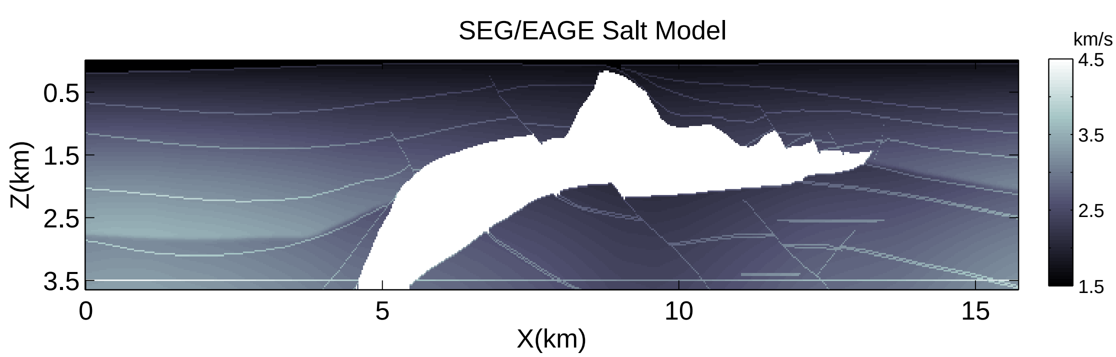

Figure 2: The 2D SEG/EAGE velocity model.

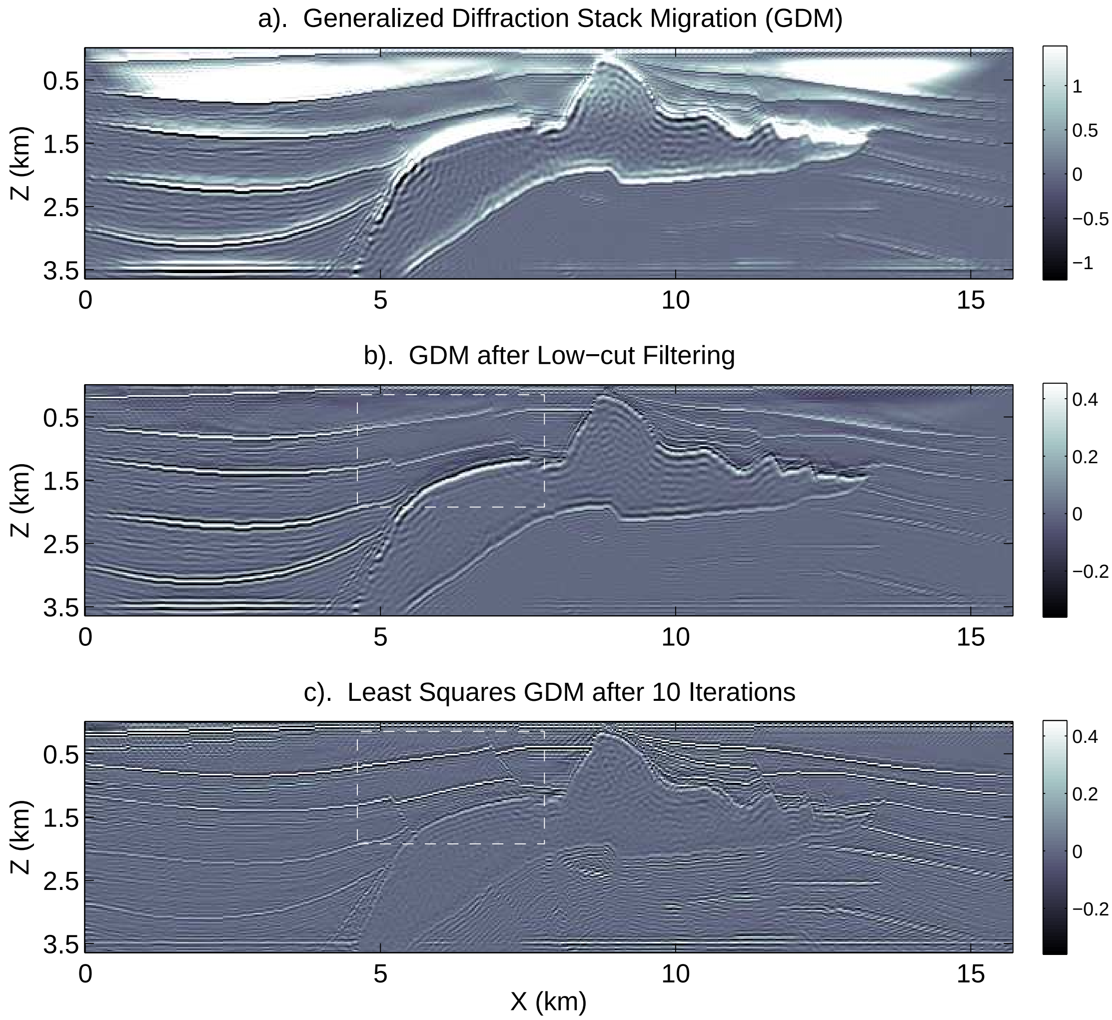

Figure 3: Migration results for all 162 shot gathers of the SEG/EAGE salt model: a) GDM result which is identical to standard RTM image; b) low cut filtered version of a); c) skeletonized least squares GDM image after 10 iterations.

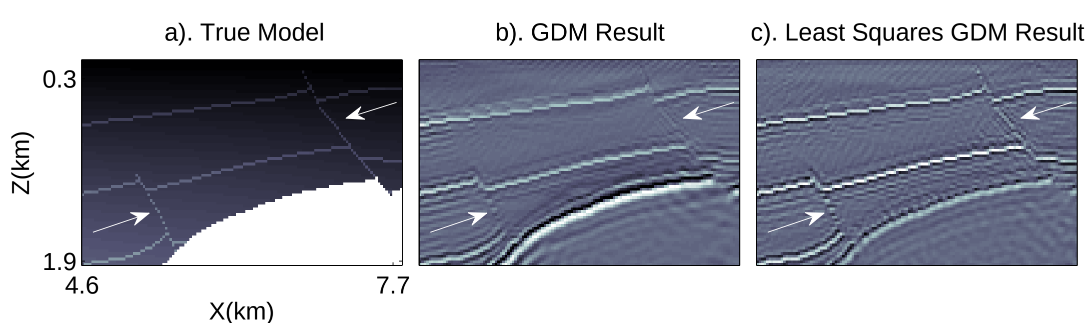

Figure 4: Zoom view of Figure 3.

EDITED REFERENCES

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