The possibilities of linearized inversion of internally scattered seismic data

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Summary

Least-square migration is an iterative linearized inversion scheme that tends to suppress the migration artifacts and enhance the spatial resolution of the migrated image. However, standard least-square migration, based on imaging single scattering energy, may not be able to enhance events that are mainly illuminated by internal multiples such as vertical and nearly vertical faults. To alleviate this problem, we propose a linearized inversion framework to migrate internally multiply scattered energy. We applied this least-square migration of internal multiples to image a vertical fault. Tests on synthetic data demonstrate the ability of the proposed method to resolve a vertical fault plane that is poorly resolved by least-square imaging using primaries only. We, also, demonstrate the robustness of the proposed scheme in the presence of white Gaussian random observational noise and in the case of imaging the fault plane using inaccurate migration velocities.

Introduction

Imaging internal multiples has the potential to retrieve seismic events, which are poorly illuminated by single-scattering energy such as nearly vertical fault planes. Malcolm et al. (2011) proposed a methodology to image internal multiples in which they image prismatic waves and other higher order internal multiples. Behura et al. (2012) proposed an iterative imaging procedure to image internal scattering along with single scattering energy. Recently, Zuberi and Alkhalifah (2013) proposed a three-step interferometric imaging procedure to image internal scattering energy.

Standard imaging of either single-scattering energy or multiple-scattering energy suffers from low spatial resolution and migration artifacts due to the coarse sampling of sources and receivers, and the band-limitedness of the source wavelet. To remedy this problem, least-square migration (LSM) can be utilized to enhance the quality of migrated images and suppress the migration artifacts when imaging primaries (LeBras and Clayton, 1988; Nemeth et al., 1999).

In this abstract, we first present the theory of least-square migration to image internally multiply scattered energy and describe how we linearize the problem of imaging internal multiples. Then, we apply the proposed linearized inversion to delineate a synthetic vertical fault that is poorly illuminated by primaries. Lastly, we demonstrate the robustness of the proposed linearized inversion framework to image this vertical fault plane in the presence of white Gaussian random observational noise and in the case of imaging the subsurface using erroneous migration velocities.

Theory

The Kirchhoff forward modeling operator **L** is a linear operator that maps the subsurface reflectivity distribution to single-scattered seismic data. Mathematically, computing synthetic seismic data corresponding to a reflectivity model of the earth, **m(x)**, is accomplished using the following forward modeling operator in the frequency domain:

$$\mathbf{d}(s,g,\omega)=\int_V \mathbf{m}(\mathbf{x})\mathrm{W}(\omega)e^{i\omega(\tau_{sx}+\tau_{xg})}d\mathbf{x}=\mathbf{Lm}, \tag{1}$$

where $\mathbf{d}(s,g,\omega)$ is the recorded single-scattered seismic data at receiver point g due to a source at s for a particular frequency ω. A point in the subsurface volume V is given by $\mathbf{x}$. $\mathrm{W}(\omega)$ is the source wavelet. In addition, $\tau_{\mathbf{sx}}$ is the travel time from the source point s to the scattering point $\mathbf{x}$ and $\tau_{\mathbf{x}g}$ is the travel time from the scattering point $\mathbf{x}$ to the receiver point g.

A standard migrated image is determined by applying the adjoint of the forward modeling operator $\mathbf{L}^{\mathbf{T}}$ to the single-scattered seismic data (Claerbout, 1992). The adjoint operator linearly maps the single-scattered seismic data to subsurface structures of the earth as follows:

$$\mathbf{m}_{\mathbf{mig}}(\mathbf{x})=\int_s\int_g\int_\omega \mathbf{d}(s,g,\omega)\mathrm{W}^*(\omega)e^{-i\omega(\tau_{sx}+\tau_{xg})}dsdgd\omega$$
$$=\mathbf{L}^{\mathbf{T}}\mathbf{d}, \tag{2}$$

where $\mathbf{m}_{\mathbf{mig}}(\mathbf{x})$ is the standard migrated image and $\mathrm{W}^*(\omega)$ is the complex conjugate of the source wavelet. The least-square solution **p** is obtained by solving the normal equation:

$$\mathbf{L}^{\mathbf{T}}\mathbf{Lp}=\mathbf{L}^{\mathbf{T}}\mathbf{d}. \tag{3}$$

This equation is usually solved iteratively by a gradient-based optimization algorithm with an initial guess given by the standard migration imaging in equation (2). At each

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iteration, the forward modeling operator is used to synthesize single-scattered seismic data to be compared with the recorded data. Then, the data residual is back-projected using the adjoint operator to determine the subsurface reflectivity model updates.

The least-square migrated image, based on single-scattering assumption, shows the reduction of the migration artifacts and the enhancement of the seismic reflector illuminated mainly by primaries. Nevertheless, least-square migration hardly delineates reflectors that are mainly illuminated by internal scattering energy. Therefore, migrating internal multiples potentially helps illuminate subsurface areas that are poorly illuminated by primaries such as vertical and nearly vertical fault planes.

The forward modeling operator **G** maps the reflectivity model to doubly-scattered seismic data as:

$$\mathbf{d_d}(s,g,\omega) = \int_V \int_V \mathbf{v(x)v(x')}\,\mathrm{W}(\omega) e^{i\omega(\tau_{s\mathbf{x}}+\tau_{\mathbf{xx'}}+\tau_{\mathbf{x'}g})} d\mathbf{x}\, d\mathbf{x'} \\ = \mathbf{G(v)}, \tag{4}$$

where $\mathbf{d_d}(s,g,\omega)$ is the recorded doubly-scattered seismic data. Moreover, **v(x)** and **v(x')** are the subsurface reflectivity functions at subsurface point **x** and **x'**, respectively. In addition, $\tau_{s\mathbf{x}}$ is the travel time from the source point *s* to the first scattering point **x**. $\tau_{\mathbf{xx'}}$ is the travel time from the first scattering point **x** to the second scattering point **x'**. $\tau_{\mathbf{x'}g}$ is the travel time from the second scattering point **x'** to the receiver point *g*.

Clearly, generating doubly-scattered data is not simply determined by a linear mapping of the reflectivity model because of the multiplication of two reflectivity functions inside this double summation modeling step. To linearize this operator, we propose replacing **v(x')** by the fixed least-square migration solution **p(x')** of the single-scattered data obtained by iteratively solving the normal equation (3); then, equation (4) becomes:

$$\mathbf{d_d}(s,g,\omega) = \int_V \int_V \mathbf{v(x)p(x')}\,\mathrm{W}(\omega) e^{i\omega(\tau_{s\mathbf{x}}+\tau_{\mathbf{xx'}}+\tau_{\mathbf{x'}g})} d\mathbf{x}\, d\mathbf{x'} \\ = \mathbf{G v}. \tag{5}$$

Equation (5) shows that the forward modeling operator **G** of doubly-scattered seismic data becomes a liner mapping of the subsurface reflectivity distribution mainly illuminated by double-scattering energy **v**. The corresponding adjoint of this forward modeling operator $\mathbf{G^T}$ can be applied to image the doubly-scattered data as follows:

$$\mathbf{v_{mig}}(\mathbf{x}) = \\ \int_s \int_g \int_{\mathbf{x}} \int_\omega \mathbf{d_d}(s,g,\omega)\mathbf{p(x')}\mathrm{W}^*(\omega) e^{-i\omega(\tau_{s\mathbf{x}}+\tau_{\mathbf{xx'}}+\tau_{\mathbf{x'}g})} ds\, dg\, d\mathbf{x}\, d\omega \\ = \mathbf{G^T d_d}. \tag{6}$$

Similar to the single-scattering case, computing the doubly-scattered migrated image is determined by a linear mapping of the doubly-scattered seismic data. We can also solve a corresponding normal equation iteratively in order to obtain a least-square migrated image $\mathbf{v_{ls}}$, which is illuminated mostly by doubly-scattered data as follows:

$$\mathbf{G^T G v_{ls}} = \mathbf{G^T d_d}. \tag{7}$$

The least-square migration of both the single-scattering and double-scattering energy would help suppress the migration artifacts. Moreover, it would enhance the spatial resolution of both horizontal reflectors and vertical fault planes.

Examples

We applied least-square migration to image a two-layer model with a vertical fault. The true model is shown in Figure 1 (top). The standard migrated image, based on single-scattering assumption, of the noise-free data, is shown in Figure 1 (middle). It yielded a blurry depiction of the true subsurface reflectivity distribution caused by the migration artifacts and the acquisition fingerprint. Also, the least-square migration, based on single-scattering assumption, suppresses these artifacts and sharpens the reflectors as shown in Figure 1 (bottom). However, this linearized inversion, after 30 quasi-Newton LBFGS iterations, mainly enhanced the horizontal reflectors, which are mostly illuminated by primaries. The vertical fault plane is not well delineated since it is primarily illuminated by multiple scattering energy.

In our double-scattering inversion framework, the least-square solution, based on single-scattering assumption in Figure 1 (bottom), is used to linearize the inversion of the doubly-scattered data. The least-square solution of the noise-free doubly-scattered data, after six quasi-Newton LBFGS iterations, is shown in Figure 2 (top). Clearly, inverting the doubly-scattered data delineates the fault and localizes it with more accurate amplitude information.

A challenging question is to find out if we can reconstruct an accurate representation of the fault plane in the presence of observational noise. For this reason, we added white Gaussian random noise to the recorded data. The least-square image, based on double-scattering assumption, of the noisy data with single-to-noise ratio (SNR) of -20 dB

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and -30 dB are shown in Figure 2 (middle) and Figure 2 (bottom), respectively. Six quasi-Newton LBFGS iterations were required to obtain these solutions. In the case of SNR = -20 dB, the least-square imaging of the doubly-scattered data still retrieves the fault with an accurate position and amplitude information. However, the quality of the recovery deteriorates and the fault plane becomes heavily blurry in the case of SNR = -30 dB.

Another question of interest is to study whether least-square migration of doubly-scattered data is robust in the case of imaging using erroneous migration velocities. What would be the effect on the image quality when the migration velocity is inaccurate? To answer this question, we used a 3% slower and faster velocities to image the fault plane in our linearized inversion framework for double-scattering energy. After seven quais-Newton LBFGS iterations, Figure 3 (top) and Figure 3 (bottom) shows the inversion results of the doubly-scattered data using the slower and faster migration velocities, respectively. The least-square migration of doubly-scattered data helps delineate the fault plane even though it becomes slightly mispositioned and defocused as we use erroneous velocities to migrate the data.

Conclusions

Standard migration and iterative least-square migration, based on imaging single-scattering energy, yield migrated images that delineate seismic events, which are mainly illuminated by primaries such as horizontal reflectors. We showed that modeling doubly-scattered data is not represented mathematically by a linear mapping of the subsurface reflectivity distribution. Thus, we linearized this step using the image, based on single-scattering assumption, as a constraint to one of the scatterers, in the forward modeling operator and the adjoint operator that models and images doubly-scattered data, respectively. We demonstrated the effectiveness of the proposed linearized inversion framework of doubly-scattered data and showed that it can localize a vertical fault plane with fairly accurate amplitude information. We, also, demonstrated the robustness of this inversion scheme in the presence of white Gaussian random observational noise. Lastly, we showed that the least-square migration of doubly-scattered data could delineate the fault plane although it becomes slightly mispositioned and defocused when inverting the data using inaccurate migration velocities.

Acknowledgement

We utilized the Matlab parallel computing toolbox and optimization toolbox to compute the least-square solutions iteratively using a quasi-Newton algorithm for the single and doubly-scattered data.

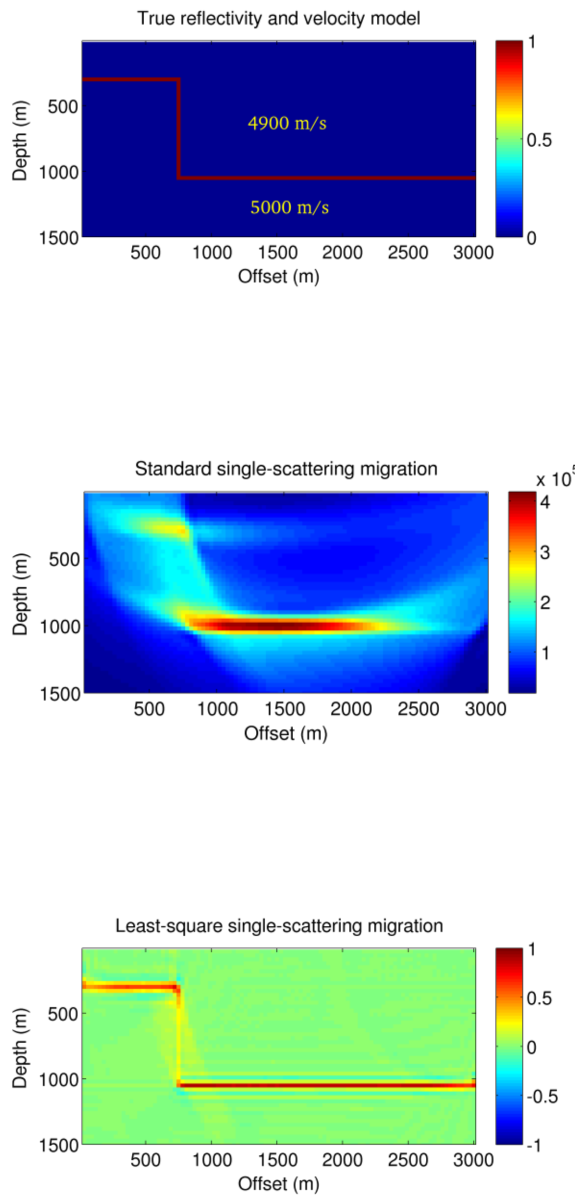

Figure 1: The true velocity and reflectivity models (top). The heavily blurred standard migrated image based on single-scattered assumption (middle). The least-square migration, based on single-scattering assumption, enhances mainly the horizontal reflectors, which are mainly illuminated by primaries.

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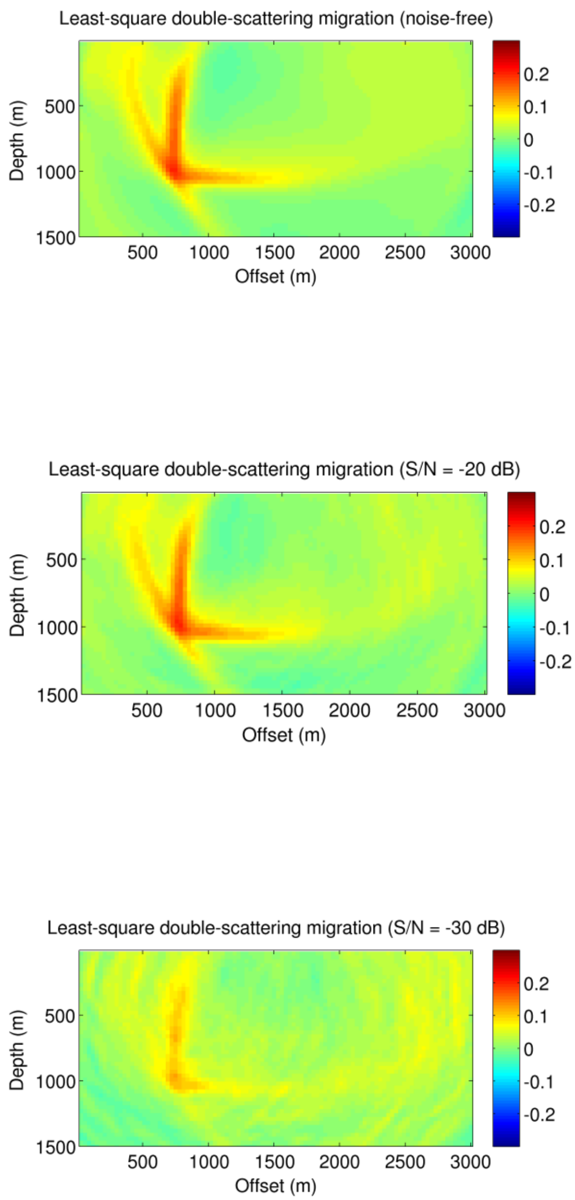

Figure 2: The least-square image of the doubly-scattered noise-free data (top). The least-square image of the doubly-scattered noisy data with SNR = -20 dB (middle) and SNR = -30 dB (bottom). As the data becomes more noisy, the fault plane becomes more smeared and blurry.

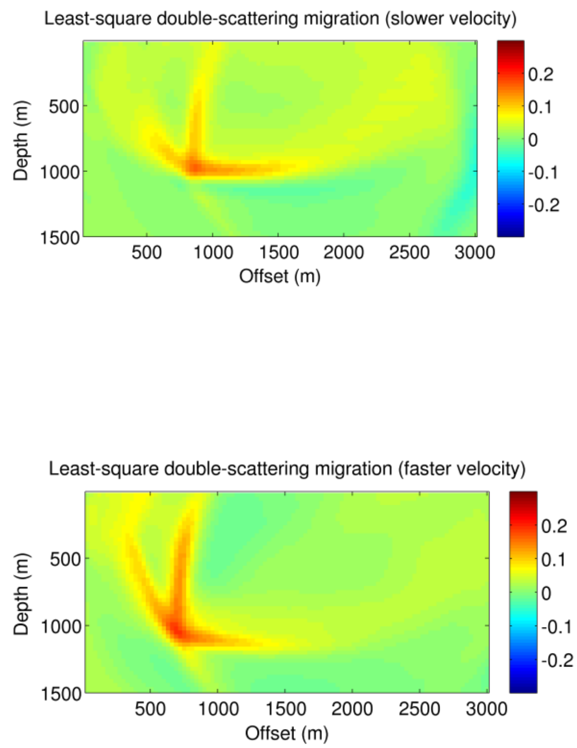

Figure 3: The least-square migration based on double-scattering assumption using inaccurate migration velocities. The doubly-scattered data was inverted using 3% slower migration velocities (top) and 3% faster migration velocities (bottom). As we use erroneous velocities, the inverted fault plane becomes slightly defocused and mispositioned.

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