Waveform inversion of lateral velocity variation from wavefield source location perturbation

Yunseok Choi and Tariq Alkhalifah, King Abdullah University of Science and Technology*

Summary

It is challenge in waveform inversion to precisely define the deep part of the velocity model compared to the shallow part. The lateral velocity variation, or what referred to as the derivative of velocity with respect to the horizontal distance, with well log data can be used to update the deep part of the velocity model more precisely. We develop a waveform inversion algorithm to obtain the lateral velocity variation by inverting the wavefield variation associated with the lateral shot location perturbation. The gradient of the new waveform inversion algorithm is obtained by the adjoint-state method. Our inversion algorithm focuses on resolving the lateral changes of the velocity model with respect to a fixed reference vertical velocity profile given by a well log. We apply the method on a simple-dome model to highlight the methods potential.

Introduction

The goal of full waveform inversion (FWI) is to find the high-resolution velocity model that produces synthetic data resuming the observed data from the field. However, because of the severe nonlinearity associated with the objective function due to the complex Earth reflectivity, converging to an accurate model is always challenging. The objective function associated with diving waves tends to be more linear, and thus, it is used more often with FWI. However, the recorded diving waves do not penetrate deep due to the limited offset range. As a result, it is still a challenge to accurately define the deep part of velocity model. Specifically, the deep part of inverted model is usually blurred. Another weakness is given by the attenuation of the energy (or amplitude) of the modeled wavefield propagating to the deep part of velocity model. The updating of the deep part of velocity model also depends on that of the shallow part. Small error in the shallow part could seriously affect the updating velocity in the deep part. Even though the Hessian matrix (or an approximate of it) can mitigate the effect of geometrical spreading effect to some degree, waveform inversion still generates a roughly or smoothly defined image in deep part.

One solution to this problem is to use the lateral velocity variation with well-log data. In this case, the velocity model is updated in lateral direction from the reference data (well-log data). Thus, the deep part of the velocity model could be precisely defined through the inversion process.

The lateral velocity variation, or what is referred to as the derivative of velocity with respect to the horizontal distance, can be obtained from the wavefield variation associated with the lateral shot location perturbation (WVSP). WVSP was introduced by Alkhalifah (2010, 2011) to extrapolate the adjacent-shot wavefields from a given shot wavefield. He first assumed a horizontal shift in the velocity model for a fixed shot location, which is equivalent to a horizontal shift in the source location but in the opposite direction for a fixed velocity model. Based on this assumption, a new wave equation is derived for WVSP with a virtual source (new source function). The obtained WVSP is used to extrapolate the adjacent-shot wavefield from a given shot wavefield using a Taylor's series expansion. On the other hand, the virtual source of the new wave equation depends mainly on the derivative of velocity with respect to the horizontal distance (DVHD), and thus, for no lateral velocity variation the virtual source is zero. This allows us to invert for lateral velocity variation from WVSP. Given the lateral velocity variation from a well, the deep velocity structure could be precisely recovered.

In this paper, we develop a new waveform inversion algorithm to find the parameter, DVHD, from WVSP. We, first, construct the misfit function using WVSP and derive the expression of the gradient of the objective function. Since the wave equation modeling for WVSP share the same modeling operator with the original wave equation, the adjoint-state method can be employed for the new waveform inversion algorithm (Tarantola, 1984; Plessix, 2006). In the inversion process, the estimated DVHD is integrated and then added to the background velocity to update the velocity model. We will describe the theory in the next section and then show the examples for the synthetic data.

Theory

Alkhalifah (2010, 2011) expressed the wave equation using a slowness term instead of velocity and then derived a new wave equation for WVSP from the original wave equation. Here, we start by expressing the 2D wave equation using a velocity parameter for convenience:

$$\frac{\partial^2 u_i(x,z)}{\partial t^2} = v^2(x,z)\Delta u_i(x,z) + f_i(x,z,t)\,, \qquad (1)$$

where u_i is the wavefield for i-th shot, v is the velocity, and f_i is the source term for i-th shot. The new wave equation for WVSP is obtained by taking the derivative of equation 1 with respect to the horizontal distance (or coordinate x) expressed as

DOI http://dx.doi.org/10.1190/segam2013-0494.1

$$\frac{\partial^2 D_i(x,z)}{\partial t^2} = v^2(x,z)\Delta D_i(x,z) + 2v(x,z)\Delta u_i(x,z)\frac{\partial v(x,z)}{\partial x}, \quad (2)$$

where $D_i = \partial u_i / \partial x$ (WVSP). Equation 2 shares the same operator with equation 1 but with a different source term, which means WVSP is obtained from the conventional modelling method, but with a source term that depends on the background velocity and DVHD.

We generate WVSP using equation 2. Figure 1a shows a part of smoothed SEG/EAGE overthrust model and Figure 1b shows the corresponding derivative of velocity with respect to the horizontal distance (DVHD). Figure 2a shows the generated WVSP using equation 2. No event associated with the direct waves is shown in Figue 2a. For the sake of contrast, in Figure 2b, we display WVSP obtained using the finite-difference method, described later. In Figure 2b, the edge effect is shown. Figure 2c shows the difference of the Figures 2a and 2b. For the finite-difference method, the effect of the non-perfect boundary implementation is severe, since it measures the difference of the modelled seismograms of adjacent shots after shifting. On the other hand, using equation 2 to compute WVSP proves to be less sensitive to the non-perfect boundary implementation.

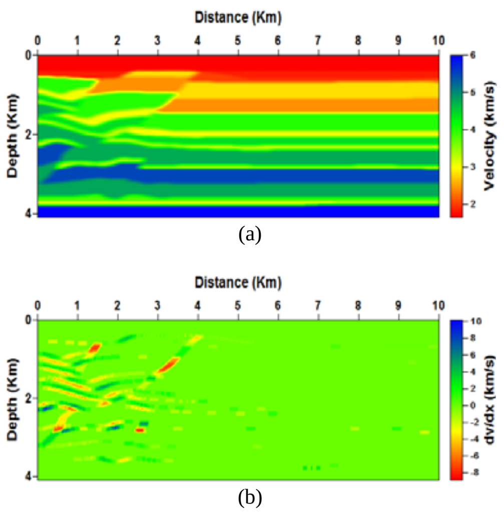


Figure 1: (a) A part of SEG/EAGE over-thrust model after smoothing, and (b) the corresponding derivative of velocity with respect to the horizontal distance (DVHD).

For the new waveform inversion, we construct the objective function using WVSP, expressed for one shot, for convenience, as:

$$E = \left\| D_i^m - D_i^d \right\|_2^2, \quad (3)$$

where D_i^m and D_i^d are WVSPs obtained from the modelled data and observed data for i-th shot, respectively. If we assume the shots are located densely and regularly on a flat surface with lateral spacing dx, D_i^m and D_i^d are obtained using the difference method as

$$D_i^m = \frac{u_i - u_{i-1}}{dx} \text{ and } D_i^d = \frac{d_i - d_{i-1}}{dx}, \quad (4)$$

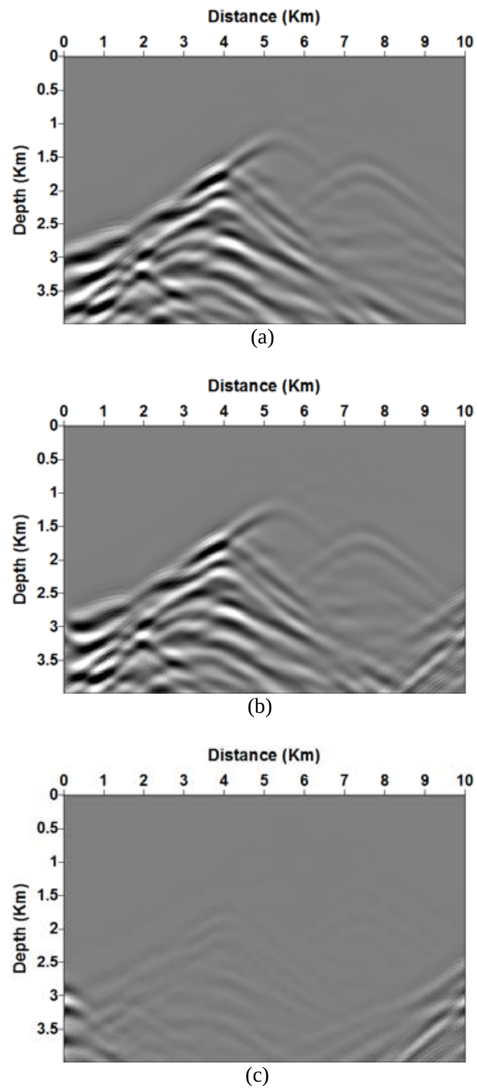


Figure 2: (a) WVSP using equation 2, (b) WVSP using the difference method, and (c) the difference between WVSPs of (a) and (b). All figures use the same grey scale in the display.

DOI http://dx.doi.org/10.1190/segam2013-0494.1

where d_i are the observed (recorded) data for i-th shot. Note that, in both D_i^m and D_i^d, one seismogram in the numerator must be shifted in horizontal direction so that the two seismograms in the numerator have the same shot position before computing the difference. D_i^m in equation 4 can be also calculated from modelling based on equation 2. Through the inversion process, we want to estimate the parameter DVHD ($p(x,z) = \partial v(x,z)/\partial x$) to update the velocity model. To obtain this parameter through inversion, we need to calculate the gradient of the new objective function with respect to DVHD (or *p*). The gradient is given by taking the derivative of equation 3 with respect to DVHD:

$$\frac{\partial E}{\partial p} = \left(\frac{\partial D_i^m}{\partial p}\right)^T \cdot (D_i^m - D_i^d)\ , \qquad (5)$$

where the superscript T in equation 5 stands for the transpose of a matrix. To calculate the gradient in equation 5 using the adjoint-state method, we first take the derivative of equation 2 with respect to DVHD:

$$\frac{\partial^2 D_{i,p}^m}{\partial t^2} = v^2 \Delta D_{i,p}^m + 2v\Delta D_i^m \frac{\partial v}{\partial p} + 2v\Delta u_i + 2\frac{\partial v}{\partial p}(\Delta u_i)\, p + 2v\left(\Delta \frac{\partial u_i}{\partial p}\right) p\ , \qquad (6)$$

where $D_{i,p}^m = \partial D_i^m / \partial p$. Equation 6 reflects the scattering theory or the Born approximation. The last four terms in the right side of equation 6 are used as forward modelled wavefields in the adjoint state method. Based on the adjoint-state method and equation (6), DVHD is obtained by back-propagating the residual of $D_i^m - D_i^d$ (WVSP) and taking the zero-lag cross-correlation of the back-propagated wavefield and forward-modelled source wavefield (the last four terms in the right hand side of equation 6). Once DVHD is obtained, the velocity update is calculated by integrating DVHD as follows:

$$v_{x,z}^n = v_{x,z}^0 + \int_{x_0}^{x} p_{x,z}^n dx\ , \qquad (7)$$

where $v_{x,z}^0$ is the background velocity model obtained from the well log information, $v_{x,z}^n$ is the velocity model at n-th iteration, and $p_{x,z}^n$ is DVHD obtained at n-th iteration. The obtained $v_{x,z}^n$ and $p_{x,z}^n$ are used for the next iteration procedure. From equation 7, the term $\partial v/\partial p$ in equation 6 is given by dx (horizontal grid interval) and consequently equation 6 is approximated by omitting the last term as:

$$\frac{\partial^2 D_{i,p}^m}{\partial t^2} \approx v^2 \Delta D_{i,p}^m + 2v\Delta D_i^m (dx) + 2v\Delta u_i + 2(\Delta u_i)\, p(dx)\ . \qquad (8)$$

We use the last three terms of the right hand side of equation 8 as a weighted forward modeled wavefield to calculate the gradient in equation 5 instead of the last four terms of the right side of equation 6.

We scale the gradient of the objective function (equation 5) with the pseudo-Hessian matrix (Shin et al., 2001) and use the conjugate-gradient method to find the optimum search direction (Gill et al., 1981).

The flow of the inversion procedure is as follow:

1. *We calculate the gradient (expressed as g) of the objective function in equation 5 from the velocity and DVHD of the previous iteration (v^{itr-1} and p^{itr-1}) using the pseudo-Hessian matrix and the conjugate-gradient method.*
2. *We update the current DVHD as follows: $p^{itr} = p^{itr-1} + \alpha g$, where α is the step length for the update of p^{itr} .*
3. *We update the velocity model by integrating the current DVHD:*

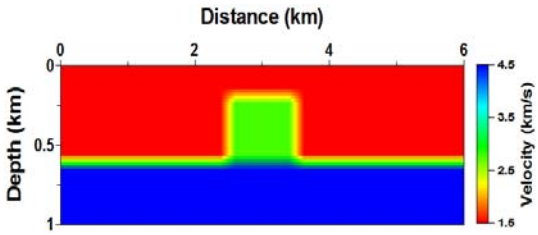
   $$v_{x,z}^{itr} = v_{x,z}^0 + \int_{x_0}^{x} p_{x,z}^{itr} dx\ ,$$

   where the background velocity profile $v_{x,z}^0$ is not changed through the inversion procedure.
4. *We repeat 1~3 until the convergence is achieved.*

Examples

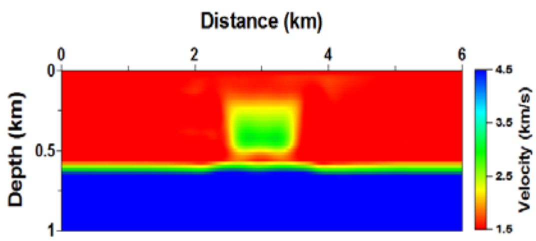
To test our algorithm, we generate synthetic data for the simple dome model shown in Figure 3a. The number of shots is 260 with an interval of 20 m and the receivers are placed at all grid points on the surface with an interval of 20 m. We use the standard time-domain finite-difference modeling technique to generate the modeled data and to calculate the adjoint state. The recording time is 4 seconds and the maximum frequency of the source wavelet is 8 Hz. We set the reference velocity profile (representing the well-log information in real applications) to the depth profile of velocity at the location 5.2 Km in the true velocity model (Figure 3b). We adopt the multi-scale approach to mitigate the local minima problem and the multi-sources algorithm to reduce the computational cost. After 600 iterations, we obtain the final inverted velocity model shown in Figure 3c. The lateral velocity variation is somewhat well defined, but there is still room for improvement in resolution. Figure 4 compares DVHDs estimated from the true model and inverted model. There are two large anomalies in the true one. DVHD of the inverted model shows the important feature of the two anomalies but their shapes do not perfectly coincide.

DOI http://dx.doi.org/10.1190/segam2013-0494.1



(a)

(b)



(c)

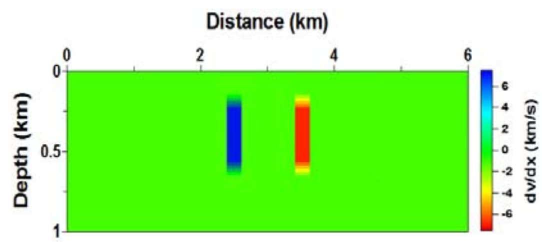
Figure 3: (a) A simple-dome velocity model, (b) the background velocity model from the reference depth profile and (c) inverted model after 600 iterations.

Discussion

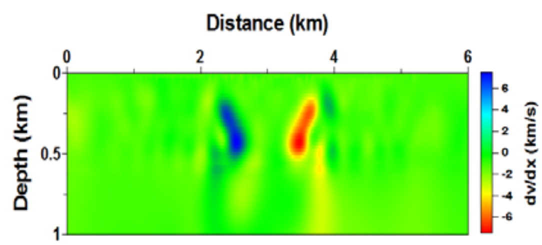
Because of the weakness of conventional FWI in inverting for reflection data, the vertical resolution, especially in the deep, of the velocity model tend to be compromised. In many FWI inversion applications well log information is introduced as a priori information that can influence the inversion results. If the well information produces admits velocities that do not fit the FWI (usually resolution wise), we end up with a smoother representation of the well log velocity at the well log. In the approach developed here, we decouple the well log information (set as a boundary) from the full waveform inversion. Specifically, we only invert for the lateral velocity variation from a fixed well log reference. Thus, the vertical resolution of the well log velocity is maintained as it is decoupled from the inversion, and the inversion is focused on the lateral resolution. The location of the well log, as we saw in the example, does not affect the lateral resolution prospective as both flanks of the dome were resolved with similar accuracy.

Conclusions

We develop a new waveform inversion algorithm to estimate the lateral velocity variation by inverting lateral perturbations in the source location of the wavefield. We can update the subsurface velocity by integrating the estimated DVHD and then adding it to the background velocity. Velocity updating using DVHD has an advantage of recovering the horizontal variation of velocity from a fixed velocity depth-profile, usually given by well-log information. We also derive the gradient of the new objective function using the adjoint-state method for waveform inversion. The numerical example shows the validity of our algorithm.



(a)



(b)

Figure 4: (a) DVHD estimated from the true model in Figure 3a and (b) DVHD estimated from the inverted model in Figure 3c.

Acknowledgments

We are grateful to the King Abdullah University of Science and Technology for the financial support.

DOI http://dx.doi.org/10.1190/segam2013-0494.1

http://dx.doi.org/10.1190/segam2013-0494.1

EDITED REFERENCES

Note: This reference list is a copy-edited version of the reference list submitted by the author. Reference lists for the 2013 SEG Technical Program Expanded Abstracts have been copy edited so that references provided with the online metadata for each paper will achieve a high degree of linking to cited sources that appear on the Web.

REFERENCES

Alkhalifah, T., 2010, Acoustic wavefield evolution as a function of source location perturbation: Geophysical Journal International, **183**, no. 3, 1324–1331, http://dx.doi.org/10.1111/j.1365-246X.2010.04800.x.

Alkhalifah, T., 2011, Wavefield dependency on virtual shifts in the source location: Geophysical Prospecting, **59**, no. 4, 627–634, http://dx.doi.org/10.1111/j.1365-2478.2010.00944.x.

Gill, P. E., W. Murray, and M. Wright, 1981, Practical optimization: Academic Press, Inc.

Plessix, R.-E., 2006, A review of the adjoint-state method for computing the gradient of a functional with geophysical applications: Geophysical Journal International, **167**, no. 2, 495–503, http://dx.doi.org/10.1111/j.1365-246X.2006.02978.x.

Shin, C., S. Jang, and D. J. Min, 2001, Improved amplitude preservation for prestack depth migration by inverse scattering theory: Geophysical Prospecting, **49**, no. 5, 592–606, http://dx.doi.org/10.1046/j.1365-2478.2001.00279.x.

Tarantola, A., 1984, Inversion of seismic reflection data in the acoustic approximation: Geophysics, **49**, 1259–1266, http://dx.doi.org/10.1190/1.1441754.