Outage Performance of Cognitive Radio Systems with Improper Gaussian Signaling

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***Abstract*—Improper Gaussian signaling has proved its ability to improve the achievable rate of the systems that suffer from interference compared with proper Gaussian signaling. In this paper, we first study impact of improper Gaussian signaling on the performance of the cognitive radio system by analyzing the outage probability of both the primary user (PU) and the secondary user (SU). We derive exact expression of the SU outage probability and upper and lower bounds for the PU outage probability. Then, we design the SU signal by adjusting its transmitted power and the circularity coefficient to minimize the SU outage probability while maintaining a certain PU quality-of-service. Finally, we evaluate the proposed bounds and adaptive algorithms by numerical results.**

I. Introduction

The innovative progress of wireless technology results in a proliferation of attractive wireless devices and diversity of services. In this era of the massive demand for data throughput and traffic, the shortage of spectrum resources can limit significantly the wireless network performance. Cognitive radio (CR) is a hierarchical dynamic spectrum access technique that can meet the market demand and solve the spectrum scarcity problem. CR allows secondary users (SU), i.e. unlicensed users, to access the spectrum as long as they do not affect the transmission quality of the primary users (PU), i.e., licensed users. The cognitive techniques are achieved by either defining transmission periods for SU as in overlay and interweave methods or by limiting the SU power to avoid unacceptable interference levels at the PU as in the underlay method [1]. Adopting the underlay technique steers the research focus to mitigate the interference received at the PU end from the SU.

Mitigating the interference in communication systems is a challenging research problem and receives a lot of attention. Recently, statistical signal characteristics are shown to affect significantly the maximum achievable rate. Different from the usual assumption of proper Gaussian signaling, which impose complex signal with uncorrelated real and imaginary components and equal power for each components, improper Gaussian signaling is shown to increase the achievable rate over interference channels systems [2].

Proper signaling term was introduced in the information theory field for the first time by Neeser and Massey in [3], where they defined new second order statistics quantity called *pseudo-covariance* to fully describe the propertness of any complex random variable besides the well known conventional covariance. To study the impact of the improper Gaussian signaling on communication systems, Taubök investigated the influence of improper Gaussian signaling on information theoretic quantities such as entropy, divergence and capacity [4].

In cognitive radio research, improper Gaussian signaling is employed in [5], where the PU is assumed to have proper Gaussian signaling, since there is no control upon it. On the other hand, the SU is assumed to use improper Gaussian signaling and have access on instantaneous channel state information of both the PU and SU communication channels. The instantaneous achievable rate of both PU and SU are studied, then the SU power and the circularity coefficient are adjusted to maximize its rate while achieving the PU quality-of-service (QoS).

In this paper, we study the outage probability of underlay CR with improper Gaussian signaling assumptions at the SU. We derive a closed form expression for the SU outage probability and upper and lower bounds of the outage probability for the PU. Different from [5], where perfect CSI for all links are assumed to be known at the SU nodes, we assume a practical scenario, where only average CSI is available at the SU nodes. Then we adjust the SU power and circularity coefficient to maximize the SU rate while satisfying PU QoS.

II. System Description

A. Preliminaries

Consider a zero mean scalar random variable x whose conventional variance is defined as $\sigma_x^2 = \mathbb{E}[|x|^2]$, and its pseudo-variance is defined as $\tilde{\sigma}_x^2 = \mathbb{E}[x^2]$ [3].

Definition 1: [3], [6] A complex random variable is called proper if its pseudo-variance is equal to zero, otherwise it is called improper.

Definition 2: [7] The impropriety degree of x is measured by the circularity coefficient that is defined as $\mathcal{C}_x = \left|\tilde{\sigma}_x^2\right|/\sigma_x^2$, where $0 \leq \mathcal{C}_x \leq 1$, $\mathcal{C}_x = 0$ denotes proper signal and $\mathcal{C}_x = 1$ denotes maximally improper signal.

B. Underlay Cognitive Radio System

We assume a spectrum sharing system consisting of SU pair of transmitter and secondary receiver that coexist with another licensed communication pair of the PU. The communication channels via all links are modelled as Rayleigh fading channels and the noise at the receivers end is modelled an additive white zero-mean circularly symmetric complex Gaussian with variance σ^2. The SU transmitter needs to adjust its power p_{s} without exceeding the maximum allowable interference level

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at the PU receiver. The received signal at the PU is expressed as

$$y_{\mathrm{p}} = \sqrt{p_{\mathrm{p}}}h_{\mathrm{p}}x_{\mathrm{p}} + \sqrt{p_{\mathrm{s}}}g_{\mathrm{s}}x_{\mathrm{s}} + n_{\mathrm{p}}, \quad (1)$$

where p_{p} is the PU transmitted power, x_{p} is the PU transmitted symbols, which is assumed to be proper zero mean Gaussian signal with unity variance, x_{s} is the SU signal with circularity coefficient $\mathcal{C}_x$, h_{p} is the fading channel of the PU transmission, g_{s} is the SU interfering channel to the PU and n_{p} is the noise at the PU receiver. Similarly, the SU received signal is expressed as,

$$y_{\mathrm{s}} = \sqrt{p_{\mathrm{s}}}h_{\mathrm{s}}x_{\mathrm{s}} + \sqrt{p_{\mathrm{p}}}g_{\mathrm{p}}x_{\mathrm{p}} + n_{\mathrm{s}}, \quad (2)$$

where h_{s} is the SU direct link channel, g_{p} is the PU interfering channel to the SU and n_{s} is the noise at the SU.

As a result of the improper Gaussian signaling, the achievable rate of the PU is expressed as [5], [8],

$$R_{\mathrm{p}}\left(p_{\mathrm{s}}, \mathcal{C}_x\right) = \log_2\left(1 + \frac{p_{\mathrm{p}}|h_{\mathrm{p}}|^2}{\sigma^2 + p_{\mathrm{s}}|g_{\mathrm{s}}|^2}\right) + \frac{1}{2}\log_2\left(\frac{1-\mathcal{C}_{y_{\mathrm{p}}}^2}{1-\mathcal{C}_{I_{\mathrm{p}}}^2}\right), \quad (3)$$

where $\mathcal{C}_{y_{\mathrm{p}}}$ and $\mathcal{C}_{I_{\mathrm{p}}}$ are the circularity coefficients of the received and interference-plus-noise signals at the PU, respectively, which are given by

$$\mathcal{C}_{y_{\mathrm{p}}} = \frac{p_{\mathrm{s}}|g_{\mathrm{s}}|^2\mathcal{C}_x}{p_{\mathrm{s}}|g_{\mathrm{s}}|^2 + p_{\mathrm{p}}|h_{\mathrm{p}}|^2 + \sigma^2}, \quad \mathcal{C}_{I_{\mathrm{p}}} = \frac{p_{\mathrm{s}}|g_{\mathrm{s}}|^2\mathcal{C}_x}{p_{\mathrm{s}}|g_{\mathrm{s}}|^2 + \sigma^2}. \quad (4)$$

From (3) and (4), we observe that the PU rate in improper Gaussian signaling is always higher than the proper Gaussian signaling case, i.e., $\mathcal{C}_x = 0$. In the later, the second term in (3) vanishes, while in the former it gives always a positive value which increases the rate of the PU. After some manipulations, $R_{\mathrm{p}}\left(p_{\mathrm{s}}, \mathcal{C}_x\right)$ can be written as

$$R_{\mathrm{p}}\left(p_{\mathrm{s}}, \mathcal{C}_x\right) = \frac{1}{2}\log_2\left(\frac{\left(p_{\mathrm{s}}|g_{\mathrm{s}}|^2 + p_{\mathrm{p}}|h_{\mathrm{p}}|^2 + \sigma^2\right)^2 - p_{\mathrm{s}}^2|g_{\mathrm{s}}|^4\mathcal{C}_x^2}{\left(\sigma^2 + p_{\mathrm{s}}|g_{\mathrm{s}}|^2\right)^2 - p_{\mathrm{s}}^2|g_{\mathrm{s}}|^4\mathcal{C}_x^2}\right), \quad (5)$$

where the circularity coefficient of the interference-plus-noise terms equals zero. As for the SU, the circularity coefficient of the interference term equals zero, thus the SU achievable rate is expressed as

$$R_{\mathrm{s}}\left(p_{\mathrm{s}}, \mathcal{C}_x\right) = \frac{1}{2}\log_2\left(\frac{p_{\mathrm{s}}^2|h_{\mathrm{s}}|^4\left(1-\mathcal{C}_x^2\right)}{\left(p_{\mathrm{p}}|g_{\mathrm{p}}|^2 + \sigma^2\right)^2} + \frac{2p_{\mathrm{s}}|h_{\mathrm{s}}|^2}{p_{\mathrm{p}}|g_{\mathrm{p}}|^2 + \sigma^2} + 1\right). \quad (6)$$

III. Outage Probability Analysis

The error performance achieved by the optimal coding and decoding strategies is limited by the so-called *outage probability*. In this section, the overall outage probability of our system is analyzed in details.

A. Secondary User Outage Probability

Let $R_{0,s}$ be defined as the target rate of the SU channel. The outage probability of the SU, $P_{\mathrm{out,s}}$, is defined as

$$P_{\mathrm{out,s}}(p_s, \mathcal{C}_x) = \Pr\left[R_{\mathrm{s}}(p_s, \mathcal{C}_x) < R_{0,\mathrm{s}}\right]. \quad (7)$$

Substituting (6) in (7), we get

$$P_{\mathrm{out,s}}(p_s, \mathcal{C}_x) = \Pr\left[\frac{p_{\mathrm{s}}^2\left(1-\mathcal{C}_x^2\right)\gamma_{\mathrm{s}}^2}{\left(1+p_{\mathrm{p}}\mathcal{I}_{\mathrm{p}}\right)^2} + \frac{2p_{\mathrm{s}}\gamma_{\mathrm{s}}}{1+p_{\mathrm{p}}\mathcal{I}_{\mathrm{p}}} - \Gamma_{\mathrm{s}} < 0\right], \quad (8)$$

where $\Gamma_{\mathrm{s}} = 2^{2R_{0,\mathrm{s}}} - 1$, $\gamma_{\mathrm{s}} = |h_{\mathrm{s}}|^2/\sigma^2$ is an exponential random variable that represents the direct-channel-to-noise-ratio of the SU with mean $\mathbb{E}\{\gamma_{\mathrm{s}}\} = \overline{\gamma}_{\mathrm{s}}$ and $\mathcal{I}_{\mathrm{p}} = |g_{\mathrm{p}}|^2/\sigma^2$ is an exponential random variable with mean $\mathbb{E}\{\mathcal{I}_{\mathrm{p}}\} = \overline{\mathcal{I}}_{\mathrm{p}}$ that represents the interference-channel-to-noise-ratio of the PU on the SU. By solving the inequality that appears inside the probability of (8), one can show that the conditional SU outage probability (conditioned on $\mathcal{I}_{\mathrm{p}}$) is given by

$$P_{\mathrm{out,s}}(p_s, \mathcal{C}_x|\mathcal{I}_{\mathrm{p}}) = \int_0^{\gamma_{\mathrm{s_o}}} \frac{1}{\overline{\gamma}_{\mathrm{s}}}\exp\left(-\frac{x}{\overline{\gamma}_{\mathrm{s}}}\right)dx, \quad (9)$$

where $\gamma_{\mathrm{s_o}} = \frac{(p_{\mathrm{p}}\mathcal{I}_{\mathrm{p}}+1)}{p_{\mathrm{s}}(1-\mathcal{C}_x^2)}\left(\sqrt{1+\Gamma_{\mathrm{s}}\left(1-\mathcal{C}_x^2\right)} - 1\right)$ represents the non-negative root that satisfies the inequality in (8). By evaluating (9) we get

$$P_{\mathrm{out,s}}(p_s, \mathcal{C}_x|\mathcal{I}_{\mathrm{p}}) = 1 - \exp\left(-\frac{p_{\mathrm{p}}\mathcal{I}_{\mathrm{p}}+1}{1-\mathcal{C}_x^2}\Psi_{\mathrm{s}}\left(\mathcal{C}_x\right)\right), \quad (10)$$

where $\Psi_{\mathrm{s}}\left(\mathcal{C}_x\right) = \left(\sqrt{1+\Gamma_{\mathrm{s}}\left(1-\mathcal{C}_x^2\right)} - 1\right)/\left(p_{\mathrm{s}}\overline{\gamma}_{\mathrm{s}}\right)$. By averaging (10) over the exponential statistics of $\mathcal{I}_{\mathrm{p}}$, we obtain

$$\begin{aligned} P_{\mathrm{out,s}}\left(p_s, \mathcal{C}_x\right) &= \mathbb{E}_{\mathcal{I}_{\mathrm{p}}}\{P_{\mathrm{out,s}}\left(p_{\mathrm{s}}, \mathcal{C}_x|\mathcal{I}_{\mathrm{p}}\right)\} \\ &= 1 - \frac{1-\mathcal{C}_x^2}{1-\mathcal{C}_x^2 + \overline{\mathcal{I}}_{\mathrm{p}}p_{\mathrm{p}}\Psi_{\mathrm{s}}\left(\mathcal{C}_x\right)}\exp\left(-\frac{\Psi_{\mathrm{s}}\left(\mathcal{C}_x\right)}{1-\mathcal{C}_x^2}\right). \quad (11) \end{aligned}$$

For $\mathcal{C}_x = 0$, the above outage probability reduces to

$$P_{\mathrm{out,s}}(p_s, 0) = 1 - \frac{1}{1+\overline{\mathcal{I}}_{\mathrm{p}}p_{\mathrm{p}}\Psi_{\mathrm{s}}\left(0\right)}\exp\left(-\Psi_{\mathrm{s}}\left(0\right)\right) \quad (12)$$

On the other extreme, when $\mathcal{C}_x \to 1$ yields

$$P_{\mathrm{out,s}}(p_s, 1) = \lim_{\mathcal{C}_x\to 1} P_{\mathrm{out,s}}(p_s, \mathcal{C}_x) = 1 - \frac{\exp\left(-\frac{\Gamma_{\mathrm{s}}}{2p_{\mathrm{s}}\overline{\gamma}_s}\right)}{1+\frac{p_{\mathrm{p}}\overline{\mathcal{I}}_{\mathrm{p}}\Gamma_{\mathrm{s}}}{2p_{\mathrm{s}}\overline{\gamma}_s}}. \quad (13)$$

B. Primary User Outage Probability

Similar to the above subsection, our goal here is to find a closed form for the PU outage probability in terms of the signal and channels parameters. Let $R_{0,p}$ be defined as the target rate of the SU channel. The outage probability of the PU, $P_{\mathrm{out,p}}(p_s, \mathcal{C}_x)$, is defined as

$$P_{\mathrm{out,p}}(p_s, \mathcal{C}_x) = \Pr\left[R_{\mathrm{p}}(p_s, \mathcal{C}_x) < R_{0,\mathrm{p}}\right]. \quad (14)$$

Substituting (5) in (14), we get

$$\begin{aligned} P_{\mathrm{out,p}}(p_s, \mathcal{C}_x) = \Pr\Bigg[&\gamma_{\mathrm{p}}^2 + \frac{2}{p_{\mathrm{p}}}\left(p_{\mathrm{s}}\mathcal{I}_{\mathrm{s}}+1\right)\gamma_{\mathrm{p}} \\ &-\frac{\Gamma_{\mathrm{p}}}{p_{\mathrm{p}}^2}\left[\left(p_{\mathrm{s}}\mathcal{I}_{\mathrm{s}}+1\right)^2 - p_{\mathrm{s}}^2\mathcal{I}_{\mathrm{s}}\mathcal{C}_x^2\right] < 0\Bigg], \quad (15) \end{aligned}$$

where $\Gamma_{\mathrm{p}} = 2^{2R_{0,\mathrm{p}}} - 1$, $\gamma_{\mathrm{p}} = |h_{\mathrm{p}}|^2/\sigma^2$ is an exponential random variable that represents the direct-channel-to-noise-ratio of the PU with mean $\mathbb{E}\{\gamma_{\mathrm{p}}\} = \overline{\gamma}_{\mathrm{p}}$ and $\mathcal{I}_{\mathrm{s}} = |g_{\mathrm{s}}|^2/\sigma^2$ is an exponential random variable with mean $\mathbb{E}\{\mathcal{I}_{\mathrm{s}}\} = \overline{\mathcal{I}}_{\mathrm{s}}$ that represents the interference-channel-to-noise-ratio of the SU on the PU. By solving the inequality that appears inside the probability of (15), one can show that the conditional SU outage probability (conditioned on $\mathcal{I}_{\mathrm{s}}$) is given by

$$P_{\mathrm{out,p}}(p_s, \mathcal{C}_x|\mathcal{I}_{\mathrm{s}}) = 1 - \exp\left(-\gamma_{\mathrm{p_o}}/\overline{\gamma}_{\mathrm{p}}\right), \quad (16)$$

where $\gamma_{\mathrm{p_o}}=\frac{p_{\mathrm{s}}\mathcal{I}_{\mathrm{s}}+1}{p_{\mathrm{p}}}\left(\sqrt{1+\Gamma_{\mathrm{p}}\left[1-\left(p_{\mathrm{s}}\mathcal{I}_{\mathrm{s}}\mathcal{C}_x/\left(p_{\mathrm{s}}\mathcal{I}_{\mathrm{s}}+1\right)\right)^2\right]}-1\right)$ represents the root that satisfies the inequality in (15). Averaging (16) over the statistics of $\mathcal{I}_{\mathrm{s}}$, we get

$$\begin{aligned}P_{\mathrm{out,p}}(p_s,\mathcal{C}_x)&=\mathbb{E}_{\mathcal{I}_{\mathrm{s}}}\{P_{\mathrm{out,p}}(p_s,\mathcal{C}_x|\mathcal{I}_{\mathrm{s}})\}\\&=1-\int\limits_0^\infty\frac{\exp(-\frac{z}{\overline{\mathcal{I}}_{\mathrm{s}}})}{\overline{\mathcal{I}}_{\mathrm{s}}}\exp\left(-\left(p_{\mathrm{s}}z+1\right)\Psi_{\mathrm{p}}\left(\frac{\mathcal{C}_xp_{\mathrm{s}}z}{1+p_{\mathrm{s}}z}\right)\right)dz,\end{aligned}\tag{17}$$

where $\Psi_{\mathrm{p}}\left(x\right)=(\sqrt{1+\Gamma_{\mathrm{p}}\left[1-x^2\right]}-1)/(p_{\mathrm{p}}\overline{\gamma}_{\mathrm{p}})$. Obtaining a closed form expression for the aforementioned integral is very difficult except for $\mathcal{C}_x=0$, where it reduces to

$$P_{\mathrm{out,p}}\left(p_{\mathrm{s}},0\right)=1-\frac{\exp\left(-\Psi_{\mathrm{p}}\left(0\right)\right)}{1+\overline{\mathcal{I}}_{\mathrm{s}}p_{\mathrm{s}}\Psi_{\mathrm{p}}\left(0\right)}.\tag{18}$$

For all values of $0\leq\mathcal{C}_x\leq1$, i.e., for improper Gaussian signaling, we will resort to deriving lower and upper bounds of the PU outage probability to provide us with PU outage probability behavior limits.

1) Lower Bound of the PU Outage Probability: One way to lower bound the PU outage probability is by using the fact that for $z\geq0$, we have $p_{\mathrm{s}}z/\left(1+p_{\mathrm{s}}z\right)\leq1$. In this case, we may lower bound $P_{\mathrm{out,p}}(p_{\mathrm{s}},\mathcal{C}_x)$ for any value of $\mathcal{C}_x$ as

$$\begin{aligned}P_{\mathrm{out,p}}(p_{\mathrm{s}},\mathcal{C}_x)\geq&1-\int\limits_0^\infty\frac{\exp(\frac{-z}{\overline{\mathcal{I}}_{\mathrm{s}}})}{\overline{\mathcal{I}}_{\mathrm{s}}}\exp\left(-\left(p_{\mathrm{s}}z+1\right)\Psi_{\mathrm{p}}\left(\mathcal{C}_x\right)\right)dz\\=&1-\frac{\exp\left(-\Psi_{\mathrm{p}}\left(\mathcal{C}_x\right)\right)}{1+\overline{\mathcal{I}}_{\mathrm{s}}p_{\mathrm{s}}\Psi_{\mathrm{p}}\left(\mathcal{C}_x\right)}\triangleq P_{\mathrm{out,p}}^{\mathrm{LB}}(p_{\mathrm{s}},\mathcal{C}_x).\end{aligned}\tag{19}$$

2) Upper Bound of the PU Outage Probability: An upper bound can be obtained for $z\geq0$, where we have that $p_{\mathrm{s}}z/\left(1+p_{\mathrm{s}}z\right)\geq0$. However, the drawback of using the above bound is that the dependency of the outage expression on the important improper parameter $\mathcal{C}_x$ will vanish and result in a loose PU outage probability upper bound. To obtain a tighter bound, we split the integral in (17) as follows

$$\begin{aligned}P_{\mathrm{out,p}}(p_{\mathrm{s}},\mathcal{C}_x)=&\int\limits_0^\alpha\frac{1}{\overline{\mathcal{I}}_{\mathrm{s}}}\exp\left(-\frac{z}{\overline{\mathcal{I}}_{\mathrm{s}}}\right)P_{\mathrm{out,p}}(p_s,\mathcal{C}_x|\mathcal{I}_{\mathrm{s}}=z)dz\\&+\int\limits_\alpha^\infty\frac{1}{\overline{\mathcal{I}}_{\mathrm{s}}}\exp\left(-\frac{z}{\overline{\mathcal{I}}_{\mathrm{s}}}\right)P_{\mathrm{out,p}}(p_s,\mathcal{C}|\mathcal{I}_{\mathrm{s}}=z)dz\\\overset{(a)}{\leq}&1-\int\limits_0^\alpha\frac{1}{\overline{\mathcal{I}}_{\mathrm{s}}}\exp\left(-\frac{z}{\overline{\mathcal{I}}_{\mathrm{s}}}\right)\exp\left(-\left(p_{\mathrm{s}}z+1\right)\Psi_{\mathrm{p}}\left(0\right)\right)dz\\&-\int\limits_\alpha^\infty\frac{1}{\overline{\mathcal{I}}_{\mathrm{s}}}\exp\left(-\frac{z}{\overline{\mathcal{I}}_{\mathrm{s}}}\right)\exp\left(-\left(p_{\mathrm{s}}z+1\right)\Psi_{\mathrm{p}}\left(\frac{\mathcal{C}_xp_{\mathrm{s}}\alpha}{1+p_{\mathrm{s}}\alpha}\right)\right)dz\\\triangleq&P_{\mathrm{out,p}}^{\mathrm{UB}}(p_{\mathrm{s}},\mathcal{C}_x,\alpha),\end{aligned}\tag{20}$$

where (a) follows from using the fact that for $z\geq0$ we have $(\frac{p_{\mathrm{s}}z}{1+p_{\mathrm{s}}z})\geq0$ in the first integral, and the fact that for $z\geq\alpha$, we have $(\frac{p_{\mathrm{s}}z}{1+p_{\mathrm{s}}z})\geq(\frac{p_{\mathrm{s}}\alpha}{1+p_{\mathrm{s}}\alpha})$ in the second integral. Evaluating both integrals we get

$$\begin{aligned}P_{\mathrm{out,p}}^{\mathrm{UB}}(p_{\mathrm{s}},\mathcal{C}_x,\alpha)=&1+\frac{\exp\left(-\frac{\alpha}{\overline{\mathcal{I}}_{\mathrm{s}}p_{\mathrm{s}}}-\left(\alpha+1\right)\Psi_{\mathrm{p}}\left(0\right)\right)}{1+p_{\mathrm{s}}\overline{\mathcal{I}}_{\mathrm{s}}\Psi_{\mathrm{p}}\left(0\right)}\\-\frac{\exp\left(\Psi_{\mathrm{p}}\left(0\right)\right)}{1+p_{\mathrm{s}}\overline{\mathcal{I}}_{\mathrm{s}}\Psi_{\mathrm{p}}\left(0\right)}&-\frac{\exp\left(-\frac{\alpha}{\overline{\mathcal{I}}_{\mathrm{s}}p_{\mathrm{s}}}-\left(\alpha+1\right)\Psi_{\mathrm{p}}\left(\frac{\mathcal{C}_x\alpha}{1+\alpha}\right)\right)}{1+p_{\mathrm{s}}\overline{\mathcal{I}}_{\mathrm{s}}\Psi_{\mathrm{p}}\left(\frac{\mathcal{C}_x\alpha}{1+\alpha}\right)}.\end{aligned}\tag{21}$$

The best upper bound can be obtained by finding the value of α that minimizes $P_{\mathrm{out,p}}^{\mathrm{UB}}(p_{\mathrm{s}},\mathcal{C}_x)$,

$$P_{\mathrm{out,p}}^{\mathrm{UB}}(p_{\mathrm{s}},\mathcal{C}_x)=\min_{\alpha\geq0}\quad P_{\mathrm{out,p}}^{\mathrm{UB}}(p_{\mathrm{s}},\mathcal{C}_x,\alpha),\tag{22}$$

which can equivalently found from

$$\begin{aligned}&\exp\left(\left(\alpha+1\right)\left[\Psi_{\mathrm{p}}\left(\frac{\mathcal{C}_x\alpha}{\alpha+1}\right)-\Psi_{\mathrm{p}}\left(0\right)\right]+\frac{1}{p_{\mathrm{p}}\overline{\gamma}_{\mathrm{p}}}\right)+\frac{\frac{p_{\mathrm{s}}\overline{\mathcal{I}}_{\mathrm{s}}}{p_{\mathrm{p}}\overline{\gamma}_{\mathrm{p}}}\Gamma_{\mathrm{p}}\left(\frac{\mathcal{C}_x^2\alpha}{(1+\alpha)^2}\right)}{1+p_{\mathrm{s}}\overline{\mathcal{I}}_{\mathrm{s}}\Psi_{\mathrm{p}}\left(\frac{\mathcal{C}_x\alpha}{\alpha+1}\right)}\\&\times\left(1+\frac{\left(\frac{p_{\mathrm{s}}\overline{\mathcal{I}}_{\mathrm{s}}}{1+\alpha}\right)}{1+p_{\mathrm{s}}\overline{\mathcal{I}}_{\mathrm{s}}\Psi_{\mathrm{p}}\left(\frac{\mathcal{C}_x\alpha}{\alpha+1}\right)}\right)\left(\frac{1}{1+p_{\mathrm{p}}\overline{\gamma}_{\mathrm{p}}\Psi_{\mathrm{p}}\left(\frac{\mathcal{C}_x\alpha}{\alpha+1}\right)}\right)-1=0.\end{aligned}\tag{23}$$

Assuming $\alpha\gg1$, the we obtain the approximate expression

$$\begin{aligned}\tilde{P}_{\mathrm{out,p}}(p_{\mathrm{s}},\mathcal{C}_x)=&1+\frac{\exp\left(-\frac{\alpha}{\overline{\mathcal{I}}_{\mathrm{s}}p_{\mathrm{s}}}-\alpha\Psi_{\mathrm{p}}\left(0\right)\right)}{1+p_{\mathrm{s}}\overline{\mathcal{I}}_{\mathrm{s}}\Psi_{\mathrm{p}}\left(0\right)}\\&-\frac{\exp\left(-\Psi_{\mathrm{p}}\left(0\right)\right)}{1+p_{\mathrm{s}}\overline{\mathcal{I}}_{\mathrm{s}}\Psi_{\mathrm{p}}\left(0\right)}-\frac{\exp\left(-\frac{\alpha}{\overline{\mathcal{I}}_{\mathrm{s}}p_{\mathrm{s}}}-\alpha\Psi_{\mathrm{p}}\left(\mathcal{C}_x\right)\right)}{1+p_{\mathrm{s}}\overline{\mathcal{I}}_{\mathrm{s}}\Psi_{\mathrm{p}}\left(\mathcal{C}_x\right)}.\end{aligned}\tag{24}$$

IV. SU Adaptive Signal Adjustment

In this section, we aim to adjust the SU signal parameters p_{s} and $\mathcal{C}_x$ to improve the SU performance measured by the outage probability while maintaining predetermined PU QoS represented in an outage probability threshold of a target rate. According to the PU perspective, the system is designed to achieve a target QoS considering an acceptable interference margin. In this case, $P_{\mathrm{out,th}}$ is expressed in term of the maximum interference power $\mathcal{P}_{\mathrm{int}}$ as

$$\begin{aligned}P_{\mathrm{out,th}}&=\Pr\left[\log_2\left(1+\frac{p_{\mathrm{p}}|h_{\mathrm{p}}|^2}{\sigma^2+\mathcal{P}_{\mathrm{int}}}\right)<R_{0,\mathrm{p}}\right]\\&=1-\exp\left(-\frac{1+\mathcal{I}_{\max}}{p_{\mathrm{p}}\overline{\gamma}_{\mathrm{p}}}\left(\sqrt{1+\Gamma_{\mathrm{p}}}-1\right)\right),\end{aligned}\tag{25}$$

where $\mathcal{I}_{\max}=\mathcal{P}_{\mathrm{int}}/\sigma^2$ is the maximum allowable interference-to-noise ratio at the PU receiver end. Thus, the PU should adjust its power according to

$$p_{\mathrm{p}}=\left(\frac{1+\mathcal{I}_{\max}}{\overline{\gamma}_{\mathrm{p}}\log\left(1-P_{\mathrm{out,th}}\right)}\right)\left(1-\sqrt{1+\Gamma_{\mathrm{p}}}\right).\tag{26}$$

As a result, the SU has to consider the PU design conditions while adjusting its signal parameters. For the design purpose, we consider different cases in the following subsections.

A. Proper Gaussian signaling Design

For the proper signaling case, the SU system adjusts p_{s} to suppress its interference to the PU to be within the acceptable margin. In other words, p_{s} needs to be computed such that $P_{\mathrm{out,p}}\left(p_{\mathrm{s}},0\right)$ satisfy a predefined outage threshold $P_{\mathrm{out,th}}$ for a given rate $R_{0,\mathrm{p}}$, which is expressed as

$$P_{\mathrm{out,th}}=1-\frac{1}{1+\overline{\mathcal{I}}_{\mathrm{s}}p_{\mathrm{s}}\Psi_{\mathrm{p}}\left(0\right)}\exp\left(-\Psi_{\mathrm{p}}\left(0\right)\right).\tag{27}$$

After some simplifications, p_{s} is found to be

$$p_{\mathrm{s}}=\frac{\exp\left(-\Psi_{\mathrm{p}}\left(0\right)\right)-\left(1-P_{\mathrm{out,th}}\right)}{\Psi_{\mathrm{p}}\left(0\right)\left(1-P_{\mathrm{out,th}}\right)\mathcal{I}_{\mathrm{s}}}.\tag{28}$$

From (28) and (27), the SU can operate while satisfying the PU QoS requirements if $\exp\left(-\Psi_{\mathrm{p}}\left(0\right)\right)>\left(1-P_{\mathrm{out,th}}\right)$, which is valid as long as the maximum marginal interference-to-noise ratio does not equal zero, otherwise, the SU remains silent.

B. Improper Gaussian signaling Design

For the improper Gaussian signaling case, we have an additional design parameter, i.e., $\mathcal{C}_x$, which controls the signal impropriety. Thus, we expect to have an infinite $(p_{\rm s}, \mathcal{C}_x)$ set that satisfy the PU QoS. This design flexibility gives us the opportunity to consider another objective to improve in addition to achieving the PU performance requirement. In this work, the SU signal parameters $p_{\rm s}$ and $\mathcal{C}_x$ is adaptably computed to minimize the SU outage probability while satisfying the PU QoS, i.e., $P_{\rm out,p}\left(p_{\rm s}, \mathcal{C}_x\right) \le P_{\rm out,th}$. For this purpose, we formulate the following optimization problem,

$$\min_{p_{\rm s},\, \mathcal{C}_x} \quad P_{\rm out,s}\left(p_{\rm s},\, \mathcal{C}_x\right)$$
$$\text{s.t. } P_{\rm out,p}\left(p_{\rm s},\, \mathcal{C}_x\right) \le P_{\rm out,th},\; p_{\rm s} \le p_{\rm s,max},\; 0 \le \mathcal{C}_x \le 1,$$

where $p_{\rm s,max}$ is the maximum SU transmitter power budget. To simplify this problem, we use the bounds derived in the previous section and obtain the following simplified problems.

1) Lower-bound-based design: Thanks to the simplified PU lower bound expression obtained in (19), we can obtain $p_{\rm s}$ that can satisfy the PU QoS, i.e., $P^{\rm LB}_{\rm out,p}(p_{\rm s}, \mathcal{C}_x) = P_{\rm out,th}$, in terms of $\mathcal{C}_x$ and $P_{\rm out,th}$ as

$$p_{\rm s}\left(\mathcal{C}_x\right) = \frac{\exp\left(-\Psi_{\rm p}\left(\mathcal{C}_x\right)\right) - \left(1 - P_{\rm out,th}\right)}{\Psi_{\rm p}\left(\mathcal{C}_x\right)\left(1 - P_{\rm out,th}\right)\overline{\mathcal{I}}_{\rm s}}. \tag{29}$$

In this case, we observe that $\exp\left(-\Psi_{\rm p}\left(\mathcal{C}_x\right)\right) > \left(1 - P_{\rm out,th}\right)$ is always valid, which means that the SU may always transmit. In the same time, increasing the signal impropriety results in a large increase of $p_{\rm s}$, which should be considered carefully to meet the maximum SU power budget.

Based on (29) and (11), the outage probability of the SU can be expressed in terms of $\mathcal{C}_x$. Therefore, we obtain the following simplified optimization problem

$$\min_{\mathcal{C}_x} \; P_{\rm out,s}\left(\mathcal{C}_x\right)$$
$$\text{subject to } \; 0 \le \mathcal{C}_x \le 1, \quad 0 < p_{\rm s}\left(\mathcal{C}_x\right) \le p_{\rm s,max}, \tag{30}$$

to compute $\mathcal{C}_x$, then we can find $p_{\rm s}$ from (29).

To solve (30), first we can prove that $P_{\rm out,s}\left(\mathcal{C}_x\right)$ is monotonically decreasing in $\mathcal{C}_x$, while $p_{\rm s}\left(\mathcal{C}_x\right)$ is monotonically increasing in $\mathcal{C}_x$. Thus, the SU can achieve lower outage if it uses maximally improper signal with the maximum possible transmitted power. As a result the lower bound power solution is $p_{\rm s,max}$ and the corresponding $\mathcal{C}_x$ is computed from Algorithm I, where $\mathcal{C}_{\rm o}$ is computed from

$$\mathcal{C}_{\rm o} = \sqrt{1 + \frac{1}{\Gamma_{\rm p}} - \frac{1}{\Gamma_{\rm p}}\left[1 + p_{\rm p}\overline{\gamma}_{\rm p}\mathbb{W}\left\{\frac{1/(\overline{\mathcal{I}}_{\rm s}p_{\rm s})}{1 - P_{\rm out,th}}\exp\left(\frac{1}{\overline{\mathcal{I}}_{\rm s}p_{\rm s}}\right)\right\} - \frac{p_{\rm p}\overline{\gamma}_{\rm p}}{\overline{\mathcal{I}}_{\rm s}p_{\rm s}}\right]^2},$$

and $\mathbb{W}\{.\}$ is the Lambert-W function [9].

Algorithm I

Input $p_{\rm s,max}$, $p_{\rm p}$, $\overline{\gamma}_{\rm p}$, $\overline{\mathcal{I}}_{\rm s}$, $\Gamma_{\rm p}$ and $P_{\rm out,th}$.
if $p\left(0\right) \ge p_{\rm s,max}$ **then**
  $\mathcal{C}_x \leftarrow 0$
else
  $\mathcal{C}_x \leftarrow \mathrm{C}_{\rm o}$
end if

2) Upper-bound-based design: Upper bound expression can be used to simplify the design problem by imposing the PU outage constraint on the derived upper bound in (24), i.e., $P^{\rm UB}_{\rm out,p}(p_{\rm s}, \mathcal{C}_x) = P_{\rm out,th}$. As a result, $\mathcal{C}_x$ can be expressed in terms of $p_{\rm s}$ as

$$\mathcal{C}_x^2 = \left(\frac{1+\alpha}{\alpha}\right)^2\left(1 + \frac{1}{\Gamma_{\rm p}} - \frac{1}{\Gamma_{\rm p}}\left[1 - \frac{p_{\rm p}\overline{\gamma}_{\rm p}}{\overline{\mathcal{I}}_{\rm s}p_{\rm s}} + \frac{p_{\rm p}\overline{\gamma}_{\rm p}}{\alpha+1}\mathbb{W}\left\{\frac{\left(\alpha+1\right)/(\overline{\mathcal{I}}_{\rm s}p_{\rm s})}{1 - P_{\rm out,th} - \Lambda_{p_{\rm s}}}\exp\left(\frac{1}{\overline{\mathcal{I}}_{\rm s}p_{\rm s}}\right)\right\}\right]^2\right), \tag{31}$$

where $\Lambda_{p_{\rm s}}$ is defined as

$$\Lambda_{p_{\rm s}} = \frac{\exp\left(-\Psi_{\rm p}(0)\right)}{1 + p_{\rm s}\overline{\mathcal{I}}_{\rm s}\Psi_{\rm p}\left(0\right)} - \frac{\exp\left(-\frac{\alpha}{p_{\rm s}\overline{\mathcal{I}}_{\rm s}} - \left(\alpha+1\right)\Psi_{\rm p}\left(0\right)\right)}{1 + p_{\rm s}\overline{\mathcal{I}}_{\rm s}\Psi_{\rm p}\left(0\right)}. \tag{32}$$

Algorithm II

Input $p_{\rm s,max}$, $p_{\rm p}$, $\overline{\gamma}_{\rm p}$, $\overline{\mathcal{I}}_{\rm s}$, $\Gamma_{\rm p}$ and $P_{\rm out,th}$.
if $p\left(0\right) \ge p_{\rm s,max}$ **then**
  $\mathcal{C}_x \leftarrow 0$
  $p_{\rm s} \leftarrow p_{\rm s,max}$
else
  if $\mathcal{C}_x\left(p_{\rm s,max}\right) > 1$ **then**
    $\mathcal{C}_x \leftarrow 1$
    $p_{\rm s} \leftarrow \mathcal{C}_x^{-1}\left(p_{\rm s}\right)$
  else
    $p_{\rm s} \leftarrow p_{\rm s,max}$
    $\mathcal{C}_x \leftarrow \mathcal{C}_x\left(p_{\rm s,max}\right)$
  end if
end if

Alternatively, if we wish to use the PU outage probability approximate expression to obtain another simplified optimization problem, we follow similar steps to that of the upper bound and obtain $\mathcal{C}_x$ in terms of $p_{\rm s}$ as

$$\mathcal{C}_x^2\left(p_{\rm s}\right) = 1 + \frac{1}{\Gamma_{\rm p}} - \frac{1}{\Gamma_{\rm p}}\left[1 + \frac{p_{\rm p}\overline{\gamma}_{\rm p}}{\alpha}\mathbb{W}\left\{\frac{\alpha/\left(\overline{\mathcal{I}}_{\rm s}p_{\rm s}\right)}{1 - P_{\rm out,th} - \mu_{p_{\rm s}}}\right\} - \frac{p_{\rm p}\overline{\gamma}_{\rm p}}{\overline{\mathcal{I}}_{\rm s}p_{\rm s}}\right]^2, \tag{33}$$

where $\mu_{p_{\rm s}}$ is defined as

$$\mu_{p_{\rm s}} = \frac{\exp\left(-\Psi_{\rm p}\left(0\right)\right)}{1 + p_{\rm s}\overline{\mathcal{I}}_{\rm s}\Psi_{\rm p}\left(0\right)} - \frac{\exp\left(-\frac{\alpha}{p_{\rm s}\overline{\mathcal{I}}_{\rm s}} - \alpha\Psi_{\rm p}\left(0\right)\right)}{1 + p_{\rm s}\overline{\mathcal{I}}_{\rm s}\Psi_{\rm p}\left(0\right)}. \tag{34}$$

Then, the SU signal parameters can be obtained using the approximate formula (33) in Algorithm II.

V. Numerical Results

In this section, we compare the PU outage probability bounds with the exact expressions. Then, we use these bounds to design the SU signal parameters to minimize the SU outage while satisfying the PU QoS.

Example 1: This example assumes that the PU is working to achieve $R_{0,\rm p} = 1$ b/sec with $p_{\rm p} = 1\ W$. Fig. 1 plots the PU outage probability versus $\overline{\gamma}_{\rm p}$ based on the exact value computed from (17), lower bound (19), upper bound (21) and the approximate expression (24). The SU is assumed to have $p_{\rm s} = 1\ W$, $\mathcal{C}_x = 0.5$ and $\overline{\mathcal{I}}_{\rm s} = 0, 5, 10$ dB. α is assumed to be computed from (23). We observe that the bounds is tight

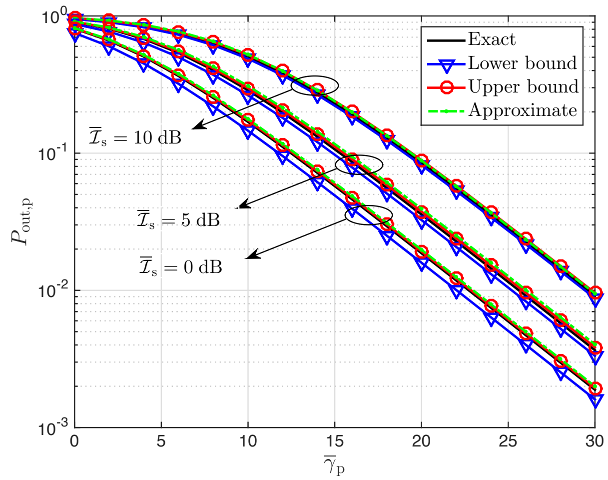

Fig. 1. A comparison between the exact PU outage probability, lower bound, upper bound and approximate expressions versus $\overline{\gamma}_s$ for $\overline{\mathcal{I}}_s = 0,\ 5,\ 10$ dB.

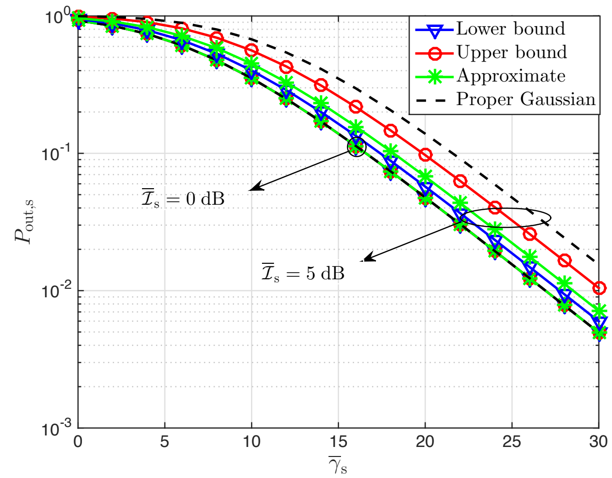

Fig. 2. SU outage probability for proper and improper Gaussian signaling versus $\overline{\gamma}_s$ for $\overline{\mathcal{I}}_s = 0, 5$ dB.

to the exact PU outage probability for different ranges of $\overline{\gamma}_p$ and $\overline{\gamma}_p$. Similar results are observed at different target rates.

Example 2: To examine the SU improper Gaussian signal design, we first assume the PU power is adjusted to achieve a rate of 1 b/sec with outage of 0.01 considering $\mathcal{I}_{\max} = 0$ dB. The communication channels are assumed to have $\overline{\gamma}_p = 20$ dB, $\overline{\mathcal{I}}_p = 3$ dB, and $\overline{\mathcal{I}}_s = 0$ dB and 5 dB. The SU proper Gaussian design adjusts the power according to (28) without violating the SU power budget $p_{s,\max} = 1$ W and the PU QoS requirements. The improper SU signal design is based on either Algorithm I for lower bound or Algorithm II upper bound and approximate expression. Fig. 2 shows the SU outage probability versus $\overline{\gamma}_s$ for different $\overline{\mathcal{I}}_s$ values. For $\overline{\mathcal{I}}_s = 0$ dB, all bounds reduces to the proper Gaussian signaling design, because the SU interference signal power is comparable to maximum allowable interference margin at the PU, hence proper signaling tends to use the maximum power. Therefore the improper Gaussian signaling system cannot increase the impropriety degree because it will be at the cost of increasing the transmitted power which cannot be achieved. On the other hand, when the SU interference channel to the PU is strong, the proper signaling design tends to use less power to meet the PU QoS, while improper signaling design uses more power to improve its outage performance and compensate its effect on the PU by increasing the signal impropriety. Fig. 2 shows a 1-3 dB improvement of improper Gaussian signaling over the proper Gaussian signaling design.

VI. Conclusion

In this paper, we studied the outage probability of underlay cognitive radio system with improper Gaussian signaling. We derived closed form expression for the SU outage probability and tight bounds for the PU. Based on the derived expressions and using the average CSI, we adjusted the SU power and circularity coefficient to improve its performance measured in terms of the outage probability while satisfying the PU QoS and meeting the SU power budget. The simulation results show that the benefit of improper Gaussian signaling system over the proper Gaussian signaling increases as the interference-to-noise ratio of the SU to the PU increases.

References

[1] Q. Zhao and B. M. Sadler, "A survey of dynamic spectrum access," *IEEE Signal Process. Mag.*, vol. 24, no. 3, pp. 79–89, May 2007.

[2] Y. Zeng, C. M. Yetis, E. Gunawan, Y. L. Guan, and R. Zhang, "Transmit optimization with improper Gaussian signaling for interference channels," *IEEE Trans. Signal Process.*, vol. 61, no. 11, pp. 2899–2913, Jun. 2013.

[3] F. D. Neeser and J. L. Massey, "Proper complex random processes with applications to information theory," *IEEE Trans. Inf. Theory*, vol. 39, no. 4, pp. 1293–1302, Jul. 1993.

[4] G. Tauböck, "Complex-valued random vectors and channels: entropy, divergence, and capacity," *IEEE Trans. Inf. Theory*, vol. 58, no. 5, pp. 2729–2744, May 2012.

[5] C. Lameiro, I. Santamaria, and P. Schreier, "Benefits of improper signaling for underlay cognitive radio," *IEE Wireless Commun. Lett.*, vol. 4, no. 1, pp. 22–25, Feb. 2015.

[6] P. J. Schreier and L. L. Scharf, *Statistical signal processing of complex-valued data: the theory of improper and noncircular signals*. Cambridge University Press, 2010.

[7] S. Lagen, A. Agustin, and J. Vidal, "Improper Gaussian signaling for the Z-interferece channel," in *in Proc. IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), 2014*, 2014, pp. 1140–1144.

[8] Y. Zeng, R. Zhang, E. Gunawan, and Y. L. Guan, "Optimized transmission with improper Gaussian signaling in the K-user MISO interference channel," *IEEE Trans. Wireless Commun.*, vol. 12, no. 12, pp. 6303–6313, Dec. 2013.

[9] R. M. Corless, G. H. Gonnet, D. E. Hare, D. J. Jeffrey, and D. E. Knuth, "On the lambertw function," *Advances in Computational Mathematics*, vol. 5, no. 1, pp. 329–359, Dec. 1996.