A dichotomy for upper domination in monogenic classes

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Abstract. An upper dominating set in a graph is a minimal (with respect to set inclusion) dominating set of maximum cardinality. The problem of finding an upper dominating set is NP-hard for general graphs and in many restricted graph families. In the present paper, we study the computational complexity of this problem in monogenic classes of graphs (i.e. classes defined by a single forbidden induced subgraph) and show that the problem admits a dichotomy in this family. In particular, we prove that if the only forbidden induced subgraph is a P_4 or a $2K_2$ (or any induced subgraph of these graphs), then the problem can be solved in polynomial time. Otherwise, it is NP-hard.

1 Introduction

In a graph $G = (V, E)$, a *dominating set* is a subset of vertices $D \subseteq V$ such that any vertex outside of D has a neighbour in D. A dominating set D is *minimal* if no proper subset of D is dominating. An *upper dominating set* is a minimal dominating set of maximum cardinality. The UPPER DOMINATING SET problem (i.e. the problem of finding an upper dominating set in a graph) is known to be NP-hard [2]. On the other hand, in some restricted graph families, the problem can be solved in polynomial time, which is the case for bipartite graphs [3], chordal graphs [8], generalized series-parallel graphs [7] and graphs of bounded clique-width [4].

In the present paper, we study the complexity of the problem in monogenic classes of graphs, i.e. classes defined by a single forbidden induced subgraph. Our main result is that the problem admits a dichotomy in this family: for each class in the family the problem is either NP-hard or can be solved in polynomial

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time. Up to date, a complete dichotomy in monogenic classes was available only for VERTEX COLORING [11], MINIMUM DOMINATING SET [10] and MAXIMUM CUT [9].

The organization of the paper is as follows. In Section 2, we introduce basic definitions and notations related to the topic of the paper and prove some preliminary results about minimal dominating sets. In Sections 3 and 4, we prove some NP-hardness and polynomial-time results, respectively. In Section 5, we summarize our arguments in a final statement.

2 Preliminaries

All graphs in this paper are simple, i.e. undirected, without loops and multiple edges. The *girth* of a graph G is the length of a shortest cycle in G. As usual, we denote by K_n, P_n and C_n a complete graph, a chordless path and a chordless cycle with n vertices, respectively. Also, $2K_2$ is the disjoint union of two copies of K_2 and a *star* is a connected graph in which all edges are incident to the same vertex, called the *center* of the star.

Let $G = (V, E)$ be a graph with vertex set V and edge set E, and let u and v be two vertices of G. If u is adjacent to v, we write $uv \in E$ and say that u and v are neighbours. The neighbourhood of a vertex $v \in V$ is the set of its neighbours; it is denoted by $N(v)$. The degree of v is the size of its neighbourhood. If the degree of each vertex of G equals 3, then G is called *cubic*.

The *complement* of a graph G, denoted $\overline{G}$, is the graph with the same vertex set in which two vertices are adjacent if and only if they are not adjacent in G. A subgraph of G is *induced* if two vertices of the subgraph are adjacent if and only if they are adjacent in G. If a graph H is isomorphic to an induced subgraph of a graph G, we say that G contains H. Otherwise we say that G is H-free.

In a graph, a *clique* is a subset of pairwise adjacent vertices, and an *independent set* is a subset of vertices no two of which are adjacent. A graph is *bipartite* if its vertices can be partitioned into two independent sets. It is well-known that a graph is bipartite if and only if it is free of odd cycles.

We say that an independent set I is *maximal* if no other independent set properly contains I. The following simple lemma connects the notion of a maximal independent set and that of a minimal dominating set.

Lemma 1. *Every maximal independent set is a minimal dominating set.*

Proof. Let $G = (V, E)$ be a graph and let I be a maximal independent set in G. Then every vertex $u \notin I$ has a neighbour in I (else I is not maximal) and hence I is dominating.

The removal of any vertex $u \in I$ from I leaves u undominated. Therefore, I is a minimal dominating set. □

Definition 1. *Given a dominating set D and a vertex $x \in D$, we say that a vertex $y \notin D$ is a private neighbour of x if x is the only neighbour of y in D.*

Lemma 2. *Let D be a minimal dominating set in a graph G. If a vertex $x \in D$ has a neighbour in D, then it also has a private neighbour outside of D.*

Proof. If a vertex $x \in D$ is adjacent to a vertex in D and has no private neighbour outside of D, then D is not minimal, because the set $D - \{x\}$ is also dominating. □

Lemma 3. *Let G be a connected graph and D a minimal dominating set in G. If there are vertices in D that have no private neighbour outside of D, then D can be transformed in polynomial time into a minimal dominating set D' with $|D'| \leq |D|$ in which every vertex has a private neighbour outside of D'.*

Proof. Assume D contains a vertex x which has no private neighbours outside of D. Then x is isolated in D (i.e. it has no neighbours in D) by Lemma 2. On the other hand, since G is connected, x must have a neighbour y outside of D. As y is not a private neighbour of x, it is adjacent to a vertex z in D. Consider now the set $D_0 = (D - \{x\}) \cup \{y\}$. Clearly, it is a dominating set. If it is a minimal dominating set in which every vertex has a private neighbour outside of the set, then we are done. Otherwise, it is either not minimal, in which case we can reduce its size by deleting some vertices, or it has strictly fewer isolated vertices than D. Therefore, by iterating the procedure, in at most $|V(G)|$ steps we can transform D into a minimal dominating set D' with $|D'| \leq |D|$ in which every vertex has a private neighbour outside of the set. □

3 NP-hardness results

Theorem 1. *The* UPPER DOMINATING SET *problem restricted to the class of planar graphs with maximum vertex degree 6 and girth at least* 6 *is NP-hard.*

Proof. We use a reduction from the MAXIMUM INDEPENDENT SET problem (IS for short) in planar cubic graphs, where IS is NP-hard [6]. The input of the decision version of IS consists of a simple graph $G = (V, E)$ and an integer k and asks to decide if G contains an independent set of size at least k.

Let $G = (V, E)$ and an integer k be an instance of IS, where G is a planar cubic graph. We denote the number of vertices and edges of G by n and m, respectively. We build an instance $G' = (V', E')$ of the UPPER DOMINATING SET problem by replacing each edge $e = uv \in E$ with two induced paths $u - v_e - u_e - v$ and $u - v'_e - u'_e - v$, as shown in Figure 1.

Clearly, G' can be constructed in time polynomial in n. Moreover, it is not difficult to see that G' is a planar graph with maximum vertex degree 6 and girth at least 6.

We claim that G contains an independent set of size at least k if and only if G' contains a minimal dominating set of size at least $k + 2m$.

Suppose G contains an independent set S with $|S| \geq k$ and without loss of generality assume that S is maximal with respect to set-inclusion (otherwise, we greedily add vertices to S until it becomes a maximal independent set). Now we consider a set $D \subset V'$ containing

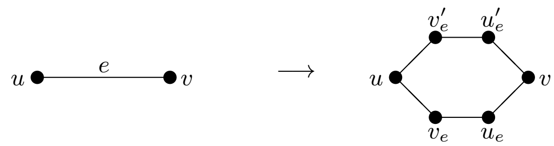

Fig. 1. Replacement of an edge by two paths

- all vertices of S,
- vertices v_e and v'_e for each edge $e = uv \in E$ with $v \in S$,
- exactly one vertex in $\{u_e, v_e\}$ (chosen arbitrarily) and exactly one vertex in $\{u'_e, v'_e\}$ (chosen arbitrarily) for each edge $e = uv \in E$ with $u, v \notin S$.

It is not difficult to see that D is a maximal independent, and hence, by Lemma 2, a minimal dominating, set in G'. Moreover, $|D| = |S| + 2m \geq k + 2m$.

To prove the inverse implication, we first observe the following:

- *Every minimal dominating set in G' contains either exactly two vertices or no vertex in the set $\{u_e, v_e, u'_e, v'_e\}$ for every edge $e = uv \in E$.* Indeed, assume a minimal dominating set D in G' contains at least three vertices in $\{u_e, v_e, u'_e, v'_e\}$, say u_e, v_e, u'_e. But then D is not minimal, since u_e can be removed from the set. If D contains one vertex in $\{u_e, v_e, u'_e, v'_e\}$, say u_e, then both u and v must belong to D (otherwise it is not dominating), in which case it is not minimal (u_e can be removed).
- *If a minimal dominating set D in G' contains exactly two vertices in the set $\{u_e, v_e, u'_e, v'_e\}$, then*
  - *one of them belongs to $\{u_e, v_e\}$ and the other to $\{u'_e, v'_e\}$.* Indeed, if both vertices belong to $\{u_e, v_e\}$, then both u and v must also belong to D (to dominate u'_e, v'_e), in which case D is not minimal (u_e and v_e can be removed).
  - *at most one of u and v belongs to D.* Indeed, if both of them belong to D, then D is not minimal dominating, because u and v dominate the set $\{u_e, v_e, u'_e, v'_e\}$ and any vertex of this set can be removed from D.

Now let $D \subseteq V'$ be a minimal dominating set in G' with $|D| \geq k + 2m$. If D contains exactly two vertices in the set $\{u_e, v_e, u'_e, v'_e\}$ for every edge $e = uv \in E$, then, according to the discussion above, the set $D \cap V$ is independent in G and contains at least k vertices, as required.

Assume now that there are edges $e = uv \in E$ for which the set $\{u_e, v_e, u'_e, v'_e\}$ contains no vertex of D. We call such edges *D-clean*. Obviously, both endpoints of a D-clean edge belong to D, since otherwise this set is not dominating. To prove the theorem in the situation when D-clean edges are present, we transform D into another minimal dominating set D' with no D'-clean edges and with $|D'| \geq |D|$. To this end, we do the following. For each vertex $u \in V$ incident to at least one D-clean edge, we first remove u from D, and then for each D-clean edge $e = uv \in E$ incident to u, we introduce vertices v_e, v'_e to D. Under this transformation vertex v may become redundant (i.e. its removal may result in a dominating set), in which case we remove it. It is not difficult to see that the set D' obtained in this way is a minimal dominating set with no D'-clean edges and with $|D'| \geq |D|$. Therefore, $D' \cap V$ is an independent set in G of cardinality at least k. □

Theorem 2. *The* UPPER DOMINATING SET *problem restricted to the class of complements of bipartite graphs is NP-hard.*

Proof. We use a reduction from the MINIMUM DOMINATING SET problem, which is known to be NP-hard [5]. The input of the decision version of this problem consists of a simple graph $G = (V, E)$ and an integer k. The problem asks to determine if G contains a dominating set of size at most k.

Assume an instance of the MINIMUM DOMINATING SET problem is given by a graph $G = (V, E)$ with n vertices and m edges and an integer $k \leq n - 3$. Without loss of generality, we may further assume that G is connected. We build an instance $G' = (V', E')$ of the UPPER DOMINATING SET problem where G' is the complement of a bipartite graph as follows.

- $V' = V \cup V_E \cup \{a, b\}$, where $V_E = \{v_e \ : \ e \in E\}$;
- $V \cup \{a\}$ and $V_E \cup \{b\}$ are cliques. Also, a vertex $v \in V$ is connected to a vertex $v_e \in V_E$ if and only if v is incident to $e \in E$ in G. Finally, a is connected to every vertex of $V_E \cup \{b\}$.

Clearly, this construction can be done in time polynomial in n. We claim that there is a dominant set in G of size at most k if and only if there is a minimal dominating set in G' of size at least $n - k$.

Suppose G contains a dominating set D with $|D| \leq k$. Without loss of generality, we assume that D is a minimal dominating set (otherwise we can remove some vertices from D to make it minimal). Moreover, we will assume that D satisfies Lemma 3, i.e. every vertex of D has a private neighbour outside of the set. Since D is a dominating set, for every vertex u outside of D, there is an edge e_u connecting it to a vertex in D. We claim that the set $D' = \{v_{e_u} \ : \ u \notin D\}$ is a minimal dominating set in G'. By construction, D' dominates $V_E \cup \{a, b\} \cup (V - D)$. To show that it also dominates D, assume by contradiction that a vertex $w \in D$ is not dominated by D' in G'. By Lemma 3 we know that w has a private neighbour u outside of D. But then the edge $e = uw$ is the only edge connecting u to a vertex in D. Therefore, v_e belongs to D' and hence it dominates w, contradicting our assumption. In order to show that D' is a minimal dominating set, we observe that if we remove from D' a vertex v_{e_u} with $e_u = uv$, $u \notin D$, $v \in D$, then u becomes undominated in G'. Finally, since $|D'| = n - |D|$, we conclude that $|D'| \geq n - k$.

Conversely, let $D' \subseteq V'$ be a minimal dominating set in G' with $|D'| \geq n - k$ and $n - k \geq 3$ (by assumption $k \leq n - 3$). Then D' cannot intersect both $V \cup \{a\}$ and $V_E \cup \{b\}$, since otherwise it contains exactly one vertex in each of these sets (else it is not minimal, because each of them is a clique), in which case $|D'| = 2$. Also, D' cannot be a subset of $V \cup \{a\}$, since otherwise it contains a (because a is the only vertex of $V \cup \{a\}$ dominating b) and hence it coincides with $\{a\}$ (else it is not minimal, because a dominates the graph), in which case $|D'| = 1$. Therefore, $D' \subseteq V_E \cup \{b\}$. Also, $b \notin D'$, since otherwise D' is not minimal (i.e. b can be removed from D'). Therefore, there exists a subset of edges $F \subseteq E$ such that $D' = \{v_e \;:\; e \in F\}$. Let us denote the subgraph of G formed by the edges of F (and all their endpoints) by G_F and prove the following:

- *G_F is a spanning forest of G*, because F covers V (else D' is not dominating) and G_F is acyclic (else D' is not minimal).
- *G_F is P_4-free, i.e. each connected component of G_F is a star*, since otherwise D' is not minimal, because any vertex of D' corresponding to the middle edge of a P_4 in G_F can be removed from D'.

Let D be the set of the centers of the stars of G_F. Then D is dominating in G (since F covers V) and $|D| = n - |F| = n - |D'| \leq k$, as required. □

4 Polynomial-time results

As we have mentioned in the introduction, the UPPER DOMINATING SET problem can be solved in polynomial time for bipartite graphs [3], chordal graphs [8] and generalized series-parallel graphs [7]. It also admits a polynomial-time solution in any class of graphs of bounded clique-width [4]. Since P_4-free graphs have clique-width at most 2 (see e.g. [1]), we make the following conclusion.

Proposition 1. *The* UPPER DOMINATING SET *problem can be solved for P_4-free graphs in polynomial time.*

In what follows, we develop a polynomial-time algorithm to solve the problem in the class of $2K_2$-free graphs.

We start by observing that the class of $2K_2$-free graphs admits a polynomial-time solution to the MAXIMUM INDEPENDENT SET problem (see e.g. [12]). By Lemma 2 every maximal (and hence maximum) independent set is a minimal dominating set. These observations allow us to restrict ourselves to the analysis of minimal dominating sets X such that

- X contains at least one edge,
- $|X| > \alpha(G)$,

where $\alpha(G)$ is the independence number, i.e. the size of a maximum independent set in G.

Let G be a $2K_2$-free graph and let ab an edge in G. Assuming that G contains a minimal dominating set X containing both a and b, we first explore some properties of X. In our analysis we use the following notation. We denote by

- N the neighbourhood of $\{a, b\}$, i.e. the set of vertices outside of $\{a, b\}$ each of which is adjacent to at least one vertex of $\{a, b\}$,

- A the anti-neighbourhood of $\{a, b\}$, i.e. the set of vertices adjacent neither to a nor to b,
- $Y := X \cap N$,
- $Z := N(Y) \cap A$, i.e. the set of vertices of A each of which is adjacent to at least one vertex of Y.

Since a and b are adjacent, by Lemma 2 each of them has a private neighbour outside of X. We denote by

- a^* a private neighbour of a,
- b^* a private neighbour of b.

By definition, a^* and b^* belong to $N - Y$ and have no neighbours in Y. Since G is $2K_2$-free, we conclude that

Claim 1. A is an independent set.

We also derive a number of other helpful claims.

Claim 2. $Z \cap X = \emptyset$ and $A - Z \subseteq X$.

Proof. Assume a vertex $z \in Z$ belongs to X. Then $X - \{z\}$ is a dominating set, because z does not dominate any vertex of A (since A is independent) and it is dominated by its neighbor in Y. This contradicts the minimality of X and proves that $Z \cap X = \emptyset$. Also, by definition, no vertex of $A - Z$ has a neighbour in $Y \cup \{a, b\}$. Therefore, to be dominated $A - Z$ must be included in X. □

Claim 3. If $|X| > \alpha(G)$, then $|Y| = |Z|$ and every vertex of Z is a private neighbor of a vertex in Y.

Proof. Since every vertex y in Y belongs to X and has a neighbour in X (a or b), by Lemma 2 y must have a private neighbor in Z. Therefore, $|Z| \geq |Y|$. If $|Z|$ is strictly greater than $|Y|$, then $|X| \leq |A \cup \{a\}| \leq \alpha(G)$ (since A is independent), which contradicts the assumption $|X| > \alpha(G)$. Therefore, $|Y| = |Z|$ and every vertex of Z is a private neighbor of a vertex in Y. □

Claim 4. If $|Y| > 1$ and $|X| > \alpha(G)$, then $Y \subseteq N(a) \cap N(b)$.

Proof. Let y_1, y_2 be two vertices in Y and let z_1, z_2 be two vertices in Z which are private neighbours of y_1 and y_2, respectively.

Assume a is not adjacent to y_1, then b is adjacent to y_1 (by definition of Y) and a^* is adjacent to z_1, since otherwise the vertices a, a^*, y_1, z_1 induce a $2K_2$ in G. Also, a^* is adjacent to z_2, since otherwise a $2K_2$ is induced by a^*, z_1, y_2, z_2. But now the vertices a^*, z_2, b, y_1 induce a $2K_2$. This contradiction shows that a is adjacent to y_1. Since y_1 has been chosen arbitrarily, a is adjacent to every vertex of Y, and by symmetry, b is adjacent to every vertex of Y. □

Claim 5. If $|Y| > 1$ and $|X| > \alpha(G)$, then a^* and b^* have no neighbours in Z.

Proof. Assume by contradiction that a^* is adjacent to a vertex $z_1 \in Z$. By Claim 3, z_1 is a private neighbour of a vertex $y_1 \in Y$. Since $|Y| > 1$, there exists another vertex $y_2 \in Y$ with a private neighbor $z_2 \in Z$. From Claim 4, we know that b is adjacent to y_2. But then the set $\{b, y_2, a^*, z_1\}$ induces a $2K_2$. This contradiction shows that a^* has no neighbours in Z. By symmetry, b^* has no neighbours in in Z. □

The above series of claims leads to the following conclusion, which plays a key role for the development of a polynomial-time algorithm.

Lemma 4. *If* $|X| > \alpha(G)$*, then* $|Y| = 1$ *and* $Y \subseteq N(a) \cap N(b)$.

Proof. First, we show that $|Y| \leq 1$. Assume to the contrary that $|Y| > 1$. By definition of a^* and Claim 2, vertex a^* has no neighbours in $A - Z$, and by Claim 5, a^* has no neighbours in Z. Therefore, $A \cup \{a^*, b\}$ is an independent set of size $|X| = |Y| + |A - Z| + 2$. This contradicts the assumption that $|X| > \alpha(G)$ and proves that $|Y| \leq 1$.

Suppose now that $|Y| = 0$. Then, by Claim 3, $|Z| = 0$ and hence, by Claim 2, $X = A \cup \{a, b\}$. Also, by definition of a^*, vertex a^* has no neighbours in A. But then $A \cup \{a^*, b\}$ is an independent set of size $|X|$, contradicting that $|X| > \alpha(G)$.

From the above discussion we know that Y consists of a single vertex, say y. It remains to show that y is adjacent to both a and b. By definition, y must be adjacent to at least one of them, say to a. Assume that y is not adjacent to b. By definition of a^*, vertex a^* has no neighbours in $\{y\} \cup (A - Z)$, and by definition of Z, vertex y has no neighbours in $A - Z$. But then $(A - Z) \cup \{a^*, b, y\}$ is an independent set of size $|X| = |Y| + |A - Z| + 2$. This contradicts the assumption that $|X| > \alpha(G)$ and shows that y is adjacent to both a and b. □

Corollary 1. *If a minimal dominating set in a* $2K_2$*-free graph* G *is larger than* $\alpha(G)$*, then it consists of a triangle and all the vertices not dominated by the triangle.*

In what follows, we describe an algorithm $\mathcal{A}$ to find a minimal dominating set M with maximum cardinality in a $2K_2$-free graph G in polynomial time. In the description of the algorithm, given a graph $G = (V, E)$ and a subset $U \subseteq V$, we denote by $A(U)$ the anti-neighbourhood of U, i.e. the subset of vertices of G outside of U none of which has a neighbour in U.

Algorithm $\mathcal{A}$

Input: A $2K_2$-free graph $G = (V, E)$.
Output: A minimal dominating set M in G with maximum cardinality.

1. Find a maximum independent set M in G.
2. For each triangle T in G:
   - Let $M' := T \cup A(T)$.
   - If M' is a minimal dominating set and $|M'| > |M|$, then $M := M'$.
3. Return M.

Theorem 3. *Algorithm $\mathcal{A}$ correctly solves the* UPPER DOMINATING SET *problem for $2K_2$-free graphs in polynomial time.*

Proof. Let G be a $2K_2$-free graph with n vertices. In $O(n^2)$ time, one can find a maximum independent set M in G (see e.g. [12]). Since M is also a minimal dominating set (see Lemma 1), any solution of size at most $\alpha(G)$ can be ignored.

If X is a solution of size more than $\alpha(G)$, then, by Corollary 1, it consists of a triangle T and its anti-neighbourhood $A(T)$. For each triangle T, verifying whether $T \cup A(T)$ is a minimal dominating set can be done in $O(n^2)$ time. Therefore, the overall time complexity of the algorithm can be estimated as $O(n^5)$. □

5 Main result

Theorem 4. *Let H be a graph. If H is a $2K_2$ or P_4 (or any induced subgraph of $2K_2$ or P_4), then the* UPPER DOMINATING SET *problem can be solved for H-free graphs in polynomial time. Otherwise the problem is NP-hard for H-free graphs.*

Proof. Assume H contains a cycle C_k, then the problem is NP-hard for H-free graphs

– either by Theorem 1 if $k \leq 5$, because in this case the class of H-free graphs contains all graphs of girth at least 6,
– or by Theorem 2 if $k \geq 6$, because in this case the class of H-free graphs contains the class of $\overline{K}_3$-free graphs and hence all complements of bipartite graphs.

Assume now that H is acyclic, i.e. a forest. If it contains a claw (a star whose center has degree 3), then the problem is NP-hard for H-free graphs by Theorem 2, because in this case the class of H-free graphs contains all $\overline{K}_3$-free graphs and hence all complements of bipartite graphs.

If H is a claw-free forest, then every connected component of H is a path. If H contains at least three connected components, then the class of H-free graphs contains all $\overline{K}_3$-free graphs, in which case the problem is NP-hard by Theorem 2. Assume H consists of two connected components P_k and P_t.

– If $k + t \geq 5$, then the class of H-free graphs contains all $\overline{K}_3$-free graphs and hence the problem is NP-hard by Theorem 2.
– If $k + t \leq 3$, then the class of H-free graphs is a subclass of P_4-free graphs and hence the problem can be solved in polynomial time in this class by Proposition 1.
– If $k + t = 4$, then
  - either $k = t = 2$, in which case $H = 2K_2$ and hence the problem can be solved in polynomial time by Theorem 3,
  - or $k = 4$ and $t = 0$, in which case $H = P_4$ and hence the problem can be solved in polynomial time by Proposition 1,
  - or $k = 3$ and $t = 1$, in which case the class of H-free graphs contains all $\overline{K}_3$-free graphs and hence the problem is NP-hard by Theorem 2.

□

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