The possibilities of compressed sensing based migration

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Summary

Linearized waveform inversion or Least-square migration helps reduce migration artifacts caused by limited acquisition aperture, coarse sampling of sources and receivers, and low subsurface illumination. However, least-square migration, based on L2-norm minimization of the misfit function, tends to produce a smeared (smoothed) depiction of the true subsurface reflectivity. Assuming that the subsurface reflectivity distribution is a sparse signal, we use a compressed-sensing (Basis Pursuit) algorithm to retrieve this sparse distribution from a small number of linear measurements. We applied a compressed-sensing algorithm to image a synthetic fault model using dense and sparse acquisition geometries. Tests on synthetic data demonstrate the ability of compressed-sensing to produce highly resolved migrated images. We, also, studied the robustness of the Basis Pursuit algorithm in the presence of Gaussian random noise.

Introduction

Standard migrated images suffer from migration artifacts and low spatial resolution due to poor acquisition geometry of seismic experiments. Least-square migration is applied to reduce the artifacts and enhance the resolution of subsurface migrated images (LeBras and Clayton, 1988; Nemeth et al., 1999). Least-square migration is a linearized waveform inversion that provides an approximate solution for subsurface reflectivity distributions (Crase et al., 1986; Landa et al., 1989; Snieder et al., 1990; Roth and Tarantola, 1992).

Fomel and Guitton (2006) showed that adding a regularization term to the objective function (i.e. data misfit) tends to reduce the artifacts and sharpen the subsurface image. However, most regularization techniques assume that the inverted model parameters are smooth and continuous resulting in a smooth version of the true reflectivity distribution that is sparse and discontinuous.

Compressed sensing (CS) is widely used to solve linear systems when the desired signal (or solution) is sparse (Eldar and Kutyniok, 2012). It relies on the fact that many signals or images can be well represented by a linear combination of suitable basis or dictionaries (e.g. Fourier or wavelet) with only a small number of non-zero coefficients (Donoho, 2006). Hence, with only a small number of linear measurements, compressed sensing techniques should be able to reconstruct a compressed version of the true signal. In this abstract, we formulate the seismic imaging problem as a Basis Pursuit denoise (BPDN) problem (Van den Berg and Friedlander, 2008) to retrieve the sparsest reflectivity distribution possible that explains the recorded data. Our objective is to test if we can accurately recover a highly resolved subsurface image using a smaller number of measurements by deploying fewer sources and receivers. This could lead to huge savings in both data acquisition and processing time. We also show that compressed sensing is robust and provide spatially focused migrated images with remarkably reduced migration artifacts in the presence of Gaussian random noise.

Theory

The Kirchhoff forward modeling operator $\mathbf{L}$ maps the subsurface reflectivity distribution to scattered seismic data. Generating synthetic seismic data by Kirchhoff modeling is represented by a matrix-vector multiplication as follows:

$$\mathbf{d} = \mathbf{Lm}, \quad (1)$$

where $\mathbf{d}$ is the modeled scattered seismic data and $\mathbf{m}$ is the true subsurface reflectivity distribution. A standard migrated image is obtained by applying the adjoint of the forward modeling operator to the scattered data (Claerbout, 1992):

$$\mathbf{m_{mig}} = \mathbf{L^T d}, \quad (2)$$

where $\mathbf{m_{mig}}$ is the migrated image. Substituting the expression for $\mathbf{d}$ in equation (1) into equation (2) yields:

$$\mathbf{m_{mig}} = \mathbf{L^T L m}, \quad (3)$$

where $\mathbf{L^T L}$ is known as the model resolution matrix, which measures how smeared is the migrated image. Therefore, the migrated image is a linear combination of the true reflectivity model. A more accurate solution is obtained by least-square migration, which results from solving the normal equation. Hence, the migrated image is estimated as following:

$$\mathbf{m_{mig}} = \mathbf{(L^T L)^{-1} L^T d} \quad (4)$$

Gradient-based algorithms are often used to solve this equation iteratively where data residuals are imaged to obtain model updates. Although least-square imaging reduces the migration artifacts and helps focus the migrated image, L2-norm minimization tends to produce a smeared depiction of the true reflectivity model.

Since the subsurface reflectivity distribution is known to be sparse and discontinuous (especially with respect to the wavelengths we deal with), compressed sensing can be utilized to produce a highly resolved (i.e. compressed) migrated image from the acquired data. The sparsest reflectivity distribution is found by solving the following optimization problem (Donoho, 2006):

$$\min \|\mathbf{m}\|_0 \text{ subject to } \mathbf{Lm} = \mathbf{d}, \quad (5)$$

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where $\|\mathbf{m}\|_0$ counts the number of non-zero elements in the vector **m**. This optimization problem has a non-convex objective function and it is NP-hard and computationally intractable. The L0-norm can be relaxed to the L1-norm giving a convex optimization problem that is computationally tractable (Chen et al., 2001):

$$\min \|\mathbf{m}\|_1 \text{ subject to } \mathbf{Lm} = \mathbf{d}. \quad (6)$$

This L1-minimization problem is known as the Basis Pursuit problem and is used to reconstruct sparse signals. Chen et al. (2001) showed that this L1-minimization problem can be cast as a linear program. However, in the presence of noise, we cannot satisfy the constraint exactly. In other words, there is no feasible reflectivity model **m** that perfectly maps to the scattered data **d**. Hence, we reformulate the Basis Pursuit problem into the Basis Pursuit denoise (BPDN) problem which is defined as (Van den Berg and Friedlander, 2008):

$$\min \|\mathbf{m}\|_1 \text{ subject to } \|\mathbf{Lm} - \mathbf{d}\|_2 < \sigma, \quad (7)$$

where the tolerance σ is a non-negative small scalar. Van den Berg and Friedlander (2007) presented an algorithm that solves this problem for any tolerance value. They showed that there is a trade-off curve between the two objectives: minimizing the least-square fit and minimizing the L1-norm of the solution that promotes sparsity. Hence, giving more weight to the data fitting can only happen at the expense of getting a less sparse solution and vice versa.

This curve is known as the Pareto-optimal curve. Van den Berg and Friedlander (2008) showed that this curve is convex and continuously differentiable over all points of interest and their root-finding algorithm can find an arbitrary point on the Pareto curve.

Van den Berg and Friedlander (2007) basically cast the BPDN problem as a problem of finding the root of a single-variable nonlinear equation $\phi(\tau) = \sigma$ parameterizing the Pareto curve. At each iteration, an estimate of τ is used to define a convex optimization problem whose solution yields derivative information used by a Newton-based root-finding algorithm to probe the Pareto curve and find the optimal trade-off between the two competing objectives. SPGL1 is a free Matlab package, which implements their algorithm that solves the BPDN problem, and we use it to solve the Kirchhoff imaging problem.

Examples

In the first example, we study whether we can reconstruct the subsurface reflectivity distribution of a fault model using a smaller number of measurements. The velocity model and the true reflectivity model are shown in Figure 1. We generated 75 shot gathers using the Kirchhoff forward modeling operator along the Earth surface using a Ricker wavelet with a 20-Hz peak frequency as a source. The acquisition geometry is composed of 75 sources and 75 receivers placed along the Earth surface with a spacing interval of 10 meters. The standard migrated image obtained by applying the adjoint operator to the scattered data is shown in Figure 1 (bottom). It is a heavily blurred representation of the true subsurface reflectivity distribution. Clearly, the fault plane cannot be easily delineated and poor illumination near the edges of the model deteriorates the quality of the migrated section.

Implementing least-square migration aims to rectify these problems caused by the limited acquisition aperture, coarse source-receiver sampling, and poor subsurface illumination, especially near the edges. After 30 iterations of gradient descent, Least-square migration helps suppress migration artifacts and enhance the spatial resolution as shown in Figure 2 (top). As a result, the fault plane is more pronounced and can be delineated more easily. However, the solution is still smooth and smeared due to the nature of L2-minimization.

Solving the BPDN problem provides a highly resolved subsurface image with more accurate positioning and amplitude of the events as shown in Figure 2 (middle). The challenge is to find out if we can reconstruct an accurate representation of the subsurface reflectivity model using a fewer number measurements.

For this reason, we significantly reduced the number of sources and receivers to only 15 instead of 75 in the previous case (i.e. utilizing only 20% of the available data). The spacing interval is now 50 meters compared to 10 meters in the dense geometry. The compressed sensing solution using the new geometry is shown in Figure 2 (bottom) and it is remarkably better than the standard migrated image and the least-square image using the dense geometry shown in Figure 1 (bottom) and Figure 2 (top), respectively. The compressed-sensing solution using the sparse geometry is quite comparable with the compressed-sensing one using the dense acquisition shown in Figure 2 (middle) even though the image is slightly blurred and the amplitude information of the scatterers is less accurate.

Hence, with only 20% of the available data and resources, compressed sensing managed to retrieve a highly focused migrated image. The computational time is reduced significantly compared to computing solutions with the dense acquisition geometry.

Another question of interest is to study whether sparse reconstruction is robust in the presence of white Gaussian random noise. What would be the effect on the image quality when the data is noisy? To answer these questions, we added white Gaussian random noise to the synthetic noise-free scattered data used in the previous examples with the dense acquisition geometry. The white Gaussian random noise distribution has a zero mean and a variance of γ^2 and about 68% of the noise lies within one standard deviation (1γ) of the mean. We used the compressed-sensing algorithm to image different datasets with standard deviations γ equals to 5, 10, and 40. Given that the signal-

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to-noise ratio (SNR) is the average power of the signal divided by the average power of the noise, these datasets have signal-to-noise ratios of 9, 2.27, and 0.142, respectively. The migrated images for these different variances are shown in Figure 3. In the case of the smallest standard deviation, the compressed sensing solution is quite similar to the compressed sensing solution in the noise-free case. However, when the standard deviation is equal to 10, the solution tends to be smoother but it is not as heavily blurred as the noise-free least-square solution in Figure 2 (top) and has more accurate amplitude information. The robustness of the compressed sensing algorithm deteriorates dramatically as shown in Figure 3 (bottom) when the average noise power is larger than the average power of the signal (SNR ~ 0.142). This represents an extreme case where the forward modeling operator fails to explain the considerable amount of noise living in its null space.

Conclusion

We showed that formulating the Kirchhoff imaging problem as a compressed-sensing Basis Pursuit denoise problem yields subsurface reflectivity distributions that are highly focused with more accurate amplitude information. The migration artifacts observed in the standard migrated image, and the smearing effect observed in least-square solution, are heavily compressed. Also, by deploying only 20% of the available sources and receivers, compressed sensing retrieves a fairly focused and accurate subsurface reflectivity model using a smaller number of linear measurements. This could potentially lead to huge savings in resources and computational time. We also demonstrated the robustness of compressed sensing in the presence of white Gaussian random noise. Overall, the compressed sensing algorithm still retrieves a sparse reflectivity model if the noise variance is small. The quality of the image deteriorates and becomes smoothed and less sparse as we increase the noise variance resulting in migrated images that are comparable to the noise-free least-square solution.

Acknowledgements

Spectral Projected Gradient (SPGL1) Matlab toolbox is a large-scale solver of the Basis Pursuit denoise problem, which we used to reconstruct the sparse subsurface reflectivity distributions.

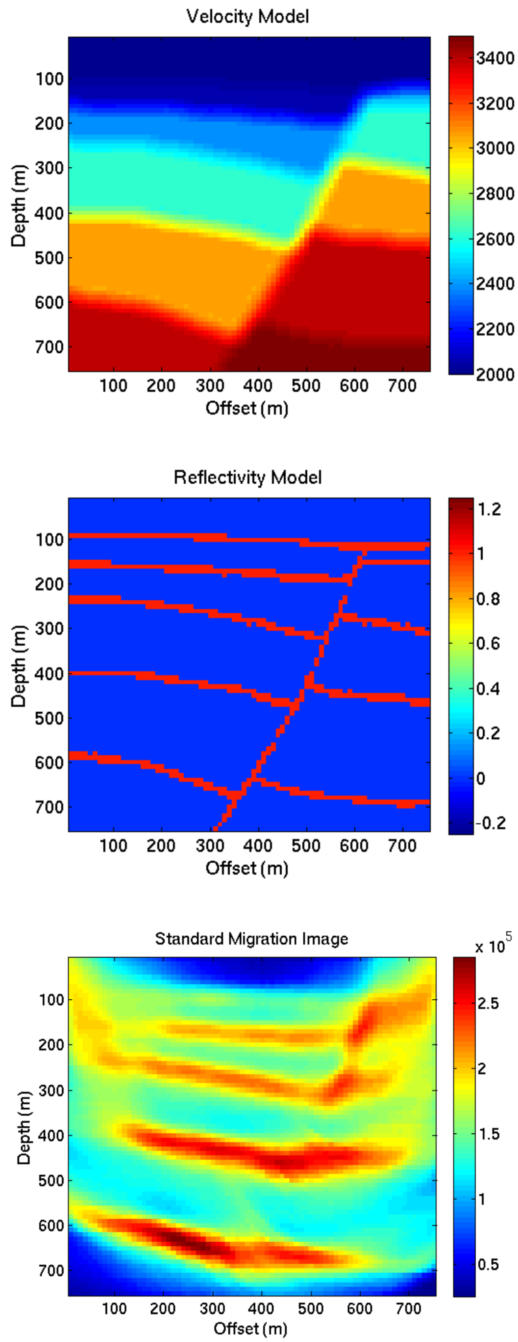

Figure 1: The velocity model (top), the true reflecitivity model (middle), and the standard migrated image showing a heavily blurred version of the true reflectivity model.

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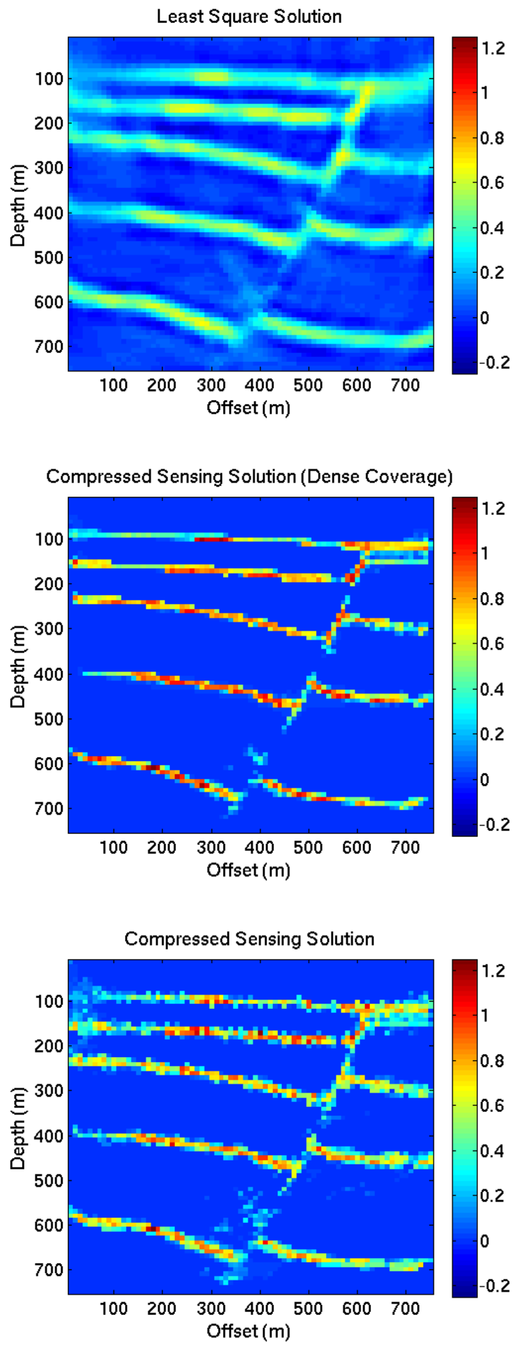

Figure 2: Least-square solution with dense acquisition geometry (top), the compressed sensing solution with dense acquisition geometry (middle), and the compressed sensing solution with the sparse acquisition geometry.

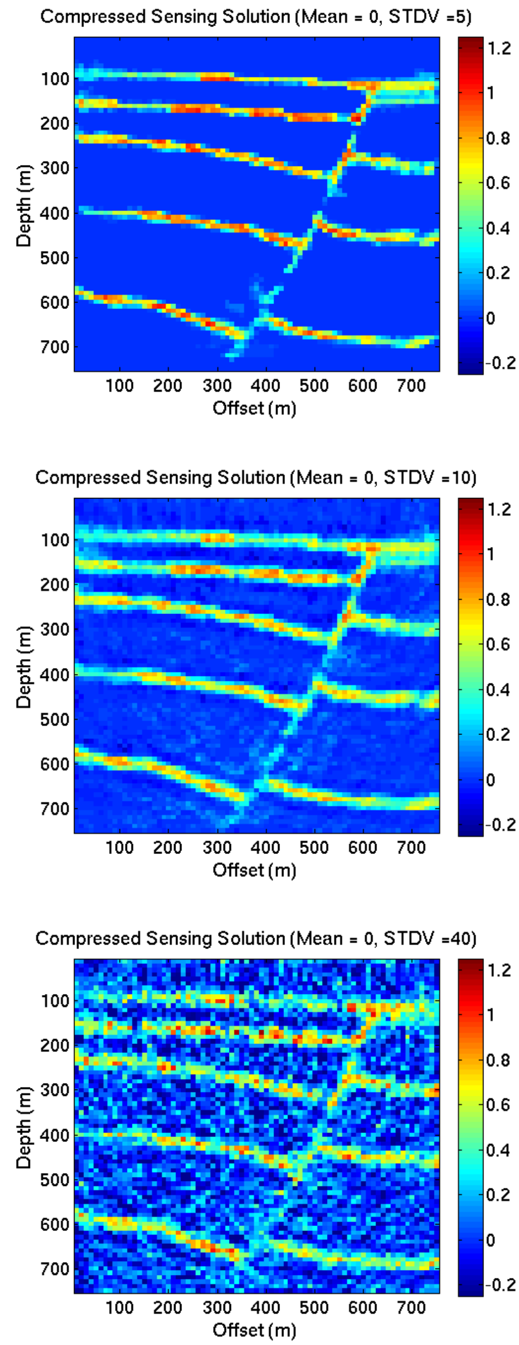

Figure 3: The compressed sensing solutions obtained from the noisy data after the addition of whie Gaussian random noise with different standard deviations (STDV): γ =5 (top), γ =10 (middle), and γ = 40 (bottom).

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