

Application of the unwrapped phase inversion to land data without source estimation

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Summary

Unwrapped phase inversion with a strong damping was developed to solve the phase wrapping problem in frequency-domain waveform inversion. In this study, we apply the unwrapped phase inversion to band-limited real land data, for which the available minimum frequency is quite high. An important issue of the data is a strong ambiguity of source-ignition time (or source shift) shown in a seismogram. A source-estimation approach does not fully address the issue of source shift, since the velocity model and the source wavelet are updated simultaneously and interact with each other. We suggest a source-independent unwrapped phase inversion approach instead of relying on source-estimation from this land data. In the source-independent approach, the phase of the modeled data converges not to the exact phase value of the observed data, but to the relative phase value (or the trend of phases); thus it has the potential to solve the ambiguity of source-ignition time in a seismogram and work better than the source-estimation approach. Numerical examples show the validation of the source-independent unwrapped phase inversion, especially for land field data having an ambiguity in the source-ignition time.

Introduction

Full waveform inversion (FWI) is widely studied since it can theoretically provide an accurate and detailed subsurface velocity model. However, FWI still faces many challenges, one of which is phase wrapping in the frequency-domain (or cycle-skipping in the time-domain) resulting in local minima problems in FWI (Virieux and Operto, 2009). If the observed data lacks low frequency information or the initial velocity model is not accurate enough, the phase wrapping problem must be addressed well for a successful implementation of FWI.

One solution for the phase wrapping problem is to explicitly unwrap the phases in two-dimensional space coordinates (phase map). Two-dimensional phase unwrapping has been studied in area of the topography mapping from the synthetic-aperture radar and optical interferometers (Ghiglia and Pritt, 1998). A big issue in two-dimensional phase unwrapping is the existence of residues in a phase map. Residue in a phase map is evaluated through a simple closed-path integral of the gradient of phase with respect to the space variable. If a phase map includes residues, the phase unwrapping process is path-dependent and becomes quite complicated, and the

phase unwrapping results depend on the applied algorithms (Ghiglia and Pritt, 1998).

Choi and Alkhalifah (2014, 2015) introduced an unwrapped phase inversion with an exponential damping. They constructed a phase map of shot and receiver coordinates and unwrapped the two-dimensional phases. They also applied a strong exponential damping to the wavefields to remove residues in a phase map. In this case, the unwrapping process is path-independent and provides a unique solution. They finally inverted the unwrapped phases, where the back-propagation algorithm is used to calculate the gradient. The exponential damping applied to the wavefield has a trade-off between reducing the number of residues in a phase map and losing the possibility to define a deeper structure. Progressively loosening the damping factor moves tomographic inversion in the direction of FWI.

When the unwrapped phase inversion is applied to land data, source information must be carefully considered. Source-ignition time shown in a seismogram of land data is sometimes uncertain and not given, which results in unrealistically low or high velocity update in FWI. Source estimation (Pratt, 1999) during the inversion could solve the issue of the unknown source. However, if the minimum frequency for the inversion is quite high or the starting model is quite far from the true one, source estimation during the inversion could fail to converge to the true one, since the source wavelet and subsurface velocity model are updated simultaneously during the inversion and affect each other.

In this study, we suggest an alternative to source estimation in the unwrapped phase inversion, in which we estimate the phase difference between general receiver and a reference receiver, and then invert the phase differences instead of the original phase value. In an unwrapped phase inversion with source estimation, we try to match the phase of the modeled data to the exact phase value of the observed data, whereas we push the phase difference (relative phase value) of the modeled data to approach to the phase difference of the observed data in the alternative method. Since we invert the relative phase value (phase difference) instead of the exact phase, we can solve the previously mentioned ambiguous situation of source signature. Actually, this alternative method is a source-independent version of the unwrapped phase inversion. The preferred reference position would be as close to the shot position as possible and changed according to each shot gather.

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We apply the alternative (source-independent) approach of the unwrapped phase inversion to real land data in this study. The data is elastic data and provided separately as P- and SH-wavefields. Here, we invert only the P-wavefield using the unwrapped phase inversion based on the acoustic modeling technique. The usable frequency range of the data is 13 ~ 100 Hz, where the minimum frequency is quite high. Another feature of the data is a strong ambiguity of source-ignition time in the seismogram. We compare the inversion results of source-estimation and source-independent methods in the unwrapped phase inversion.

In the following sections, we provide a description of the source-independent unwrapped phase inversion with an exponential damping. Then we show the inversion results of the real land data.

Theory

The modeled and observed data in the frequency-domain at a certain frequency are expressed respectively as

$$u_{i,j} = A_{i,j}^u e^{i\theta_{i,j}^u} \quad \text{and} \quad d_{i,j} = A_{i,j}^d e^{i\theta_{i,j}^d}, \quad (1)$$

where subscripts i and j are the source and receiver index, superscripts u and d stands for the modeled and observed data, and A and θ are the amplitude and phase of the data, respectively. The phases of the modeled and observed wavefields are sum of phases of the Greens function and source wavelet:

$$\begin{aligned} \theta_{i,j}^u &= \theta_{i,j}^{ug} + \theta_i^{us} \\ \theta_{i,j}^d &= \theta_{i,j}^{dg} + \theta_i^{ds}, \end{aligned} \quad (2)$$

where $\theta_{i,j}^{ug}$ and $\theta_{i,j}^{dg}$ are the phases of the Greens functions of the modeled and observed wavefields, and θ_i^{us} and θ_i^{ds} are the phases of source wavelet of the modeled and observed wavefields, respectively. Note that θ_i^{us} and θ_i^{ds} are not functions of the receiver index j .

For the phase unwrapping process, we construct the phase residual map. Since we apply a strong damping, the phase map does not include residue points. If the phase map has no residue, the unwrapping process becomes path-independent (Ghiglia and Pritt, 1998). We first set a path line in the phase map, which does not cross itself, and then we unwrap the phase values following the path. If there is a sudden jump of phase values on the path, we add $\pm 2\pi$ to the phase values (Choi and Alkhalifah, 2014, 2015).

The objective function of the unwrapped phase inversion is expressed for a single frequency as follows (Choi and Alkhalifah, 2014, 2015):

$$\begin{aligned} E &= \sum_i \sum_j \left[\text{UW}(\theta_{i,j}^u - \theta_{i,j}^d) \right]^2 \\ &= \sum_i \sum_j \left[\text{UW}(\theta_{i,j}^{ug} + \theta_i^{us} - \theta_{i,j}^{dg} - \theta_i^{ds}) \right]^2, \end{aligned} \quad (3)$$

where $\text{UW}()$ stands for the unwrapped value of phase. In this case, the phase of source wavelet (θ_i^{us}) must be estimated during the phase inversion. However, if the minimum frequency for the inversion is high and the starting model is quite far from the true model, the estimation of phase of source wavelet could fail to converge to the true one since updating velocity and source wavelet interact with each other.

As an alternative, we use the phase difference between the regular receivers and reference one for the new objective function:

$$\begin{aligned} E &= \sum_i \sum_j \left[\text{UW} \left\{ (\theta_{i,j}^u - \theta_{i,ref}^u) - (\theta_{i,j}^d - \theta_{i,ref}^d) \right\} \right]^2 \\ &= \sum_i \sum_j \left[\text{UW} \left\{ (\theta_{i,j}^{ug} - \theta_{i,ref}^{ug}) - (\theta_{i,j}^{dg} - \theta_{i,ref}^{dg}) \right\} \right]^2. \end{aligned} \quad (4)$$

In equation 4, the source wavelet terms (θ_i^{us} and θ_i^{ds}) are canceled, and we therefore don't need to estimate them. In this approach, we do not match the phase of the modeled data ($\theta_{i,j}^u$) directly to that of the observed data ($\theta_{i,j}^d$), but rather try to match the relative phase value (phase difference) of the modeled data ($\theta_{i,j}^u - \theta_{i,ref}^u$) to that of the observed data ($\theta_{i,j}^d - \theta_{i,ref}^d$). Actually, this approach is the source-independent version of the unwrapped phase inversion. This source-independent approach could work better than the source-estimation method when the ignition time of source (or source shift) in a seismogram is considerably ambiguous and not given. In equation 4, the reference position denoted by 'ref' could be changed for each shot gather.

The gradient of the new objective function in equation 4 is obtained by taking derivative of the objective function with respect to the k th model parameter, p_k :

$$\begin{aligned} \frac{\partial E}{\partial p_k} &= \sum_i \sum_j \left[\left(\frac{\partial \theta_{i,j}^u}{\partial p_k} - \frac{\partial \theta_{i,ref}^u}{\partial p_k} \right) q_{i,j} \right] \\ &= \sum_i \sum_j \text{Im} \left[\left(\frac{1}{u_{i,j}} \frac{\partial u_{i,j}}{\partial p_k} - \frac{1}{u_{i,ref}} \frac{\partial u_{i,ref}}{\partial p_k} \right) q_{i,j} \right], \end{aligned} \quad (5)$$

where $q_{i,j} = \text{UW} \left\{ (\theta_{i,j}^u - \theta_{i,ref}^u) - (\theta_{i,j}^d - \theta_{i,ref}^d) \right\}$ and 'Im' stands for the imaginary part of a complex value.

Finally, the gradient is expressed in vector and matrix form using the back-propagation algorithm:

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$$\frac{\partial E}{\partial p_k} = \text{Im} \left\{ (\mathbf{v}_k)^T \mathbf{S}^{-1} \mathbf{r} \right\}, \quad (6)$$

where \mathbf{S} is a modeling operator matrix, \mathbf{v}_k is a virtual source vector for the k th model parameter (weighted forward wavefield) and \mathbf{r} is a residual vector expressed for i th shot as

$$\mathbf{r}^T = \begin{bmatrix} \frac{1}{u_{i,1}} q_{i,1} & \frac{1}{u_{i,1}} q_{i,2} & \cdots & \frac{1}{u_{i,n}} q_{i,n} \\ 0 & \cdots & 0 & \frac{1}{u_{i,ref}} \sum_j q_{i,j} & 0 & \cdots & 0 \end{bmatrix}. \quad (7)$$

In equation 6, to calculate the gradient, we back-propagate the residual vector, \mathbf{r} , and then multiply the back-propagated wavefield and weighted forward wavefield (\mathbf{v}_k).

Examples

We apply an acoustic-based inversion algorithm to the P-wavefield of land data. To address the irregular topography, we employ a finite-element method for modeling. The number of shots is 257 with an interval of 16.75 m and the number of receivers for each shot is 242 with an interval of 16.75 m. The recording and sampling time of the data are 3 and 0.002 sec, respectively. We choose a grid interval of 2.5 m and model size is 1.04 km x 8.345 km for the inversion.

Figure 1a shows a representative seismogram. The data has a bandwidth of approximately 13 ~ 100 Hz. We started by picking the first arrival traveltimes on the seismograms (red lines in Figure 1a). Since we apply an acoustic-based inversion algorithm to an elastic data, we need to mute Rayleigh waves shown in Figure 1a. To mute the Rayleigh waves and later arrival events, which violate the acoustic assumption, we set a line for muting (blue lines in Figure 1a). Finally, we mute signals prior to the red line and below the two blue lines.

An important issue in this seismogram is the strong ambiguity of source-ignition time (or source shift). Picked traveltimes in Figure 1a appear shifted along the time axis (i.e., the source-ignition time is not zero and is shifted in the seismogram). The ambiguity of source shift must be addressed in the inversion.

One velocity profile from well-log data is given. We apply a low-pass filter to the well-log velocity profile and get a smooth version of it. By extending the smoothed velocity profile along the horizontal axis, we generated the two-dimensional starting model (referred to “given starting model”) shown in Figure 1b. The shallow part of the given

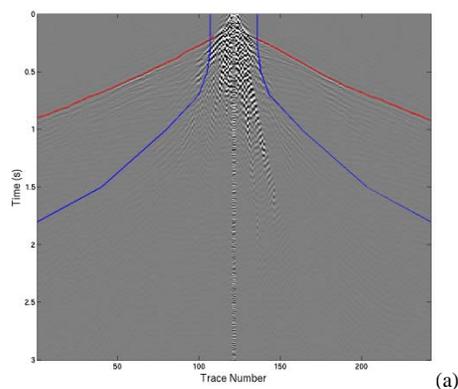
starting model has a minimum velocity around 2 km/s and the deeper part has high velocities around 6 km/s.

We apply the unwrapped phase inversion with a high damping factor of 30 using both source-estimation and source-independent approaches and compare the results. We start from the given starting model in Figure 1b. The frequency used in the inversion is 3 Hz. Figure 2 shows the inverted models derived from the unwrapped phase inversion after 200 iterations using source-estimation and source-independent approaches, respectively. Figure 2a shows an unrealistically low velocity zone in the shallow part and some artifact-looking structures in the shallow part, whereas Figure 2b shows generally smooth velocity structures.

Figure 3 shows the comparison of phases of the observed data and modeled data after the final iteration. Even though the phases of the modeled data in the source-estimation approach converge well to the observed data (Figure 3a), they suffer from the aforementioned ambiguity of source-ignition time (or source shift). On the other hand, the phases of the modeled data in the source-independent approach converge to the relative phase value (or trend of phase) of the observed data. This relative convergence (or convergence to the trend) of the phase in the source-independent approach could help avoid the ambiguity of source shift.

Figure 4 shows the reverse-time migration (RTM) images generated using the given starting model (Figure 1b) and the inverted models (Figure 2), where we focus on horizontal range of 4 ~ 7 km. From Figure 4, we note that the RTM image using the source-independent result shows the most focused and flat images among the RTM images.

The inverted models and RTM images prove that the source-independent approach is working better for this example of land field data with ambiguous and variable source shifts than the source-estimation approach.



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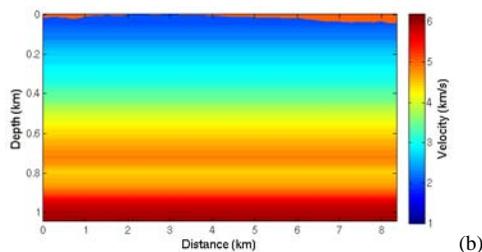


Figure 1: (a) A representative seismogram with a picked traveltimes (red line) and muting line (blue line) and (b) given starting model.

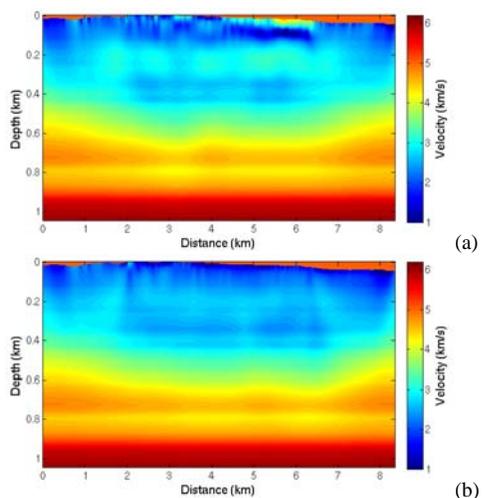


Figure 2: The inverted models obtained by the unwrapped phase inversion with a strong damping using (a) source-estimation and (b) source-independent approaches.

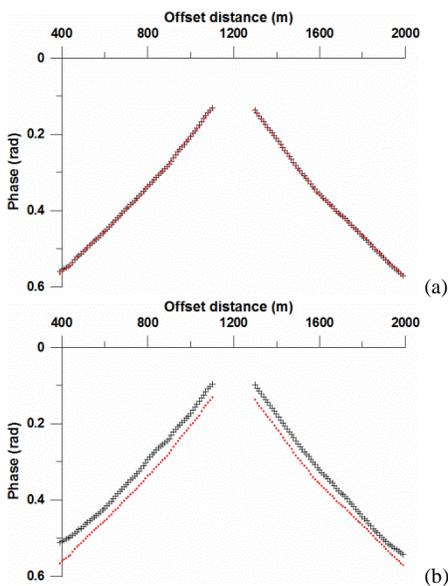


Figure 3: Comparison of phases of the observed data (red dot) and the modeled data at final iteration (black cross) for (a) the source-estimation approach and (b) source-independent approach.

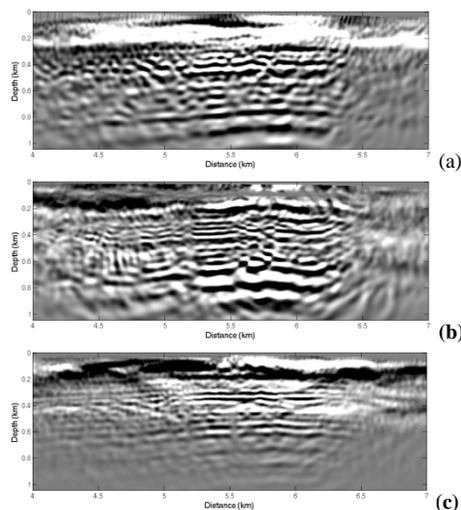


Figure 4: The reverse time migration images using (a) the given initial model (Figure 1b), (b) inverted model by the source-estimation approach (Figure 2a) and (c) inverted model by the source-independent approach (Figure 2b).

Conclusions

Use of an unwrapped phase inversion with a strong damping can avoid the well-known phase wrapping problem, which often causes a convergence of FWI to local minima. We apply the unwrapped phase inversion to band-limited real land data, for which the minimum frequency is quite high. An important issue typical of real land field data is an ambiguity of the source-ignition time (or source shift). A source-estimation approach does not fully address this issue. We, thus, developed a source-independent approach for successful implementation of the unwrapped phase inversion of the land data. In the source-independent approach, the phase of the modeled data attempts to converge to the relative phase value (or the trend of phases) of the observed data; thus, it could solve the ambiguity of the source-ignition time better than the source-estimation approach. The numerical examples demonstrate the validation of the source-independent unwrapped phase inversion for land data having ambiguous source-ignition timing.

Acknowledgments

We are grateful to King Abdullah University of Science and Technology for financial support. We are also grateful to Vecta Oil and Gas Ltd. for providing the field data.

EDITED REFERENCES

Note: This reference list is a copyedited version of the reference list submitted by the author. Reference lists for the 2015 SEG Technical Program Expanded Abstracts have been copyedited so that references provided with the online metadata for each paper will achieve a high degree of linking to cited sources that appear on the Web.

REFERENCES

- Choi, Y., and T. Alkhalifah, 2014, Two-dimensional unwrapped phase inversion with damping and a Gaussian Filter: 76th Annual International Conference and Exhibition, EAGE, Extended Abstracts, Tu-E106-03.
- Choi, Y., B. Devault, and T. Alkhalifah, 2015, Application of the unwrapped phase inversion to land field data with irregular topography: 77th Annual International Conference and Exhibition, EAGE, Extended Abstracts, We-N104-07.
- Ghiglia, D. C., and M. D. Pritt, 1998, Two-dimensional phase unwrapping: Theory, algorithms, and software: John & Sons, Inc.
- Pratt, R. G., 1999, Seismic waveform inversion in the frequency domain, Part 1: Theory and verification in a physical scale model: *Geophysics*, **64**, 888–901. <http://dx.doi.org/10.1190/1.1444597>.
- Virieux, J., and S. Operto, 2009, An overview of full-waveform inversion in exploration geophysics: *Geophysics*, **74**, no. 6, WCC1–WCC26.