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Impact of Users Identities and Access Conditions on Downlink Performance in Closed Small-Cell Networks

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***Abstract*—This paper investigates the effect of various operation parameters on the downlink user performance in overlaid small-cell networks. The case study considers closed-access small cells (e.g., femtocells), wherein only active authorized user equipments (UEs) can be served, and each of which is allocated single downlink channel at a time. On the other hand, the macrocell base station can unconditionally serve macrocell UEs that exist inside its coverage space. The available channels can be shared simultaneously in the macrocell network and the femtocell network. Moreover, a channel can be reused only at the macrocell base station. The analysis provides quantitative approaches to model UEs identities, their likelihoods of being active, and their likelihoods of producing interference, considering UEs classifications, locations, and access capabilities. Moreover, it develops models for various interference sources observed from effective interference femtocells, considering femtocells capacities and operation conditions. The associated formulations to describe a desired UE performance and the impact of the number of available channels as well as the adopted channel assignment approach are thoroughly investigated. The results are generally presented for any channel models of interference sources as well as the desired source of the served UE. Moreover, specific channel models are then adopted, for which generalized closed-form analytical results for the desired UE outage probability performance are obtained. Numerical and simulation results are presented to further clarify the main outcomes of the developed analysis.**

***Index Terms*—overlaid cellular networks, closed-access small cells, co-channel interference, users identities, access control, channel assignment, statistical modeling, downlink performance.**

I. Introduction

Among the main restrictions that limit the sustainable operation of existing macrocell networks are the expanded demand on high data rate services and the coverage limitations. Specifically, due to the relatively high indoors losses at high frequencies, the current supported data rates on such networks are only fractions of their theoretical limits [1]. Short-range small cells, such as femtocells in macrocell networks, have been proposed to overcome some limitations associated with indoors operation of existing macrocell infrastructure. They can enhance the service coverage in specific spatial locations inside the macrocell coverage space via a distributed radio processing approach [2]. The use of femtocells reduces the separation distances between user equipments (UEs) and their serving access points, which improves the system power efficiency and its reliability [2]–[5].

The realization of expected benefits with the use of femtocells technology requires a careful consideration of various technical issues. Among these issues is the distribution of radio resources among access points inside macrocell coverage space (see, e.g. [6]–[15] for different approaches of modeling and analysis). Specifically, network operators may prefer to establish the concurrent deployment of femtocells inside a macrocell while avoiding additional expansion on expensive radio resources. This scenario may be achieved through newly developed physical layer and/or hybrid data processing techniques. However, such techniques can either create additional issues or demand comprehensive updates of existing network specifications. In any case, an ultimate design objective that is anticipated to provide the best long-term system coverage and capacity is to have macrocell radio resources available to any femtocell access point anywhere anytime.

Femtocells may be configured to operate using various access control strategies. Among these strategies are closed-access, open-access, and hybrid-access strategies [16]–[20]. Open-access femtocells can be suitable for small public places to unconditionally serve active UEs by any nearby femtocell, but at the expense of extra burden on femtocell backhaul. They can also result in a large number of macrocell/femtocell and femtocell/femtocell handoff requests, and consequently an increased overhead [17], [21]. On the other hand, closed-access femtocells can be installed for private or home use. In this case, only authorized femtocell UEs can have access to radio resources through their private femtocell access point. However, this approach may limit the long-term system capacity and reliability. This limitation can be due to the expected increase in cross-tier interference because of the presence of active macrocell UEs, which can not access radio resources through a nearby femtocell access point, inside the coverage

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space of a femtocell.

The network-level models for relatively large coverage spaces of overlaid femtocells and macrocells in [22]–[24], which are based on Poisson point processes (PPPs), can be suitable when the number of network terminals (or nodes) is very large (e.g., number of access points, users, etc.). However, they can not be adopted for relatively limited coverage spaces with finite number of nodes and to study the effect of various operation parameters, such as users identities and femtocell operation conditions, on downlink performance of UEs. Therefore, there is a need for detailed formulations for various operation parameters that affect the downlink performance of UEs in an overlaid operation of open-access macrocell and multiple closed-access femtocells inside a limited macrocell coverage space.

The adopted system model in this paper focuses on unconstrained resource sharing in the macrocell network and the femtocell network without any complicated resource management. The developed analysis incorporates the use of multiple physical channels, wherein an active UE can access only one channel at a time, and a channel reuse is not allowed in closed-access femtocells. On the other hand, the macrocell base station can be accessed unconditionally by any active macrocell UE that is present inside the macrocell coverage space (i.e. open-access macrocell base station) in order to control the rate of macrocell UE service interruption.

The analysis aims to provide new analytical approaches to model the statistical behavior of interference sources considering the variation of UEs identities, UEs access limitations, and macrocell base station and femtocell access points operation conditions. Particularly, the effect of UEs identities (macrocell or authorized femtocell), UEs densities and their likelihoods to be active, active UEs likelihoods to access channels at their serving stations (macrocell base station or femtocell access point), femtocells operation conditions (under-loaded or overloaded) that affect femtocell interference sources, the number of available channels, and the adopted channel assignment approach (e.g., arbitrary or sequential) are incorporated into the analytical formulations. Therefore, the obtained results herein provide insights into an involved and practical scenario.

The obtained results can be generally used for any small-scale and large scale channel fading models for desired link as well as interference sources. Considering the outage probability performance as an important performance metric, the analysis proceeds to address the general formulations for the outage performance of an active UE of interest for specific channel models, for which closed-form analytical results are obtained. Numerical examples, which are supported by simulations, are provided to clarify the main outcomes of the developed analytical models.

The rest of the paper is organized as follows. Section II presents preliminary discussion on the system model, highlights the interference effect from femtocell network, and classifies UEs identities and access conditions. Section III discusses interference sources, and explains the femtocells operation conditions. Section IV presents analytical and statistical models of important terms that affect downlink performance. Section V deals with the general formulations of outage performance of the desired UE and discusses some important limiting cases. Section VI provides closed-form analytical results of the outage performance under specific channel models. Section VII presents some numerical and simulation results, and finally section VIII provides concluding remarks. Table I contains a list of the main notations that are used throughout this paper.

TABLE I
LIST OF MAIN NOTATIONS.

Notation	Definition
K	Number of physical channels
L	Number of effective interference femtocells
N_0	Number of classified macrocell UEs
N_ℓ	Number of classified authorized UEs in the ℓth femtocell
N'_0	Number of macrocell UEs outside the coverage spaces of femtocells
$N''_{\ell,0}$	Number of macrocell UEs inside the coverage space of ℓth femtocell
N'_ℓ	Number of authorized femtocell UEs inside the coverage space of their ℓth femtocell
$N''_{p,\ell}$	Number of authorized ℓth femtocell UEs inside the coverage space of the pth femtocell
$N''_{0,\ell}$	Number of authorized ℓth femtocell UEs inside the macrocell space uncovered by any femtocell
$s_{\text{I},k,0}$	Total macrocell interference power on the kth channel at the desired macrocell UE
$N_{\text{act},0}$	Number of active macrocell UEs
$\mathcal{G}_1$	Set of under-loaded femtocells
$\mathcal{G}_2$	Set of over-loaded femtocells
$N'_{\text{act},g}$	Number of active authorized femtocell UEs in the gth femtocell
$\widetilde{N}'_{g_2}$	Number of *excess* active authorized femtocell UEs that are not granted an access in their g_2th femtocell
$\mathcal{G}'_2$	Set of fully-occupied and over-loaded femtocells
$\mathcal{G}''_2$	Set of partially-occupied and over-loaded femtocells
$s_{\text{I},k,\text{macrocell UE}}$	Aggregate interference power observed at the desired macrocell UE on the kth channel
$s_{\text{D},k,0}$	Received desired power on the kth channel at the macrocell UE
σ_k^2	Average power of the background white noise on the kth channel
$N_{\text{eq},0}$	The *equivalent* number of macrocell UEs that can be served by the macrocell base station
$p_{\text{act},0,i}$	Probability that the ith macrocell UE is active
$W_{0,k}$	Number of *excess* active macrocell UEs that access the kth channel allocated to the desired macrocell UE
$p_{0,k}$	Probability that an active macrocell UE from $W_{0,k}$ accesses the kth channel
$p_{\text{act},g,i}$	Probability that the ith authorized femtocell UE is active in the gth femtocell
Q	Number of femtocells each of which contains at least one active authorized UE
$p_{\widetilde{g},k,i}$	Probability that the ith active authorized UE accesses the kth channel in the $\widetilde{g}$th femtocell

II. PRELIMINARY DISCUSSION

This section contains three parts. The first part presents an overview on radio coverage spaces under consideration, and discusses the radio resource sharing between macrocell network and femtocell network. The second part highlights the density of interference observed from the femtocell network. Finally, the third part classifies UEs along with their access identities in the coverage space of interest.

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A. Coverage Spaces and Resource Sharing

The adopted system model considers a macrocell coverage space that contains multiple femtocell access points, which are deployed to cover specific locations. It is assumed that a total of K physical channels (e.g., subcarriers as in OFDM and/or time slots as in TDM) are available to serve UEs in the macrocell coverage space. The channels can be used simultaneously in the macrocell network and the femtocell network. Moreover, they are known to femtocell access points, macrocell base station, and UEs in the two networks.

A closed-access femtocell access point in the macrocell coverage space can only serve authorized femtocell UEs inside the coverage space of that femtocell. On the other hand, the macrocell base station operates according to the open-access control strategy, and hence, it has to serve any active macrocell UE inside the macrocell coverage space. The access to downlink channels by active UEs in the two networks is arranged such that an active UE can be allocated one channel at a time. This approach is motivated by the need to enhance the average number of served UEs per packet duration as well as to improve resource sharing among active UEs.

The active UEs requests for downlink service can be arranged in time domain, and then rearranged over successive packet durations to achieve long-term fairness[1]. The allocation of downlink channels during each packet duration at each femtocell access point attempts to exploit unoccupied channels (if available), which can be known via a radio sensing algorithm. It terminates when downlink requests by active authorized UEs within the allowed uplink period are handled according to the aforementioned approach, taking into consideration the capacity limit of each closed-access femtocell. On the other hand, the same channel may be reused at the macrocell base station when it is found that all channels are occupied. In this case, the channel assignment to active macrocell UEs can be managed either arbitrarily or sequentially. While arbitrary channel assignment does not consider the load balancing on available channels, the sequential channel assignment aims to establish a balanced load distribution on available channels, which can provide a noticeable performance improvement, as will be treated in subsection V-A.

B. Effective Interference Femtocells

For the sake of tractable discussion, the subsequent presentations consider a macrocell UE as being the desired UE (i.e., the UE of interest)[2]. In this case, cross-tier interference sources from femtocell network as well as co-tier macrocell interference sources from macrocell network may vary according to several issues, such as the macrocell sectorization[3] and the mobility of the desired macrocell UE. However, since the average transmit power of femtocells access points is relatively low as compared to that of the macrocell base station, the desired macrocell UE is likely to experience cross-tier interference from a limited number of femtocells.

The operating femtocell access points that can produce de-correlated interference sources are referred to as *effective* interference femtocells. The number of these effective interference femtocells during a packet duration at specific desired macrocell UE location, which is denoted by L, satisfies $L \ll L_{\text{tot}}$. Under low mobility, the term L can be treated as a fixed quantity over at least two packet durations (i.e., quasi-static operation conditions). On the other hand, under high mobility, the term L can be periodically predicted based on large-scale fading measurements of interference powers from nearby femtocells. In this regard, a femtocell will be considered as a source of cross-tier interference if it generates interference power level that exceeds a specified threshold, which may be adjusted relative to the background noise floor. In both cases, the spatial distribution of effective interference femtocell access points relative to the desired macrocell UE location can be incorporated into the large-scale channel fading models of interference sources. The results in this paper are generally applicable for any channel models. These issues will be revisited in sections III and IV.

C. Access Identities of UEs

The UEs can be classified according to their locations inside the coverage space of the macrocell as well as their access conditions. The following discussions involve terms in which the meanings of subscripts/superscripts are described as follows. For $N_{a,b}^{\prime/\prime\prime}$, the term N refers to the number of UEs, the first subscript index a denotes the macrocell index ($a = 0$) or the femtocell index ($a = 1, 2, \ldots$) in which a UE is present, and the second subscript index b denotes the macrocell index ($b = 0$) or the femtocell index ($b = 1, 2, \ldots$) to which a UE belongs to. For the superscripts, $\prime$ denotes a UE that is present inside the coverage space of its serving access point (or base station space that is uncovered by femtocells), while $\prime\prime$ denotes a UE that is present outside the coverage space of its serving access point (or base station that is uncovered by femtocells). Moreover, the term N_c refers to the total number of classified UEs as being macrocell UEs ($c = 0$) or femtocell UEs ($c = 1, 2, \ldots$).

As per Fig. 1, the total number of UEs inside the macrocell coverage space, which is denoted by N_{tot}, can be classified as macrocell UEs and authorized femtocell UEs. For L effective interference femtocells inside the macrocell coverage space, as defined in subsection II-B, the quantity N_{tot} can be expressed as $N_{\text{tot}} = N_0 + \sum_{\ell=1}^{L} N_\ell$, where N_0 refers to the number of classified macrocell UEs and N_ℓ denotes the number of classified authorized femtocell UEs that are associated with the ℓth femtocell.

Since the classified macrocell UEs can be anywhere inside the macrocell coverage space, and from Fig. 1 (cases 1 and 2), it be written that $N_0 = N_0' + \sum_{\ell=1}^{L} N_{\ell,0}''$, where N_0' is the number of macrocell UEs that are present inside the macrocell space uncovered by any femtocell (case 1 in Fig. 1), and $N_{\ell,0}''$

[1]Here, multi-user scheduling can be exploited to further enhance the scheduled UE downlink performance. However, this case is not treated herein, but it will be a topic of future work.

[2]The developed analysis and results in this paper can be generally used to characterize any identity of the desired UE (e.g., macrocell or authorized femtocell UE). Related discussions are presented in subsections V-B–V-D.

[3]With macrocell sectorization using directional antennas, the number of macrocell UEs and the number of femtocell access points per sector decreases almost linearly with the number of created sectors.

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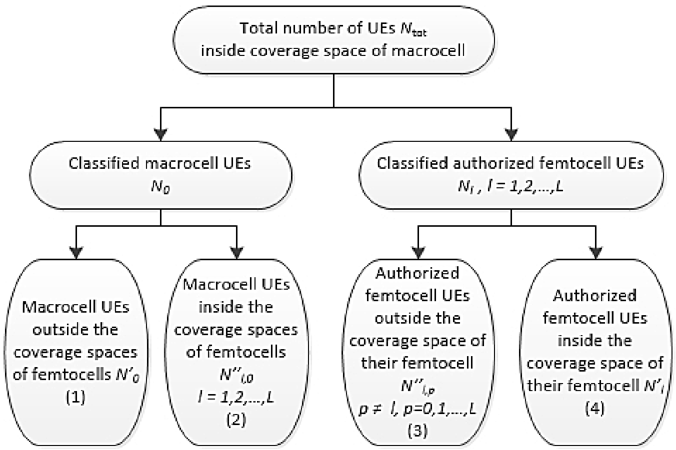

Fig. 1. Access identities of UEs in the macrocell coverage space.

is the number of macrocell UEs that exist inside the coverage space of the ℓth femtocell, but can not be served by that femtocell access point (case 2 in Fig. 1). Moreover, for classified authorized femtocell UEs, the authorized femtocell UEs in the coverage space of their ℓth femtocell, whose number is denoted by N_ℓ, can be anywhere inside the macrocell coverage space. Specifically, a portion of the N_ℓ authorized femtocell UEs can be inside the coverage space of their ℓth femtocell, which is denoted by N'_ℓ (case 4 in Fig. 1), while others may be in the coverage space of the pth femtocell, which is denoted by $N''_{p,\ell}$, for $p = 1, 2, \ldots, L$ and $p \neq \ell$ as well as in the macrocell space uncovered by any femtocell, which is denoted by $N''_{0,\ell}$ (case 3 in Fig. 1). Therefore, it can be written that $N_\ell = N'_\ell + N''_{0,\ell} + \sum_{p=1,p\neq\ell}^{L} N''_{p,\ell}$, for $\ell = 1, 2, \ldots, L$.

Based on the preceding classifications, the total number UEs with different identities that exist in the macrocell space uncovered by any femtocell can be expressed as $N'_0+\sum_{\ell=1}^{L} N''_{0,\ell}$. On the other hand, the total number UEs with different identities that exist in the coverage space of the jth femtocell access point can be expressed as $N'_j + N''_{j,0} + \sum_{\ell=1,\ell\neq j}^{L} N''_{j,\ell}$, for $j = 1, 2, \ldots, L$. The jth closed-access femtocell access point can serve any the N'_j authorized UEs.

A closed-access femtocell is usually deployed to serve a limited number of authorized femtocell UEs that aim to unconditionally access available channels while avoiding intra-cell interference from their serving access point. To meet this objective, it is considered that each channel at each closed-access femtocell can not be reused concurrently (i.e., allocated to more than one active authorized femtocell UE at a time). This condition has to be satisfied, particularly when perfect separation between simultaneous and independent data streams to serve multiple active authorized femtocell UEs on the same channel can not be achieved, which is the case treated herein. Therefore, as discussed in subsection II-A, each femtocell access point can distribute the requests for downlink service on the channels, whose number is denoted by K above, such that intra-cell interference is avoided. This approach can result in no intra-cell interference when the number of concurrently active authorized femtocell UEs in each femtocell does not exceed the number of channels, under the condition that an active UE can be served on single channel at a time. Further explanations on the impact of this approach on femtocell interference sources will be presented in subsection III-B.

III. Interference Sources and Femtocells Operation Conditions

This section characterizes interference sources and the operation conditions of effective closed-access femtocells. The treatment herein is applicable for any spatial location of the desired macrocell UE inside the coverage space of the macrocell. However, the results are first presented when the desired macrocell UE is present outside the coverage spaces of the deployed femtocells, which will be then extended to other possible spatial locations (e.g., far away from the macrocell base station and very close to femtocell access point(s), very close to the macrocell base station and far away from coverage spaces of femtocells, etc. (refer to subsections V-B–V-D)).

Capitalizing on subsections II-B and II-C, the density of authorized UEs in each femtocell and the density of macrocell UEs in macrocell space uncovered by any femtocell may vary from time to time according to UEs modes (i.e., active or inactive), mobility, channel conditions, and traffic demand. This variation lead to dynamic behavior of cross-tier interference and co-tier interference observed at the desired macrocell UE, as discussed in the following three parts of this section.

A. Macrocell Interference

Since the macrocell base station can unconditionally reuse the channels, the desired macrocell UE may undergo co-tier interference if its allocated channel is simultaneously reused to serve other active macrocell UEs (i.e., case when the serving macrocell base station is overloaded and fully-occupied) or the same channel is used at adjacent macrocell base stations that produce noticeable inter-cell interference effect (i.e., case of adjacent macrocell interference). Herein, the term $s_{\mathrm{I},k,0}$ is used to refer to the total (accumulated) interference power on the kth channel due to intra-cell and/or inter-cell macrocell interference effect at the desired macrocell UE when at least one active macrocell UE is supported by the serving (or adjacent) macrocell base station on the same channel that is allocated to the desired UE.

The level of $s_{\mathrm{I},k,0}$ may be significant, specifically due to the relatively high average transmit power of the macrocell base station(s), possible high inter-cell interference occasions (e.g., when the desired macrocell UE is at the edge of its macrocell coverage space), and the adopted macrocell open-access control and channel reuse strategies. Moreover, the level of $s_{\mathrm{I},k,0}$ can vary with the number of active macrocell UEs that can access the channels at the macrocell base station, which is referred to as $N_{\mathrm{act},0}$, and the likelihoods that these active UEs use the same channel that is allocated to the desired macrocell UE. Since the number of classified macrocell UEs can vary according to the operation conditions of femtocells, as will be discussed in the following part below, the detailed analysis of $s_{\mathrm{I},k,0}$, which is directly related to $N_{\mathrm{act},0}$, is provided in section IV.

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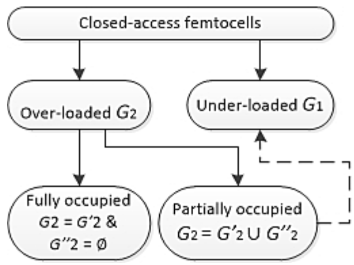

Fig. 2. Closed-access femtocells operation conditions.

B. Femtocells Interference

Since femtocells can use the same channels of the macrocell base station, there can be cross-tier interference observed from an effective number of femtocells (refer to subsection II-B). As shown in Fig. 2, these femtocells can be classified into two disjoint sets of over-loaded and under-loaded femtocells. Specifically, when a femtocell is over-loaded and fully-occupied, and since the reuse of channels is not permitted in closed-access femtocells, then it will result in a single cross-tier interference source with probability one. On the other hand, when a femtocell is under-loaded or partially-occupied, there is a certain likelihood of observing one cross-tier interference source if the channel assigned to the desired macrocell UE is simultaneously used in that femtocell to serve an authorized femtocell UE. The following two parts treat the aforementioned cases.

1) Case of under-loaded femtocells: Let $\mathcal{G}_1$ be a set that contains the indexes of under-loaded femtocells. Recall that N'_ℓ denotes the number of authorized femtocells UEs inside the coverage space of their ℓth serving femtocell. Then the femtocells whose indexes belong to the set $\mathcal{G}_1$ satisfy the inequality that $N'_{g_1} < K$, for $g_1 \in \mathcal{G}_1$ (the subscript g_1 refers to a femtocell index). In general, an interference source will be observed from a femtocell whose index belongs to a subset of the set $\mathcal{G}_1$, which is denoted by $\mathcal{G}'_1$, where $\mathcal{G}'_1 \subset \mathcal{G}_1$, in which the same channel allocated to the desired macrocell UE is also occupied by an active authorized femtocell UE whose serving femtocell index belongs to $\mathcal{G}'_1$. Therefore, the cardinality of $\mathcal{G}'_1$, which is denoted by $|\mathcal{G}'_1|$, is a variable, which takes on values from $\{0, 1, \ldots, |\mathcal{G}_1|\}$.

The aggregate interference power observed at the desired macrocell UE due to interference sources from under-loaded femtocells can be written as

$$s_{\mathrm{I},k,\text{under-loaded}} = \sum_{g_1 \in \mathcal{G}'_1} s_{\mathrm{I},k,g_1}, \tag{1}$$

where s_{I,k,g_1} is the single interference power component that is observed from the g_1th under-loaded femtocell whose index belongs to $\mathcal{G}'_1$. Clearly, the best case scenario from the desired macrocell UE point occurs when $|\mathcal{G}'_1| = 0$. The statistical modeling of $|\mathcal{G}'_1|$ will be presented in subsection IV-D.

2) Case of over-loaded femtocells: The other class of femtocells include the over-loaded femtocells whose indexes are considered to be in the set $\mathcal{G}_2$, where $\mathcal{G}_1 \cap \mathcal{G}_2 = \emptyset$. In this case, the condition that $N'_{g_2} \geq K$, for $g_2 \in \mathcal{G}_2$, takes place (the subscript g_2 refers to a femtocell index). Since the number of active authorized femtocell UEs in an over-loaded femtocell can vary from time to time, let N'_{act,g_2} be the number of active authorized femtocell UEs in the g_2th over-loaded femtocell, which takes on values from $\{0, 1, 2, \ldots, N'_{g_2}\}$. Due to the random nature of N'_{act,g_2}, and as shown in Fig. 2, the over-loaded femtocells may be fully-occupied (defined as the case of fixed $\mathcal{G}_2$) or they have variable occupancy (defined as the case of variable $\mathcal{G}_2$).

The likelihood that the set $\mathcal{G}_2$ be fixed is the probability that the number of active authorized UEs in each femtocell whose index belongs to $\mathcal{G}_2$ exceeds the number of channels (i.e., the case of $N'_{\mathrm{act},g_2} \geq K$, for $g_2 \in \mathcal{G}_2$), which results in

$$\Pr\{\mathcal{G}_2 \text{ is fixed}\} = \prod_{g_2 \in \mathcal{G}_2} \Pr\{N'_{\mathrm{act},g_2} \geq K\}. \tag{2}$$

In this case, the desired macrocell UE will observe with probability one an interference source from each over-loaded femtocell. Moreover, since the closed-access femtocell can support a maximum number of K active authorized femtocell UEs at a time, the number of *excess* active authorized femtocell UEs that can not be granted an access to their serving femtocell access point, which is denoted by $\widetilde{N}'_{g_2}$, may be served by the macrocell base station. The total number of these UEs is given as

$$\sum_{g_2 \in \mathcal{G}_2} \widetilde{N}'_{g_2} = \sum_{g_2 \in \mathcal{G}_2} (N'_{\mathrm{act},g_2} - K). \tag{3}$$

These active UEs become active macrocell UEs that are present inside the coverage spaces of femtocells. Therefore, the co-tier interference is expected to increase due to the increase in the population of active macrocell UEs.

The aggregate interference power that is observed with probability one from fully-occupied and over-loaded femtocells when $\mathcal{G}_2$ is fixed reaches its maximum limit:

$$s_{\mathrm{I},k,\text{over-loaded,fixed } \mathcal{G}_2} = \sum_{g_2 \in \mathcal{G}_2} s_{\mathrm{I},k,g_2}, \tag{4}$$

where s_{I,k,g_2} is the single interference power component that is observed from the g_2th fully-occupied and over-loaded femtocell whose index belongs to the fixed set $\mathcal{G}_2$. Note that, under the condition in (2), the set of under-loaded femtocells remains as treated in subsection III-B1, and result in (1) is still applicable.

For the other case of variable set $\mathcal{G}_2$, the likelihood that $\mathcal{G}_2$ is variable can be obtained as the probability that at least one femtocell whose index belongs to $\mathcal{G}_2$ has a number of active authorized UEs that is below K, which gives

$$\Pr\{\mathcal{G}_2 \text{ is variable}\} = 1 - \prod_{g_2 \in \mathcal{G}_2} \Pr\{N'_{\mathrm{act},g_2} \geq K\}. \tag{5}$$

Under the condition in (5), the set of over-loaded femtocells $\mathcal{G}_2$ can be divided into two disjoint sets, which are denoted by $\mathcal{G}'_2$ and $\mathcal{G}''_2$, such that $\mathcal{G}_2 = \mathcal{G}'_2 \cup \mathcal{G}''_2$ and $\mathcal{G}'_2 \cap \mathcal{G}''_2 = \emptyset$. Then the set $\mathcal{G}'_2$ contains the indexes of over-loaded femtocells that are fully-occupied, for which the condition $N'_{\mathrm{act},g_2} \geq K$, for $g_2 \in \mathcal{G}'_2$ is satisfied. On the other hand, the set $\mathcal{G}''_2$ contains the indexes of over-loaded femtocells that are partially-occupied, for which the condition $N'_{\mathrm{act},g_2} < K$, for $g_2 \in \mathcal{G}''_2$ is satisfied.

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The probability that an interference source will be observed from a fully-occupied femtocell whose index belongs $\mathcal{G}_2'$ is one. However, a partially-occupied femtocell whose index belongs to $\mathcal{G}_2''$ may produce a single interference source with a certain likelihood. Therefore, and based on (5), the set $\mathcal{G}_2''$ can be combined with $\mathcal{G}_1$, which results in the new set of under-loaded femtocells.

Further clarifications on the operation conditions of femtocells and their impact on interference sources are now described. When the set $\mathcal{G}_2$ is fixed (i.e., $\mathcal{G}_2'' = \emptyset$), the index g_1 associated with the result in (1) draws its values from $\mathcal{G}_1' \subset \mathcal{G}_1$ whose cardinality satisfies $|\mathcal{G}_1'| \leq |\mathcal{G}_1|$. On the other hand, when $\mathcal{G}_2'' \neq \emptyset$, the set from which the index g_1 in (1) is drawn becomes $\mathcal{G}_1' \cup \widetilde{\mathcal{G}}_2''$, where $\widetilde{\mathcal{G}}_2''$ is a subset of the set $\mathcal{G}_2''$ (i.e., $\widetilde{\mathcal{G}}_2'' \subset \mathcal{G}_2''$) that defines the over-loaded partially-occupied femtocells, which use the same channel allocated to the desired macrocell UE to support an active authorized femtocell UE per femtocell. Since $\mathcal{G}_1'$ and $\widetilde{\mathcal{G}}_2''$ are disjoint subsets, the cardinality of their union is $|\mathcal{G}_1'| + |\widetilde{\mathcal{G}}_2''|$, with the condition that $N'_{\text{act},g_1} < K$, for $g_1 \in \mathcal{G}_1' \cup \widetilde{\mathcal{G}}_2''$, is satisfied. Then it can be written that

$$s_{\text{I},k,\text{under-loaded,varaible }\mathcal{G}_2} = \sum_{g_1 \in \mathcal{G}_1' \cup \widetilde{\mathcal{G}}_2''} s_{\text{I},k,g_1}, \tag{6}$$

$$s_{\text{I},k,\text{over-loaded,variable }\mathcal{G}_2} = \sum_{g_2 \in \mathcal{G}_2'} s_{\text{I},k,g_2}. \tag{7}$$

The decrease in $|\mathcal{G}_2|$ is expected to decrease the level of cross-tier interference due to the reduction in the likelihood that the kth channel that is allocated to the desired macrocell UE be used in effective interference femtocells. Moreover, the reduction in $|\mathcal{G}_2|$ decreases the total number of excess active authorized femtocells UE that can potentially use the channels at the macrocell base station, which results in a lower level of co-tier interference from the macrocell network. In any case, the sum of cardinalities of $\mathcal{G}_2$ and $\mathcal{G}_1$ satisfies $|\mathcal{G}_1| + |\mathcal{G}_2| = L$ (refer to subsection II-B).

C. Aggregate Interference Power

Based on the results in (1)–(7), the aggregate interference power observed at the desired macrocell UE that is allocated the kth channel at the macrocell base station, which is referred to as $s_{\text{I},k,\text{macrocell UE}}$, can be now expressed as shown in (8), where $s_{\text{I},k,0}$ is defined in subsection III-A. Moreover, as shown in subsection III-B, $\mathcal{G}_2 = \mathcal{G}_2' \cup \mathcal{G}_2''$, $\mathcal{G}_2' \cap \mathcal{G}_2'' = \emptyset$, $\mathcal{G}_1' \subset \mathcal{G}_1$, and $\widetilde{\mathcal{G}}_2'' \subset \mathcal{G}_2''$.

The findings in the preceding subsections will be used in the following section to develop statistical models of important terms that affect the performance of the desired macrocell UE.

IV. Analytical Formulations and Statistical Models

This section contains four parts. In the first part, interference bounding scenarios are presented. Then a preliminary formulation of an important performance metric, which is the outage probability of the desired macrocell UE, is presented. The final two parts present statistical models of important terms that are associated with the outage performance. The findings of this section are important to obtain the general formulations presented in the next section.

A. Interference Bounding Scenarios

From subsections III-A and III-B, it is noted that the worst case scenario for cross-tier interference takes place when $N'_{\text{act},g_2} \geq K$, for $g_2 \in \mathcal{G}_2$, and $|\mathcal{G}_2| = L$ (i.e., all femtocells are over-loaded and fully-occupied). In this case, it follows that $\mathcal{G}_1 = \emptyset$ and $\mathcal{G}_2'' = \emptyset$, and the desired macrocell UE will experience the largest number of possible cross-trier interference sources as well as an increased total number of excess authorized femtocell UEs (refer to (3) and to the first entry of (8) with $\mathcal{G}_1 = \emptyset$). On the other hand, the best possible scenario from the desired macrocell UE point of view takes place when $|\mathcal{G}_1| = L$ (i.e., all femtocell are under-loaded) and the kth channel is not allocated to any active authorized UE in any femtocell (i.e., the channel allocated to the desired macrocell UE is not allocated to any active authorized UE in any under-loaded femtocell). In this case, cross-tier interference will be avoided and there will be no excess authorized femtocell UEs (i.e., $\sum_{g_2 \in \mathcal{G}_2} \widetilde{N}'_{g_2} = 0$ in (3)). Using the preceding results into (8) gives the results in (9). Moreover, an upper bound to the case when the set $\mathcal{G}_2$ is variable for any cardinality $|\mathcal{G}_2|$, which also includes the two limits in (9) as special cases, follows the same form of the first entry of (8) for fixed $\mathcal{G}_2$ with the conditions that $|\mathcal{G}_2| < L, |\mathcal{G}_1| = L - |\mathcal{G}_2|, \mathcal{G}_1' \subset \mathcal{G}_1 \neq \emptyset, \mathcal{G}_2'' = \emptyset$.

As discussed in subsection III-A, the total macrocell interference power $s_{\text{I},k,0}$ may be significant. The first step towards characterizing $s_{\text{I},k,0}$ is to provide statistical modeling for the number of active macrocell UEs that can access channels at the macrocell base station, $N_{\text{act},0}$. Noting that $N_{\text{act},0}$ can take its values from $\{0, 1, \ldots, N_{\text{eq},0}\}$, where $N_{\text{eq},0}$ is the *equivalent* number of macrocell UEs that can be served by the macrocell base station, but can not access the resources of femtocell access points (refer to subsection II-C and the result in (3)). For the cases in (9), it follows that $N_{\text{eq},0}$ is defined as shown in (10)[4]. Moreover, for the result in (8) with fixed $\mathcal{G}_2$, it can be written that

$$N_{\text{eq},0}\Big|_{|\mathcal{G}_2|<L, |\mathcal{G}_1| = L - |\mathcal{G}_2|, \mathcal{G}_1' \subset \mathcal{G}_1 \neq \emptyset, \mathcal{G}_2'' = \emptyset} = N_0 + \sum_{\ell=1}^{L} N''_{0,\ell} + \sum_{g_2 \in \mathcal{G}_2} \widetilde{N}'_{g_2}. \tag{11}$$

According to (10) and (11), the term $s_{\text{I},k,0} = 0$ when the condition that $N_{\text{act},0} \leq K - 1$ is satisfied. Therefore, the best possible scenario from the desired macrocell UE point of view takes place when at least one of the channels at the serving macrocell base station is not assigned to any other active macrocell UE and it is not used by active macrocell UE in adjacent macrocells. Moreover, no interference from femtocells will be observed if the desired UE channel is unoccupied by active authorized femtocell UEs and there is no

[4]The effect of adjacent macrocell interference sources can be incorporated into the value of $N_{\text{eq},0}$ without any loss of generality. For instance, this effect can be added to both sides of the result in (10) and to the result in (11).

$$s_{\mathrm{I},k,\text{macrocell UE}} = \begin{cases} s_{\mathrm{I},k,0} + \sum_{g_1 \in \mathcal{G}_1'} s_{\mathrm{I},k,g_1} + \sum_{g_2 \in \mathcal{G}_2} s_{\mathrm{I},k,g_2}, & \text{for fixed } \mathcal{G}_2 \\ s_{\mathrm{I},k,0} + \sum_{g_1 \in \mathcal{G}_1' \cup \widetilde{\mathcal{G}}_2''} s_{\mathrm{I},k,g_1} + \sum_{g_2 \in \mathcal{G}_2'} s_{\mathrm{I},k,g_2}, & \text{for variable } \mathcal{G}_2 \end{cases} \tag{8}$$

$$\left(s_{\mathrm{I},k,0}\right)\big|_{|\mathcal{G}_1|=L,\mathcal{G}_1'=\emptyset,\mathcal{G}_2=\emptyset} \leq s_{\mathrm{I},k,\text{macrocell UE}} \leq \left(s_{\mathrm{I},k,0} + \sum_{\ell=1}^{L} s_{\mathrm{I},k,\ell}\right)\big|_{|\mathcal{G}_2|=L,\mathcal{G}_1=\emptyset,\mathcal{G}_2''=\emptyset} \tag{9}$$

$$\left(N_0 + \sum_{\ell=1}^{L} N_{0,\ell}''\right)\big|_{|\mathcal{G}_1|=L,\mathcal{G}_1'=\emptyset,\mathcal{G}_2=\emptyset} \leq N_{\mathrm{eq},0} \leq \left(N_0 + \sum_{\ell=1}^{L}\left(N_{0,\ell}'' + \widetilde{N}_\ell'\right)\right)\big|_{|\mathcal{G}_2|=L,\mathcal{G}_1=\emptyset,\mathcal{G}_2''=\emptyset}. \tag{10}$$

fully-occupied and over-loaded femtocells. Under these conditions, the desired macrocell UE will experience interference-free service (i.e., $s_{\mathrm{I},k,\text{macrocell UE}} = 0$).

B. Preliminary Formulation for Outage Performance

This subsection presents preliminary formulation for the performance of the desired macrocell UE, which is needed herein to clarify the importance of developing results for some terms that can significantly affect the achieved performance. Considering the outage probability as an important performance measure[5], it can be written that

$$\begin{aligned} P_{\mathrm{OUT}} &= \Pr\{\gamma_{\mathrm{SINR},k,\text{macrocell UE}} < x\} \\ &= \Pr\left\{\frac{s_{\mathrm{D},k,0}}{s_{\mathrm{I},k,\text{macrocell UE}} + \sigma_k^2} < x\right\}, \end{aligned} \tag{12}$$

where $\gamma_{\mathrm{SINR},k,\text{macrocell UE}}$ is the received signal-to-interference-plus-noise ratio (SINR) at the desired macrocell UE, x is the received SINR threshold, $s_{\mathrm{D},k,0}$ is the received desired power from the serving macrocell base station on the kth allocated channel, and σ_k^2 is the average power of the background white noise observed on the kth channel. Then, based on the results in subsection IV-A, the results shown in (13) are obtained[6], where $N_{\mathrm{eq},0}$ is given in (11)[7]. Note that (13) exploits the independence between $N_{\mathrm{act},0}$ and $|\mathcal{G}_1'|$ whose statistical models are the topics of the following two subsections.

C. Statistical modeling of $N_{act,0}$

In this part, the statistics of the term $N_{\mathrm{act},0}$, which are needed in (13), are presented. The impact of this term on macrocell interference sources is also explained. The treatment herein covers the following:

- The macrocell UEs conditions that may result in *potential* macrocell interference sources at a time.
- The active macrocell UEs that can access the same channel allocated to the desired macrocell UE.

[5]The consideration of other performance metrics follow the same principles adopted herein for the outage performance.

[6]Again, the general formulations for the outage probability will be treated in Section V. However, its preliminary form is presented herein to clarify the developments of subsections IV-C and IV-D.

[7]Other formulations of the outage probability based on the second entry of (8), (9), or (10) can be also obtained following the same footsteps of (13).

Each macrocell UE has a certain probability to be active at a time. Therefore, the term $p_{\mathrm{act},0,i}$ is used to denote the probability that the ith macrocell UE be active, and it attempts to receive downlink service from the macrocell base station on one of the channels. The probability that there will be exactly $n_{\mathrm{act},0}$ number of active UEs that attempt to access the macrocell base station resources simultaneously per a packet duration can be characterized using Poisson binomial distribution [25], which gives

$$\begin{aligned} \Pr\{N_{\mathrm{act},0} = n_{\mathrm{act},0}\} &= \prod_{i=1}^{N_{\mathrm{eq},0}} (1 - p_{\mathrm{act},0,i}) \\ &\times \left[\sum_{\mathcal{A} \in \mathcal{F}_{n_{\mathrm{act},0}}} \prod_{i \in \mathcal{A}} p_{\mathrm{act},0,i}(1 - p_{\mathrm{act},0,i})^{-1}\right], \end{aligned} \tag{14}$$

where $\mathcal{F}_{n_{\mathrm{act},0}}$ denotes the set of all subsets of $n_{\mathrm{act},0}$ integers that can be selected from $\{1, 2, \ldots, N_{\mathrm{eq},0}\}$, and $\mathcal{A} \in \mathcal{F}_{n_{\mathrm{act},0}}$ represents an ordered set of $(j_1, \ldots, j_{n_{\mathrm{act},0}})$ such that $j_q < j_k$ if $q < k$. There will be a total number of $\binom{N_{\mathrm{eq},0}}{n_{\mathrm{act},0}}$ subsets each of size $n_{\mathrm{act},0}$ in $\mathcal{F}_{n_{\mathrm{act},0}}$. The term $\Pr\{N_{\mathrm{act},0} = 0\} = \prod_{i=1}^{N_{\mathrm{eq},0}}(1 - p_{\mathrm{act},0,i})$ represents the case when all macrocell UEs are non-active at the time the desired macrocell UE requests downlink service. For the case when $p_{\mathrm{act},0,i} = p_{\mathrm{act},0}$, for $i = 1, 2, \ldots, N_{\mathrm{eq},0}$, it follows that

$$\begin{aligned} &\Pr\{N_{\mathrm{act},0} = n_{\mathrm{act},0}\}|_{p_{\mathrm{act},0,i}=p_{\mathrm{act},0},\ i\in\{1,2,\ldots,N_{\mathrm{eq},0}\}} \\ &= \binom{N_{\mathrm{eq},0}}{n_{\mathrm{act},0}} (p_{\mathrm{act},0})^{n_{\mathrm{act},0}} (1 - p_{\mathrm{act},0})^{N_{\mathrm{eq},0} - n_{\mathrm{act},0}}. \end{aligned} \tag{15}$$

When $N_{\mathrm{eq},0} \gg 1$ and $p_{\mathrm{act},0} \ll 1$, the distribution of $N_{\mathrm{act},0}$ converges to Poisson distribution with a mean value of $N_{\mathrm{eq},0} p_{\mathrm{act},0}$.

It is now required to quantify the number of active macrocell UE that can access the same channel allocated to the desired macrocell UE. Specifically, since the channels are de-separated in frequency and/or time, the desired macrocell UE will experience co-tier interference from the serving macrocell base station (or adjacent macrocells) only if its channel is reused simultaneously to serve other active macrocell UEs, irrespective to their density. For a given number of active macrocell UEs (i.e., $N_{\mathrm{act},0} = n_{\mathrm{act},0}$), let $W_{0,k}$ be the number of *excess* active macrocell UEs that can access the same kth channel allocated to the desired macrocell UE, where

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$$P_{\text{OUT}}|_{\mathcal{G}_2\neq\emptyset,\,\mathcal{G}_2''=\emptyset} = \sum_{p=0}^{|\mathcal{G}_1|}\sum_{n_{\text{act},0}=0}^{N_{\text{eq},0}} \Pr\left\{\gamma_{\text{SINR},k,\text{macrocell UE}} < x \Big| |\mathcal{G}_1'| = p, N_{\text{act},0} = n_{\text{act},0}\right\}\Pr\{N_{\text{act},0} = n_{\text{act},0}\}\Pr\{|\mathcal{G}_1'| = p\}$$
$$= \sum_{p=0}^{|\mathcal{G}_1|}\sum_{n_{\text{act},0}=0}^{N_{\text{eq},0}} \Pr\left\{\frac{s_{\text{D},k,0}}{s_{\text{I},k,0|N_{\text{act},0}=n_{\text{act},0}} + \sum_{g_1\in\mathcal{G}_1'} s_{\text{I},k,g_1} + \sum_{g_2\in\mathcal{G}_2} s_{\text{I},k,g_2} + \sigma_k^2} < x\right\}\Pr\{N_{\text{act},0} = n_{\text{act},0}\}\Pr\{|\mathcal{G}_1'| = p\}, \quad (13)$$

$W_{0,k}$ takes on values from $\{0, 1, 2, \ldots, n_{\text{act},0} - K\}$. The upper limit on $W_{0,k}$, which is $n_{\text{act},0} - K$, is a result of the consideration that the macrocell base station can distribute active macrocell UEs requests on available channels such that co-tier interference is avoided when the channels are partially occupied. Moreover, the $n_{\text{act},0}-K$ active macrocell UEs (when all channels occupied) may be served by the kth channel (i.e., case of arbitrary channel assignment for the excess active macrocell UEs, as considered in subsection V-A1).

For an active macrocell UE, it may be granted an access to any of the K channels at the macrocell base station. In this regard, let the term $p_{0,k}$ be the probability that an active macrocell UE from those counted into $n_{\text{act},0} - K$ accesses the kth channel, where $\sum_{k=1}^{K} p_{0,k} = 1$. It follows that the distribution of $W_{0,k}$ can be expressed as

$$\Pr\{W_{0,k} = w_{0,k}\}|_{p_{0,k,j}=p_{0,k},\ j=1,2,\ldots,n_{\text{act},0}-K} = \binom{n_{\text{act},0} - K}{w_{0,k}}(p_{0,k})^{w_{0,k}}(1 - p_{0,k})^{n_{\text{act},0}-K-w_{0,k}}. \quad (16)$$

Based on (15) and (16), the statistics of the conditional total macrocell interference power (conditioned on $N_{\text{act},0} = n_{\text{act},0}$) observed due to the concurrent access of some active macrocell UEs of the same channel allocated to the desired macrocell UE, which is denoted by $s_{\text{I},k,0|N_{\text{act},0}=n_{\text{act},0}}$, can be written as shown in (17), where the term z refers to an arbitrary threshold and $\{s_{\text{I},k,0,n}\}$, for $n = 1, 2, \ldots, n_{\text{act},0} - K + 1$, refer to the independent and spatially de-correlated co-tier macrocell interference powers. The preceding result counts for the presence of a single interference source due to an active macrocell UE that accesses the kth channel of the macrocell base station with probability one when $n_{\text{act},0} = K$, as discussed above.

D. Statistical Modeling of $|\mathcal{G}_1'|$

This part presents the statistics of the term $|\mathcal{G}_1'|$, which appear in (13). The impact of this term on femtocell interference sources is also clarified. The analysis presented below treats the following:

- The active authorized femtocell UEs per femtocell.
- The femtocells, each of which contains at least one active authorized femtocell UE.
- The femtocells, each of which produces a cross-tier interference source.

For the class of under-loaded femtocells, which is considered in subsection III-B, let the term $p_{\text{act},g_1,i}$ be the probability that the ith authorized femtocell UE is active, and it attempts to access its serving g_1th femtocell access point, for $g_1 \in \mathcal{G}_1$. When authorized UEs in each femtocell access point have identical probabilities to be active (i.e. $p_{\text{act},g_1,i} = p_{\text{act},g_1}$, for $i = 1, 2, \ldots, N'_{g_1}$), the number of active authorized femtocell UEs in the g_1th femtocell, which is denoted by N'_{act,g_1}, takes on values from $\{0, 1, \ldots, N'_{g_1}\}$, where N'_{g_1} is defined in subsection II-C. Then the probability that $N'_{\text{act},g_1} = n'_{\text{act},g_1}$, for $g_1 \in \mathcal{G}_1$, is given by

$$\Pr\{N'_{\text{act},g_1} = n'_{\text{act},g_1}\}|_{p_{\text{act},g_1,i}=p_{\text{act},g_1},\ i\in\{1,2,\ldots,N'_{g_1}\}} = \binom{N'_{g_1}}{n'_{\text{act},g_1}}(p_{\text{act},g_1})^{n'_{\text{act},g_1}}(1 - p_{\text{act},g_1})^{N'_{g_1}-n'_{\text{act},g_1}}. \quad (18)$$

From the result in (18), it is clear that the probability that all authorized UEs in the g_1th femtocell will be inactive is given by $\Pr\{N'_{\text{act},g_1} = 0\} = (1-p_{\text{act},g_1})^{N'_{g_1}}$, and the probability that at least one authorized UE is active is given by

$$\Pr\{N'_{\text{act},g_1} > 0\} = 1-\Pr\{N'_{\text{act},g_1} = 0\} = 1-(1-p_{\text{act},g_1})^{N'_{g_1}}. \quad (19)$$

It is now required to quantify the number of under-loaded femtocells wherein each of which contains at least one active authorized femtocell UE at a time. Let the term Q be the number of femtocells wherein each of which contains at least one active authorized UE. Then Q can take on values from $\{0, 1, 2, \ldots, |\mathcal{G}_1|\}$. The probability that $Q = q$ can be drawn from Poisson binomial distribution, where the probabilities of success of the independent trials are $\{1 - \Pr\{N'_{\text{act},g_1} = 0\}\}$, for $g_1 \in \mathcal{G}_1$. For identical probabilities of success (i.e., $\Pr\{N'_{\text{act},g_1} = 0\} = \Pr\{N'_{\text{act},1} = 0\}$, for $g_1 \in \mathcal{G}_1$), it follows that

$$\Pr\{Q = q\}|_{\Pr\{N'_{\text{act},g_1}=0\}=\Pr\{N'_{\text{act},1}=0\},g_1\in\mathcal{G}_1} = \binom{|\mathcal{G}_1|}{q}(1 - \Pr\{N'_{\text{act},1} = 0\})^q(\Pr\{N'_{\text{act},1} = 0\})^{|\mathcal{G}_1|-q}. \quad (20)$$

The active authorized UEs in each of the under-loaded femtocells that are quantified through (20) can be granted an access to any of the K channels at their serving femtocell access point. To characterize the random access of active authorized UEs for this Q number of femtocells, let $p_{\widetilde{g}_1,k,i}$ be the probability that the ith active authorized UE will access the kth channel in the $\widetilde{g}_1$th femtocell access point, where $\sum_{k=1}^{K} p_{\widetilde{g}_1,k,i} = 1$, and $\widetilde{g}_1$ is a counter that is drawn from $\{1, 2, \ldots, Q = q\}$. Then, based on the definition of the subset $\mathcal{G}_1'$ in subsection III-B1, it follows that the event that $|\mathcal{G}_1'| = 0$ takes place when the active authorized UEs in all femtocells whose indexes in the set $\mathcal{G}_1$ are inactive, or when there are some active authorized UEs in different femtocells but none of

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$$\Pr\{s_{\mathrm{I},k,0|N_{\mathrm{act},0}=n_{\mathrm{act},0}} < z\} = \sum_{w_{0,k}=0}^{n_{\mathrm{act},0}-K} \Pr\left\{ s_{\mathrm{I},k,0|N_{\mathrm{act},0}=n_{\mathrm{act},0}} = \sum_{n=1}^{w_{0,k}+1} s_{\mathrm{I},k,0,n}, s_{\mathrm{I},k,0|N_{\mathrm{act},0}=n_{\mathrm{act},0}} < z \right\}$$
$$= \sum_{w_{0,k}=0}^{n_{\mathrm{act},0}-K} \Pr\left\{ \sum_{n=1}^{w_{0,k}+1} s_{\mathrm{I},k,0,n} < z \right\} \Pr\{W_{0,k} = w_{0,k}\} \quad (17)$$

$$\Pr\{|\mathcal{G}_1'| = 0\} = \left(\Pr\{N'_{\mathrm{act},1} = 0\}\right)^{|\mathcal{G}_1|} + \sum_{q=1}^{|\mathcal{G}_1|} \binom{|\mathcal{G}_1|}{q} \left(1 - \Pr\{N'_{\mathrm{act},1} = 0\}\right)^{q} \left(\Pr\{N'_{\mathrm{act},1} = 0\}\right)^{|\mathcal{G}_1|-q} \prod_{\widetilde{g}_1=1}^{q} (1 - p_{\widetilde{g}_1,k,i}). \quad (21)$$

them is granted an access to the kth channel that is allocated to the desired macrocell UE. These two scenarios result in (21). Note that when $p_{\widetilde{g}_1,k,i} = 0$, for $\widetilde{g}_1 = \{1, 2, \ldots, Q = q\}$, the likelihood of the event $|\mathcal{G}_1'| = 0$ becomes unity, which represents the event wherein the cross-tier interference from under-loaded femtocells is avoided. In this case, none of the femtocells whose indexes belong to the set $\mathcal{G}_1$ grants an access to the same channel of the desired macrocell UE.

The event that $|\mathcal{G}_1'| = 1$ takes place when there is at least one femtocell in which the number of active authorized UEs is not zero, but only one authorized UE in one of the femtocell access points will access the kth channel. In general, the event that $|\mathcal{G}_1'| = p$ takes place when at least p femtocell access points each of which has a non-zero number of active authorized UEs, but only an active UE per femtocell in p different femtocells of them will access the kth channel. For the case when the probability that an active authorized UE can access the kth channel at a femtocell access point is insensitive to the both the UE index the femtocell index (i.e. $p_{\widetilde{g}_1,k,i} = p_{1,k}$), the term $\Pr\{|\mathcal{G}_1'| = p\}$, which is associated with (13), can be expressed as shown in (22).

Specific cases of the results in (22), which carry important physical interpretation, are now explained. For the case when an authorized femtocell UE in a specific femtocell is active with probability one, it follows that $p_{\mathrm{act},c_1,i} = 1$, for $c_1 \in \mathcal{C}_1$, where the set $\mathcal{C}_1$ contains the indexes of femtocells where each of which satisfies the condition that at least one authorized femtocell UE is active with probability one for specific values of these UEs indexes $i \in \{1, 2, \ldots, N'_{c_1}\}$. In this case, $\Pr\{N'_{\mathrm{act},c_1} = 0\} = 0$ and $\Pr\{N'_{\mathrm{act},c_1} > 0\} = 1$. On the other hand, it can be a possible case that all authorized femtocell UEs in the remaining femtocells are inactive with probability one, which gives $p_{\mathrm{act},c_2,i} = 0$, for $c_2 \in \mathcal{C}_2$, where the set $\mathcal{C}_2$ contains the indexes of femtocells, where each of which has no active authorized femtocell UEs for specific values of these UEs indexes $i \in \{1, 2, \ldots, N'_{c_2}\}$. In this case, $\Pr\{N'_{\mathrm{act},c_2} = 0\} = 1$ and $\Pr\{N'_{\mathrm{act},c_2} > 0\} = 0$. The sum of the cardinalities of the sets $\mathcal{C}_1$ and $\mathcal{C}_2$ satisfies $|\mathcal{C}_1| + |\mathcal{C}_2| = |\mathcal{G}_1|$, considering the fixed set $\mathcal{G}_2$, as per subsection III-B2. For the preceding specific cases, the statistics of $|\mathcal{G}_1'|$ in (22) reduces to those given in (23).

The results in the preceding two subsections can now be incorporated into (13). Hence, the following section presents the general formulations of the desired macrocell UE outage performance, and describes some limiting cases of these general formulations.

V. Performance Characterization

This section contains four parts. In the first part, general formulations for the outage probability of the desired macrocell UE are presented, considering arbitrary and sequential channel assignment approaches. The remaining parts discuss some limiting cases of the presented general results. Specifically, the second part comments the effect of the spatial location of the desired macrocell UE on the observed interference sources. The third part treats an explicit limiting case when the desired macrocell UE undergoes dominant macrocell interference. Finally, the fourth part discusses the possible extension of the results when the desired UE is an authorized femtocell UE.

A. General Formulations of Outage Performance

This part is divided into two subparts, which treat the outage probability of the desired macrocell UE for arbitrary and sequential channel assignment approaches, respectively.

1) Arbitrary Channel Assignment: Based on the preliminary formulation in (13) and using the results in subsections IV-C and IV-D, and for the case of arbitrary channel assignment for the *excess* active macrocell UEs when the channels at the macrocell base station are fully-occupied (i.e., the case of $W_{0,k} > 0$), the results in (24) can be obtained. The following comments can be drawn from (24):

- The interference-free outage performance of the desired macrocell UE can be obtained as a limiting case when the conditions that $|\mathcal{G}_1'| = 0$ (no cross-tier interference from partially-occupied under-loaded femtocells), $N_{\mathrm{act},0} \leq K - 1$ (at least one channel at the macrocell base station is free), and $\mathcal{G}_2 = \emptyset$ (the fixed set of fully-occupied over-loaded femtocells is empty) are realized.
- The total macrocell interference power is proportional to the number of *excess* active macrocell UEs. Hence, arbitrary channel assignment can not utilize the channels to further mitigate this interference.

2) Sequential Channel Assignment: Note that the result in (24) considers that the kth channel that is allocated to the desired macrocell UE can be assigned to as many as

$$\begin{aligned}\Pr\{|\mathcal{G}_1'|=p\}|_{p_{\tilde{g}_1,k,i}=p_{1,k}} &= \sum_{q=0}^{|\mathcal{G}_1|}\Pr\{|\mathcal{G}_1'|=p, Q=q\}\\ &= \sum_{q=p}^{|\mathcal{G}_1|}\binom{|\mathcal{G}_1|}{q}\left(1-\Pr\{N'_{\text{act},1}=0\}\right)^q\left(\Pr\{N'_{\text{act},1}=0\}\right)^{|\mathcal{G}_1|-q}\binom{q}{p}(p_{1,k})^p(1-p_{1,k})^{q-p}. \end{aligned} \tag{22}$$

$$\Pr\{|\mathcal{G}_1'|=p\} = \begin{cases} \binom{|\mathcal{C}_1|}{p}(p_{1,k})^p(1-p_{1,k})^{|\mathcal{C}_1|-p}, & |\mathcal{C}_1|>0,\ p=0,1,2,\ldots,|\mathcal{C}_1| \\ 0, & |\mathcal{C}_2|=|\mathcal{G}_1|,\ p>0 \\ 1, & |\mathcal{C}_2|=|\mathcal{G}_1|,\ p=0 \end{cases} \tag{23}$$

$$\begin{aligned} P_{\text{OUT}}|_{\mathcal{G}_2\neq\emptyset,\,\mathcal{G}_2''=\emptyset} &= \Pr\{|\mathcal{G}_1'|=0\}\Pr\{N_{\text{act},0}\le K-1\}\Pr\left\{\frac{s_{\text{D},k,0}}{\sum_{g_2\in\mathcal{G}_2}s_{\text{I},k,g_2}+\sigma_k^2}<x\right\}\\ &+\Pr\{|\mathcal{G}_1'|=0\}\sum_{n_{\text{act},0}=K}^{N_{\text{eq},0}}\Pr\{N_{\text{act},0}=n_{\text{act},0}\}\sum_{w_{0,k}=0}^{n_{\text{act},0}-K}\Pr\{W_{0,k}=w_{0,k}\}\\ &\times\Pr\left\{\frac{s_{\text{D},k,0}}{\sum_{n=1}^{w_{0,k}+1}s_{\text{I},k,0,n}+\sum_{g_2\in\mathcal{G}_2}s_{\text{I},k,g_2}+\sigma_k^2}<x\right\}\\ &+\Pr\{N_{\text{act},0}\le K-1\}\sum_{p=1}^{|\mathcal{G}_1|}\Pr\{|\mathcal{G}_1'|=p\}\Pr\left\{\frac{s_{\text{D},k,0}}{\sum_{\ell=1}^{p}s_{\text{I},k,\ell}+\sum_{g_2\in\mathcal{G}_2}s_{\text{I},k,g_2}+\sigma_k^2}<x\right\}\\ &+\sum_{p=1}^{|\mathcal{G}_1|}\Pr\{|\mathcal{G}_1'|=p\}\sum_{n_{\text{act},0}=K}^{N_{\text{eq},0}}\Pr\{N_{\text{act},0}=n_{\text{act},0}\}\sum_{w_{0,k}=0}^{n_{\text{act},0}-K}\Pr\{W_{0,k}=w_{0,k}\}\\ &\times\Pr\left\{\frac{s_{\text{D},k,0}}{\sum_{n=1}^{w_{0,k}+1}s_{\text{I},k,0,n}+\sum_{\ell=1}^{p}s_{\text{I},k,\ell}+\sum_{g_2\in\mathcal{G}_2}s_{\text{I},k,g_2}+\sigma_k^2}<x\right\}. \end{aligned} \tag{24}$$

$W_{0,k}$ *excess* active macrocell UEs, regardless of the occupancy of other channels at the macrocell base station. A possible approach to further reduce the total macrocell interference power is to utilize the channels such that a balanced load distribution among channels can be achieved through assigning channels to active UEs sequentially when the macrocell base station is fully-occupied.

To clarify this approach, it can be written that $N_{\text{eq},0} = \mu K + t$, where $t \le K-1$ and $\mu \triangleq \lfloor N_{\text{eq},0}/K \rfloor$. With the sequential channel assignment at the macrocell base station, it follows that $\hat{N}_{\text{act},0} \triangleq \lfloor N_{\text{act},0}/K \rfloor$, where $\hat{N}_{\text{act},0}$ now represents the number of active macrocell UEs that can be granted an access to the kth channel that is allocated to the desired macrocell UE, which takes on values from $\{0,1,2,\ldots,\mu\}$. Therefore, the result in (24) becomes as shown in (25). The sequential channel assignment can reduce the macrocell interference effect on the desired macrocell UE as the number of channels at the macrocell base station increases, but this requires more complexity for channel synchronization and overhead. Note that this approach has no impact on cross-tier interference since closed-access femtocell access points can not reuse fully-occupied channels.

B. *Effect of Desired Macrocell UE Location*

The results in (24) and (25) are applicable for any active macrocell UE that may be present anywhere inside the coverage space of the macrocell. This includes active UEs whose sum is $\sum_{\ell=1}^{L} N''_{\ell,0}$, and they are present in femtocells coverage spaces.

When the desired macrocell UE exists inside a femtocell coverage space that is relatively far away from the serving macrocell base station and other active macrocell UEs, it may observe cross-tier interference that is likely to be dominated by the co-channel service provided to active authorized UEs in nearby femtocells, which is feasible when the macrocell coverage is poor. On the other hand, strong co-tier interference due to co-channel macrocell access becomes dominant, specifically in dense macrocell spaces that are well-covered by the macrocell base station and far away from femtocells. The following part treats the latter case.

C. *Dominant Macrocell Interference*

In this case, the desired macrocell UE will experience dominant co-tier macrocell interference due to the reuse of

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$$\begin{aligned}P_{\rm OUT}|_{\mu=\lfloor N_{\rm eq,0}/K\rfloor} =& \Pr\{|\mathcal{G}_1'|=0\}\Pr\{\hat{N}_{\rm act,0}=0\}\Pr\left\{\frac{s_{{\rm D},k,0}}{\sum_{g_2\in\mathcal{G}_2} s_{{\rm I},k,g_2}+\sigma_k^2}<x\right\}\\
&+\Pr\{|\mathcal{G}_1'|=0\}\sum_{\hat{n}_{\rm act,0}=1}^{\mu}\Pr\{\hat{N}_{\rm act,0}=\hat{n}_{\rm act,0}\}\sum_{w_{0,k}=0}^{\hat{n}_{\rm act,0}-1}\Pr\{W_{0,k}=w_{0,k}\}\\
&\times\Pr\left\{\frac{s_{{\rm D},k,0}}{\sum_{n=1}^{w_{0,k}+1} s_{{\rm I},k,0,n}+\sum_{g_2\in\mathcal{G}_2} s_{{\rm I},k,g_2}+\sigma_k^2}<x\right\}\\
&+\Pr\{\hat{N}_{\rm act,0}=0\}\sum_{p=1}^{|\mathcal{G}_1|}\Pr\{|\mathcal{G}_1'|=p\}\Pr\left\{\frac{s_{{\rm D},k,0}}{\sum_{\ell=1}^{p} s_{{\rm I},k,\ell}+\sum_{g_2\in\mathcal{G}_2} s_{{\rm I},k,g_2}+\sigma_k^2}<x\right\}\\
&+\sum_{p=1}^{|\mathcal{G}_1|}\Pr\{|\mathcal{G}_1'|=p\}\sum_{\hat{n}_{\rm act,0}=1}^{\mu}\Pr\{\hat{N}_{\rm act,0}=\hat{n}_{\rm act,0}\}\sum_{w_{0,k}=0}^{\hat{n}_{\rm act,0}-1}\Pr\{W_{0,k}=w_{0,k}\}\\
&\times\Pr\left\{\frac{s_{{\rm D},k,0}}{\sum_{n=1}^{w_{0,k}+1} s_{{\rm I},k,0,n}+\sum_{\ell=1}^{p} s_{{\rm I},k,\ell}+\sum_{g_2\in\mathcal{G}_2} s_{{\rm I},k,g_2}+\sigma_k^2}<x\right\}. \quad (25)\end{aligned}$$

channels at the macrocell base station. Hence, the result in (24) reduces to

$$\begin{aligned}&P_{\rm OUT}|_{\text{Dominant Macrocell}}\\
&=\Pr\{N_{\rm act,0}\le K-1\}\Pr\left\{\frac{s_{{\rm D},k,0}}{\sigma_k^2}<x\right\}\\
&+\sum_{n_{\rm act,0}=K}^{N_{\rm eq,0}}\sum_{w_{0,k}=0}^{n_{\rm act,0}-K}\Pr\left\{\frac{s_{{\rm D},k,0}}{\sum_{n=1}^{w_{0,k}+1} s_{{\rm I},k,0,n}+\sigma_k^2}<x\right\}\\
&\times\Pr\{W_{0,k}=w_{0,k}\}\Pr\{N_{\rm act,0}=n_{\rm act,0}\}. \quad (26)\end{aligned}$$

For the case when the sequential channel assignment at macrocell base station is adopted, it follows that

$$\begin{aligned}&P_{\rm OUT}|_{\mu=\lfloor N_{\rm eq,0}/K\rfloor,\text{ Dominant Macrocell}}\\
&=\Pr\{\hat{N}_{\rm act,0}=0\}\Pr\left\{\frac{s_{{\rm D},k,0}}{\sigma_k^2}<x\right\}\\
&+\sum_{\hat{n}_{\rm act,0}=1}^{\mu}\sum_{w_{0,k}=0}^{\hat{n}_{\rm act,0}-1}\Pr\left\{\frac{s_{{\rm D},k,0}}{\sum_{n=1}^{w_{0,k}+1} s_{{\rm I},k,0,n}+\sigma_k^2}<x\right\}\\
&\times\Pr\{W_{0,k}=w_{0,k}\}\Pr\{\hat{N}_{\rm act,0}=\hat{n}_{\rm act,0}\}. \quad (27)\end{aligned}$$

D. Extension to Desired Femtocell UE

This part discusses the applicability of the preceding analytical and statistical formulations to quantify the outage probability performance of an authorized femtocell UE. Specifically, when the desired femtocell UE exists outside its serving femtocell access point (say the ℓth femtocell) coverage space (i.e., inside the coverage space of another femtocell (the pth femtocell; one of $N''_{p,\ell}$, for $p\neq\ell$) or the macrocell base station (one of the femtocell UEs whose total number is $\sum_{\ell=1}^{L} N''_{0,\ell}$)), it can be only served by the macrocell base station. Therefore, the analysis for the outage probability of that UE follows similar procedures to those described above for the desired macrocell UE case, after taking into consideration the changes in co-tier interference sources (co-channel active UEs served by the macrocell base station) and cross-tier interference sources (co-channel active authorized UEs served by their femtocell access points).

The preceding formulations can also quantify the outage performance of an authorized femtocell UE that is present inside the coverage space of its serving femtocell access point, but it is not granted an access. Such active UEs have their femtocell indexes belong to the set $\mathcal{G}_2$ (over-loaded and fully-occupied) and their total number is referred to as $\sum_{g_2\in\mathcal{G}_2}\widetilde{N}'_{g_2}$ (refer to (3)). In any case, the application of the results above to a particular UE identity has to count for the changes in the desired UE access condition, interference sources, and the desired link.

VI. Results for Specific Channel Models

In this section, the analysis considers specific channel models of the desired link as well as interference links, and provides closed-form analytical results for the general formulations in (24) and (25).

A. Models for $\{s_{D,k,0}\}$, $\{s_{I,k,0,n}\}$, and $\{s_{I,k,\ell}\}$

Based on (24) and (25), specific channel models are needed to quantify the statistics of $\{s_{{\rm D},k,0}\}$, $\{s_{{\rm I},k,0,n}\}$, and $\{s_{{\rm I},k,\ell}\}$. Downlink channels associated with different UEs undergo both large-scale and small-scale fading. The large-scale fading counts for the effects of separation distances, path loss, and shadowing. On the other hand, the small-scale fading reflects the small-scale channel fluctuations, and it is herein assumed to follow a non-identically distributed Nakagami-m model, which includes the Gaussian model as a special case. In this case, the received power at the desired UE on the kth channel, $\{s_{{\rm D},k,0}\}$ as defined in (24) and (25), will have a probability

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density function (PDF) and a cumulative distribution function (CDF) that are given by [26]

$$f_{s_{\mathrm{D},k,0}}(x) = \frac{1}{\Gamma(m_{\mathrm{D},k,0})} \left(\frac{1}{\bar{s}_{\mathrm{D},k,0}}\right)^{m_{\mathrm{D},k,0}} x^{m_{\mathrm{D},k,0}-1} e^{-\frac{x}{\bar{s}_{\mathrm{D},k,0}}}, \tag{28}$$

$$F_{s_{\mathrm{D},k,0}}(x) = 1 - \sum_{v=0}^{m_{\mathrm{D},k,0}-1} \frac{1}{\Gamma(v+1)} \left(\frac{x}{\bar{s}_{\mathrm{D},k,0}}\right)^{v} e^{-\frac{x}{\bar{s}_{\mathrm{D},k,0}}}, \tag{29}$$

where $m_{\mathrm{D},k,0}$ is the fading parameter and $\bar{s}_{\mathrm{D},k,0}$ is the average value of the received signal power (normalized by the associated fading parameter and reflects the effect of large-scale fading), both associated with the desired link. Moreover, the interference powers $\{s_{\mathrm{I},k,0,n}\}$ and $\{s_{\mathrm{I},k,\ell}\}$ will have PDFs that are given by

$$f_{s_{\mathrm{I},k,0,n}}(x) = \frac{1}{\Gamma(\alpha_{k,0,n})} \left(\frac{1}{\bar{s}_{\mathrm{I},k,0,n}}\right)^{\alpha_{k,0,n}} x^{\alpha_{k,0,n}-1} e^{-\frac{x}{\bar{s}_{\mathrm{I},k,0,n}}}, \tag{30}$$

$$f_{s_{\mathrm{I},k,\ell}}(x) = \frac{1}{\Gamma(\alpha_{k,\ell})} \left(\frac{1}{\bar{s}_{\mathrm{I},k,\ell}}\right)^{\alpha_{k,\ell}} x^{\alpha_{k,\ell}-1} e^{-\frac{x}{\bar{s}_{\mathrm{I},k,\ell}}}, \tag{31}$$

where $\alpha_{k,0,n}$ and $\alpha_{k,\ell}$ are the fading parameters, and $\bar{s}_{\mathrm{I},k,0,n}$ and $\bar{s}_{\mathrm{I},k,\ell}$ are the average values of interference powers (normalized by their fading parameters) that undergo the large-scale fading effect.

B. Distribution of Aggregate Interference Power

In this part, the statistics of the aggregate interference power, which is denoted by $s_{\mathrm{I},k}$ for simplicity, that is associated with the results in (24)–(27) are presented. For the general case in (24)–(27), the term $s_{\mathrm{I},k}$ for various interference sources can be written as

$$s_{\mathrm{I},k} = \sum_{n=1}^{L_1} s_{\mathrm{I},k,0,n} + \sum_{\ell=1}^{L_2} s_{\mathrm{I},k,\ell} + \sum_{g_2 \in \mathcal{G}_2} s_{\mathrm{I},k,g_2}, \tag{32}$$

where L_1 and L_2 take values from specific sets with certain likelihoods of occurrence as discussed in the preceding sections. By exploiting the independence among interference power sources, the characteristic function (CF) of $s_{\mathrm{I},k}$, which is denoted by $\psi_{s_{\mathrm{I},k}}(t, L_1, L_2) = \mathbb{E}\{e^{-\mathrm{j}\,t s_{\mathrm{I},k}}\}$, where $\mathbb{E}\{\cdot\}$ is the expectation operator and $\mathrm{j}^2 = -1$, is given by

$$\begin{aligned} \psi_{s_{\mathrm{I},k}}(t, L_1, L_2) &= \prod_{n=1}^{L_1} \psi_{s_{\mathrm{I},k,0,n}}(t) \prod_{\ell=1}^{L_2} \psi_{s_{\mathrm{I},k,\ell}}(t) \prod_{g_2 \in \mathcal{G}_2} \psi_{s_{\mathrm{I},k,g_2}}(t) \\ &= \prod_{i=1}^{K_D} (1 - \mathrm{j}\,t\beta_i)^{-\gamma_i}, \end{aligned} \tag{33}$$

where $K_D \triangleq L_1 + L_2 + |\mathcal{G}_2|$, $\beta = \{\beta_i\}_{i=1}^{K_D}$, $\beta = \left\{ \{\bar{s}_{\mathrm{I},k,0,n}\}_{n=1}^{L_1}, \{\bar{s}_{\mathrm{I},k,\ell}\}_{\ell=1}^{L_2}, \{\bar{s}_{\mathrm{I},k,g_2}\}_{g_2 \in \mathcal{G}_2} \right\}$, $\gamma = \{\gamma_i\}_{i=1}^{K_D}$, and $\gamma = \left\{ \{\alpha_{k,0,n}\}_{n=1}^{L_1}, \{\alpha_{k,\ell}\}_{\ell=1}^{L_2}, \{\alpha_{k,g_2}\}_{g_2 \in \mathcal{G}_2} \right\}$. The second equality in (33) is introduced to unify the presentation and ease the analysis below, with no loss of generality.

The PDF of $s_{\mathrm{I},k}$ can now be obtained with the help of [27], [28]. The resulting PDF of $s_{\mathrm{I},k}$, which is denoted by $f_{s_{\mathrm{I},k}}(x, L_1, L_2)$, can be written as

$$\begin{aligned} f_{s_{\mathrm{I},k}}(x, L_1, L_2) &= \frac{1}{2\pi} \int_{\mathbf{R}} e^{\mathrm{j}\,tx} \psi_{s_{\mathrm{I},k}}(t, L_1, L_2) dt \\ &= \left(\prod_{i=1}^{K_D} (\beta_i)^{-\gamma_i} \right) \frac{x^{K_T-1}}{\Gamma(K_T)} \\ &\quad \times \Phi_2^{(K_D)} \Bigg(\underbrace{\gamma_1, \ldots, \gamma_{K_D}}_{K_D \text{ terms}}; K_T; \underbrace{\frac{-x}{\beta_1}, \ldots, \frac{-x}{\beta_{K_D}}}_{K_D \text{ terms}} \Bigg), \end{aligned} \tag{34}$$

where $K_T \triangleq \sum_{i=1}^{K_D} \gamma_i$ and $\Phi_2^{(n)}(a_1, \ldots, a_n; b; x_1, \ldots, x_n)$ is the Lauricella confluent hypergeometric function of n variables.

C. Statistics of Received SINRs

The objective of this part is to provide a generic formulation for the statistics of received SINRs that can be applied for any statistical combinations of interference power sources as per the results in (24)–(27). To proceed in the analysis, define a generic SINR form as $\widetilde{\gamma}_{\mathrm{SINR},k} = \frac{s_{\mathrm{D},k,0}}{s_{\mathrm{I},k} + \sigma_k^2}$, where the statistics of $s_{\mathrm{D},k,0}$ are given in (28) and (29), the distribution of $s_{\mathrm{I},k}$ is given in (34), and σ_k^2 is the average power of the background white noise power, as defined previously. The CDF of $\widetilde{\gamma}_{\mathrm{SINR},k}$, which is denoted by $\mathcal{F}(x, L_1, L_2)$, can now be computed using the results in (29) and (34). Capitalizing on the findings in [28], it can be shown that

$$\begin{aligned} \mathcal{F}(x, L_1, L_2) &= \int_0^{+\infty} F_{s_{\mathrm{D},k,0}}(x(\nu + \sigma_k^2)) f_{s_{\mathrm{I},k}}(\nu, L_1, L_2) d\nu \\ &= 1 - \left(\prod_{i=1}^{K_D} (\beta_i)^{-\gamma_i} \right) \frac{1}{\Gamma(K_T)} e^{-\frac{x\sigma_k^2}{\bar{s}_{\mathrm{D},k,0}}} \\ &\quad \times \sum_{v=0}^{m_{\mathrm{D},k,0}-1} \frac{1}{\Gamma(v+1)} \left(\frac{x}{\bar{s}_{\mathrm{D},k,0}}\right)^{v} \\ &\quad \times \sum_{u=0}^{v} \binom{v}{u} \sigma_k^{2(v-u)} \mathcal{R}(x, L_1, L_2, u), \end{aligned} \tag{35}$$

where the term $\mathcal{R}(x, L_1, L_2, u)$ in (35) is given by

$$\begin{aligned} &\mathcal{R}(x, L_1, L_2, u) \\ &= \int_0^{+\infty} \nu^{K_T+u-1} e^{-\frac{x\nu}{\bar{s}_{\mathrm{D},k,0}}} \\ &\times \Phi_2^{(K_D)} \left(\gamma_1, \ldots, \gamma_{K_D}; K_T; \frac{-\nu}{\beta_1}, \ldots, \frac{-\nu}{\beta_{K_D}} \right) d\nu \\ &= \Gamma(K_T + u) \left(\frac{\bar{s}_{\mathrm{D},k,0}}{x} \right)^{K_T+u} \\ &\times F_D^{(K_D)} \Bigg(K_T + u, \underbrace{\gamma_1, \ldots, \gamma_{K_D}}_{K_D \text{ terms}}; K_T; \underbrace{\frac{-\bar{s}_{\mathrm{D},k,0}}{x\beta_1}, \ldots, \frac{-\bar{s}_{\mathrm{D},k,0}}{x\beta_{K_D}}}_{K_D \text{ terms}} \Bigg), \end{aligned} \tag{36}$$

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where $F_D^{(n)}(a, b_1, ..., b_n; c; x_1, ..., x_n)$ is the Lauricella hypergeometric function of n variables. Following the procedure used in [29, Appendix], the hypergeometric function in (36) can be expressed in terms of finite series that can be efficiently computed as

$$F_D^{(K_D)}\left(K_T+u, \gamma_1, \ldots, \gamma_{K_D}; K_T; \frac{-\bar{s}_{\mathrm{D},k,0}}{x\beta_1}, \ldots, \frac{-\bar{s}_{\mathrm{D},k,0}}{x\beta_{K_D}}\right)$$
$$= \prod_{\breve{i}=1}^{K_D}\left(1+\frac{\bar{s}_{\mathrm{D},k,0}}{x\beta_{\breve{i}}}\right)^{-\gamma_{\breve{i}}} \sum_{\substack{u_1,\ldots,u_{K_D}=0 \\ u\geq\widetilde{u}}}^{u} \frac{\Gamma(u+1)}{\Gamma(u-\widetilde{u}+1)} \frac{1}{(K_T)_{\widetilde{u}}}$$
$$\times \prod_{\breve{i}=1}^{K_D} \frac{(\gamma_{\breve{i}})_{u_{\breve{i}}}}{\Gamma(u_{\breve{i}}+1)} \left(\frac{-\bar{s}_{\mathrm{D},k,0}}{x\beta_{\breve{i}}+\bar{s}_{\mathrm{D},k,0}}\right)^{u_{\breve{i}}}, \qquad (37)$$

where $\widetilde{u} = \sum_{i=1}^{n} u_i$, and $(x)_y = \Gamma(x+y)/\Gamma(x)$, with $(x)_0 = 1$ and $(x)_1 = x$. Substituting (37) into (36) gives the desired form of the term $\mathcal{R}(x, L_1, L_2, u)$ that is required in (35) to provide the desired result for $\mathcal{F}(x, L_1, L_2)$ of the generic SINR $\widetilde{\gamma}_{\mathrm{SINR},k}$ defined above[8].

D. Closed-form Result for Outage Probability Performance

Using the results for the generic CDF $\mathcal{F}(x, L_1, L_2)$ above, the outage probability in (24) can now be reexpressed as given in (38). Following the same procedure above, the results in (25)–(27) can be also reexpressed in terms of $\mathcal{F}(x, L_1, L_2)$. Further details are omitted for brevity.

VII. Numerical and Simulation Results

This section presents selected numerical examples that aim to quantify the impact of various parameters and statistical models described in the previous sections on the outage performance of the desired macrocell UE. The numerical results focus on the outage performance that is depicted versus the outage threshold as shown in Figs. 3–8, excluding the results in Fig. 9. The numerical results in Figs. 3–8 have been obtained using the general formulations in (24) and (38), whereas they have been obtained from (26) and (35) in Fig. 9. To simplify the presentations on the effect of various parameters on the outage performance of the desired UE, it is assumed that $\bar{s}_\mathrm{I} \triangleq \bar{s}_{\mathrm{I},k,0,n} = \bar{s}_{\mathrm{I},k,\ell} = \bar{s}_{\mathrm{I},k,g_2}$ and $\alpha \triangleq \alpha_{k,0,n} = \alpha_{k,\ell} = \alpha_{k,g_2}$ (all interference sources have identical average power levels and they experience the same level fading severity, regardless of their origins). The latter assumption is useful to investigate either the worst-case scenario with significant interference or the best-case scenario with marginal interference effect.

Fig. 3 shows the outage probability versus the outage threshold in dB for different values of the equivalent number of macrocell interference sources ($N_{\mathrm{eq},0} = 5$, 10, and 15), and different values of the average power per interference source (case 1: $\bar{s}_\mathrm{I} = 0$ dBm, case 2: $\bar{s}_\mathrm{I} = 10$ dBm). The results are generated for the values of $m_{\mathrm{D},k,0} = 2$, $\bar{s}_{\mathrm{D},k,0} = 20$ dBm, $|\mathcal{G}_1| = 5$ (cardinality of under-loaded femtocells), $|\mathcal{G}_2| = 5$ (cardinality of over-loaded femtocells), $K = 4$ (number of channels), $\alpha = 2$, $N'_\ell = 3$ (number of authorized UEs in the ℓth femtocell), $p_{1,k} = 1$ (probability that an active authorized femtocell UE accesses the kth channel that is allocated to the desired macrocell UE), $p_{0,k} = 1$ (probability that an active macrocell UE accesses the kth channel), $p_{\mathrm{act},0} = 1$ (probability that a macrocell UE is active), $p_{\mathrm{act},\ell} = 1$ (probability that an authorized femtocell UE in the ℓth femtocell is active), and $\sigma_k^2 = 1$. The selected values above reveal that the worst-case scenario of interference effect is considered, with all potential interference sources are active and they access the same channel that is allocated to the desired macrocell UE. It is clear from the figure that the increase in $N_{\mathrm{eq},0}$ under such conditions decreases the outage performance. Moreover the increase in the average power per interference source reduces the outage performance, as expected. For the considered worst-case interference scenario, the effect of the average power per interference source exceeds that due to the number of active macrocell UEs that result in the co-tier interference effect.

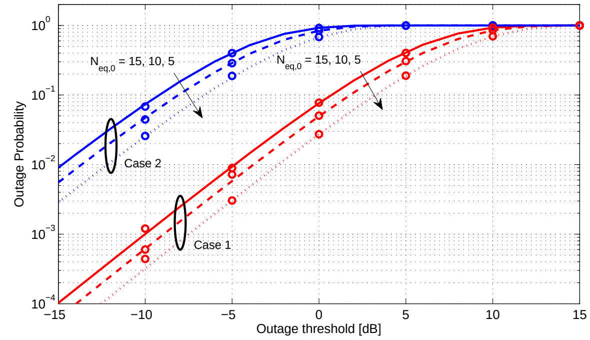

Fig. 3. Outage probability versus the outage threshold in dB for different values of the equivalent number of macrocell UEs, $N_{\mathrm{eq},0}$, and different values of the average received power per interference source, $\bar{s}_\mathrm{I}$, (Case 1: $\bar{s}_\mathrm{I} = 0$ dBm, Case 2: $\bar{s}_\mathrm{I} = 10$ dBm). Circle points are simulation results.

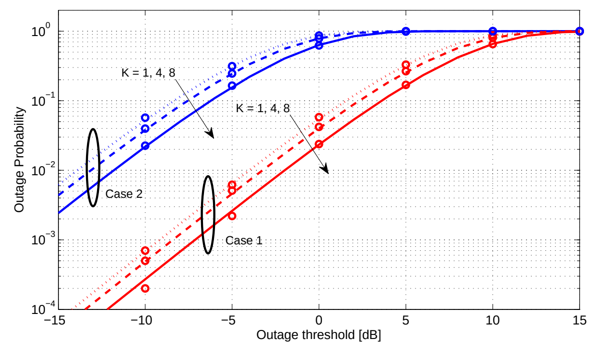

Fig. 4. Outage probability versus the outage threshold in dB for different values of the number of available channels, K, and different values of the average received power per interference source, $\bar{s}_\mathrm{I}$, (Case 1: $\bar{s}_\mathrm{I} = 0$ dBm, Case 2: $\bar{s}_\mathrm{I} = 10$ dBm). Circle points are simulation results.

To comment of the effect of sequential channel assignment approach as per the result in (25), and for the same cases in Fig. 3, the case when $K = 4$ and $N_{\mathrm{eq},0} = 5$ using sequential channel assignment is anticipated to give an outage performance that corresponds to $N_{\mathrm{eq},0} = 1$. Moreover, a relatively

[8] This closed-from result for $\mathcal{F}(x, L_1, L_2)$ is valid for both cases of integer as well as non-integer fading parameters of the desired link and/or any interference link.

$$\begin{aligned} P_{\text{OUT}}|_{\mathcal{G}_2\neq\emptyset,\,\mathcal{G}_2''=\emptyset} &= \Pr\{|\mathcal{G}_1'|=0\}\Pr\{N_{\text{act},0}\leq K-1\}\mathcal{F}(x,0,0) \\ &+ \Pr\{|\mathcal{G}_1'|=0\}\sum_{n_{\text{act},0}=K}^{N_{\text{eq},0}}\Pr\{N_{\text{act},0}=n_{\text{act},0}\}\sum_{w_{0,k}=0}^{n_{\text{act},0}-K}\Pr\{W_{0,k}=w_{0,k}\}\mathcal{F}(x,w_{0,k}+1,0) \\ &+ \Pr\{N_{\text{act},0}\leq K-1\}\sum_{p=1}^{|\mathcal{G}_1|}\Pr\{|\mathcal{G}_1'|=p\}\mathcal{F}(x,0,p) \\ &+ \sum_{p=1}^{|\mathcal{G}_1|}\Pr\{|\mathcal{G}_1'|=p\}\sum_{n_{\text{act},0}=K}^{N_{\text{eq},0}}\Pr\{N_{\text{act},0}=n_{\text{act},0}\}\sum_{w_{0,k}=0}^{n_{\text{act},0}-K}\Pr\{W_{0,k}=w_{0,k}\}\mathcal{F}(x,w_{0,k}+1,p). \end{aligned} \quad (38)$$

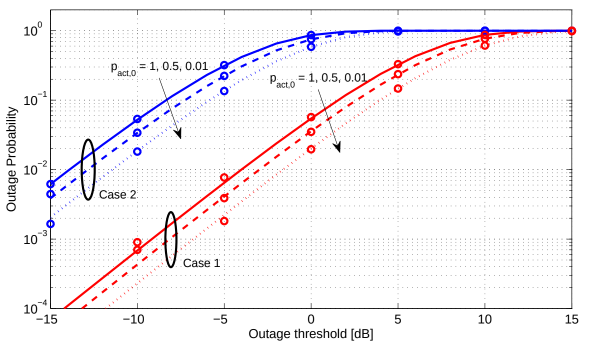

Fig. 5. Outage probability versus the outage threshold in dB for different values of the probability that a macrocell UE is active, $p_{\text{act},0}$, and different values of the average received power per interference source, $\bar{s}_{\text{I}}$, (Case 1: $\bar{s}_{\text{I}}=0$ dBm, Case 2: $\bar{s}_{\text{I}}=10$ dBm). Circle points are simulation results.

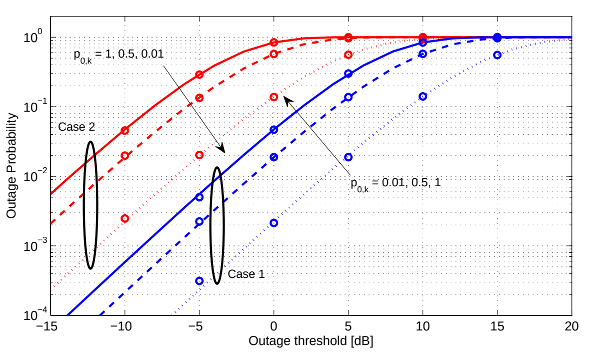

Fig. 6. Outage probability versus the outage threshold in dB for different values of the probability that an active macrocell UE accesses the same channel of the desired macrocell UE, $p_{0,k}$, and different values of the average received desired power, $\bar{s}_{\text{D}}$, (Case 1: $\bar{s}_{\text{D}}=30$ dBm, Case 2: $\bar{s}_{\text{D}}=20$ dBm). Circle points are simulation results.

large performance gain due to the use of the sequential channel assignment is expected as $N_{\text{eq},0}$ increases. Specifically, the sequential channel assignment when $N_{\text{eq},0}=15$ results in an outage performance that corresponds to the results obtained with the use of the arbitrary channel assignment when $N_{\text{eq},0}=3$, which is bounded above by the results shown in the Fig. 3 for $N_{\text{eq},0}=5$.

Fig. 4 studies the effect of the number of the channels ($K=1$, 4, and 8) on the results shown in Fig. 3 for a given value of $N_{\text{eq},0}=8$. The figure considers the same parameters that are used in Fig. 3, which are: different values of $\bar{s}_{\text{I}}$ (case 1: $\bar{s}_{\text{I}}=0$ dBm, case 2: $\bar{s}_{\text{I}}=10$ dBm), $m_{\text{D},k,0}=2$, $\bar{s}_{\text{D},k,0}=20$ dBm, $|\mathcal{G}_1|=5$, $|\mathcal{G}_2|=5$, $\alpha=2$, $N_\ell'=3$, $p_{1,k}=1$, $p_{0,k}=1$, $p_{\text{act},0}=1$, $p_{\text{act},1}=1$, and $\sigma^2=1$. The figure clarifies the effect of having more channels on the outage performance of the desired macrocell UE under the worst-case interference scenario. It is clear that the increase in K improves the outage performance as this reduces the number of excess active macrocell UEs that can access the same channel of the desired UE. The change in the average power per interference source does not vary the behavioral impact of channels, mainly due to the considered values of likelihoods of active UEs and their likelihoods of accessing the same channel of the desired UE.

Fig. 5 shows the effect of the probability that a macrocell UE is active ($p_{\text{act},0}=1$, 0.5, and 0.01) on the outage performance of the desired UE for given values of $K=1$ and $N_{\text{eq},0}=8$, and different values of $\bar{s}_{\text{I}}$ (case 1: $\bar{s}_{\text{I}}=0$ dBm, case 2: $\bar{s}_{\text{I}}=10$ dBm). The values of the remaining parameters are identical to the ones used in the preceding two figures. The figure treats the worst case of femtocell interference sources when there is only one channel and for specific number of potential interference macrocell UEs with different probabilities that each of them will be active. It is shown that the decrease in $p_{\text{act},0}$ reduces the effective value of interference observed from the macrocell UEs. The observation clarifies the importance of the likelihood that an interference macrocell UE is active in determining the amount of outage performance loss due to concurrent access of the same channel. This scenario can be exploited to control the level of macrocell interference at the expense of service coverage, particularly when the number of interference macrocell UEs is relatively large and the number of channels is small.

Fig. 6 addresses the effects of the probability that an active macrocell UE accesses the same channel of the desired macrocell UE ($p_{0,k}=1$, 0.5, and 0.01) and the average value of the received desired power (case 1: $\bar{s}_{\text{D},k,0}=30$ dBm, case 2: $\bar{s}_{\text{D},k,0}=20$ dBm) on the outage performance of the desired UE. The results have been obtained using the parameters: $m_{\text{D},k,0}=2$, $|\mathcal{G}_1|=1$, $|\mathcal{G}_2|=1$, $N_{\text{eq},0}=15$, $K=1$, $\bar{s}_{\text{I}}=10$ dBm, $\alpha=2$, $N_\ell'=3$, $p_{1,k}=1$, $p_{\text{act},0}=1$, $p_{\text{act},1}=1$, and $\sigma^2=1$. With these parameters,

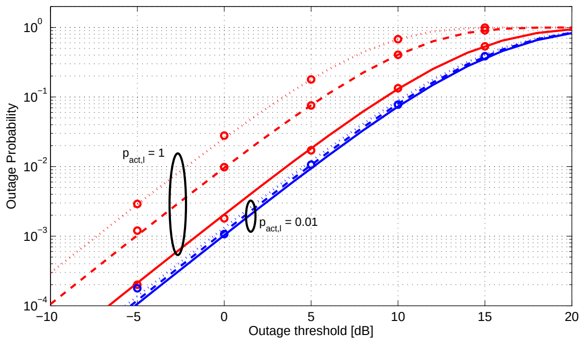

Fig. 7. Outage probability versus the outage threshold in dB for different values of the probability that an authorized femtocell UE in a given femtocell is active, $p_{\text{act},\ell}$, and different cardinalities of the under-loaded femtocells, $|\mathcal{G}_1|$ (solid-line curve $|\mathcal{G}_1| = 1$, dashed-line curve $|\mathcal{G}_1| = 5$, dotted-line curve $|\mathcal{G}_1| = 10$). Circle points are simulation results.

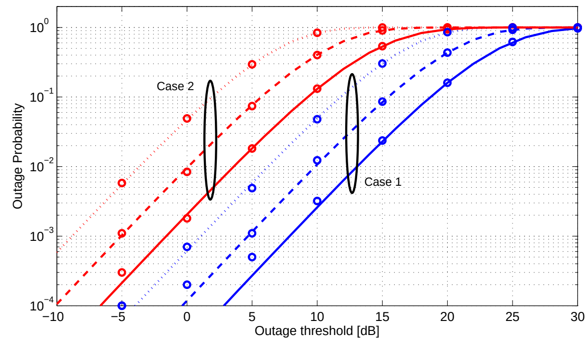

Fig. 8. Outage probability versus the outage threshold in dB for different values of the average received power per interference source (Case 1: $\bar{s}_{\text{I}} = 0$ dBm, Case 2: $\bar{s}_{\text{I}} = 10$ dBm), and different cardinalities of the fixed set of over-loaded femtocells, $|\mathcal{G}_2|$ (solid-line curve $|\mathcal{G}_2| = 1$, dashed-line curve $|\mathcal{G}_2| = 5$, dotted-line curve $|\mathcal{G}_2| = 15$). Circle points are simulation results.

the case study herein shows also the impact of a relatively large $N_{\text{eq},0} = 15$ when there is only one channel at each femtocell access point and at the macrocell base station. It clarifies that the effect of the probability that a relatively large number of active macrocell UEs access a single channel that is allocated to the desired macrocell UE with probability one. Specifically, with the assumption that all the $N_{\text{eq},0} = 15$ macrocell UEs are active (i.e., $p_{\text{act},0} = 1$), the probability that each macrocell UE accesses the single channel that is allocated to the desired macrocell UE can incur significant effect on the outage performance. Therefore, the channel assignment can be modified to avoid significant effect of worst-case macrocell interference even when all macrocell UEs are active. On the other hand, the improvement of the desired macrocell UE channel quality improves the outage performance, and hence, it can be an efficient approach to compensate for deep interference events. However, this improvement does not vary the behavior of the probability that an active macrocell UE accesses the same channel of the desired macrocell UE.

From the results presented in Figs. 3–6, it is seen that the outage probability converges to the maximum value of unity when the SINR outage threshold is above 10 dB. This observation is justified by noting that the increase in the outage threshold will reduce the likelihood that the desired macrocell UE SINR be greater than that threshold. Therefore, the desired macrocell UE will experience an outage event, which indicates that the desired quality of data reception that is related to the specified outage threshold is not satisfied.

Fig. 7 shows the outage probability of the desired macrocell UE versus the outage threshold in dB for different values of the probability that an authorized femtocell UE in a given femtocell is active ($p_{\text{act},\ell} = 1$ and 0.01) and different cardinalities of under-loaded femtocells (solid-line curve $|\mathcal{G}_1| = 1$, dashed-line curve $|\mathcal{G}_1| = 5$, dotted-line curve $|\mathcal{G}_1| = 10$). The results are generated using: $m_{\text{D},k,0} = 2$, $\bar{s}_{\text{D},k,0} = 30$ dBm, $|\mathcal{G}_2| = 1$, $N_{\text{eq},0} = 5$, $K = 5$, $\bar{s}_{\text{I}} = 10$ dBm, $\alpha = 2$, $N'_\ell = 3$, $p_{1,k} = 1$, $p_{0,k} = 1$, $p_{\text{act},0} = 1$, and $\sigma^2 = 1$. The figure clarifies that the increase in the cardinality of under-loaded femtocells have a noticeable impact on the outage performance of the

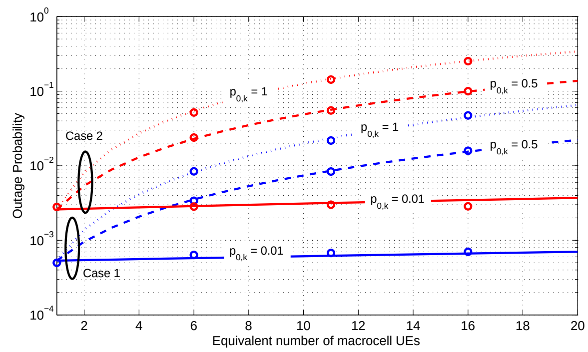

Fig. 9. Outage probability versus the equivalent number of macrocell UEs, $N_{\text{eq},0}$, for different values of the probability that an active macrocell user accesses that same channel of the desired macrocell UE under dominant macrocell interference effect ($p_{0,k} = 1, 0.5, 0.01$), and different values of the fading parameter per interference source (Case 1: $\alpha = 2$, Case 2: $\alpha = 6$). Circle points are simulation results.

desired macrocell UE only if the probability that an authorized femtocell UE has a relatively large likelihood to be active. When this probability is relatively small, the increase in the cardinality of under-loaded femtocells has a marginal effect on the outage performance, even if the probability that an authorized femtocell UEs accesses the same channel of the desired macrocell UE is unity (i.e., $p_{1,k} = 1$).

Fig. 8 investigates the impact of the cardinality of the set of over-loaded femtocells (solid-line curve $|\mathcal{G}_2| = 1$, dashed-line curve $|\mathcal{G}_2| = 5$, dotted-line curve $|\mathcal{G}_2| = 15$) on the outage performance of the desired macrocell UE. These results are shown for different values of the average power per interference source (case 1: $\bar{s}_{\text{I}} = 0$ dBm, case 2: $\bar{s}_{\text{I}} = 10$ dBm), and using the same values of other parameters as in Fig. 7, with $m_{\text{D},k,0} = 2$, $\bar{s}_{\text{D},k,0} = 30$ dBm, $N_{\text{eq},0} = 5$, $K = 5$, $\alpha = 2$, $N'_\ell = 3$, $p_{1,k} = 1$, $p_{0,k} = 1$, $p_{\text{act},0} = 1$, and $\sigma^2 = 1$, and given value of $|\mathcal{G}_1| = 1$ and its associated parameter $p_{\text{act},\ell} = 1$. In this scenario, there can be only one interference macrocell UE, one interference authorized femtocell UE from a single under-loaded femtocell. However, the increase in the number of over-

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loaded femtocells wherein an interference source is observed with probability one has a noticeable effect on the outage performance. Specifically the increase of $|\mathcal{G}_2|$ from 1 to 15 induces around 7 dB loss in terms of the outage threshold at an outage probability of 10^{-3}. The importance of this observation shows that the presence of the desired macrocell UE in vicinity of fully-occupied and over-loaded femtocells can significantly degrade the performance even when the macrocell interference is relatively marginal. The average power per interference source appears to degrade the outage performance by inducing specific shifts to the outage probability curves, without altering the aforementioned observations.

Fig. 9 shows the outage probability of the desired macrocell UE versus the equivalent number of macrocell UEs under dominant macrocell interference scenario. It considers the effects of different values of the probability that an active macrocell UE accesses the same channel of the desired macrocell UE ($p_{0,k} = 1$, 0.5, 0.01) and different values of the fading parameter per interference source (case 1: $\alpha = 2$, case 2: $\alpha = 6$). The results have been generated using $m_{\mathrm{D},k,0} = 2$, $\bar{s}_{\mathrm{D},k,0} = 20$ dBm, $K = 1$ (single channel case), $\bar{s}_{\mathrm{I}} = 0$ dBm, $p_{\mathrm{act},0} = 1$ (all macrocell UEs are active), $\sigma^2 = 1$, and SINR threshold of 0 dB. When there is only one available channel to support active macrocell UEs, the increase in the number of active macrocell UEs degrades the outage performance at different slopes according to the probability that an active macrocell UE accesses the same channel of the desired macrocell UE. The relatively small value of $p_{0,k} = 0.01$ results in a marginal effect for the number of active macrocell UEs. The increase in the fading parameter per interference source reduces the outage performance as it represents an improved condition of interference channels.

VIII. Conclusions

This paper has investigated the impact of UEs identities and their access conditions on the downlink performance of overlaid cellular networks. It considers closed-access femtocells that are deployed inside a macrocell coverage space and share the same resources that are available to the macrocell network. The analysis has treated the case when active UEs with different identities can potentially interfere on any of the channels, wherein the interference sources can be from femtocell network and/or macrocell network. Statistical models for interference sources from femtocells, considering their occupancies and access conditions, have been presented. Moreover, the developed results for a desired UE performance have incorporated various operation parameters that have shown to incur noticeable impact on downlink performance. Specifically, the effect of the number of potential interference macrocell UEs, the likelihood that a macrocell UE is active, the likelihood that an active macrocell UE accesses a given channel, the occupancy of closed-access femtocells, the likelihood that an authorized femtocell UE is active, the likelihood that an active authorized femtocell UE accesses a given channel, the number of available channels, the channel assignment approach at serving stations, the location of the desired UE relative to interference sources, and the fading conditions of the desired link and interference links have been thoroughly investigated. The general formulations for the outage performance of a desired UE have been presented for any channel models of the desired link and interference links. These general results have been then treated for specific channel models, which have resulted in closed-form results for any channel fading parameters and average powers. The presented numerical examples have clarified the impact of different operation parameters on the outage performance of a desired UE. The findings in this paper can be extended for any performance measure and/or any desired UE identity. Moreover, they can be utilized to study related topics, such as multi-user scheduling, interference mitigation, power control, and traffic load distribution in the two networks.

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