

Multiuser Switched Diversity Scheduling Schemes

Mohammad Shaqfeh¹, Hussein Alnuweiri¹, and Mohamed-Slim Alouini²

¹ Texas A&M University at Qatar, Doha, Qatar.

Email: {Mohammad.Shaqfeh, Hussein.Alnuweiri}@qatar.tamu.edu

² King Abdullah University of Science and Technology (KAUST),

Thuwal, Mekkah Province, Saudi Arabia.

Email: slim.alouini@kaust.edu.sa

Abstract

Multiuser switched-diversity scheduling schemes were recently proposed in order to overcome the heavy feedback requirements of conventional opportunistic scheduling schemes by applying a threshold-based, distributed, and ordered scheduling mechanism. The main idea behind these schemes is that slight reduction in the prospected multiuser diversity gains is an acceptable trade-off for great savings in terms of required channel-state-information feedback messages. In this work, we characterize the achievable rate region of multiuser switched diversity systems and compare it with the rate region of full feedback multiuser diversity systems. We propose also a novel proportional fair multiuser switched-based scheduling scheme and we demonstrate that it can be optimized using a practical and distributed method to obtain the feedback thresholds. We finally demonstrate by numerical examples that switched-diversity scheduling schemes operate within 0.3 bits/sec/Hz from the ultimate network capacity of full feedback systems in Rayleigh fading conditions.

Index Terms

Opportunistic scheduling, reduced feedback, multiuser switched diversity, achievable rate region, proportional fairness.

I. INTRODUCTION

The concept of *multiuser diversity* (MUD) has been well studied in the literature, e.g. [1, Chapter 6], and exploited in the design of channel-aware “opportunistic” scheduling schemes that control in a dynamic

This paper was made possible by NPRP grant # 08-577-2-241 from the Qatar National Research Fund (a member of Qatar Foundation). The statements made herein are solely the responsibility of the authors.

This work was partially presented in 7th IEEE Broadband Wireless Access Workshop (BWA2011) in Houston, Texas, US.

way how the users access the shared air-link resources in wireless systems. This concept was originally initiated in [2] where it was shown that in order to maximize the sum capacity (bits/sec) of the network, we should always schedule the user with the best instantaneous channel quality. The design of opportunistic schedulers has been further studied in the literature taking into consideration key factors such as fairness among users and maintaining the quality-of-service (QoS) constraints, e.g. [3].

In virtually all modern wireless communication systems, explicit training sequences (i.e. pilot signals) are used to enable the receivers to measure and/or predict (e.g. [4]) the instantaneous channel conditions in order to use it in the coherent detection of the transmitted signals. Opportunistic schedulers that are capable of exploiting the full MUD gains are based on having continuously-updated channel state information (CSI) of all back-logged mobile users in the network at the central scheduler (i.e. at the base station). Thus, all mobile terminals inform the central scheduler about their CSI using explicit feedback messages. As a result, a considerable portion of the air-link resources and a significant share of the battery energy of the mobile terminals are used for the CSI feedback instead of useful data traffic. This fact has motivated many researchers to examine the feedback load of opportunistic scheduling schemes¹ and to search for alternative schemes which can trade off some of the MUD gains for considerable savings of the feedback load. In [6] and [7], extensive surveys on feedback reduction methods are provided. Note that the CSI feedback load is a common challenge in wireless communication systems [8]. At present there is no general theory of single or multiuser wireless feedback communication networks [7]. We can classify the solutions for the multiuser case into two main approaches: (i) compression of the CSI messages by using quantization methods or source coding techniques to exploit the channel correlation across the air-link resource units, and (ii) reduction of the feedback load by selectively choosing when to acquire a CSI feedback message based on its likelihood to be useful in obtaining MUD gains. The latter approach is generally more effective in reducing the feedback load significantly and it is less complex to implement.

Under the theme of reduced-feedback opportunistic scheduling, Holter *et al.* proposed the multiuser “switched-diversity” (MUSwiD) scheduling scheme [9]. The basic principle in MUSwiD scheduling schemes is to find *any acceptable user* (i.e. having good channel condition) instead of finding *the best user* among all. The term “multiuser switched diversity” was suggested in [9], because the proposed scheduling scheme has a similar principle of operation to the “switch-based” antenna selection scheme used long-time ago in multiple-antenna receivers [10]. It was suggested in [9] to use a scheduling strategy

¹Similar to other papers in the literature such as [5], we refer to the systems that are based on full CSI feedback as multiuser selection diversity (MUSelD) scheduling schemes.

based on *examining the CSI of the users sequentially instead of jointly*. Once a “good-channel” user is found, the process of examining the channel conditions terminates, and that user is scheduled. The decision whether the channel condition of a specific user is acceptable or not is assessed by a predefined threshold. After the pioneer work [9], several modifications and enhancements have been proposed in the literature (e.g. [11], [12], [13] and [14]). The state-of-the-art in this field are the recent works in [13] and [14] in which fundamental concepts were suggested to enhance the performance of the MUSwiD schemes; namely the per-user thresholds [13] and the post-user selection strategy [14]. In this paper, we basically build upon the per-user threshold approach adopted in [13].

The operation mode (i.e. protocol) of the MUSwiD scheduling schemes [13] is based on using a tiny-slotted feedback channel that is shared by all active users in the network. The shared feedback channel was called the guard period in [12], [13]. Each mini time-slot of the shared feedback channel can be used to send a 1-bit flag signal². Furthermore, each mini-slot can be firmly accessed by a single user. The users are ordered into a sequence and assigned access to the mini-slots of the shared feedback channel accordingly. Per-user channel state thresholds are used. After a pilot signal is detected and a channel measurement is done, each user compares its current channel condition with respect to its associated channel threshold. A user sends a flag signal in its associated mini time-slot if it has above threshold channel condition, and all users before it in the feedback sequence have not sent flag signals. The first user to send a flag signal is the scheduled user to access the next resource unit. If the system adopts adaptive modulation and coding transmission [15], the selected user sends a full CSI message after the 1-bit flag signal in order for the base station to adapt the transmission rate accordingly.

The feedback in MUSwiD systems is reduced significantly into only one feedback channel per resource unit instead of per-user feedback channels due to the distributed scheduling mechanism that makes the mobile terminals participate in the scheduling process by comparing their channel condition locally against a pre-defined threshold, and sending feedback flag signals using an ordered strategy which resolves contention. Another advantage of the system is that a user sends CSI feedback only ahead of the resource units that it will be allocated instead of sending feedback for all resource units, and this provides considerable savings in terms of battery life of mobile terminals.

Despite the evident feedback-reduction advantage of the state-of-the-art MUSwiD schemes, there are some fundamental technical challenges that should be addressed adequately before MUSwiD schemes can

²The time duration of the feedback channel is not long, and hence the MUSwiD scheduling scheme does not cause additional delay to the scheduling process.

lend themselves for practical implementation. In our opinion, there are mainly three technical challenges:

- **Fairness:** Maximizing the sum capacity is not always an appropriate optimization criterion for realistic network scenarios since users usually have asymmetric channel statistics. Furthermore, in MUSwiD schemes, the users' ordering strategy gives an advantage to the users who are placed in the first positions in the feedback sequence. It becomes likely that users placed in the latter positions of the sequence may not get channel access despite having very strong channel. So, is it possible to achieve fairness in MUSwiD schemes? and how? The current proposals in the topic (e.g. [14], [11]) suggest to keep changing the feedback sequence continuously in order to achieve fairness. We demonstrate in this paper that we can maintain fairness without this requirement.
- **Centralized optimization:** As discussed in [13], the optimization of the feedback thresholds in MUSwiD systems is done at the central scheduler and it requires the knowledge of the statistics (i.e. probability density functions (PDF)) of all users' channels. However, due to the CSI feedback reduction, the central scheduler will not be able to have accurate estimates of the PDFs of the users' channels. This will affect the optimality of the assigned per-user thresholds and will consequently degrade the system performance.
- **Capacity-feedback tradeoff:** A comparison of MUSwiD schemes with full-feedback (MUSelD) opportunistic scheduling schemes is needed to evaluate how much rate do we lose due to the feedback savings. Such analysis is not provided in the available literature.

In this paper we provide a comprehensive study to answer the aforementioned technical challenges. Furthermore, we aim in this work to persuade that MUSwiD scheduling systems are actually attractive options for practical implementation in emerging mobile broadband communication systems. Toward this end, we take the following steps; We provide detailed discussions to enhance our understanding about the attributes of the system and how to optimize its performance. In particular, we characterize the achievable rate region of MUSwiD systems. Also, we show that the achievable rates in MUSwiD systems are comparable with selection-based systems although they are significantly more economic in terms of CSI feedback load. Furthermore, we propose a novel MUSwiD scheduling scheme that achieves the proportional fairness criterion ([16], [17]), which is preferable for practical implementation [18]. We show that this can be achieved by proper per-user threshold optimization based on the objective function of maximizing the sum of the logarithms of the achievable rates. We demonstrate that our proposed scheme has a special interesting feature that the solution of the corresponding optimization problem

yields independent equations for each user, and hence the threshold optimization can be decentralized, which overcomes the centralized optimization challenge.

The remainder of this paper is organized as follows. We provide in Section II detailed discussion about the achievable rates using MUSwiD scheduling schemes and their optimization procedure. We, then, provide in Section III a motivation case study of the achievable rate region in a 2-user scenario. After that, we propose in Section IV a novel proportional fair MUSwiD scheduling scheme and we discuss its optimization procedure and demonstrate its practical advantages. Next, we provide in Section V several numerical examples to compare the performance of MUSwiD schemes with respect to full-feedback MUSelD scheduling schemes. Finally, we summarize the main conclusions in Section VI.

II. ACHIEVABLE RATES USING MUSWID SYSTEMS

A. System Model and General Assumptions

We consider the downlink³ in a single cell of a wireless communication system, and we consider best-effort services so that delay constraints are not taken into consideration in the scheduling decisions. The base station communicates with the users through wireless block-fading channels. We assume orthogonal access scheme in which the air-link resource units (i.e. the channel blocks) are slotted in time and possibly in frequency as well. One user only can be scheduled per resource unit. The time duration and the frequency bandwidth of one resource unit are assumed to be less than the coherence time and the coherence bandwidth of the fading channels so that the channels can be modeled as constant additive white Gaussian noise (AWGN) channels within one resource unit and varies randomly and independently from one resource unit to another. Furthermore, we assume that the base station transmits with constant power over all resource units.

Assume that we have a number M of active users in the network. The users are ordered according to a strategy π which is an injective (one-to-one) function. User i has the position $\pi(i)$ within the feedback sequence which defines the order by which the users can send flag signals to request being scheduled. For simplicity of notation, we assume that the users indices are consistent with their locations within the feedback sequence (i.e. $\pi(i) = i$). A user is scheduled if (i) its current channel condition is better than its associated channel threshold, and (ii) all users ahead of it in the feedback sequence have below-threshold

³The proposed scheduling schemes can be applied to the uplink as well based on the reciprocity of the uplink and downlink. The receiver (i.e. base station) transmits pilot signals prior to every resource block, and the users (i.e. mobile terminals) estimate the uplink CSI from their measurements of the downlink channel condition.

channel condition. In a mathematical context, the two conditions for a user i to be scheduled⁴ are:

$$r_i \geq r_i^* \quad (1a)$$

$$\mathbf{r} \in \mathcal{S}_i, \quad (1b)$$

where r_i is the achievable rate by user i , r_i^* is the channel threshold associated with user i , and $\mathbf{r} = [r_1 \ r_2 \ \dots \ r_M]$ is the vector of achievable rates of all users. The per-user thresholds were presented in terms of SNR in [13]. However, we prefer in this paper to present the thresholds in terms of achievable rates (i.e. channel capacity) because it is a more generic framework and enables extending the results into multiple antenna scenarios. The event \mathcal{S}_i in (1b) is defined as:

$$\mathcal{S}_i \doteq \{\mathbf{r} \in \mathcal{R}_+^M \mid r_j < r_j^* \quad \forall j < i\}. \quad (2)$$

Note that $\mathcal{S}_1 = \mathcal{R}_+^M$ and that $\mathcal{S}_M \subset \mathcal{S}_{M-1} \subset \dots \subset \mathcal{S}_1$. As an alternative mathematical representation, we can combine the two conditions in (1) into one expression $\mathbf{r} \in \mathcal{S}_i \setminus \mathcal{S}_{i+1}$. We assume that the fading processes of the users' channels are stationary, independent of each other and have continuous PDF of the achievable rates ($f_R(r)$). As such, we can write $f_{R_1 \dots R_M}(r_1, \dots, r_M) = \prod_{i=1}^M f_{R_i}(r_i)$.

In the numerical examples in the paper, we assume that the base station and the users' terminals are equipped with a single antenna to transmit/receive and thus the relation of the achievable rate – i.e. capacity – (denoted by r) and the SNR (denoted by γ) is given by the classical capacity relation of AWGN channels $r_i = \log(1 + \gamma_i)$. We can show using simple steps that the PDF of achievable rate $f_R(r)$ can be expressed in terms of the PDF of SNR $f_\Gamma(\gamma)$ in this case as $f_R(r) = \exp(r) \cdot f_\Gamma(\exp(r) - 1)$.

The extension to multiple antennas is straightforward as long as a single user only is served per resource unit. The per-user thresholds can still be presented using a single value instead of multiple SNR thresholds for every transmit/receive antenna pair. The appropriate capacity formulas should be used in the derivation of the PDF of the achievable rates in this case.

B. Statistical Analysis of the Users Expected Achievable Rates

The long-term expected (i.e. average) achievable rate by each user in MUSwiD systems was analyzed in [13] in terms of $f_{\Gamma_i}(\gamma_i)$. In this section, we briefly review these results in a variant representation

⁴The scheduling decision for a resource unit is based on the channel conditions of the users in this particular resource unit only and it is independent of the channel conditions in other resource units. Thus, in multi-carrier systems, the scheduling for each carrier (i.e. frequency band) is done independently.

using $f_{R_i}(r_i)$. We denote the conditional expected value of the achievable rate by user i given that (1b) is satisfied as R_i^c defined mathematically as

$$R_i^c \doteq E[r_i | \mathbf{r} \in \mathcal{S}_i] = \int_{r_i^*}^{\infty} r f_{R_i}(r) dr, \quad (3)$$

where $E[\cdot]$ is the expectation operator. Since we assume that the fading channels of the users are independent, the event $\mathbf{r} \in \mathcal{S}_i$ happens with probability $\Pr\{\mathbf{r} \in \mathcal{S}_i\} = \prod_{j < i} F_{R_j}(r_j^*)$. Furthermore, the unconditional expected value of the achievable rate by user i , denoted as R_i , equals

$$R_i \doteq E[r_i] = E[r_i | \mathbf{r} \in \mathcal{S}_i] \cdot \Pr\{\mathbf{r} \in \mathcal{S}_i\} = \int_{r_i^*}^{\infty} r f_{R_i}(r) dr \cdot \prod_{j < i} F_{R_j}(r_j^*) \quad (4)$$

Similarly, the expected percentage of resource units scheduled to user i (i.e. channel access ratio AR_i) is given as $\text{AR}_i = (1 - F_{R_i}(r_i^*)) \cdot \prod_{j < i} F_{R_j}(r_j^*)$. The expected achievable rates in single-input-single-output (SISO) channels can be presented equivalently in terms of SNR threshold values (γ_j^*) as [13]:

$$R_i = \int_{\gamma_i^*}^{\infty} f_{\Gamma_i}(\gamma) \log(1 + \gamma) d\gamma \cdot \prod_{j < i} F_{\Gamma_j}(\gamma_j^*). \quad (5)$$

C. Per-User Thresholds Optimization

From (4), it is clear that the system performance is dependent on (i) the chosen strategy to order the users in the feedback sequence and (ii) the channel thresholds r^* of all users. The channel threshold of one user does not only affect its achievable rate alone, but additionally all other users placed next in the feedback sequence. In this Section we discuss the joint optimization of the per-user thresholds. However, we assume first that the feedback sequence is fixed beforehand. We discuss in later sections the selection of the feedback sequence. In [13], the optimization of the per-user thresholds was derived with the objective of maximizing the aggregate (sum) capacity (achievable rate) of all users in the network. We first summarize these results in the context of this paper based on representing the per-user thresholds in terms of achievable rates. We then provide a generalized framework to optimize the per-user thresholds taking fairness into consideration.

The optimal per-user thresholds are obtained by solving the following optimization problem:

$$\{\hat{r}_1^*, \dots, \hat{r}_M^*\} = \arg \max_{\{r_1^*, \dots, r_M^*\}} \Phi, \quad (6)$$

where we use the notation \hat{r}_i^* to denote the optimal value for the threshold r_i^* under the objective function

Φ . In the special case of maximizing the sum achievable rate, Φ is defined as

$$\Phi = \sum_{i=1}^M R_i, \quad (7)$$

where the expected achievable rates R_i follow (4). In order to solve (6) with (7), we search at the points at which the gradient equals zero (i.e. the stationary points):

$$\frac{\partial \Phi}{\partial r_i^*} = 0, \quad \forall i \leq M. \quad (8)$$

The derivative $\frac{\partial R_j}{\partial r_i^*}$ is obtained as follows:

$$\frac{\partial R_j}{\partial r_i^*} = \begin{cases} 0 & : i > j \\ -r_i^* f_{R_i}(r_i^*) \prod_{k<i} F_{R_k}(r_k^*) & : i = j \\ \frac{R_j}{F_{R_i}(r_i^*)} f_{R_i}(r_i^*) & : i < j \end{cases} \quad (9)$$

The derivative of R_i with respect to r_i^* (second line in (9)) is obtained by applying the first fundamental theorem of calculus (e.g. [19]). We can alternatively write $\frac{\partial R_j}{\partial r_i^*} : i < j$ as:

$$\frac{\partial R_j}{\partial r_i^*} = R_j^c f_{R_i}(r_i^*) \prod_{k<j, k \neq i} F_{R_k}(r_k^*) \quad : i < j. \quad (10)$$

By inserting (9) into (8), we obtain $\hat{r}_i^* f_{R_i}(\hat{r}_i^*) \prod_{k<i} F_{R_k}(\hat{r}_k^*) = \frac{f_{R_i}(\hat{r}_i^*)}{F_{R_i}(\hat{r}_i^*)} \sum_{j>i}^M R_j$, yielding

$$\hat{r}_i^* = \frac{\sum_{j>i}^M R_j}{\prod_{k<i+1} F_{R_k}(\hat{r}_k^*)}, \quad (11)$$

where the assumptions ($r_i^* \neq 0, \forall i < M$) and ($f_{R_i}(r) \neq 0, : r > 0$) are used. We can re-write (11) as

$$\hat{r}_i^* = E \left[\Phi | \mathbf{r} \in \mathcal{S}_{i+1} \text{ and } r_j^* = \hat{r}_j^* \quad \forall j > i \right]. \quad (12)$$

From (12) and using a simple intuitive explanation, we can describe the basic principle for optimizing the per-user thresholds in switched diversity systems as trying to maximize the outcome (which is the achievable rate in our case) of a random experiment (which is examining the channel condition, i.e. achievable rate, of one user in our case) with the possibility to repeat the experiment up to a limited number of trials (which is the total number of users in our case). After the experiment is executed once and its output is observed, we can either choose to accept its outcome and stop repeating the experiment, or opt to repeat the random experiment taking into consideration that we will lose the output that is

already observed and the expected output of the new trial of the experiment will be totally independent of the previous ones⁵. As an intuitive guideline to the decision making of choosing whether to repeat the experiment or to accept the observed output (which corresponds to the decision to send a flag signal by the corresponding user in our case), we will decide not to repeat the experiment if the observed output is very good so that we do not expect to obtain such a good output if we repeat the experiment. Similarly, we will decide to repeat the experiment if the observed output is low so that we expect that most likely we will obtain a better result by repeating the experiment. The optimal solution to this decision making problem is that we compare the observed output with the expected value for the outcome of the allowed number of trials to repeat the experiment. If the current outcome is higher than the expected value for repeating the experiment, we accept it and stop repeating the experiment and vice versa.

We can write (11) alternatively as

$$\hat{r}_i^* = \sum_{j>i} \left[R_j^c \prod_{i<k<j} F_{R_k}(\hat{r}_k^*) \right]. \quad (13)$$

We can see that the optimal threshold of each user depends on the optimal thresholds of all users that are placed after it according to the feedback sequence. Thus, the per-user thresholds can be obtained using a backward successive approach starting from the last user in the sequence. Note that it is intuitive to predict that the threshold of the last user in the sequence is zero since we do not apply a post-user selection strategy [14] or power control.

$$r_M^* = 0 \quad (14)$$

Furthermore, by using some mathematical manipulations [13], we can use the following formula for the backward successive approach for obtaining the per-user thresholds:

$$\hat{r}_i^* = \int_{r_{i+1}^*}^{\infty} r f_{R_{i+1}}(r) dr + r_{i+1}^* F_{R_{i+1}}(r_{i+1}^*). \quad (15)$$

Also, we can show by simple mathematical manipulations similar to (12) and (15) that the maximum achievable sum rate is:

$$\max \Phi = \int_{r_1^*}^{\infty} r f_{R_1}(r) dr + r_1^* F_{R_1}(r_1^*), \quad (16)$$

where Φ is the sum achievable rate (7).

⁵Note that (12) is invalid in case of post-user selection [14] since the last trial (i.e. post-user selection) is dependent on another trial (the one related to the post-selected user). Thus, the optimization of MUSwiD systems with post-user selection is not as straightforward as in the case of MUSwiD systems without post-user selection.

As well-known, maximizing the sum achievable rate is not always a suitable optimization criterion for multiuser networks since it creates fairness problems. Motivated by this fact, we provide here a more generic framework to optimize the performance of MUSwiD systems. From an information-theoretic point of view (e.g. [20], [21]), the objective in multiuser channels is to operate at the boundary surface of the achievable-rate region. The points on the boundary surface are Pareto-optimal, which means that we cannot increase the achievable rate of one user without decreasing the achievable rate of another user. The objective of scheduling schemes in multiuser networks should be to achieve Pareto-optimality⁶. The points on the boundary surface of the achievable rate region are obtained by maximizing a weighted sum of the rates. By varying the weights we can scan all points on the boundary surface. Thus, we propose to use a weighted sum of the achievable rate as the optimization objective for (6):

$$\Phi = \sum_{i=1}^M \mu_i R_i. \quad (17)$$

Note that another common approach is to maximize the sum of concave and monotonically increasing utility functions of the achievable rates of the users $\Phi = \sum_{i=1}^M U(R_i)$. As discussed in [3], this is interlinked with the objective of maximizing a weighted sum of the rates by using $\mu_i = U'(R_i)$.

By repeating the same procedure used for maximizing the sum achievable rates case, we obtain the following results for optimizing the per-user thresholds with the objective of maximizing a weighted sum of the achievable rates. Equations (11), (13) and (15) are replaced by (18), (19) and (20) respectively.

$$\mu_i \hat{r}_i^* = \frac{\sum_{j>i}^M \mu_j R_j}{\prod_{k<i+1} F_{R_k}(\hat{r}_k^*)}, \quad (18)$$

$$\mu_i \hat{r}_i^* = \sum_{j>i} \left[\mu_j R_j^c \prod_{i<k<j} F_{R_k}(\hat{r}_k^*) \right], \quad (19)$$

$$\mu_i \hat{r}_i^* = \mu_{i+1} \left[\int_{\hat{r}_{i+1}^*}^{\infty} r f_{R_{i+1}}(r) dr + \hat{r}_{i+1}^* F_{R_{i+1}}(\hat{r}_{i+1}^*) \right]. \quad (20)$$

⁶As discussed in [22], there is no contradiction between the two objectives of (i) efficient resource allocation by designing scheduling schemes leading to operating at the points on the boundary surface of the achievable rate region, and (ii) achieving fairness among the users as well as maintaining the QoS requirements, which can be done by controlling the operating point of the system based on proper selection of $\boldsymbol{\mu}$ (the vector of the users' weighting factors). The specific selection of $\boldsymbol{\mu}$ to meet fairness requirements or QoS constraints is a different topic that is not specific to this work on MUSwiD schedulers. Few examples of the many possible approaches suggested in the literature to select the specific operating point of the system are (i) the fairness-based approach, such as the proportional fairness scheduler [17] and the flexible resource-sharing constraints scheduler [23], (ii) the utility-maximization-based approach [24], and (iii) the QoS constraints based approach [3].

Note that (14) is also valid for the generic case of maximizing a weighted sum of the achievable rates (17). Equation (12) is replaced by:

$$\mu_i \hat{r}_i^* = E \left[\Phi | \mathbf{r} \in \mathcal{S}_{i+1} \text{ and } r_j^* = \hat{r}_j^* \quad \forall j > i \right]. \quad (21)$$

The maximum weighted sum of the achievable rates can be expressed as

$$\max \Phi = \mu_1 \left[\int_{\hat{r}_1^*}^{\infty} r f_{R_1}(r) dr + \hat{r}_1^* F_{R_1}(\hat{r}_1^*) \right], \quad (22)$$

where Φ is defined in (17).

For SISO channels, the optimal per-user thresholds in terms of SNR are computed according to

$$\mu_i \log(1 + \hat{\gamma}_i^*) = \mu_{i+1} \left[\int_{\hat{\gamma}_{i+1}^*}^{\infty} f_{\Gamma_{i+1}}(\gamma) \log(1 + \gamma) d\gamma + \log(1 + \hat{\gamma}_{i+1}^*) F_{\Gamma_{i+1}}(\hat{\gamma}_{i+1}^*) \right], \quad (23)$$

which is done in a backward successive approach starting with $\hat{\gamma}_M^* = 0$.

In the numerical examples used in this paper, we assume SISO Rayleigh block-faded channels. We show in Table I the closed-form formulas to characterize the performance of the system and the optimization of the thresholds. In order to obtain simple closed-form expressions, the formulas are presented in terms of the SNR-based thresholds $\hat{\gamma}_i^*$.

III. MOTIVATION CASE STUDY – ACHIEVABLE RATE REGION IN 2 USER SCENARIO

Studying the achievable rate region in 2-user scenario is a useful tool in order to get basic insights regarding the performance limits of the system and the tradeoff between maximizing the sum capacity and maintaining fairness among the users. In order to characterize the achievable rate region in MUSwiD schemes, we solve (6) with the objective function (17) for different values of the weighting factors μ , ranging from $(\mu_1 = 1, \mu_2 = 0)$ to $(\mu_1 = 0, \mu_2 = 1)$.

In the numerical example of 2-user scenario shown in Fig. 1, we assume that both users as well as the base station are equipped with single antennas. Furthermore, we assume that both users have Rayleigh block-fading channels but with different expected average values. Table I summarizes the main formulas under these particular assumptions. We show in Fig. 1 the achievable rate region for the two possible feedback sequences. In the first case, the user with better average SNR is placed in the first position of the sequence. While in the second case, the user with lower average SNR is placed first. Furthermore, we compare with the achievable rate region of the full feedback selection-based MUSelD scheme, which

is known from the literature (e.g. [25], [22]). A summary of the formulas to characterize the achievable rates in MUSelD schemes is provided in Section V-A. We show in Fig. 1 some special cases including the maximum sum rate and the proportional fairness operating points. Detailed discussion about proportional fair MUSwiD scheduling is provided in Section IV. The main conclusions obtained from this motivation case study carry over to the general case of M users. We summarize below the key learnt messages.

The achievable rates by MUSwiD scheduling schemes are always close to the achievable rates with full feedback MUSelD scheduling schemes. The little loss in the achievable rates is an acceptable trade-off for evident reductions in the CSI feedback load. Also, the maximum sum rate is an unfair operating point in MUSelD scheme as well as in MUSwiD schemes. Changing the feedback sequence in MUSwiD systems, while optimizing the thresholds to maximize the sum rate, does not solve the fairness issues. Furthermore, unlike the common belief in early works in MUSwiD schemes such as [14] and [11], Fig. 1 demonstrates that we can actually achieve fairness in MUSwiD schemes without the need of alternating between feedback sequences. However, the per-user channel thresholds should be adjusted properly to allow achieving fairness. Also, we can achieve fairness regardless of the used feedback sequence.

We observe also that alternating between feedback sequences can in some cases (the line BC in Fig. 1) be the optimal solution. However, the optimization of a MUSwiD scheduler including alternating between the feedback sequences as a degree of freedom is not simple and require complex algorithms with significant computation load in order to find the optimal per-user thresholds for each used sequence as well as the average time percentage of each used sequence since some sequences may need to be used more frequently than others. Furthermore, the real-time implementation of MUSwiD schedulers with the option of mixing up between different feedback sequences adds more control messages communication since the base station should inform the users about all used feedback sequences and their associated per-user thresholds. On the other hand, Fig. 1 demonstrates that it is almost sufficient to use one sequence to operate on or close to the achievable rate region limits. Furthermore, it provides a practical scheduler design with low computation complexity and feasible implementation procedure. The loss in terms of performance will be void for most operating points and negligible for some ranges of Pareto-optimal operating points.

Finally, we observe that choosing the proper feedback sequence is important in order to operate at the boundary of the achievable rate region. However, for a number M of users, we have a number $M!$ of possible feedback sequences. Thus, even for a relatively small number of users, comparing the

performance for all possible feedback sequences in order to find the optimal one is computationally expensive. To simplify this task, we propose instead a very simple rule based on Fig. 1 and the numerical results in Section V-C. This rule is that when the objective is to maximize the sum achievable rate in the network, we should use a feedback sequence in which the users are sorted in descending order of their expected channel condition. On the other hand, when fairness is taken into consideration, we should use a feedback sequence in which the users are sorted in ascending order of their expected channel condition.

IV. PROPOSED SCHEME – PROPORTIONAL FAIR SCHEDULER

Proportional fairness [16] is a well-known fairness criterion that provides a good trade-off between the aggregate rate over the network and fairness among users. Proportional fairness resolves this conflict by allocating to each user a transmission rate relative to its channel condition without affecting the rates of other users. Proportional fairness was suggested for full-feedback MUSelD scheduling schemes in [17], and it was applied in industry such as in the IS-856 standard [18]. In this paper, we propose to apply proportional fairness into MUSwiD scheduling schemes.

The optimization objective function Φ in case of proportional fairness is to maximize the product of the expected achievable rates of the users $\prod_{i=1}^M R_i$, or equivalently, to maximize the sum of the logarithms of the expected achievable rates:

$$\Phi = \sum_{i=1}^M \log(R_i). \quad (29)$$

In order to optimize the per-user thresholds to achieve proportional fairness, we solve (6) with the objective function (29). We find the points at which the gradient equals zero, yielding

$$\frac{\partial \Phi}{\partial r_i^*} = \sum_{j=1}^M \frac{\partial \log(R_j)}{\partial r_i^*} = \sum_{j=1}^M \frac{\frac{\partial R_j}{\partial r_i^*}}{R_j} = 0, \quad \forall i \leq M, \quad (30)$$

where $\frac{\partial R_j}{\partial r_i^*}$ is obtained in (9). By solving (30) we obtain:

$$\frac{\hat{r}_i^* f_{R_i}(\hat{r}_i^*) \prod_{k<i} F_{R_k}(\hat{r}_k^*)}{R_i^c \prod_{k<i} F_{R_k}(\hat{r}_k^*)} = \sum_{j>i}^M \frac{f_{R_i}(\hat{r}_i^*)}{F_{R_i}(\hat{r}_i^*)}. \quad (31)$$

We can simplify (31) as $\frac{\hat{r}_i^* f_{R_i}(\hat{r}_i^*)}{R_i^c} \frac{\prod_{k<i} F_{R_k}(\hat{r}_k^*)}{\prod_{k<i} F_{R_k}(\hat{r}_k^*)} = \frac{f_{R_i}(\hat{r}_i^*)}{F_{R_i}(\hat{r}_i^*)} (M - i)$. With the assumptions that $(r_i^* \neq 0, \forall i < M)$ and $(f_{R_i}(r) \neq 0, : r > 0)$ and by substituting for R_i^c using (3) we obtain

$$\frac{\hat{r}_i^* F_{R_i}(\hat{r}_i^*)}{\int_{\hat{r}_i^*}^{\infty} r f_{R_i}(r) dr} = M - i \quad (32)$$

We can observe from (32) that the optimization of the proportional fair scheduler has a very interesting and unique feature. The optimal achievable rate threshold of any user is only dependent on its channel statistics alone and its location (index) within the feedback sequence. Thus, we can optimize the system by solving M independent equations instead of solving dependent equations successively as in the general case of MUSwiD scheduling schemes which was discussed in Section II-C. Among all Pareto-optimal operating points, the independent equations feature is uniquely valid in the case of the proportional fair operating point. This feature has a significant advantage from practical implantation perspective because it enables every user to obtain its optimal threshold value locally. This overcomes the technical challenge of centralized threshold optimization of conventional MUSwiD schemes since every user can have accurate prediction of its channel statistics while the base station cannot have such accurate measures of the PDFs of the users' channels without explicit feedback from all users. This feature is compatible with the main theme of MUSwiD schemes, which is to limit the feedback load.

Note that the optimization of the proportional fair scheduler (32) is consistent with the optimization procedure of the generic scheduling criteria of maximizing a weighted sum of the achievable rates discussed in Section II-C. In the case of proportional fairness, the weighting factor of each user is inversely proportional with its expected achievable rate [17]. By substituting $\mu_i^{\text{PF}} = \frac{1}{R_i}$ into (18) we obtain (32).

In the case of SISO channels, We can alternatively present (32) in terms of SNR thresholds as:

$$\frac{\log(1 + \hat{\gamma}_i^*) F_{\Gamma_i}(\hat{\gamma}_i^*)}{\int_{\hat{\gamma}_i^*}^{\infty} f_{\Gamma_i}(\gamma) \log(1 + \gamma) d\gamma} = M - i \quad (33)$$

The left hand side of equation (32) is a monotonically increasing function of \hat{r}_i^* . Thus, the solution of (32) always exists and it is unique. The solution can be obtained using simple numerical methods such as the bisection method. Alternatively, the results can be obtained for standard channel models and stored in the mobile terminals using look-up tables versus the user index within the feedback sequence. Fig. 2 and Fig. 3 show the optimal per-users thresholds in the proportional fair scheduler in terms of achievable rates and SNR respectively for SISO Rayleigh block-faded channels. The per-user thresholds in both figures are normalized with respect to the average achievable rate and average SNR respectively.

We observe from Fig. 2 and Fig. 3 that as the number of next users in the sequence increases (meaning being placed in the first positions in the sequence), the corresponding per-user threshold increases. Thus, the users in the first places of the sequence are requested to achieve high rates (MUD gains) with low expected success ratio, while the users at the last places is expected to achieve lower rate gains but with

higher success ratio. It is intuitive to predict that placing the users who have wider dynamic range of channel variations in the first positions of the sequence is advantageous because these users are more capable of achieving high rate gains when their channel condition is at its peak. Thus, it is better in the feedback sequence to sort the users in ascending order of their expected (average) SNR. This is due to the fact that at low SNR, the achievable rate formula $r = \log(1 + \gamma)$ becomes almost linear and consequently more sensitive to the variations in SNR. Fig. 4 shows the PDF of the normalized achievable rates for SISO Rayleigh block-fading channels with different average values. We can see from Fig. 4 that at high average SNR ($\bar{\gamma} = 20\text{dB}$), the user can get a maximum gain of around 50% of the achievable rate when the channel condition is at its peak. On the other hand, at low SNR ($\bar{\gamma} = -10\text{dB}$), the achievable rate at peak channel conditions can exceed four times its average value. The intuitive suggestion of sorting the users in ascending order of their expected SNR is also supported by the numerical examples shown in Section V-C.

In order to solve (32) numerically at the mobile terminal, $f_R(r)$ should be estimated from the continuously measured channel conditions (after every pilot signal transmitted by the base station). The practical implementation steps of channel statistics (PDF) estimators is out of the scope of this work and was discussed in the signal processing literature. As an example, the PDF estimation using order statistic filter bank was suggested in [26].

A major concern in distributed systems in general is the effect of ill-behaving mobile terminals. In our proposed proportional fair MUSwiD scheme, the users obtain their thresholds locally. However, if one mobile terminal uses lower threshold than its correct threshold, the performance of all next users in the sequence will be affected and degraded. We demonstrate here that it is possible to assign a monitoring task to the base station in order to detect ill-behaving users without the need of knowing the channel PDF of every user. The suggested centralized monitoring mechanism works as follows; The users compute their channel thresholds locally and update the main scheduler at the base station about their thresholds. This does not produce significant feedback load as the threshold values are re-computed only after sound variations in the channel PDF. The base station makes sure that the requested rates by the scheduled users are above their thresholds, and it tracks two quantities (measures) for each mobile user that are updated in real-time whenever the user has the opportunity to request transmission (i.e. condition (1b)): (i) R_i^c : the average requested rate of user i when condition (1b) is valid, (ii) P_i : the success ratio of exceeding the channel threshold when condition (1b) is valid. The base station makes sure that the measured quantities

are consistent with (32).

In a mathematical context, R_i^c is defined in (3) and P_i is defined as $P_i = \Pr\{r_i \geq r_i^* | \mathbf{r} \in \mathcal{S}_i\}$. Note that tracking R_i^c and P_i does not require any additional feedback load. The base station can detect a wrongly used threshold if the following condition is true:

$$\left| \frac{r_i^* (1 - P_i)}{R_i^c} - (M - i) \right| > \epsilon, \quad (34)$$

where ϵ is the tolerance value for the accuracy of achieving condition (32).

V. PERFORMANCE ANALYSIS – COMPARISON WITH FULL-FEEDBACK SCHEMES

We provide different numerical examples to compare the performance of MUSwiD scheduling schemes with the performance of full-feedback MUSelD scheduling schemes. We briefly summarize the achievable rates using MUSelD schemes which were studied in the literature such as in [5] and [25].

A. Review of Achievable Rates of Full-Feedback Selection-Based Systems

In MUSelD scheduling schemes, the users continuously update the centralized scheduler at the base station about their instantaneous achievable rates r_i , and the scheduler chooses the user that maximizes the scheduler metric. There are many scheduling metrics suggested in the literature. A survey and comparison between different schemes is provided in [22]. In a generic form that enables achieving all Pareto-optimal points, the scheduling criterion is to select a user m with maximum weighted rate metric [3], i.e. $m = \arg \max_i \mu_i r_i$, where μ_i is a weighting factor assigned to user i . In full feedback MUSelD scheduling, the expected achievable rates in terms of $(f_{\Gamma}(\gamma))$ is known from the literature (e.g. [22], [25]):

$$R_i = \int_0^{\infty} f_{\Gamma_i}(\gamma) \cdot \prod_{j \neq i} F_{\Gamma_j} \left((1 + \gamma)^{\frac{\mu_i}{\mu_j}} - 1 \right) \cdot \log(1 + \gamma) d\gamma. \quad (35)$$

The average channel access ratio (percentage of being scheduled) is:

$$\text{AR}_i = \int_0^{\infty} f_{\Gamma_i}(\gamma) \cdot \prod_{j \neq i} F_{\Gamma_j} \left((1 + \gamma)^{\frac{\mu_i}{\mu_j}} - 1 \right) d\gamma. \quad (36)$$

We present (35) and (36) in an alternative form using the PDF of the achievable rates $f_R(r)$:

$$R_i = \int_0^{\infty} r f_{R_i}(r) \prod_{j \neq i} F_{R_j} \left(\frac{\mu_i r}{\mu_j} \right) dr, \quad (37)$$

$$\text{AR}_i = \int_0^\infty f_{R_i}(r) \prod_{j \neq i} F_{R_j} \left(\frac{\mu_i r}{\mu_j} \right) dr. \quad (38)$$

B. Network Models and Fairness Measures

We compare MUSwiD and MUSelD schemes using different network scenarios (in terms of the distribution of the expected channel conditions of the users) and for different number of users. We analyze the case of independent and identically distributed (i.i.d.) Rayleigh block-faded channels as well the case of independent and non-identically distributed Rayleigh channels which is more realistic from practical perspective. We provide numerical examples for the asymmetric channel distribution case using two models:

$$\text{Model 1: } \bar{\gamma}_i = \gamma_{\min} + (2i - 1) \cdot \frac{\gamma_{\max} - \gamma_{\min}}{2M} \quad (39a)$$

$$\text{Model 2: } \bar{\gamma}_i = \left[\sqrt{\gamma_{\min}} + \frac{2i - 1}{2M} \cdot (\sqrt{\gamma_{\max}} - \sqrt{\gamma_{\min}}) \right]^2, \quad (39b)$$

where γ_{\max} and γ_{\min} in (39a) and (39b) define respectively the upper and lower limits for the average SNR in the network. We used in our numerical results 20 dB and 0 dB respectively.

We compare two variants of the scheduling criteria: (i) the maximum sum achievable rate, and (ii) our proposed proportional fair scheduler. We use two performance measures in our comparisons: (i) the sum achievable rate in the network, and (ii) the degree of fairness (DOF) among the users. There are several fairness measures suggested in the literature. We opt in this work to use the well-known Jain's fairness index [27]:

$$\text{DOF} \equiv \frac{\left(\sum_{i=1}^M x_i \right)^2}{M \sum_{i=1}^M x_i^2}, \quad (40)$$

where x_i is a user-related metric. In our numerical examples we used two metrics for x_i :

- Resource sharing fairness: x_i is selected to be the expected channel access ratio AR_i .
- Multiuser diversity gains fairness: we propose the following metric as well for the fairness measure:

$$x_i \equiv \frac{R_i}{\int_0^\infty r f_{R_i}(r) dr}, \quad (41)$$

where R_i is the achievable rate of user i according to the applied scheduling scheme.

C. Numerical Results

Fig. 5 shows the comparison between MUSwiD and MUSelD schemes under i.i.d. Rayleigh block-fading conditions for different values of the identical average SNR of the users. The feedback sequence of the MUSwiD scheme is irrelevant in this case as the channels are identical. The maximum sum achievable rates are used for the comparison. The sum capacity⁷ were computed using (28) for the MUSwiD scheme where the per-user thresholds optimization follows (27). Fig. 5 shows that switched-diversity scheduling schemes operate within 0.3 bits/sec/Hz from the ultimate network capacity of full feedback systems in Rayleigh block-fading conditions over wide range of SNR and for any number of users. This rate loss is compensated by significant savings in the CSI feedback load. At high SNR conditions, the ratio between the sum capacity (i.e. achievable rates) of MUSwiD schemes with respect to the sum capacity of MUSelD schemes decreases as shown in Fig. 6. Fig 7 and Fig. 8 show a comparison in terms of sum capacity and degree of fairness under asymmetric channel conditions according to (39a). Both maximum sum capacity and proportional fairness are used in the comparison. We used the derived analytical formulas in this paper to calculate the achievable rates for MUSwiD schemes. Another numerical example is provided in Fig. 9 and Fig. 10 for the proportional fair scheduler under the assumption of asymmetric channel distribution according to (39b). The fairness results in Fig. 8 are based on using $x_i = AR_i$ in (40), while (41) is used for the fairness measure in Fig. 10.

The results in this section support the key messages learnt by studying the achievable rate region in Section III and demonstrate that they are valid for higher number of users. The performance of switched diversity is always within 0.3 bits/sec/Hz from the ultimate performance of full feedback schedulers. This is true for both maximum sum capacity and proportional fairness. The proportional fair scheduler provides very high degree of fairness regardless of the used feedback sequence. The differences in fairness measures of different feedback sequences are negligible. The assessment of the performance of the sequence strategies is better judged based on achievable rates which demonstrates that sorting the users in a descending average SNR order is better for maximizing the sum achievable rate, while the opposite sequence is better for the proportional fair scheduler. These results are consistent with our results in section III and with our perceptive analysis in section IV.

⁷In the special case of i.i.d. Rayleigh block-fading channels with identical average SNR, the maximum sum capacity of the MUSelD can be computed as [25]:

$$\sum_{i=1}^M R_i = \sum_{i=1}^M (-1)^{(i-1)} \binom{M}{i} \exp\left(\frac{i}{\bar{\gamma}}\right) E_1\left(\frac{i}{\bar{\gamma}}\right),$$

where E_1 is the exponential integral function (25).

VI. CONCLUSIONS

In this paper, we have proposed novel reduced-feedback scheduling schemes that provides significant reduction of channel state information feedback load at the cost of slight reduction in the achievable multiuser diversity gains. Our proposed schemes are based on the concept of multiuser switched diversity that has been recently introduced in the literature. We have provided rigorous mathematical treatment to analyze the performance of switched diversity scheduling schemes as well as to optimize their performance. We have also characterized the achievable rate region of these scheduling schemes and provided a case study to understand their main attributes and useful design options. We proposed a proportional fair scheduler that overcomes major technical challenges of the state-of-the-art proposals in the field. Mainly, our proposed scheduler maintains fairness among users and interestingly enables simpler optimization procedure. we have demonstrated that, unlike other schedulers, the optimization procedure of our proposed proportional fair scheduler can be distributed among the users. We have shown that the distributed optimization mechanism can be supported by a monitoring mechanism of the base station that enables the detection of ill-behaving users based on real-time performance measurements. Due to their features and performance, multiuser switched diversity scheduling systems are actually attractive options for practical implementation in emerging mobile broadband communication systems.

REFERENCES

- [1] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*, Cambridge University Press, May 2005.
- [2] R. Knopp and P. A. Humblet, "Information capacity and power control in single-cell multiuser communications," in *Proceedings IEEE International Conference on Communications (ICC)*, Seattle, WA, USA, June 1995, pp. 331–335.
- [3] X. Wang, G. Giannakis, and A. Marques, "A unified approach to QoS-guaranteed scheduling for channel-adaptive wireless networks," *Proceedings of the IEEE*, vol. 95, no. 12, pp. 2410–2431, Dec. 2007.
- [4] A. Dual-Hallen, "Fading channel prediction for mobile radio adaptive transmission systems," *Proceedings of the IEEE*, vol. 95, no. 12, pp. 2299–2313, Dec. 2007.
- [5] L. Yang and M.-S. Alouini, "Performance analysis of multiuser selection diversity," *IEEE Transactions on Vehicular Technology*, vol. 55, no. 6, pp. 1848–1861, Nov. 2006.
- [6] T. Eriksson and T. Ottosson, "Compression of feedback for adaptive transmission and scheduling," *Proceedings of the IEEE*, vol. 95, no. 12, pp. 2314–2321, Dec. 2007.
- [7] D. Love, R. Heath, V. Lau, D. Gesbert, B. Rao, and M. Andrews, "An overview of limited feedback in wireless communication systems," *IEEE Journal on Selected Areas in Communications*, vol. 26, no. 8, pp. 1341–1365, Oct. 2008.
- [8] IEEE Journal on Selected Areas in Communications, *Special Issue on Exploiting Limited Feedback in Tomorrows Wireless Communication Networks*, vol. 26, no. 8, October 2008.

- [9] B. Holter, M.-S. Alouini, G. E. Oien, and H.-C. Yang, "Multiuser switched diversity transmission," in *Proceedings IEEE Vehicular Technology Conference (VTC)*, Los Angeles, C.A., Sept. 2004, vol. 3, pp. 2038–2043.
- [10] W. Shortall, "A switched diversity receiving system for mobile radio," *IEEE Transactions on Communications*, vol. 21, no. 7, pp. 1269–1275, Nov. 1973.
- [11] Y. Al-Harathi, A. Tewfik, and M.-S. Alouini, "Multiuser diversity with quantized feedback," *IEEE Transactions on Wireless Communications*, vol. 6, no. 1, pp. 330–337, Jan. 2007.
- [12] H. Nam, Y. Ko, and M.-S. Alouini, "Performance analysis of joint switched diversity and adaptive modulation," *IEEE Transactions on Wireless Communications*, vol. 7, no. 10, pp. 3780–3790, Oct. 2008.
- [13] H. Nam and M.-S. Alouini, "Multiuser switched diversity scheduling systems with per-user threshold," *IEEE Transactions on Communications*, vol. 58, no. 5, pp. 1321–1326, May 2010.
- [14] H. Nam and M.-S. Alouini, "Multiuser switched diversity scheduling systems with per-user threshold and post-user selection," in *Proceedings IEEE International Symposium on Information Theory (ISIT)*, Austin, Texas, U.S.A., June 2010, pp. 2173–2177.
- [15] Proceedings of the IEEE, *Special Issue on Adaptive Modulation and Transmission in Wireless Systems*, vol. 95, no. 12, December 2007.
- [16] F. Kelly, "Charging and rate control for elastic traffic," Available at <http://www.statslab.cam.ac.uk/frank/elastic.pdf>. This is a (corrected version) of a paper that appeared in *European Transactions on Telecommunications*, volume 8 (1997) pages 33-37.
- [17] P. Viswanath, D. Tse, and R. Laroia, "Opportunistic beamforming using dumb antennas," *IEEE Transactions on Information Theory*, vol. 48, no. 6, pp. 1277–1294, June 2002.
- [18] P. Bender, P. Black, M. Grob, R. Padovani, N. Sindhusayana, and A. Viterbi, "CDMA/HDR: A bandwidth-efficient high-speed wireless data service for nomadic users," *IEEE Communications Magazine*, vol. 38, no. 7, pp. 70–77, July 2000.
- [19] T. Apostol, *Calculus, Vol. 1: One-Variable Calculus with an Introduction to Linear Algebra*, Wiley, second edition, June 1967.
- [20] D. Tse and S. Hanly, "Multiaccess fading channels – Part 1: Polymatroid structure, optimal resource allocation and throughput capacities," *IEEE Transactions on Information Theory*, vol. 44, no. 7, pp. 2796–2815, Nov. 1998.
- [21] L. Li and A. Goldsmith, "Capacity and optimal resource allocation for fading broadcast channels – Part 1: Ergodic capacity," *IEEE Transactions on Information Theory*, vol. 47, no. 3, pp. 1083–1102, Mar. 2001.
- [22] M. Shaqfeh and N. Goertz, "Performance analysis of scheduling policies for delay-tolerant applications in centralized wireless networks," in *Proceedings IEEE International Symposium on Performance Evaluation of Computer and Telecommunication Systems (SPECTS 2008)*, Edinburgh, Scotland, June 2008, pp. 309–316.
- [23] M. Shaqfeh and N. Goertz, "Channel-aware scheduling with resource-sharing constraints in wireless networks," in *Proceedings IEEE International Conference on Communications (ICC)*, Beijing, China, May 2008, pp. 4149–4153.
- [24] G. Song and Y. Li, "Utility-based resource allocation and scheduling in OFDM-based wireless broadband networks," *IEEE Communications Magazine*, vol. 43, no. 12, pp. 127–134, Dec. 2005.
- [25] M. Shaqfeh, N. Goertz, and J. Thompson, "Ergodic capacity of block-fading Gaussian broadcast and multi-access channels for single-user-selection and constant-power," in *Proceedings European Signal Processing Conference (EUSIPCO)*, Glasgow, Scotland, Aug. 2009, pp. 784–788.
- [26] R. Suoranta, K. Estola, S. Rantala, and H. Vaataja, "PDF estimation using order statistic filter bank," in *Proceedings IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP)*, Adelaide, SA, Apr. 1994, vol. 3, pp. 625–628.
- [27] R. Jain, D. Chiu, and W. Hawe, "A quantitative measure of fairness and discrimination for resource allocation in shared computer systems," *DEC Research Report TR-301*, Available at <http://www.cs.wustl.edu/jain/papers/ftp/fairness.pdf>, pp. 1–37, Sept. 1984.

TABLE I
MULTIUSER SWITCHED DIVERSITY SYSTEM - SISO RAYLEIGH BLOCK-FADING

Expected achievable rates:

$$R_i = \left[\exp\left(\frac{-\gamma_i^*}{\bar{\gamma}_i}\right) \log(1 + \gamma_i^*) + \exp\left(\frac{1}{\bar{\gamma}_i}\right) E_1\left(\frac{1 + \gamma_i^*}{\bar{\gamma}_i}\right) \right] \cdot \prod_{j < i} \left(1 - \exp\left(\frac{-\gamma_j^*}{\bar{\gamma}_j}\right)\right), \quad (24)$$

where $\bar{\gamma}_i$ is the average SNR of user i and E_1 is the exponential integral function:

$$E_1(x) \equiv \int_x^\infty \frac{\exp(-u)}{u} du. \quad (25)$$

Expected access ratio:

$$\text{AR}_i = \exp\left(\frac{-\gamma_i^*}{\bar{\gamma}_i}\right) \cdot \prod_{j < i} \left(1 - \exp\left(\frac{-\gamma_j^*}{\bar{\gamma}_j}\right)\right). \quad (26)$$

The optimal feedback thresholds are computed in a backward successive approach, starting from $\hat{\gamma}_M^* = 0$, according to:

$$\mu_i \log(1 + \hat{\gamma}_i^*) = \mu_{i+1} \left[\exp\left(\frac{1}{\bar{\gamma}_{i+1}}\right) E_1\left(\frac{1 + \hat{\gamma}_{i+1}^*}{\bar{\gamma}_{i+1}}\right) + \log(1 + \hat{\gamma}_{i+1}^*) \right]. \quad (27)$$

In the special case of maximizing the sum achievable rates, all weighting factors μ_i in (27) are equal, and the maximum sum achievable rate can be expressed as:

$$\max \sum_{i=1}^M R_i = \exp\left(\frac{1}{\bar{\gamma}_1}\right) E_1\left(\frac{1 + \hat{\gamma}_1^*}{\bar{\gamma}_1}\right) + \log(1 + \hat{\gamma}_1^*). \quad (28)$$

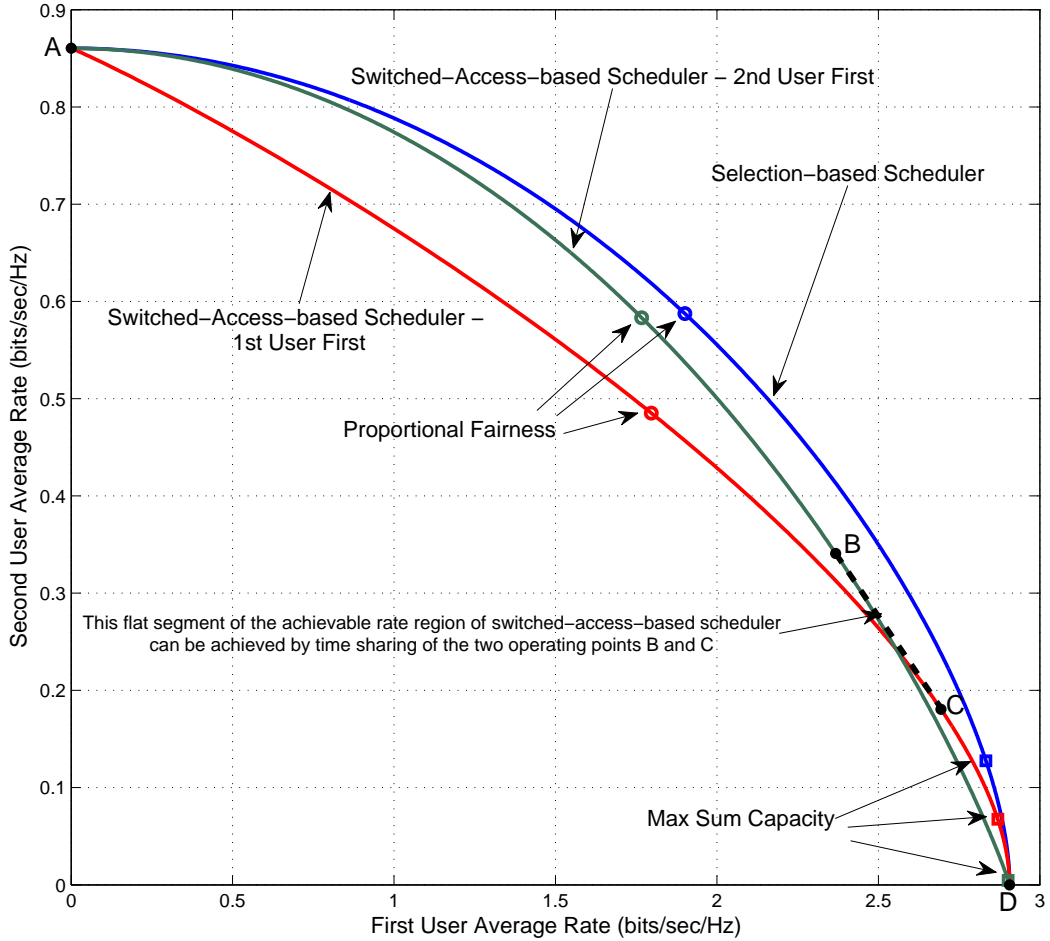


Fig. 1. Achievable rate region of selection-based and switched-access-based diversity systems in a two-user scenario. The channels are Rayleigh block-faded with average SNR of 10 dB and 0 dB for the first user and second user, respectively.

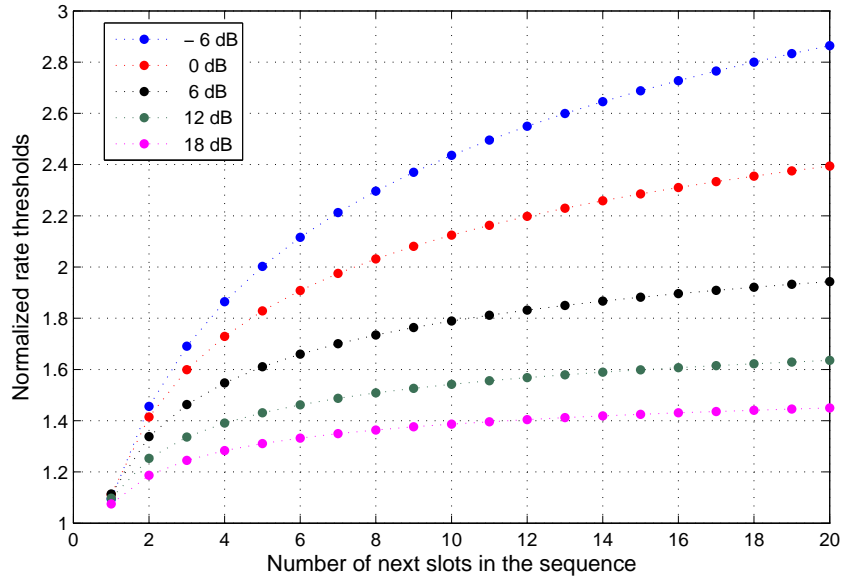


Fig. 2. Normalized achievable rate thresholds to achieve proportional fairness for SISO Rayleigh block-fading conditions with different values for the average SNR plotted versus the number of next users in the sequence.

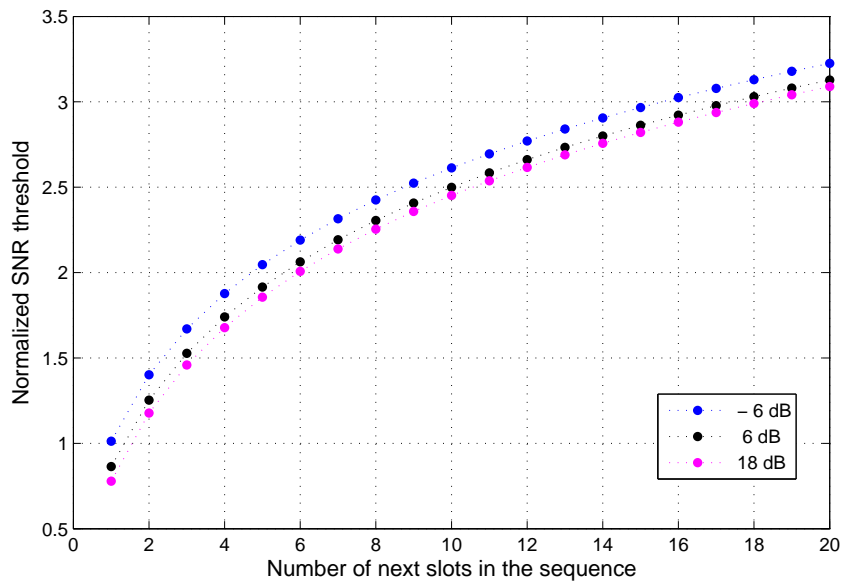


Fig. 3. Normalized SNR thresholds to achieve proportional fairness for SISO Rayleigh block-fading conditions with different values for the average SNR plotted versus the number of next users in the sequence.

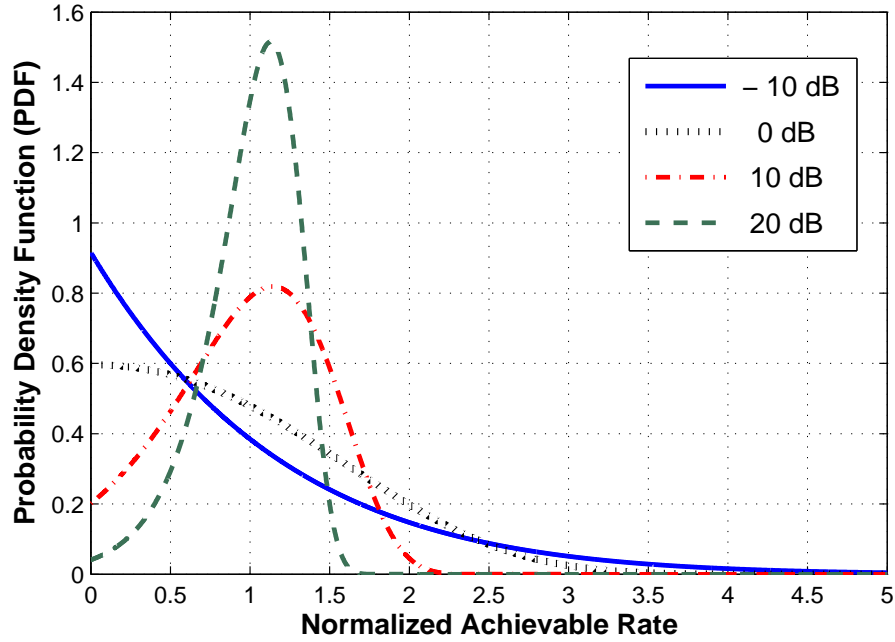


Fig. 4. The PDF of the normalized achievable rates over Rayleigh block-fading channels. The achievable rates are normalized with respect to the average achievable rate over the channel.

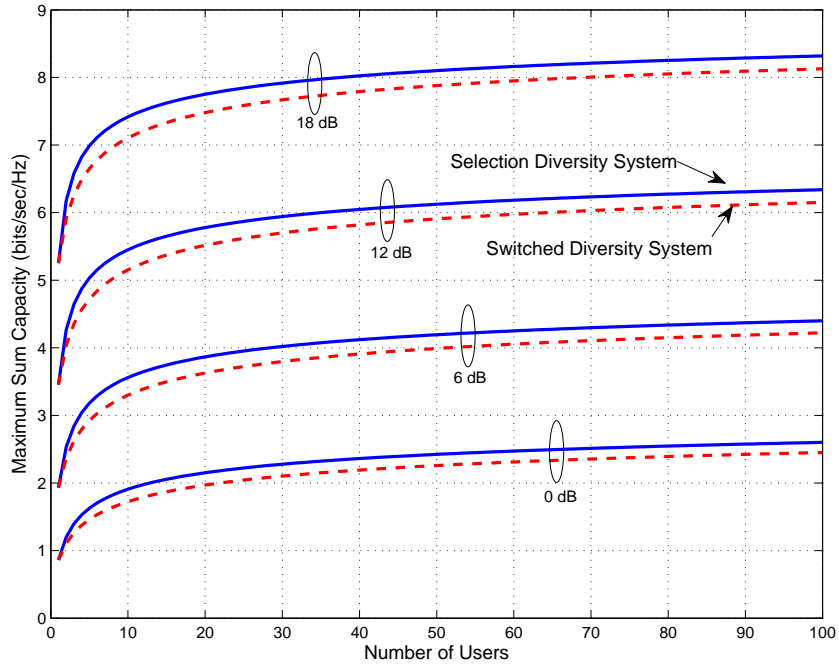


Fig. 5. Maximum sum achievable rate (capacity) comparison between the selection diversity system (solid blue lines) and the switched diversity system (dashed red lines) as a function of the number of users over i.i.d. Rayleigh block-fading channels. Results are based on average SNR of 0, 6, 12 and 18 dB.

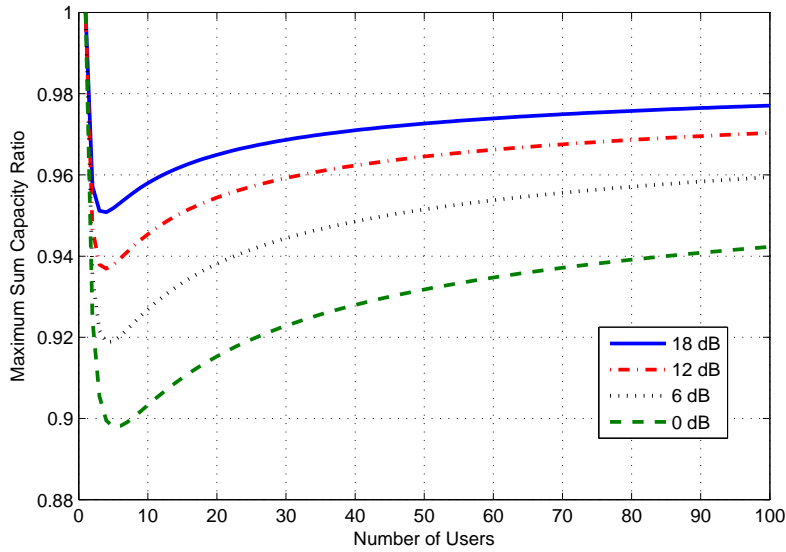


Fig. 6. The ratio between the maximum sum achievable rate (capacity) of switched diversity system and the maximum sum capacity of the selection diversity system as a function of the number of users over i.i.d. Rayleigh block-fading channels.

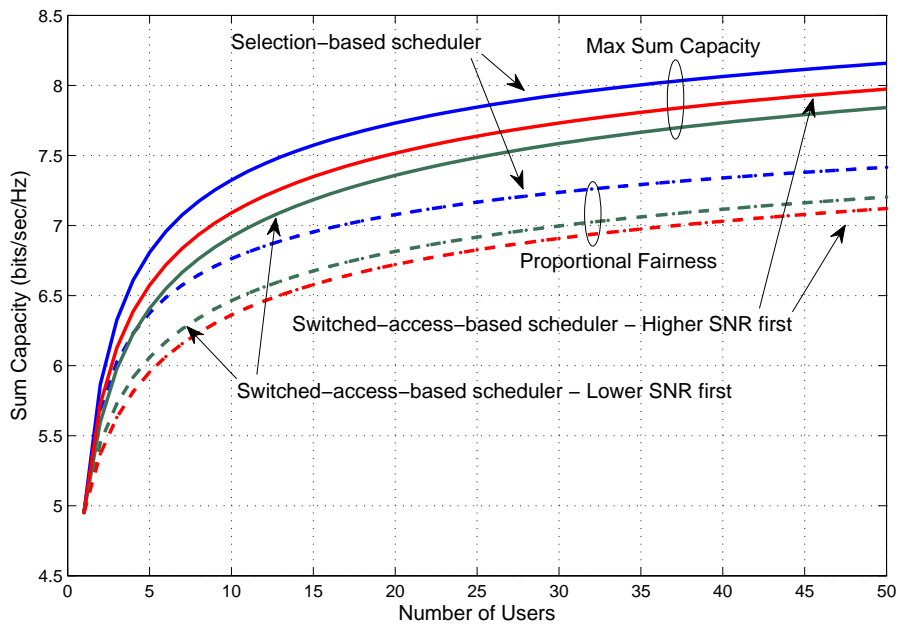


Fig. 7. Sum achievable rate comparison between the selection diversity system (blue lines) and the switched diversity system (red and green lines) as a function of the number of users for maximum sum rate scheduling (solid lines) and proportional fairness scheduling (dashed lines). The users have Rayleigh block-fading channels with average SNR distributed according to (39a). Two feedback sequence strategies are examined: ascending (green lines) and descending (red lines) average SNR order.

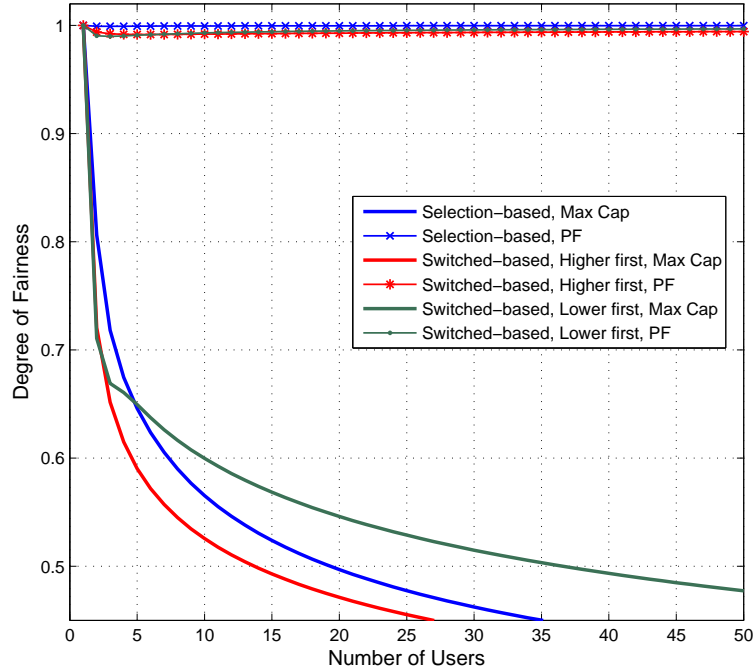


Fig. 8. Fairness measure by applying Jain's index (40) with $(x_i = AR_i)$ the average channel access ratio. The users have Rayleigh block-fading channels with average SNR distributed according to (39a).

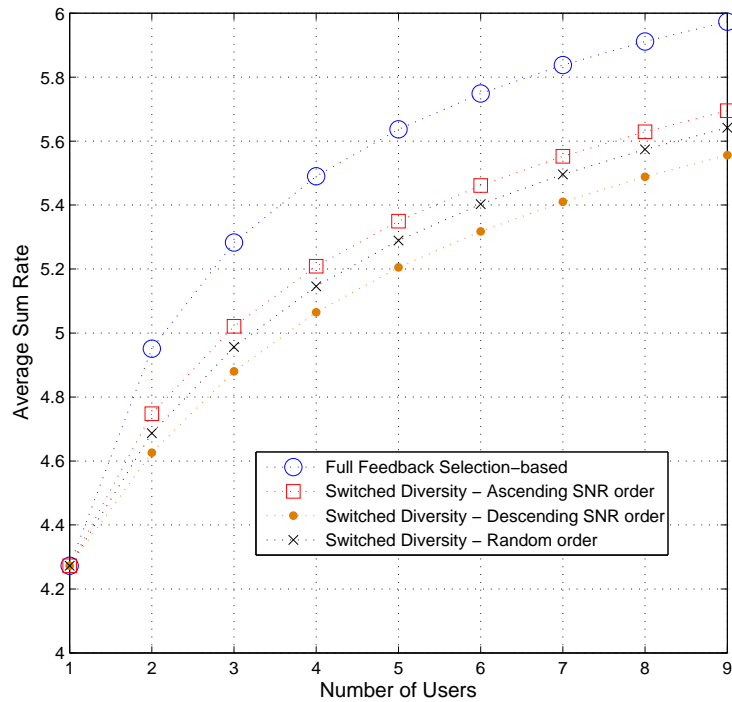


Fig. 9. Sum achievable rate comparison between the selection diversity system and the switched diversity system as a function of the number of users for proportional fairness scheduling. The users have Rayleigh block-fading channels with average SNR distributed according to (39b).

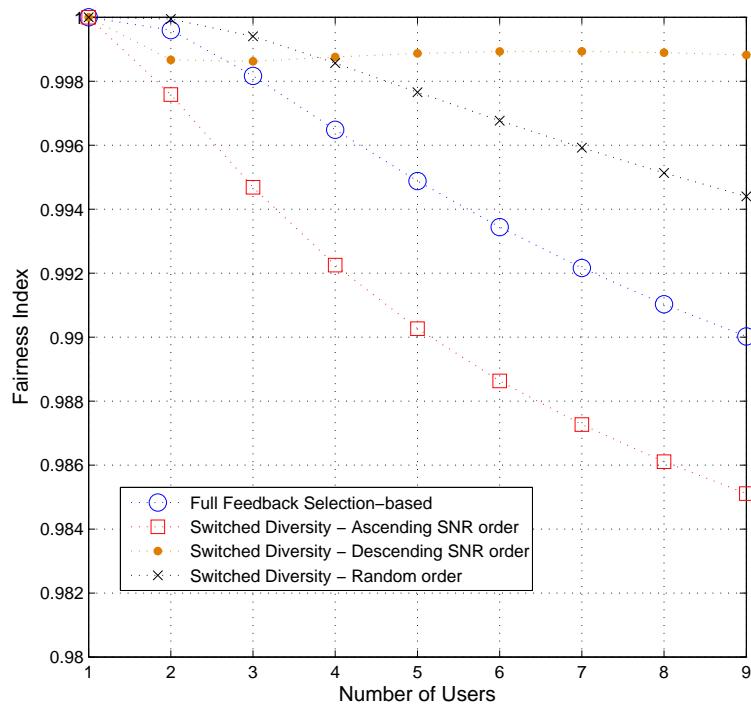


Fig. 10. Fairness measure by applying Jain's index (40) with (41). The users have Rayleigh block-fading channels with average SNR distributed according to (39b).