Multi-parameters scanning in HTI media

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SUMMARY

Building credible anisotropy models is crucial in imaging. One way to estimate anisotropy parameters is to relate them analytically to traveltime, which is challenging in inhomogeneous media. Using perturbation theory, we develop traveltime approximations for transversely isotropic media with horizontal symmetry axis (HTI) as explicit functions of the anellipticity parameter η and the symmetry axis azimuth ϕ in inhomogeneous background media. Specifically, our expansion assumes an inhomogeneous elliptically anisotropic background medium, which may be obtained from well information and stacking velocity analysis in HTI media. This formulation has advantages on two fronts: on one hand, it alleviates the computational complexity associated with solving the HTI eikonal equation, and on the other hand, it provides a mechanism to scan for the best fitting parameters η and ϕ without the need for repetitive modeling of traveltimes, because the traveltime coefficients of the expansion are independent of the perturbed parameters η and ϕ. The accuracy of our expansion is further enhanced by the use of shanks transform. We show the effectiveness of our scheme with tests on a 3D model and we propose an approach for multi-parameters scanning in TI media.

INTRODUCTION

With wide-azimuth 3D surveys becoming more common, it is important to realize that azimuthal anisotropy affects our data many ways, and that extracting this information will considerably improve the accuracy of our subsurfaces images. Luckily, the horizontal symmetry axis TI (HTI), can predict many of azimuth variations features including the strength of the azimuthal anisotropy (Grechka and Tsvankin, 1999), and its general direction. Developing simple traveltime formulations for this model can help in many applications, including traveltime tomography and integral-based kirchhoff imaging.

Traveltimes are generally evaluated by solving a nonlinear partial differential equation referred to as the eikonal equation. In anisotropic media, traveltime computation depends on more than one parameter. However, through specific parametrization of HTI media, P-wave traveltimes in 3D, under the acoustic assumption, become dependent on only three parameters and the symmetry axis azimuth. These parameters include the vertical velocity υ_υ, the normal moveout velocity $\upsilon_{nmo} = \upsilon_\upsilon\sqrt{1+2\delta}$, and the anellipticity parameter $\eta = \frac{\varepsilon-\delta}{1+2\delta}$ (where δ and ε are Thomsen parameters (Thomsen, 1986)). Numerically, solving the HTI eikonal equation using finite difference is generally hard, especially because such a process requires finding the root of a quartic equation at each computational step. However, traveltime computation for a simple elliptically anisotropic model is far more efficient, requiring solving a quadratic equation at each computational step. Although, an elliptically anisotropic medium is uncommon in nature, it has the same order of complexity as isotropic media in terms of solving the eikonal equation. Thus, it is used here as the background medium for perturbing traveltime for the more practical HTI model.

Alkhalifah (2011a) developed an eikonal-based scanning scheme to search for the anisotropy parameter η, which provides the best traveltime fit to the data in a general inhomogeneous background medium. He also used this concept to derive a multi-parameters traveltime expansion for the symmetry axis media (TI) in terms of η and tilt θ of the symmetry axis (Alkhalifah, 2011b), as well as, to derive an expansion for HTI media in terms η and the azimuth angle ϕ (Alkhalifah, 2013). He showed the accuracy of these traveltime expansions for homogeneous model cases. Here, we further apply this concept to derive multi-parameters traveltime expansion in terms of the anellipticity parameter η and the symmetry axis azimuth ϕ and assess its efficiency on a 3D model containing a salt structure. We apply the approach to invert for the perturbed parameters, and suggest a scheme to apply the scanning approach for the more general TI media.

THE HTI EIKONAL EQUATION FOR ARBITRARY SYMMETRY-AXIS AZIMUTH

The eikonal equation for HTI media under the acoustic approximation, can be extracted from the VTI version (Alkhalifah, 1998) by a rotation of the axis of symmetry, has the form :

$$\upsilon_\upsilon^2\left(1+2\delta\right)\left(1+2\eta\right)\left(\left(\frac{\partial\tau}{\partial y}\right)^2+\left(\frac{\partial\tau}{\partial z}\right)^2\right)+\upsilon_\upsilon^2\left(\frac{\partial\tau}{\partial x}\right)^2 \left(1-2\eta\upsilon_\upsilon^2\left(1+2\delta\right)\left(\left(\frac{\partial\tau}{\partial y}\right)^2+\left(\frac{\partial\tau}{\partial z}\right)^2\right)\right)=1, \quad (1)$$

where $\tau(x,y,z)$ is the traveltime measured from the source to a point with coordinates (x,y,z). For an arbitrary symmetry axis azimuth in the x - y plane, the traveltime derivatives in equation 1 are taken with respect to the azimuthal angle ϕ. Thus, we use the following rotation operator :

$$\begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (2)$$

The numerical solution of equation 1 requires solving a quartic equation at each time step of the finite difference implementation, or can be solved approximately by solving a series of simpler equations using perturbation theory. By considering η and ϕ to be constant and small, we expand the traveltime solution as a series expansion in η and ϕ. The constant η and ϕ assumption assumes a factorized medium useful for model-development applications. However, all other velocities, including, υ_υ and υ_{nmo} (or δ), are allowed to vary freely.

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Therefore, we substitute the following trial solution

$$\tau(x,y,z) \approx \tau_0(x,y,z) + \tau_\eta(x,y,z)\,\eta + \tau_\phi(x,y,z)\sin\phi + \tau_{\eta_2}(x,y,z)\,\eta^2 + \tau_{\eta\phi}(x,y,z)\,\eta\sin\phi + \tau_{\phi_2}(x,y,z)\sin^2\phi, \quad (3)$$

where τ_0, τ_η, τ_ϕ, τ_{η_2} τ_{ϕ_2} and $\tau_{\eta\phi}$ are coefficients of the expansion with units of traveltime, into the eikonal equation 1. As a result, and as shown in appendix A, τ_0 satisfies the eikonal equation for elliptical anisotropy, which is easy solvable, whereas the coefficients of the expansion satisfy linear partial differential equations having, as shown in Appendix A, the following general form :

$$\frac{\partial\tau_0}{\partial x}\frac{\partial\tau_i}{\partial x} + (1+2\delta)\left(\frac{\partial\tau_0}{\partial y}\frac{\partial\tau_i}{\partial y} + \frac{\partial\tau_0}{\partial z}\frac{\partial\tau_i}{\partial z}\right) = f_i(x,y,z), \quad (4)$$

with $i = \eta, \phi, \eta_2, \eta\phi, \phi_2$. The function $f_i(x,y,z)$ becomes more complicated for i corresponding to the second-order term and it depends on the terms for the first order and background medium solutions. Therefore, the linear partial differential equations must be solved in succession starting with $i = \eta$ and $i = \phi$. Once we compute all τ_i coefficients, we use them as (Alkhalifah, 2013) showed, to estimate the traveltime using shanks transform (Bender and Orszag, 1978), which has the following form :

$$\tau(x,y,z) \approx \tau_0(x,y,z) + \tau_\phi(x,y,z)\sin\phi + \tau_{\phi_2}(x,y,z)\sin^2\phi + \frac{\eta\left(\tau_\eta(x,y,z) + \tau_{\eta\phi}(x,y,z)\sin\phi\right)^2}{\tau_\eta(x,y,z) + \tau_{\eta\phi}(x,y,z)\sin\phi - \eta\,\tau_{\eta_2}(x,y,z)}. \quad (5)$$

To scan for η and ϕ, the coefficients τ_i need to be evaluated only once and can be used with equation 5 to search for the best traveltime fit to the traveltime extracted from the data. As we will see in the next section, the accuracy of our expansion depends on the azimuth of the axis symmetry as the expansion is with respect to $\phi = 0$. However, as (Alkhalifah, 2013) proposed, we can use a similar expansion around $\phi = 90^\circ$ to extend the accuracy limit to angles near $\phi = 90^\circ$.

SCANNING FOR EFFECTIVE η AND ϕ

In this section, we test the accuracy of the shanks transform formulation and show its utility in estimating effective η and ϕ values in complex media. Specifically, we test our approach on a 3D model containing a salt structure. This model is interesting, since most of the azimuthal anisotropy exists near salt diapairs, where radial faulting caused by salt emplacement may be observed. Figure 1 shows the model parameters: the vertical velocity (a), the tilt θ of the axis symmetry (b), the anelipticity parameter η (c) and the azimuthal angle ϕ (d). First, notice the sharp velocity discontinuity especially around the salt body which jumps from $1500 m/s$ at the edge of the salt structure to $4000 m/s$ inside the salt body. Second, as the tilt angle θ is measured from the vertical direction, notice that θ varies from 85° to 95° which means that the symmetry axis does not completely remain in the horizontal plane. The azimuthal angle varies considerably from 10° to 90°. For the two experiments we present, we consider the Thomsen parameter δ to be homogeneous and equals 0.1.

To assess the accuracy of equation 5 and its scanning capabilities, we present two examples. The idea behind these experiments is first to choose some source locations inside the model likewise the exploding reflector assumption. Then, we solve for the HTI eikonal equation with the model parameters, using a fast marching type eikonal solver (Sethian and Popovici, 1999). Afterwards, we assume complete ignorance of η and ϕ model and we scan for the effective η and ϕ that best fit the traveltime, using the formulation given in equation 5. Specifically, we compute the Root-Mean-Square-Error (RMSE) between the exact (observed) and approximated (predicted) traveltimes. Thus the minimum RMSE, given as follows

$$\min_{\phi,\eta} RMSE(\eta,\phi) = \min_{\phi,\eta} \sqrt{\frac{\sum_{i,j}^{n_1,n_2}\left(\widehat{\tau_{ij}}(\eta,\phi) - \tau_{ij}\right)^2}{n_1 \times n_2}}, \quad (6)$$

provides the criteria to choose the best effective η and ϕ values. In equation 6, the summation is made over the number of receivers $(n_1 \times n_2)$ which are considered to be on the surface $(z = 0)$, with $\widehat{\tau_{ij}}$ and τ_{ij}, respectively, as the predicted and observed traveltimes.

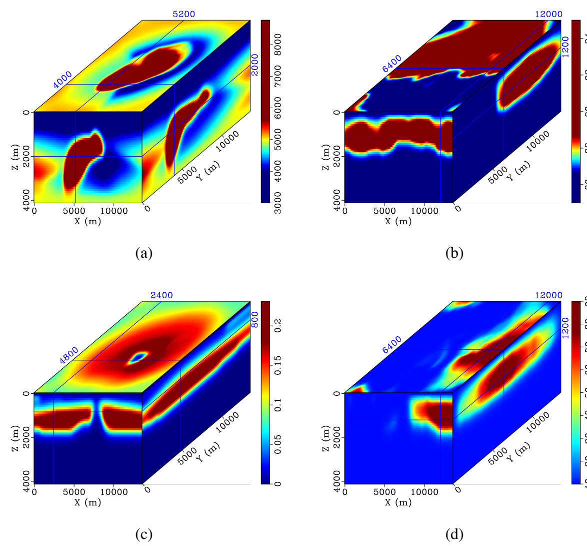

Figure 1: The 3D salt model parameters, (a) the vertical velocity υ_υ, (b) the tilt angle θ, (c) η and (d) the azimuthal angle ϕ.

Example 1 - Homogeneous η and ϕ models

In the first experiment, we will not use the η and ϕ models shown in figure 1(c) and 1(d), but we consider homogeneous η and ϕ models and let υ_υ and θ vary as they are in Figure 1(a) and 1(b). We compute the exact traveltimes using a fast marching eikonal solver. Then, we invert for η and ϕ using the perturbation formulations and we apply our scanning scheme to check if we can retrieve the constant η and ϕ values. We tested this experiment for different constant η and ϕ models and we show the results in Table 1. For all cases, we placed only one source at the bottom of the model located at $(X = 7km, Y = 7km, Z = 4km)$.

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The first half of table 1 in blue corresponds to experiments with $\eta = 0.1$ and different ϕ angles. We notice that the smaller the actual ϕ values, the more accurate estimate of the angle we obtain. This is evident since our expansion is around $\phi = 0$. We also notice that η cannot be retrieved exactly. The tilt of the symmetry axis is one reason that prevents us to obtain exact values for the model parameters. However, the results we got, considering the varying tilt is reasonable. The second half of table 1 in red, shows examples with $\phi = 10^\circ$ and different η cases. Here, as well, the estimated η and ϕ are close to the model ones. We check the accuracy of the traveltime expansion with the effective η and ϕ by computing the absolute relative error. Figure 2 shows such error for two cases taken from table 1. We can see that the maximum error is around 5%.

Model parameters		Estimated parameters	
η	ϕ°	η	ϕ°
0.1	5	0.11	6
0.1	10	0.12	11
0.1	15	0.12	17
0.1	20	0.13	21
0.1	30	0.09	36
0.05	10	0.07	12
0.1	10	0.12	11
0.15	10	0.18	11
0.2	10	0.23	12
0.25	10	0.28	13

Table 1: Scanning for constant η and ϕ.

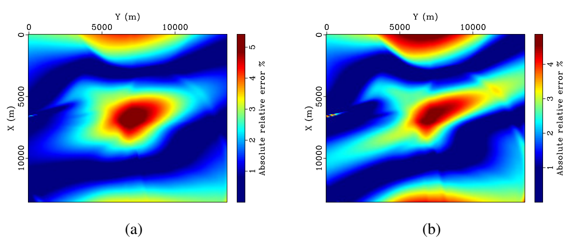

Figure 2: Maps of the absolute relative error in traveltimes as a function of X and Y for the case the model parameters are ($\eta = 0.1,\ \phi = 15^\circ$) and the estimated parameters are ($\eta = 0.12,\ \phi = 17^\circ$) (a) and for the case the model parameters are ($\eta = 0.15,\ \phi = 10^\circ$) and the estimated parameters are ($\eta = 0.18,\ \phi = 11^\circ$) (b).

Example 2 - Inhomogeneous η and ϕ models

In the second experiment, we compute the traveltimes with the inhomogeneous model parameters as shown in Figure 1. Then, we assume complete ignorance of η and ϕ and we scan for the effective η and ϕ that gives the minimum RMSE. In this experiment, we use 4 sources located at different locations as shown in table 2. The number of receivers (placed at $z = 0$) used to compute the RMSE correspond to a $4km$ lateral aperture away from the source location. Figure 3 shows maps of the RMSE as a function of η and ϕ for the 4 sources. The effective values found are given in table 2. We notice that the 4 sources do not give us the same effective values. This is expected since the exact traveltime is computed with inhomogeneous η and ϕ models, which will affect the results of scanning using different source locations.

Source location in Km	Effective η	Effective ϕ°
X=3 Y=3 Z=4	0.11	12
X=5 Y=5 Z=4	0.1	15
X=7 Y=7 Z=4	0.1	16
X=9 Y=9 Z=4	0.11	19

Table 2: Effective η and ϕ values from the minimum RMSE for 4 different source locations.

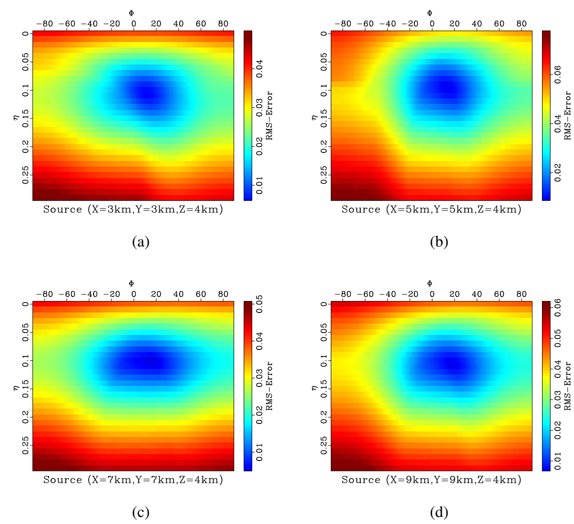

Figure 3: Maps of the RMSE as a function of η and ϕ for 4 different source locations.

DISCUSSION

The main objective of the traveltime approximation is multi-parameter search in complex media. Although, the multi-parameters expansion we present is in terms of η and ϕ, the approach could be easily extended to any combination of anisotropy parameters, specifically to scan for an effective η and δ, which we plan to show in the presentation. If prior information of the azimuth angle is available, we could do velocity analysis in the isotropic plane to determine an inhomogeneous velocity field. We could then scan for effective η and δ. Also, by approximating the tilt angle by the structure of the model, we can incorporate the tilt effect on the background model by the following coordinate transformation applied to the eikonal equation 1:

$$\begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}. \tag{7}$$

Because this coordinate transform is linear and the perturbation PDEs are linear, as well, then the resulting equations will remain linear. In this case, also, the background medium is elliptically anisotropic with a tilted axis of symmetry, which

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is easily solvable and has the same complexity as the isotropic eikonal equation. The challenging part in all this process is to get accurate tilt angles. In all cases, the choice of the parameters to scan for depends on the prior information we have. Their accuracy will determine our ability to estimate the rest of the parameters.

CONCLUSION

By expanding the traveltime solution of the HTI eikonal equation in a power series in the anelipticity parameter η and the azimuthal angle ϕ, we developed an efficient tool to estimate these parameters in a generally inhomogeneous background medium. Shanks transform allowed us to better represent the expansion using fewer terms, and thus fewer equations to solve. We tested our scanning scheme on a realistic 3D model containing a salt body and suggested an approach to scan for δ and η parameters.

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APPENDIX A

HIGHER ORDER EXPANSIONS

For an expansion in η and ϕ, we use the following trial solution:

$$\tau(x,y,z) \approx \tau_0(x,y,z) + \tau_\eta(x,y,z)\,\eta + \tau_\phi(x,y,z)\sin\phi + \tau_{\eta_2}(x,y,z)\,\eta^2 + \tau_{\eta\phi}(x,y,z)\,\eta\sin\phi + \tau_{\phi_2}(x,y,z)\sin^2\phi. \quad \text{(A-1)}$$

Inserting the trial solution, equation A-1, into equation 1 yields a long formula but by setting both $\sin\phi = 0$ and $\eta = 0$, we get the zeroth-order term given by

$$\upsilon_v^2(x,y,z)(1+2\delta)\left(\left(\frac{\partial\tau_0}{\partial y}\right)^2 + \left(\frac{\partial\tau_0}{\partial z}\right)^2\right) + \upsilon_v^2(x,y,z)\left(\frac{\partial\tau_0}{\partial x}\right)^2 = 1, \quad \text{(A-2)}$$

which is simply the elliptical eikonal equation. By equating the coefficients of the powers of the independent parameters $\sin\phi$ and η, in succession starting with first powers of the two parameters, we end up first with the coefficients of first power in $\sin\phi$ and zeroth power in η :

$$\frac{\partial\tau_0}{\partial x}\frac{\partial\tau_\phi}{\partial x} + (1+2\delta)\left(\frac{\partial\tau_0}{\partial y}\frac{\partial\tau_\phi}{\partial y} + \frac{\partial\tau_0}{\partial z}\frac{\partial\tau_\phi}{\partial z}\right) = 2\delta\frac{\partial\tau_0}{\partial x}\frac{\partial\tau_0}{\partial y}, \quad \text{(A-3)}$$

which is a first order linear partial differential equation in τ_ϕ. The coefficients of zero power in $\sin\phi$ and the first power in η, simplified by using A-2, is given by

$$\frac{\partial\tau_0}{\partial x}\frac{\partial\tau_\eta}{\partial x} + (1+2\delta)\left(\frac{\partial\tau_0}{\partial y}\frac{\partial\tau_\eta}{\partial y} + \frac{\partial\tau_0}{\partial z}\frac{\partial\tau_\eta}{\partial z}\right) = -\upsilon_v^2(1+2\delta)^2\left(\left(\frac{\partial\tau_0}{\partial z}\right)^2 + \left(\frac{\partial\tau_0}{\partial y}\right)^2\right)^2. \quad \text{(A-4)}$$

The coefficients of the square term in $\sin\phi$ with some manipulation, result in the following relation:

$$\frac{\partial\tau_0}{\partial x}\frac{\partial\tau_{\phi_2}}{\partial x} + (1+2\delta)\left(\frac{\partial\tau_0}{\partial y}\frac{\partial\tau_{\phi_2}}{\partial y} + \frac{\partial\tau_0}{\partial z}\frac{\partial\tau_{\phi_2}}{\partial z}\right) = -\frac{1}{2}\left((1+2\delta)\left(\left(\frac{\partial\tau_\phi}{\partial z}\right)^2 + \left(\frac{\partial\tau_\phi}{\partial y}\right)^2\right) + \left(\frac{\partial\tau_\phi}{\partial x}\right)^2\right) + 2\delta\left(\frac{\partial\tau_0}{\partial y}\frac{\partial\tau_\phi}{\partial x} + \frac{\partial\tau_0}{\partial x}\frac{\partial\tau_\phi}{\partial y}\right) + \delta\left(\left(\frac{\partial\tau_0}{\partial y}\right)^2 - \left(\frac{\partial\tau_0}{\partial x}\right)^2\right), \quad \text{(A-5)}$$

which is again a first-order linear PDE in τ_{ϕ_2} with a more complicated source function given by the right hand side. The coefficients of the square terms in η, with also some manipulation, result also in a linear first-order PDE with the following relation :

$$\frac{\partial\tau_0}{\partial x}\frac{\partial\tau_{\eta_2}}{\partial x} + (1+2\delta)\left(\frac{\partial\tau_0}{\partial y}\frac{\partial\tau_{\eta_2}}{\partial y} + \frac{\partial\tau_0}{\partial z}\frac{\partial\tau_{\eta_2}}{\partial z}\right) = -\frac{1}{2}\left((1+2\delta)\left(\left(\frac{\partial\tau_\eta}{\partial z}\right)^2 + \left(\frac{\partial\tau_\eta}{\partial y}\right)^2\right) + \left(\frac{\partial\tau_\eta}{\partial x}\right)^2\right) + 2\upsilon_v^2(1+2\delta)^2\left(\left(\frac{\partial\tau_0}{\partial z}\right)^2 + \left(\frac{\partial\tau_0}{\partial y}\right)^2\right)\left(\frac{\partial\tau_0}{\partial x}\frac{\partial\tau_\eta}{\partial x}\right) - 2\upsilon_v^2 (1+2\delta)^2\left(\left(\frac{\partial\tau_0}{\partial z}\right)^2 + \left(\frac{\partial\tau_0}{\partial y}\right)^2\right)\left(\frac{\partial\tau_0}{\partial y}\frac{\partial\tau_\eta}{\partial y} + \frac{\partial\tau_0}{\partial z}\frac{\partial\tau_\eta}{\partial z}\right). \quad \text{(A-6)}$$

Finally, the coefficients of the first-power terms in both $\sin\phi$ and η with some manipulation result into the following first order PDE :

$$\frac{\partial\tau_0}{\partial x}\frac{\partial\tau_{\eta\phi}}{\partial x} + (1+2\delta)\left(\frac{\partial\tau_0}{\partial y}\frac{\partial\tau_{\eta\phi}}{\partial y} + \frac{\partial\tau_0}{\partial z}\frac{\partial\tau_{\eta\phi}}{\partial z}\right) = -(1+2\delta)\left(\frac{\partial\tau_\eta}{\partial z}\frac{\partial\tau_\phi}{\partial z} + \frac{\partial\tau_\eta}{\partial y}\frac{\partial\tau_\phi}{\partial y}\right) - \frac{\partial\tau_\eta}{\partial x}\frac{\partial\tau_\phi}{\partial x} - 2(1+2\delta)\left(\frac{\partial\tau_0}{\partial z}\frac{\partial\tau_\phi}{\partial z} + \frac{\partial\tau_0}{\partial y}\frac{\partial\tau_\phi}{\partial y}\right) + 2(1+2\delta)\left(\frac{\partial\tau_0}{\partial y}\frac{\partial\tau_0}{\partial x}\right) + 2\delta\left(\frac{\partial\tau_\eta}{\partial y}\frac{\partial\tau_0}{\partial x} + \frac{\partial\tau_0}{\partial y}\frac{\partial\tau_\eta}{\partial x}\right) + 2(1+2\delta)\upsilon_v^2\frac{\partial\tau_0}{\partial x}\left(\frac{\partial\tau_0}{\partial z}\frac{\tau_\phi}{\partial z}\frac{\partial\tau_0}{\partial x} + \left(\frac{\partial\tau_0}{\partial y}\right)\left(\left(\frac{\partial\tau_0}{\partial y}\right)^2 + \frac{\partial\tau_0}{\partial x}\left(\frac{\partial\tau_\phi}{\partial y} - \frac{\partial\tau_0}{\partial x}\right) + \frac{\partial\tau_0}{\partial y}\frac{\partial\tau_\phi}{\partial x}\right) + \left(\frac{\partial\tau_0}{\partial z}\right)^2\left(\frac{\partial\tau_0}{\partial y} + \frac{\partial\tau_\phi}{\partial x}\right)\right). \quad \text{(A-7)}$$

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