Extended exploding reflector concept for computing prestack traveltimes for waves of different in the DSR framework

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SUMMARY

The double-square-root (DSR) equation can be viewed as a Hamilton-Jacobi equation describing kinematics of downward data continuation in depth. It describes simultaneous propagation of source and receiver rays which allows computing reflection wave prestack traveltimes (for multiple sources) in a one run thus speeding up solution of the forward problem. Here we give and overview of different alternative forms of the DSR equation which allows stepping in two-way time and subsurface offset instead of depth. Different forms of the DSR equation are suitable for computing different types of waves including reflected, head and diving waves.

We develop a WENO-RK numerical scheme for solving all mentioned forms of the DSR equation. Finally the extended exploding reflector concept can be used for computing prestack traveltimes while initiating the numerical solver as if a reflector was exploding in extended imaging space.

INTRODUCTION

The double-square-root (DSR) equation describes kinematics of downward data continuation in depth (Belonosova and Alekseev, 1967; Claerbout, 1985). It can be viewed as a characteristic equation describing recomputation of reflected wave traveltimes from one depth level to another under assumption of no turning rays (DSR assumption). Associated (pseudo)differential equation and its approximations (15 and 45 degree equation) are used for implementing prestack depth migration that has several names: sinking-survey, shot-geophone, one-way extrapolation, or DSR migration.

The DSR equation can be viewed as a Hamilton-Jacobi equation describing kinematics of propagating singularities or waves. This propagation is calculated in one run for the whole prestack data cube accounting for down- and up-going rays. Thus it should be computationally efficient for dense multi-fold seismic surveys compared to computing traveltimes shot by shot. (Note that standard eikonal equation is a particular case of a Hamilton-Jacobi equation).

Numerical solutions to Hamilton-Jacobi equations become unstable in case of multipathing. The common choice of addressing this problem is to compute a so called viscosity solution (Crandall et al., 1983) that provides first arrivals. A version of a fast marching method was developed for the DSR equation in (Li et al., 2012). In this paper we suggest using a weighted essentially nonoscillatory Runge-Kutta (WENO-RK) scheme for solving the DSR equation because of a possibility to achieve high accuracy and speed in a straight forward manner. Also note that WENO-type schemes require stepping in some evolution direction which naturally exists in the DSR approach (evolution in depth, time or offset).

Finally we make use of extended exploding reflector concept suggested in (Duchkov and de Hoop, 2010) allows computing prestack traveltimes while exploding a reflector in the extended imaging space.

Proposed WENO-RK numerical scheme was also implemented for curvilinear coordinates which allows traveltime computations taking into account surface topography as well as the topography of prominent interfaces which can produce head waves. We plan to use this numerical scheme for speeding up solution of the forward problem for iterative kinematic inversion of prestack data.

DIFFERENT VERSIONS OF THE DSR EQUATION

DSR equation can be viewed as describing downward data continuation in extended (unphysical) prestack imaging domain with coordinates (x_s, x_r, z, t) or, alternatively, (x, h, z, t), where x_s is source coordinate, x_r is receiver coordinate, $x = (x_r + x_s)/2$ is midpoint coordinate, $h = (x_r - x_s)/2$ - subsurface half-offset, z - depth, t - two-way time (both coordinate systems are equivalent). Let us write out eikonal equations describing kinematics of the DSR equation.

Classic DSR equation (evolution in depth)

The DSR equation is a Hamilton-Jacobi equation for computing a two-way traveltime function $t(x,h,z)$ with evolution variable being depth z. For upward propagation it takes the form:

$$\begin{aligned} &t_z = H^{DSR}(x,h,z,t_x,t_h), \\ &H^{DSR}(x,h,z,t_x,t_h) = -\sqrt{c(x-h,z)^{-2} - (t_x - t_h)^2/4} \\ &-\sqrt{c(x+h,z)^{-2} - (t_x + t_h)^2/4}, \end{aligned} \qquad (1)$$

where x - subsurface midpoint, h - subsurface half-offset, z - depth, $t_z = \partial t(x,h,z)/\partial z$, $t_x = \partial t(x,h,z)/\partial x$, $t_h = \partial t(x,h,z)/\partial h$, $c(y,z)$ - seismic velocity.

Prestack reflected wave traveltimes can be obtained as a slice of the solution at $z = 0$, i.e. $t(x,h,0)$.

TWT equation (evolution in two-way-time)

In (Duchkov and de Hoop, 2010) it was suggested to rewrite the DSR equation in alternative form such that function $z(x,h,t)$ is computed while stepping in t that is two-way time (TWT):

$$\begin{aligned} &z_t = H^{TWT}(x,h,z,z_x,z_h) \equiv \\ &-c_r c_s \sqrt{\frac{1 + z_x^2 z_h^2 + z_x^2 + z_h^2}{c_+^2 + z_x z_h c_-^2 + \sqrt{4c_s^2 c_r^2 - (z_x^2 + z_h^2)c_-^4 + 2 z_x z_h c_+^2 c_-^2}}}, \end{aligned} \qquad (2)$$

where $z_t = \partial z/\partial t$, $z_x = \partial z/\partial x$, $c_s = c(x-h,z)$, $c_r = c(x+h,z)$, $c_+^2 = c_r^2 + c_s^2$, and $c_-^2 = c_s^2 - c_r^2$.

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Prestack reflected wave traveltimes can be obtained by taking section $z = 0$ of the computed cube $z(x,h,t)$.

SOF equation (evolution in subsurface offset)

The DSR equation can be used for recomputing traveltimes of reflected waves from one depth level to another assuming that rays are nowhere horizontal (the DSR assumption). Diving and head waves obviously violate this assumption. In order to treat them we need to derive another version of the DSR equation for stepping in the offset h.

We follow the strategy from Duchkov and de Hoop (2010) and resolve the DSR equation explicitly with respect to k_h:

$$k_h = \frac{1}{c_r^2 c_s^2 (k_x^2 + k_z^2)} \Big\{ (c_s^2 - c_r^2) k_x \omega^2 + \sqrt{k_z^2 \left[4\omega^4 c_r^2 c_s^2 - \left(c_r^2 c_s^2 (k_x^2 + k_z^2) - (c_r^2 + c_s^2)\omega^2 \right)^2 \right]} \Big\}, \quad (3)$$

where (k_x, k_h, k_z) are wavenumbers corresponding to variables (x,h,z), ω - frequency, $c(y,z)$ - seismic velocity, $c_s \equiv c(x-h,z)$, $c_r \equiv c(x+h,z)$. We can write out corresponding Hamilton-Jacobi equation:

$$t_h = \frac{1}{t_x^2 + t_z^2} \left(n_- n_+ t_x + \sqrt{t_z^2 \left(n_+^2 - t_x^2 - t_z^2\right)\left(t_x^2 + t_z^2 - n_-^2\right)} \right) = 0, \quad (4)$$

where $n_- \equiv 1/c_r - 1/c_s$, $n_+ \equiv 1/c_r + 1/c_s$, $t_h \equiv \partial t/\partial h$, $t_x \equiv \partial t/\partial x$ etc.

This equation allows computing traveltime $t(x,z,h)$ while stepping in offset h. In order to apply it to diving waves we split corresponding ray into two branches starting from the lowest point as shown in Fig. 1. Note that we assume $t_h > 0$ and $t_z > 0$ here. This still allows one branch going down locally as shown in Fig. 1, right. Traveltime will then increase along one branch while decreasing along the other. However, it is important that combined two-way time is always increasing so that $t(h)$ is a monotonous function. This requirement is a substitution for the standard DSR condition, i.e. no turning rays.

Prestack refracted wave traveltimes can be obtained as a slice of the solution at $z = 0$, i.e. $t(x,0,h)$.

Vertically inhomogeneous medium. Let us consider vertically inhomogeneous medium $c(x,z) = c(z)$ that is $n_+ = 2/c$, $n_- = 0$. Also for small h we put $t_x = 0$. Then eq. (4) can be reduced to the form:

$$t_h - \sqrt{4c^{-2} - t_z^2} = 0. \quad (5)$$

Then for each vertical section of physical space ($x = x_1$) kinematics is described for small h by conventional eikonal equation in (h,z) domain:

$$t_h^2 + t_z^2 = 4\tilde{c}(z)^2, \;\; \tilde{c}(z) \equiv c(x_1, z). \quad (6)$$

EXPLODING REFLECTOR CONCEPT FOR INITIATION OF DSR SOLVERS

For computing kinematics of reflected (head) waves for a given reflector one can use DSR equation following the exploding reflector concept (Lowenthal et al., 1976) extended to prestack imaging domain (x,h,z,t) (cf. Duchkov and de Hoop (2010)).

Reflected waves. Assume that reflector is described by equation $z = \phi(x)$. Prestack traveltimes of reflected waves can be computed by solving equation (2) (in Cartesian coordinates) with the source term in the form of an exploding reflector at zero subsurface offset h and zero two-way time t:

$$f(x,h,z,t) = \delta(z - \phi(x))\delta(h)\delta(t). \quad (7)$$

For numerical solution of TWT equation (2) one should set initial values at some small time using analytic formulas for gradient models (near each reflector point).

Head waves. Let us consider horizontal interface $z = z_0$ which generates head waves; let v_b be seismic velocity in the lower layer just beneath the interface. Head wave traveltimes for multiple shots can be computed by solving equation (1) with the source term in the form of an exploding reflector which is ignited with a delay in t for different subsurface offsets h:

$$f(x,h,z,t) = \delta(z - z_0)\delta(h - v_b t). \quad (8)$$

Note that one can easily generalize this result for inhomogeneous velocity distribution of boundary velocity v_b along the interface.

Diving waves. We initialize this numerical solver at $h = 0$ while $t(x,z,0) = 0$. This initialization can be seen as exploding plane $t = 0$ at $h = 0$:

$$f(t,x,z,h) = \delta(h)\delta(t). \quad (9)$$

In terms of ray tracing at each point we have to initiate a couple of rays propagating horizontally in the opposite directions. Then we get $t_z = t_x = 0$ leading to zero denominator in (4). However from ray geometry we have that $k_x >> k_z$ for small h, i.e. generically $\partial k_z/\partial h \neq 0$ but $\partial k_x/\partial h = 0$ at $h = 0$.

WENO SOLVER FOR DSR EQUATIONS

We develop WENO-RK scheme for this Hamilton-Jacobi equation following (Osher and Shu, 1991). Given mesh sizes $\Delta x, \Delta h$ and Δz, let us denote by $t_{i,j}^k$ the numerical approximation of the viscosity solution $t(x,h,z)$ of equation (1) at the grid point (x^i, h^j, z^k). First we need to approximate spatial derivatives in the right-hand side of the equation (1). Define the backward $(-)$ and forward $(+)$ differences at the location (x^i, h^j, z^k) with respect to x and h as

$$\Delta_x^{\pm} = \pm\left(t_{i\pm1,j}^k - t_{i,j}^k\right), \;\; -\Delta_h^{\pm} = \pm\left(t_{i,j\pm1}^k - t_{i,j}^k\right), \quad (10)$$

The 5th order WENO approximations of the backward $(-)$ and forward $(+)$ derivatives of $t(x,h,z)$ at the location (x^i, h^j, z^k) with respect to x are (formulas for the derivative respect to h are similar):

$$D_x^{\mp W,5} t_{i,j}^k = \frac{1}{12}\left(-\frac{\Delta_x^{\pm} t_{i\mp2,j}^k}{\Delta x} + 7\frac{\Delta_x^{\pm} t_{i\mp1,j}^k}{\Delta x} + 7\frac{\Delta_x^{\pm} t_{i,j}^k}{\Delta x} - \frac{\Delta_x^{\pm} t_{i\pm1,j}^k}{\Delta x} \right) - \Phi^{WENO}\left(\frac{\Delta_x^{\mp}\Delta_x^{\pm} t_{i\mp2,j}^k}{\Delta x}, \frac{\Delta_x^{\mp}\Delta_x^{\pm} t_{i\mp1,j}^k}{\Delta x}, \frac{\Delta_x^{\mp}\Delta_x^{\pm} t_{i,j}^k}{\Delta x}, \frac{\Delta_x^{\mp}\Delta_x^{\pm} t_{i\pm1,j}^k}{\Delta x} \right), \quad (11)$$

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where

$$\Phi^{WENO}(a,b,c,d)=\frac{1}{3}w_0(a-2b-c)+\frac{1}{6}(w_1-\frac{1}{2})(b-2c+d).$$

The weights $w_0,\ w_1$ are defined by the formulas:

$$w_0=\alpha_0/(\alpha_0+\alpha_1+\alpha_2),\ w_1=\alpha_2/(\alpha_0+\alpha_1+\alpha_2),$$
$$\alpha_0=1/(\varepsilon+S_0)^2,\ \alpha_1=6/(\varepsilon+S_1)^2,\ \alpha_2=3/(\varepsilon+S_2)^2,$$
$$S_0=13(a-b)^2+3(a-3b)^2,\ S_1=13(b-c)^2+3(b+c)^2,$$
$$S_2=13(c-d)^2+3(3c-d)^2,$$

where we use $\varepsilon=10^{-6}$ to prevent the denominators from becoming zero.

To integrate equation (1) with respect to z at each step k the Riemann problem should be solved. The Hamiltonian H^{DSR}, defined in (1), is decreasing in its second and third arguments (t_x and t_h) so the use of a Godunov-type flux is reduced to the following choice between backward and forward derivatives at the grid point (x^i,h^j,z^k):

$$\left(\widehat{\frac{\partial t}{\partial x}}\right)^k_{i,j}=\text{maxmod}(\max(D_x^{-W,5}t^k_{i,j},0),\min(D_x^{+W,5}t^k_{i,j},0)),$$

where maxmod returns the modulus of the largest value; $(\widehat{\partial t/\partial h})^k_{i,j}$ is computed in a similar way.

Actually this kind of flux function is used for only for DSR equation, stepping by z in cartesian coordinates and for TWT equation. For other eqiations the modification of Lax-Friedrichs flux should be used (Osher and Shu, 1991).

The WENO Runge-Kutta 4th order scheme for equation (1) can be formulated as

$$t^{k+\frac{1}{4}}_{i,j}=t^k_{i,j}+\frac{\Delta z}{2}H^{DSR}\left(t^k_{i,j},\left(\widehat{\frac{\partial t}{\partial x}}\right)^k_{i,j},\left(\widehat{\frac{\partial t}{\partial h}}\right)^k_{i,j}\right),$$

$$t^{k+\frac{1}{2}}_{i,j}=t^k_{i,j}+\frac{\Delta z}{2}H^{DSR}\left(t^{k+\frac{1}{4}}_{i,j},\left(\widehat{\frac{\partial t}{\partial x}}\right)^{k+\frac{1}{4}}_{i,j},\left(\widehat{\frac{\partial t}{\partial h}}\right)^{k+\frac{1}{4}}_{i,j}\right),$$

$$t^{k+\frac{3}{4}}_{i,j}=t^k_{i,j}+\Delta z H^{DSR}\left(t^{k+\frac{1}{2}}_{i,j},\left(\widehat{\frac{\partial t}{\partial x}}\right)^{k+\frac{1}{2}}_{i,j},\left(\widehat{\frac{\partial t}{\partial h}}\right)^{k+\frac{1}{2}}_{i,j}\right),t^{k+1}_{i,j}=$$

$$\frac{t^{k+\frac{1}{4}}_{i,j}+t^{k+\frac{1}{2}}_{i,j}+t^{k+\frac{3}{4}}_{i,j}}{3}+\frac{\Delta z}{6}H^{DSR}\left(t^{k+\frac{3}{4}}_{i,j},\left(\widehat{\frac{\partial t}{\partial x}}\right)^{k+\frac{3}{4}}_{i,j},\left(\widehat{\frac{\partial t}{\partial h}}\right)^{k+\frac{3}{4}}_{i,j}\right).$$

For the stability of this scheme the mesh sampling $\Delta x,\Delta h$ and Δz should satisfy the CFL condition:

$$1\geq 2\Big(\frac{\Delta z}{\Delta x}\Big|\frac{\partial H^{DSR}}{\partial x}\Big|+\frac{\Delta z}{\Delta h}\Big|\frac{\partial H^{DSR}}{\partial h}\Big|\Big) \qquad (12)$$

EXAMPLES

All newly developed numerical schemes were were tested for models with known solutions (homogeneous and linear gradient) for confirming their numerical precision. Here we show examples of applications for inhomogeneous 2D synthetic velocity model shown in Fig. 2, left (reflecting interface can be seen at the bottom) and Fig. 3, left. We consider multifold seismic acquisition system of the towed streamer type.

Using eq. (2) prestack traveltimes of reflected waves were computed for the model Fig. 2, left directly on the midpoint-half-offset grid (x,h); they are shown in Fig. 2, right. using eq. (4) prestack traveltimes of diving (refracted) waves were computed for the model Fig. 3, left directly on the midpoint-half-offset grid (x,h); they are shown in Fig. 3, right.

For checking performance we extended the model and the acquisition system. Array contained 400 receivers placed 10 m apart (offsets from 0 to 4 km). Whole acquisition consisted of a little more that 700 shots (every 20 m).

First we computed prestack traveltimes using our DSR approach. As an alternative we computed reflected-wave traveltimes for each shot separately running two fast-marching solvers (for conventional eikonal equation) for each shot: computing traveltimes from the source down to the reflector, and then initiate eikonal solver at the reflector with corresponding delay.

In our example it appeared to be about 10 times faster to compute all traveltimes using the DSR equation than doing it shot by shot. Also note that traveltimes produced by fast-marching were less accurate because of two reasons: the first order fast-marching scheme was used and some 'wiggling' of traveltime curves was produced due to discrete initiation of the solver at the reflector. For diving waves the DSR approach also turns to be also faster, but not so much: about 5 times (because there is no re-initialization for fast-marching in this case).

CONCLUSIONS

In this paper we have developed the WENO-RK numerical scheme for solving the DSR eikonal equation. Using our DSR eikonal solver one can compute all prestack traveltimes (for multiple sources) in one run thus speeding up solution of the forward kinematic problem. Our numerical example shows that computing reflected-wave traveltimes for about 700 shots is 10 faster using our new approach. However, note that DSR approach requires more memory as computations are made in higher-dimensional space.

The numerical scheme was also developed for curvilinear coordinates which allows traveltime computations for meshes fitting surface topography as well as topography of interfaces. This numerical scheme can be used for speeding up forward modeling for iterative kinematic inversion of prestack data (for example, migration velocity analysis based on picking residual moveouts in common-image gathers).

Note that results presented here can be extended to curvilinear coordinates which will allow computing travel times for meshes fitting topography of reflecting interfaces as well as topography of acquizition surface.

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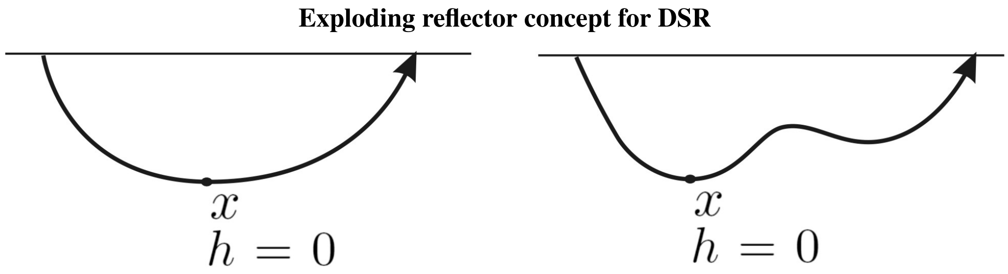

Figure 1: Diving rays that can be computed by eq. (4).

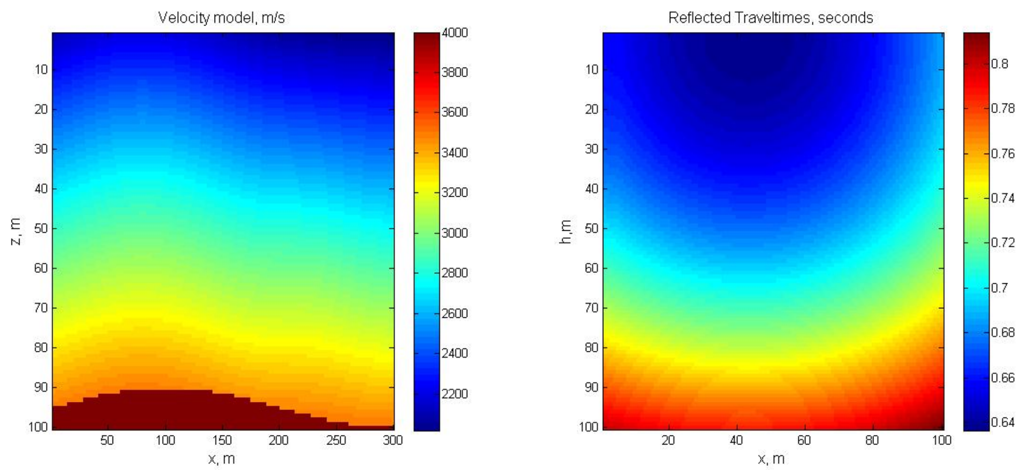

Figure 2: Computing prestack traveltimes of reflected waves using eq. (2): left - velocity model used (reflecting interface can be seen at the bottom); right - computed reflection traveltimes $t(x,h)$.

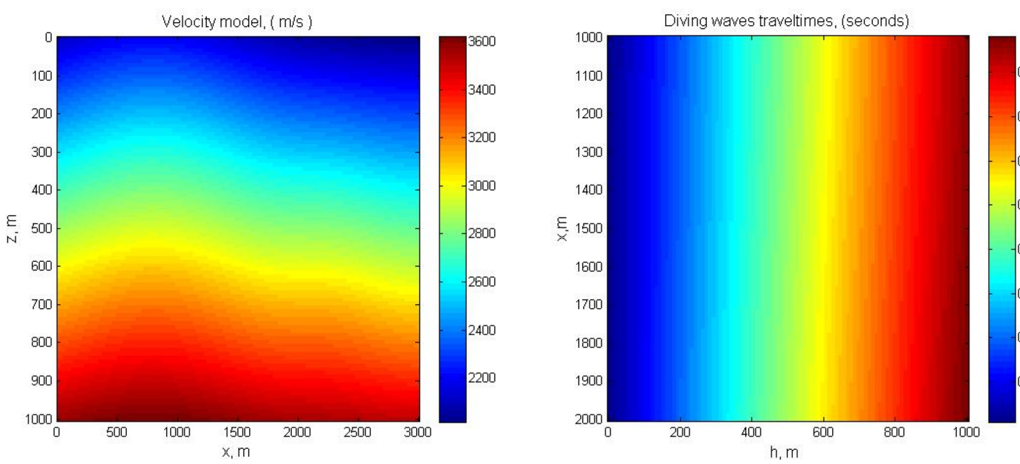

Figure 3: Computing prestack traveltimes of refracted waves using eq. (4): (a) velocity model used; (b) computed refraction wave traveltimes $t(x,h)$.

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EDITED REFERENCES

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