New Insights on the Uncertainties in Finite-Fault Earthquake Source Inversion

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ABSTRACT

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Earthquake source inversion is a non-linear problem that leads to non-unique solutions. The aim of this dissertation is to understand the uncertainty and reliability in earthquake source inversion, as well as to quantify variability in earthquake rupture models. The source inversion is performed using a Bayesian inference. This technique augments optimization approaches through its ability to image the entire solution space which is consistent with the data and prior information.

In this study, the uncertainty related to the choice of source-time function and crustal structure is investigated. Three predefined analytical source-time functions are analyzed; isosceles triangle, Yoffe with acceleration time of 0.1 and 0.3 s. The use of the isosceles triangle as source-time function is found to bias the finite-fault source inversion results. It accelerates the rupture to propagate faster compared to that of the Yoffe function. Moreover, it generates an artificial linear correlation between parameters that does not exist for the Yoffe source-time functions. The effect of inadequate knowledge of Earth’s crustal structure in earthquake rupture models is subsequently investigated. The results show that one-dimensional structure variability leads to parameters resolution changes, with a broadening of the posterior
PDFs and shifts in the peak location. These changes in the PDFs of kinematic parameters are associated with the blurring effect of using incorrect Earth structure. As an application to real earthquake, finite-fault source models for the 2009 L’Aquila earthquake are examined using one- and three-dimensional crustal structures. One-dimensional structure is found to degrade the data fitting. However, there is no significant effect on the rupture parameters aside from differences in the spatial slip extension. Stable features are maintained for both structures.

In the last part of this work, a multidimensional scaling method is presented to compare and classify earthquake slip distributions. A similarity scale to rank them are thus formulated. Dissimilarities among slip models (from various parameterizations) are computed using two different distance metrics, normalized squared and gray-scale metrics. Multidimensional scaling is then used to visualize the differences among the models. The analyzes are done for 2 case studies; one based on artificial scenarios with a known answer and another one based on the published rupture models of the 2011 Tohoku earthquake.
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Chapter 1

Introduction

An earthquake is a natural hazard resulting from a sudden release of strain energy accumulated in the earth crust over years. It can cause severe economic losses and casualties. Governments and geoscientists put significant efforts on earthquake monitoring, prediction and mitigation. However, reliable prediction (exact location, origin time, and the magnitude) is still one of the great unsolved problems that seismologists face. There is a dichotomy on the issue of earthquake prediction. Geller et al. (1997) reported that the factors influencing the growth of small earthquakes into large ones are so complex and subject to numerous variables, that earthquake prediction is severely limited. Observation limitation also is another factor that challenges earthquake prediction. Despite the occurrence of small earthquakes on a daily basis, large ones are infrequent in human time. Hence, a proper understanding of physical mapping/transition from frequently occurring small earthquakes to large ones is one of the essential elements for the earthquake prediction challenges.

Most of ‘tectonic earthquakes’ occur along the plate boundaries where one plate thrusts under the other (e.g., Cascadia subduction zone where Juan de Fuca plate subducts beneath North American plate; Japan Trench subduction zone), plates scrape horizontally past each other (e.g., San Andreas fault in California, separating the Pacific and the North American plates; North Anatolian fault in Turkey), or plates pull apart (e.g., Red Sea-Gulf of Aden rift; East-African rift). Less than 10% of all earthquakes occurs within plate interior (www.usgs.gov/).
Deformations of rocks along plate boundaries occur with two primary mechanisms. First, for creeping mechanism, the plate moves steadily (smoothly past one another) without rupturing since it does not build up the stress required to generate large earthquakes. This creeping mechanism is associated with the material property, temperature, pressure, fluids as well as the stress history in the area. It occurs in the upper crust due to the presence of unconsolidated materials (Marone & Scholtz, 1988). Such material prohibits earthquake from nucleating at very shallow depth. Some cases of creeping fault also occur deeper, in seismogenic zones. In this case, it causes slow-slip events, which have been detected mainly in subduction zones (e.g., Japan, Mexico, New Zealand; Schwartz & Rokosky, 2007; Beroza & Ide, 2011). The second case of rock deformation consists of locked fault where the plates do not slide freely past each other due to friction on the fault surface. Instead, stress is accumulated over decades or centuries. Once the stress exceeds the frictional forces, the fault ruptures (in a meter scale) causing moderate to large earthquakes. These fault-behaviors might change over time, and there are also some cases where the fault is partially locked (e.g., Hayward Fault in San Francisco Bay, California; Lienkaemper et al., 2012). Small to moderate size earthquakes could, therefore, be expected along faults known as creeping.

Damages from earthquakes are not only caused by the violent ground shaking, but in most of the cases by secondary hazards, such as tsunamis, fire, landslide, site effect, and liquefaction. In the last ten years, a series of mega-events have occurred, including the 2004 Mw 9.1 Sumatra earthquake, which is the third largest earthquake ever recorded. It ruptured over a 1000 km fault length (Shearer & Burgmann, 2010; Krüger & Ohrnberger, 2005) triggering mega-tsunami that was one of the deadliest ever to hit the globe with more than 227 000 fatalities. Another example is the 2011 Mw 9.0 Tohoku earthquake that occurred near the East Coast of Honshu, along the Japan Trench. It also generated a devastating tsunami and underwater landslide (Tappin et al., 2014) that caused more than 16 000 fatalities (e.g., Satake et al., 2013; Mori & Takahashi, 2012). Another example includes The Mw 7.8 devastating earthquake, which occurred in the mountain range of Sichuan province in China and claimed the lives of up to 80 000 people. Most of the damages were due to the heavy sedimentation in the basin that amplified the seismic shaking and
triggered landslides in the mountain range.

Many densely populated areas are located close to plate boundaries where mega-earthquakes are likely to occur. Earthquake mitigation and preparedness is thus necessary. Since the occurrence of the 2004 Sumatra earthquake, significant efforts have been made to develop earthquake early warning systems (e.g., [http://www.shakealert.org/](http://www.shakealert.org/)). Such systems estimate the location/size of the earthquake rapidly, as well as the level of ground shaking to be expected. Then, they emit a few seconds of warning prior the arrival of the strong shaking that usually causes most of the damages. This warning time obviously is not enough to evacuate an entire city. It is crucial, however, to reduce the effect of secondary hazard such as fire.

Another valuable tool to increase preparedness for earthquakes is to ensure that structures can withstand a given level of ground shaking. The first step in designing these earthquake-resistant structures is to assess seismic hazard in a specific region. This consists of estimating the level of expected ground motion in a particular region due to future earthquakes. Scenario simulations have been performed to evaluate ground shaking for specific areas (e.g., [Graves & Pitarka 2010](http://example.com), [Olsen et al. 2009](http://example.com), [Denolle et al. 2014](http://example.com)).

Ground motion prediction (GMP) is of great importance for structural engineers that design earthquake-resistant structures. GMP is also used by authorities and seismologists to establish appropriate building codes as well as studying the physical processes leading to ground-motion complexity. GMP incorporates information about the source processes, the geologic structure where the seismic wave propagates, and the local site condition. Therefore, it is subject to a number of uncertainties. A lack of knowledge in one of these three entities thus implies variability in the generated ground motion leading to underestimation/overestimation of the predicted damages. Hence, a proper understanding of the earthquake source process, the geological structure in a particular area, as well as the impact of site effect is an essential ingredient toward a robust ground motion estimation. Among these three parts, I particularly undertake an effort in understanding the earthquake source process and the associated uncertainty.
1.1 Earthquake Source Modeling

1.1.1 Earthquake Source Description

Significant efforts have been made to study the seismic source, which is a crucial ingredient in understanding the earthquake process and eventually in mitigating earthquake risks. Two distinct techniques are mainly used to model earthquake source; kinematic and dynamic modeling. These methods typically fix the fault region, and then employ a number of parameterizations to capture the temporal and spatial evolution of slip on the fault plane. Both kinematic and dynamic models can contribute to the understanding of the earthquake mechanisms and near-source ground motion complexity (e.g., Ripperger et al. 2008; Mai 2009; Bydlon & Dunham 2015). The main differences appear in the problem setup.

Kinematic (also called finite-fault) rupture model describes the spatio-temporal evolution of the slip along the fault plane that is compatible with the recorded ground motion. This model does not consider the forces and stresses that cause the movements. Therefore, the resulting model does not necessarily obey the physical principles of rock fracturing and slip. The dynamic rupture model, on the other hand, is used to learn about the general properties of earthquake faulting. It is based on the physics of the earthquake rupture process. The dynamic rupture model includes the characteristics of materials surrounding the fault and conditions (initial and boundary) for the forces and stresses acting on different parts of the fault plane. Therefore, the dynamic description gives a more physical understanding of the earthquake rupture compared to the kinematic model. The dynamic rupture model can be obtained by solving the elastodynamic equation coupled with the frictional law on the fault plane. Although the dynamic rupture model is physically more consistent, information is limited to realistically parameterize the friction model and the state of stress in the crust. This approach also is computationally expensive (e.g., Day et al. 2005). Dynamic rupture model typically abides by friction law, in which friction parameters and pre-stress are constrained by kinematic source model (Peyrat et al. 2001; Dalgner et al. 2001). Thus, reliable and precise kinematic models that are the aim of this dissertation are crucial for such research.
1.1.2 Kinematic Source Inversion

Only a few observations have been made directly from the earthquake source, which are located kilometers below the earth surface. Of these few, the drillholes across the Nojima fault and the Japan Trench fast drilling project aiming at understanding the 1995 Kobe and the 2011 Tohoku earthquakes, respectively. In most cases, however, rupture models are inferred, based on the radiated seismic wavefield recorded on the surface. Thanks to extensive efforts in earthquake/tsunami monitoring amongst other purposes, the earthquake source community has greatly benefited from the existence of various recording systems including GPS/Seismic/Tide gauges Networks, satellite imagery, and land survey.

Over the last three decades, several finite-fault inversion techniques have been developed. They have mainly been extensively applied to infer optimal finite-fault rupture model. These methods are based on the representation theorem (Aki & Richard, 1980), which relates the recorded displacement field with the slip on the fault and the response of the medium. This representation theorem can be formulated as an inverse problem. The goal is to solve for the slip parameters on the fault using information about the crustal structure and the recorded ground motion. Typically, various assumptions are made to reduce the free parameters and to ensure that the kinematic source parameters is realistic. The temporal parameters and peak slip velocity, for instance, are assumed to be positive. Even though, this problem is highly non-linear, linearized and non-linear parameterizations could be carried out.

The multi-time window inversion (Olson & Apsel, 1982; Hartzell & Heaton, 1983) that is a linearized inverse algorithm and represents the pioneer of finite-fault earthquake source inversion. In this approach, the slip-history at each point of the fault is presented as a superposition of many elementary source-time functions. Therefore, it allows the point to slip more than once, and hence to detect an episode of slip reactivation during the rupture propagation. In this inversion strategy, the source parameter to be inverted include the slip. This approach has been improved and applied to numerous earthquakes (e.g., Sekiguchi & Itawa, 2002; Zhou et al., 2004; Lee et al., 2011).

To account for the full non-linearity of the problem, optimization strategies were developed and widely applied (e.g., Hartzell et al., 1996; Bouchon et al., 2000; Delouis et al., 2002).
Liu & Archuleta, 2004; Emolo & Zollo, 2005). In these techniques, the time-dependent slip history at each point participating in the rupture process is presented in terms of one single source-time function (velocity or displacement). This source time function is modeled using a simple analytical form. A typical example of this function includes the boxcar, Brunes function (Brune, 1970), isosceles triangle, Kostrov-like function (Hisada, 2000), modified Yoffe (Tinti et al., 2005b). The non-linear formulation of the finite-source inversion estimates the space-time rupture evolution in terms of slip rate, rupture time, slip duration, and rake angle at each point of the fault plane.

Cohee & Beroza (1994) and Hartzell et al. (2007) investigated the differences between the linear and non-linear source inversions. They found that these two techniques yield consistent results in the slip amplitude and the overall temporal parameters. Cohee & Beroza (1994) particularly found that the single-time-window technique recovers the true seismic moment and the average rupture velocity more efficiently.

In most of the previous source studies, the uncertainty analyzes are not typically incorporated, even though earthquake source inversion suffers from non-uniqueness (ill-posed), as other geophysical inversions. In general, for the same earthquake, inferred rupture models are widely different. This motivates the use of Bayesian inversion, which spans the solution space and determines the well-resolved parts of the source process. Instead of generating a single slip model, this technique represents the solutions in terms of a probability density function (Tarantola, 2005). In recent years, this approach becomes widely developed for earthquake source inversion (e.g., Monelli & Mai, 2008; Minson et al., 2013; Dettmer et al., 2014). In this dissertation, we particularly use this technique to infer the finite-fault source model.

1.2 Uncertainty of Finite-Fault Inversions

Several studies are typically conducted to investigate the rupture process of a particular earthquake using various data sets and inversion algorithms. Some of the resulting models show similarity, however in general considerable intra-event variability appears in the in-
ferred rupture models (e.g., Ide et al., 2005; Page et al., 2011; Mai et al., 2007). Kinematic source models of the 2011 Mw 9.0 Tohoku earthquake are prime examples of this variability (see earthquake source database; Mai & Thingbaijam, 2014). Therefore, without a rigorous way to compare slip models quantitatively, it is difficult to analyze and to draw conclusions about the source process. The primary source of uncertainty in earthquake source inversion is the incomplete and noisy data. In addition, to formulate a resolvable and physically meaningful slip-inversion problem, a number of assumptions and constraints have to be adopted. Depending on which set of constraints is chosen and how it is implemented, the results of inversion may dramatically change. A number of studies have been conducted to investigate these non-uniqueness and uncertainty issues. Beresnev (2003) discussed the uncertainty related to incorrect assumptions about some fixed parameters in the inversions, such as rupture speed, fault geometry, and the crustal structure. Hartzell et al. (1991), on the other hand, examined the effect of different inversion norms (L1 and L2) on the derived slip distributions. They found significant discrepancies in the inverted models. However, robust features were identified to be common for any chosen norm. Variability of slip distribution could also be associated with different stations configuration (Beresnev, 2003; Zhang et al., 2015b). Sarao et al. (1998) found particularly that the azimuthal distribution and the position of the stations in the direction of rupture propagation are more important than merely the number of stations. Iida et al. (1990), on the other hand, argued that it is more important to have circular distributed stations with well-spaced azimuthal coverage. The type of data being used also controls the resolution of the rupture patch, as various data sets are sensitive to different aspects of the rupture process (e.g., Delouis et al., 2002).

1.2.1 Source Inversion Validation Project

To further understand and examine the variability in earthquake rupture model, ‘Source Inversion Validation’ project (SIV; Page et al., 2011; Mai et al., 2007) has been initiated. This project is an online cooperation (http://equake-rc.info/sivdb/) in which different exercises are available for scientists to perform. The exercise is designed to tackle major issues in earthquake source inversion such as the uncertainty and the ability of simple
inversion to recover physically computed models. It also allows scientists to test their source inversion techniques. One of the exercises, for instance, consider a crack-like spontaneous dynamic rupture to test the ability of source inversion techniques to retrieve the macroscopic source properties as well as the spatio-temporal evolution of the rupture process. We use this particular exercise in Chapter 2 to explore the effects of variability in source-time function and one-dimensional Earth structure variability in the inferred finite-fault models.

1.2.2 Importance of Realistic Crustal Structure

In earthquake source inversion, the Green’s functions play a critical role because they are essential for computing synthetic seismograms. They depend on an assumed model that includes the geological structure, the elastic parameters, density, and attenuation parameters. A number of studies have been performed to extensively analyze the nature of ground motion due to earthquakes and the related seismic hazard (e.g., Wald & Graves, 1998; Olsen et al., 2009). Topography has been found to increase the amplitude of shaking. It produces a more complex wave field since seismic waves are reflected and scattered when the waves travel through the mountains (e.g., Imperatori & Mai, 2012). The existence of Basin also amplifies the seismic waveform due to the low wave-speed in the sediment areas (e.g., Mexico City; Bard et al., 1988; Chávez-Garcia & Bard, 1994). Therefore, incomplete knowledge of crustal structure could ruin the source inversion (e.g., Sarao et al., 1998; Das & Suhadolc, 1996).

Graves & Wald (2001) and Wald & Graves (2001) addressed the effect of incomplete knowledge of crustal structure. They analyzed the resolution of the finite fault source inversion using 1-D and 3-D Green’s functions. Graves & Wald (2001) notably concluded that only gross features of slip could be recovered in case of incomplete knowledge of velocity structure. Cohee & Beroza (1994) similarly indicated that, if the Green’s function fails to explain the wave propagation in the inversion, the source-inversion results might be expected to have significant errors. In Chapter 3 of this dissertation, we analyze the uncertainty related to the choice of crustal structure.
1.2.3 Statistical Ranking

As mentioned previously, inverted kinematic models are used as input for a number of studies, including dynamic rupture model, Coulomb stress changes as well as hazard analyzes. Understanding the uniqueness of these solutions remains challenging. Therefore, it is important to develop robust metrics to quantify model variability. This mainly helps to rank and classify rupture models. In chapter 4 of this dissertation, we present some benchmarks including the comparison of the published slip models of the 2011 Tohoku earthquake.

1.3 Objectives and Contributions

This thesis focuses on understanding the complexity of kinematic models and the corresponding uncertainties. The contributions of this thesis fall into the following categories:

- Quantify the uncertainty in earthquake source inversion and explore the effects of variability in source time functions (isosceles triangle, Yoffe with acceleration time of 0.1 s and 0.3 s) and one-dimensional Earth structure.

- Further understand the earthquake source complexity and the corresponding uncertainty through using more realistic crustal structures that are of particular importance for areas where evidence of site effects have been reported, for example, L'Aquila (Italy) and Christchurch (New Zealand).

- Develop a set of benchmarks and metrics to assess and rank rupture models. In doing so, we evaluate the role of different datasets and techniques in constraining rupture models.

1.4 Thesis Outline

Four subsequent chapters follow this introduction.

In Chapter 2, we explore the effects of uncertainty due to different source time function and one-dimensional Earth structure on kinematic source inversions. Using a Bayesian approach, we infer source parameters including peak slip rate, rupture time and rise time. We present the solution space in terms of posterior probabilities. We test three source
time functions - an isosceles triangle and two asymmetric Yoffe functions with different acceleration times, and analyze how the inferred solutions capture the target model. We subsequently explore the effects of uncertainty in Earth structure. We introduce variations in wave speed only, and in both wave speed and layer depth, in two different sets of tests. This chapter is adapted from a paper published in the Bulletin of the Seismological Society of America (Razaﬁndrakoto & Mai, 2014).

Chapter 3 is a work in progress aiming at the uncertainty analysis of the L’Aquila rupture model from one- and three-dimensional crustal structure. This work will be submitted for publication.

In Chapter 4, we examine the ability of statistical techniques to compare and classify earthquake rupture models, from which we formulate a similarity scale to rank them. As case studies, we use inverted slip models from one of the Source Inversion Validation exercise and on published slip models for the 2011 Mw 9.0 Tohoku earthquake (Razaﬁndrakoto et al. (2015); Zhang et al. (2015a)).

Chapter 5 summarizes and concludes the key findings presented in this PhD dissertation.
Chapter 2

Uncertainty in Earthquake Source Imaging due to Variations in Source Time Function and Earth Structure

The work within this chapter is related to the publication:

Abstract

One way to improve the accuracy and reliability of kinematic earthquake source imaging is to investigate the origin of uncertainty and to minimize their effects. The difficulties in kinematic source inversion arise from the nonlinearity of the problem, nonunique choices in the parameterization, and observational errors. We analyze particularly the uncertainty related to the choice of the source time function (STF) and the variability in Earth structure. We consider a synthetic data set generated from a spontaneous dynamic rupture calculation. Using Bayesian inference, we map the solution space of peak slip rate, rupture time, and rise time to characterize the kinematic rupture in terms of posterior density functions. Our test to investigate the effect of the choice of STF reveals that all three tested STFs (isosceles triangle, regularized Yoffe with acceleration time of 0.1 and 0.3 s) retrieve the patch of high slip and slip rate around the hypocenter. However, the use of an isosceles triangle as STF artificially accelerates the rupture to propagate faster than the target solution. It additionally generates an artificial linear correlation between rupture onset time and rise time. These appear to compensate for the dynamic source effects that are not included in the symmetric triangular STF. The exact rise time for the tested STFs is difficult to resolve due to the small amount of radiated seismic moment in the tail of STF. To highlight the effect of Earth structure variability, we perform inversions including the uncertainty in the wavespeed only, and variability in both wavespeed and layer depth. We find that little difference is noticeable between the resulting rupture model uncertainties from these two parameterizations. Both significantly broaden the posterior densities and cause faster rupture propagation particularly near the hypocenter due to the major velocity change at the depth where the fault is located.

2.1 Introduction

An improved understanding of earthquake physics and modern seismic-hazard assessment is a key requirement for reducing earthquake risks. In this context, robust kinematic source models are needed that are realistic representations of the slip history of an earthquake on a
fault and provide information on the spacetime evolution of ruptures. In the 1980s,\cite{ Olson & Apsel 1982} and \cite{Hartzell & Heaton 1983} developed the linear multi-time window inversion method to estimate rupture speed and slip amplitude on the fault. In this approach, the slip history at each point of the fault is presented as a superposition of several elementary slip functions, which allow the point to slip more than once. Applying this multi-time window approach, \cite{Sekiguchi & Itawa 2002}, for example, analyzed the rupture process of the 1999 Kocaeli earthquake, and \cite{Zhou et al. 2004} examined the 1999 Chi-Chi earthquake; this latter study also included resolution analysis of the source history. Previous works revealed that this approach is capable of capturing details of source complexity \cite{Wald & Heaton 1994, Wald & Graves 2001, Graves & Wald 2001}. More recently, \cite{Lee et al. 2011} applied this technique and detected an episode of slip reactivation during the rupture propagation of the 2011 Tohoku-Oki earthquake.

As computational resources increased, seismologists started to consider the full non-linear formulation of source inversion without linearization. In this case, a predefined analytical source time function (STF) is required, and one estimates the space-time rupture evolution in terms of slip velocity, rupture time, slip duration, and rake angle at each point of the fault plane. This non-linear formulation may help to better understand the underlying dynamic rupture process \cite{Tinti et al. 2005b}. Optimization strategies such as simulated annealing (e.g., \cite{Hartzell et al. 1996, Bouchon et al. 2000, Delouis et al. 2002, Liu & Archuleta 2004}), neighborhood searches (e.g., \cite{Vallée & Bouchon 2004}), and genetic algorithms (e.g., \cite{Emolo & Zollo 2005}) have been applied to estimate the slip-rate, rise time, rupture time and rake angle that characterize the kinematic rupture model. These techniques consist of iterative approaches in which random or new generations of rupture models are created for each iteration. The reliability of the model is then assessed based on the fits of the corresponding synthetics with the observed data. Once convergence is achieved, all models that fit the data within some chosen limit can be accepted. These studies rely only on one model that fits the data best, or consider a limited number of models to compute a corresponding mean model.

However, a single model is never fully representative of the model space in a non-linear
inverse problem. Therefore, even though macroscopic parameters such as seismic moment and average displacement may be almost identical in all models for the same earthquake, the variability of slip on the fault may be significant. For example, kinematic models for the 1999 Izmit earthquake (Delouis et al., 2002; Bouchon et al., 2000; Sekiguchi & Itawa, 2002; Clévedé et al., 2004) obtained using various data sets, making alternative assumption on the source parameterization, and deploying different inversion algorithms clearly illustrate this non-uniqueness and the resulting variations in the final rupture models. Hence, without appropriate uncertainty assessment of each of these models, it is difficult to examine their reliability, that is, their common and stable features, as well as the limits of resolution.

Custodio et al. (2005) and Hartzell et al. (2007) used the 2004 Parkfield to show how the resulting source model depends on chosen data and assumptions such as the misfit function, and size of fault plane. Their findings call for rigorous and quantitative analysis of uncertainty to better understand the main factors that lead to variability in the models. Uncertainty analyses were also carried out for the 1989 Loma Prieta earthquakes by Hartzell et al. (1991) and Emolo & Zollo (2005) who defined a Gaussian probability density function around the best fitting model obtained through a genetic algorithm to estimate kinematic source models. Piatanesi et al. (2007) performed an uncertainty estimation from statistical analysis of the ensemble of models generated by an optimization algorithm, while Monelli & Mai (2008) proposed the use of a Bayesian inference technique to estimate the model uncertainty by mapping posterior probability density functions (PDFs) of source parameters. Producing PDFs of the model parameters defines the admissible solution space more comprehensively.

In this study, we extend the approach of Monelli et al. (2009) by incorporating the spatial correlation between neighboring nodes and using the regularized Yoffe function (Tinti et al., 2005a) as STF, that is compatible with dynamic rupture simulations. Additionally, we account for the 'epistemic' uncertainty (e.g., Abrahamson & Bommer (2005)) which is the uncertainty associated with inadequate knowledge about physical assumptions regarding a specific model under investigation. Among the different sources of uncertainty related to source inversion, we highlight the 'epistemic' uncertainty associated with Earth’s crustal
structure. We present results using synthetic tests developed in the context of the Source Inversion Validation exercise (SIV; Page et al. (2011); Mai et al. (2007)). The reference model stems from a spontaneous dynamic rupture calculation that assumes random initial stress on an 80° dipping strike-slip fault, with 90° strike, producing a right-lateral strike-slip event with a seismic moment of $M_0 = 1.06 \times 10^{19}$ Nm. We use the corresponding three-component velocity waveforms at 40 well-distributed local sites (Figure 2.1) to conduct the inversion; seismograms are filtered using a Butterworth bandpass in the range of 0.01 to 1 Hz. With this test, we assess the ability of kinematic source inversion to retrieve the physical aspects of the rupture process from a dynamic rupture simulation.

Figure 2.1: Source-station geometry. The data consists of waveforms at 40 stations (grey squares). The black star denotes the epicenter, the black line marks the surface projection of the 80°-dipping fault.

2.2 Formulation of Kinematic Source Imaging

We use the representation theorem in the formulation of Spudich & Archuleta (1987) to infer kinematic rupture parameters. The ground velocity recorded at the Earth surface is expressed as a convolution of a local STF with the corresponding tractions on the fault,
written in the frequency domain as:

\[
\hat{u}_k(y, w) = \int \int \hat{s}(x, w) T_k(x, w; y, 0) d\Sigma \tag{2.1}
\]

in which \(\hat{u}_k(y, w)\) is the Fourier transform of \(u_k(y, t)\), the \(k\)-th component of ground-velocity at an observation point \(y\); \(\hat{s}(x, w)\) is the Fourier transform of \(s(x, t)\), the slip-rate function at a point \(x\) on the fault; \(T_k(x, w; y, 0)\) is the Fourier transform of the traction vector at a point \(x\) on the fault caused by a point impulse force in the \(k\) direction at the observer location \(y\) and angular frequency \(w\). Note that this formulation obeys the reciprocity theorem, meaning that the location of source and observation points can be switched, and the exact same response will be observed. Assuming an appropriate Earth structure in the source region, we compute the traction, \(T_k\), applying a frequency-wavenumber integration method (Spudich & Xu [2002]).

In the framework of inverse theory, the representation theorem in Eq. (2.1) can be cast as \(d = g(\theta)\), where \(d\) is the data composed of the observed (velocity) waveforms, \(\theta\) contains the parameters that define the earthquake source properties (e.g., peak slip-rate, rise time, rupture onset time and rake), and \(g\) is an operator that describes the Earth structure response. The purpose in this inverse problem formulation is to infer the source parameters \(\theta\), given synthetic data \(d\) and the calculated Earth response \(g\).

One may use any optimization technique to obtain one or more models with minimum misfit. However, several models with very different source-parameter distributions may fit the data equally well (Mai et al. [2007]). This limitation of the optimization can be overcome using a Bayesian technique (Monelli & Mai [2008]; Monelli et al. [2009]) that infers posterior probability density functions to characterize the ensemble of all possible parameters (peak slip-rate, rise time and rupture time) describing the STFs \(s\) that are consistent with the data and the available prior information. This technique consists of a stochastic process in which the major sources of uncertainty include the data noise, the improper knowledge about Earth structure, the fault parameterization, and the intrinsic randomness from the inversion process. To understand the effects of these factors and to eventually minimize
them, this study introduces variability in the Earth structure and the assumed STF in the uncertainty analysis.

2.2.1 Kinematic Source Parameterization

We assume that rupture occurs on a planar fault with known length and width; for real earthquakes, these parameters can be estimated based on geodetic information and aftershock distributions, albeit with considerable uncertainty. However, the determination of fault geometry and its associated uncertainty is beyond the scope of this study and will be left for a future extension of this work. The fault plane is discretized into different nodes. The smaller the cell size, the more detailed information we may obtain on the rupture process. However, the data cannot resolve a cell size finer than the smallest wavelength corresponding to the used frequency. In this study, we use a frequency range up to 1 Hz corresponding to wavelength of 3-3.5 km in the upper crustal layers ($z < 18$ km) of the given Earth model. The shortest wavelength determines the finest useful grid spacing on the rupture plane. Therefore we choose a grid spacing of 3 km along strike and 4 km along dip. Inside each cell, source parameters (peak slip-rate, rise time, rupture onset time and rake) are assumed to be constant. The final slip on each node is not inferred, but calculated from the chosen STF. In our parameterization, the total number of parameters is then 195.

In this study, we test the effects of two different STFs, an isosceles triangle and the regularized Yoffe function. The peak slip-rate corresponds to the height of this function; rise time is defined as its duration, and the rupture onset time is the time when a particular point on the fault starts to slip. For the regularized Yoffe STF, an additional parameter needs to be considered; the duration of slip acceleration (acceleration time $T_{acc}$). We choose $T_{acc}$ deterministically, and invert for peak slip velocity, rise time, and rupture onset time.

2.3 Bayesian Inference

In a Bayesian approach, the purpose is to quantify the posterior distribution that maps the solution space. It is defined as the probability density of model parameters $\theta$ given data
Following Bayes’ theorem, the posterior PDF, $\pi(\theta|d)$, is obtained by combining prior probability distribution, $\pi(\theta)$, on model parameters with the likelihood $f(d|\theta)$ of observing these data given the model parameters $\theta$:

$$
\pi(\theta|d) = \frac{\text{likelihood}}{\pi(d)} \frac{\text{prior probability}}{\pi(\theta)} = \frac{f(d|\theta)}{\int f(d|\theta) \pi(\theta) \, d\theta}
$$

In Eq. 2.2, $\pi(d)$ is the evidence, or marginal likelihood, which is independent of the model $\theta$. Therefore, we can consider it as a normalization constant $k^{-1}$, and Eq. 2.2 can be written as

$$
\pi(\theta|d) = k f(d|\theta) \pi(\theta)
$$

We use an iterative stochastic Markov Chain Monte Carlo approach to generate samples of $\theta$ from a proposed distribution. To search the vast space of potential kinematic rupture models, we apply the Metropolis algorithm that produces a chain of random models in which each candidate only depends on the previously generated model. Each candidate model is accepted as a new model based on the acceptance probability. The procedure of the Markov Chain Monte Carlo based on the Metropolis algorithm works as follows:

(a) Draw initial state $\theta$ in model parameter space

(b) Generate a candidate sample of $\theta'$, denoted $\theta'$, from a proposal distribution $q(\theta'|\theta)$, a Gaussian centered at the current value of $\theta$ in our case. The step size of the Markov chain is proportional to the variance of $q(\theta'|\theta)$ and tuned according to the rejection rate. Too large a step size, for example, leads to many rejections. We choose the step size for the peak-slip-rate, rupture time, and rise time as 2.5 cm/s, 0.05 s, and 0.05 s, respectively, leading to acceptance rates of about 30%, in agreement with suggested acceptance rates of 30 - 50%.

(c) Accept or reject the candidate $\theta'$ based on the acceptance probability defined as
\[ \alpha(\theta, \theta') = \min \left[ 1, \frac{f(d|\theta')}{f(d|\theta)} \right] \]  

(2.4)

(d) Draw a uniform random number \( u \in [0, 1] \); if \( \alpha(\theta, \theta') > u \) accept the candidate sample, \( \theta = \theta' \); if \( \alpha(\theta, \theta') < u \), reject the candidate sample (keep the current state).

(e) Repeat the process to generate the next samples.

The samples obtained through this algorithm approximate the posterior distribution \( \pi(\theta|d) \) (Hoff, 2009). To minimize the effect of initial values and autocorrelation among samples, we discard an initial portion of the Markov Chain, and consider only every \( p \)th sample (thinning the chain). The choice of \( p \) is arbitrary, and different numerical experiments are needed to choose \( p \) appropriately. In our case, we select \( p = 100 \), as larger value of \( p \) lead to a smaller number of samples to infer the posterior, and the marginal posterior becomes non-smooth. On the other hand, smaller \( p \) returns nearly the same posterior, showing that the chain length has a larger effect on the posterior than does the thinning interval \( p \) (Arakawa et al., 2009).

2.3.1 Prior Distribution

The prior distribution incorporates our \textit{a priori} knowledge about the model parameters. We use a uniform distribution for each parameter, assuming that no further information is available aside from the range of possible values:

\[ \pi(\theta) = \frac{1}{\theta_{\text{max}} - \theta_{\text{min}}} \quad \text{for} \quad \theta_{\text{min}} \leq \theta \leq \theta_{\text{max}}, \]  

(2.5)

in which \( \pi(\theta) \) is the prior density, and \( \theta_{\text{min}} \) and \( \theta_{\text{max}} \) are the limits of the sample space for the model parameter \( \theta \) (see Table 2.1). They are chosen based on physical considerations for peak slip-rate and the temporal parameters (rupture time and rise time). Furthermore, for rupture time we define the range based on the rupture speed such that we allow the rupture to propagate at subshear and at intersonic speed (Archuleta, 1984; Dunham & Archuleta, 2004). However, we do not permit rerupturing of the fault. We choose the rupture speed...
Table 2.1: Search ranges for peak slip-rate, rise time, and rupture speed.

<table>
<thead>
<tr>
<th>Case</th>
<th>Peak slip-rate (cm/s)</th>
<th>Rise time (s)</th>
<th>Rupture speed (km/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle Yoe-0.1</td>
<td>0 - 400</td>
<td>0 - 6</td>
<td>0.5 - 10</td>
</tr>
<tr>
<td>Yoe-0.3†</td>
<td>0 - 200</td>
<td>0 - 3</td>
<td>0.5 - 10</td>
</tr>
</tbody>
</table>

* Yoffe source time function with acceleration time of 0.1 s
† Yoffe source time function with acceleration time of 0.3 s

between 0.5 and 2.0 times the shear wavespeed at the shallowest and the deepest layers of the fault zone, respectively, which correspond to a rupture speed between 1.5 km/s and 7.1 km/s.

Along the edges of the assumed rupture plane, the range for slip is chosen such that slip occurs with a maximum possible peak slip-rate of 200 cm/s, which is about half of the value at the inner nodes. This choice is more realistic than in Monelli & Mai (2008) and Monelli et al. (2009) who did not allow this area to slip at all. Monelli et al. (2009) also showed that there is a trade-off for the peak slip-rate at neighboring points, characterized by a strong anticorrelation. Hence, considering them as totally independent parameters may be problematic. Therefore, we take into account the spatial correlation between neighboring patches (Jónsson et al., 2002) using a two-dimensional spatial Laplacian filter, respecting the distance between patches and the boundary of the fault plane (see Appendix A).

Table 2.2: Spatial cross-correlation coefficient between the reference and each inverted model.

<table>
<thead>
<tr>
<th>Model</th>
<th>Peak slip-rate</th>
<th>Slip</th>
<th>Rise time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle</td>
<td>0.71</td>
<td>0.74</td>
<td>0.60</td>
</tr>
<tr>
<td>Yoffe-0.1</td>
<td>0.83</td>
<td>0.84</td>
<td>0.76</td>
</tr>
<tr>
<td>Yoffe-0.3</td>
<td>0.72</td>
<td>0.73</td>
<td>0.80</td>
</tr>
</tbody>
</table>

* Yoffe source time function with acceleration time of 0.1 s
† Yoffe source time function with acceleration time of 0.3 s
2.3.2 Likelihood Function

The likelihood function is the output of the forward modeling and is used to assess how well the model explains the data. It can be written as

\[ f(d|\theta) = c \exp[-S(\theta)], \tag{2.6} \]

in which \( c \) is a normalization constant and \( S(\theta) \) quantifies the misfit. Assuming the data uncertainty is Gaussian, we can write

\[ S(\theta) = \frac{1}{2}[(g(\theta) - d)^T C_D^{-1} (g(\theta) - d)] \tag{2.7} \]

in which \( g(\theta) \) is the forward modeling operator, \( d \) is the data, and \( C_D \) is the covariance matrix of data errors. The expressions in Equations (2.3) and (2.7) show that the covariance matrix of data errors directly determines the form of the posterior, and therefore is crucial in a Bayesian approach. However, in earthquake seismology, quantifying data errors is difficult as different sources (instrumental and processing noise) need to be included. To overcome this problem, Monelli et al. (2009) propose an empirical formulation for the likelihood function. In contrast, Yagi & Fukahata (2008) and Bodin et al. (2012) follow a Hierarchical Bayesian approach in which all parameters that define the data error are considered as unknown hyperparameters (Gelman et al., 1995). Here, we adopt the empirical formulation of the likelihood function (Monelli et al., 2009) as

\[ f(d|\theta) = c \exp[-\phi(\theta)] \tag{2.8} \]

\[ \phi(\theta) = \frac{S(\theta) - S(\theta_{\text{best}})}{S(\theta_{\text{best}})} \times 100, \tag{2.9} \]

in which \( S(\theta_{\text{best}}) \) represents the fit of the best model \( \theta_{\text{best}} \), obtained using an optimization based on the Evolutionary Algorithm Beyer (2001), and \( S(\theta) \) is the misfit function in Equation (2.7). Incorporating the spatial constraint of the parameters at neighboring nodes, the likelihood function is expressed using the penalized \( L_2 \) norm on \( S(\theta) \):
\[ S(\theta) = \|d - g(\theta)\|_2 + \alpha^2 \|D\theta\|_2, \]  

(2.10)

in which D is the Laplacian operator, and \( \alpha \) is the parameter that controls the smoothness of the model, which can be estimated deterministically from a trade-off curve.

### 2.3.3 Parameterization of Data Covariance Matrix

Following Gouveia & Scales (1998) and Tarantola (2005), the data covariance matrix \( C_D \) (Equation 2.7) represents the combination of observational errors \( C_{obs} \) and forward modeling uncertainty \( C_{theory} \):

\[ C_D = C_{obs} + C_{theory} \]  

(2.11)

Application of this formulation in seismic source inversion is described in Duputel et al. (2012). Following the representation theorem (Eq. 2.1), the kinematic source model depends on the choice of an Earth model. However, generally we do not have a complete knowledge of the true velocity structure. For this source of modeling error, we define the covariance matrix \( C_{theory} \) as the covariance describing the uncertainty in Earth structure. By computing the variability in the synthetics obtained from the combination of the best model, \( m_{best} \), and variations on crustal models (Figure 2.2), we deduce the data covariance that we insert in Equations (2.7) and (2.9).

Two cases are studied to include uncertainty of crustal structure: (a) accounting for wavespeed uncertainty only, and (b) accounting for simultaneous variations in wavespeed and layer depth. Mooney (1989) proposes an uncertainty of 3% to 4% for the seismic velocities in global crustal models. However, to be conservative on a local scale when considering heterogeneity near the surface and the trade-off between Moho depth and the crustal velocity, we assume a standard deviation of 10% for the wavespeeds near the surface and around the Moho. In all cases, we maintain the ratio \( \frac{V_p}{V_s} \) around 1.73, the standard value for rocks without significant fluid content. For layer depths, we consider a standard deviation of 10%.
Figure 2.2: Reference 1D crustal models with two different types of uncertainties. Solid black line and dashed black line show the reference S and P wavespeed, respectively, the grey lines around them correspond to random samples of $V_P$ and $V_S$ drawn from a normal distribution with 10% standard deviation. (a) wavespeed variability only (b) wavespeed and layer depth variabilities.

2.4 Modeling Results

We present here the result of the inversion using the data in the SIV exercise [Page et al., 2011; Mai et al., 2007]. The reference model (Figure 2.3a) stems from a spontaneous dynamic rupture simulation, with heterogeneous initial stress conditions and a constant slip weakening distance in the inner part of the fault plane. Rupture occurs over a rectangular fault of about 36 km x 16 km with a slip patch around the hypocenter, between 10 to 14 km depth, extending about 20 km along strike, and with a maximum of 180 cm. The peak slip-rate distribution presents two patches, with maximum values of about 250 cm/s near the hypocenter and at 12 km distance from the hypocenter in the strike direction.

We assess the quality of the synthetics based on the variance reduction [Cohee & Beroza, 1994]:

$$VR = \left(1 - \frac{(d - g(\theta))^T C_D^{-1} (d - g(\theta))}{d^T C_D^{-1} d}\right) \times 100\%,$$

(2.12)

where $g(\theta)$ is the modeling results, $d$ is the data, $C_D$ is the covariance matrix of data errors,
and superscript $T$ denotes transpose.

Using the Metropolis algorithm, we produce 600,000 samples of rupture models, each of which consisting of 65 node points at which we estimate the kinematic source parameters. Discarding an initial portion of the Markov chain and considering every 100th sample, this leads to a total sample size of 5500 to build the posterior PDF. As we incorporate the variability in crustal structure and the source time function, including two different types of Earth structure variabilities (composed of 11 Earth structures each) and three different source time functions (an isosceles triangle and two regularized Yoffe functions), respectively, we produce about $5500 \times 22$ (Earth structures) $\times 3$ (STF) rupture model samples to understand these variabilities. We then examine the posterior PDFs and the corresponding statistical estimates to assess the accuracy of the resulting source parameters at different points on the fault. We also show the 2D marginal posterior PDFs, to visualize the correlation between parameters at the same location, and between neighboring points for the same parameter.

### 2.4.1 Source time function variability

In this section we present inversion results using the input 1D crustal model (Figure 2.2) without crustal structure uncertainty. This model is used to compute the traction $T_k$ (Eq. 2.1) required in our formulation of kinematic source imaging. The posterior PDFs in this part are generated using the empirical likelihood function (Eq. 2.9) which requires the level of fit corresponding to the model that best fits the data.

In Figure 2.3 we present the optimal models obtained from an Evolutionary Algorithm (Beyer, 2001; Monelli et al., 2009), using as STF an isosceles triangle and a regularized Yoffe function. For the Yoffe function, we tested different values of acceleration time, 0.1 and 0.3 s, respectively, following Tinti et al. (2005a) who suggested for $T_{acc}$ a value between 0.1 and 0.38 s. All models show high slip and slip-rate patches around the hypocenter with maximum values between 300 and 400 cm/s for the peak slip-rate, and 100 - 200 cm for the calculated slip, respectively. For the Yoffe function with $T_{acc} = 0.1$ s (Yoffe-0.1), we find the peak slip-rate patch extends about 12 km along strike. However, for both the isosceles
triangle STF and the Yoffe function with \( T_{\text{acc}} = 0.3 \) s (Yoffe-0.3), a more compact region of high slip-rate patch is inferred, with an extension of about 6 km along strike. In terms of rupture time, there appears to be a discrepancy between source models, showing that the rupture propagates faster for the isosceles triangle than for the regularized Yoffe (see contours on final slip panels, Figure 2.3).

To examine the resolution of each inverted slip model and how it compares to the reference model, we calculate the spatial cross correlation between reference (Figure 2.3a) and inverted models, defined as

\[
\rho = \frac{\sum_{j=1}^{n} R_j I_j}{\left[ \sum_{j=1}^{n} R_j^2 \sum_{j=1}^{n} I_j^2 \right]^{1/2}} \tag{2.13}
\]

where \( n \) is number of points on the fault, \( R_j \) and \( I_j \) represent the reference and inverted rupture models, respectively, at one point \( j \) on the fault (e.g., Shao & Ji (2012); Graves & Wald (2001)). The value of \( \rho \) could vary from 0 (no correlation) to 1 (best correlation). We apply this formulation to the maps of peak slip-rate, slip, and rise time (see Table 2.3), and find that the Yoffe-0.1 rupture model is most similar to the reference model, with slip and peak slip-rate correlations of 0.83 and 0.84, respectively. The isosceles triangle and Yoffe-0.3 models have similar ability to reproduce the feature of the reference model, however, with lower spatial cross correlation, 0.74 and 0.73 for the slip correlations, and 0.71 and 0.72 for the peak slip-rate correlations. However, in terms of rise time, Yoffe-0.3 model is most similar to the reference model with cross correlation of 0.80 compared to 0.60 and 0.76 for the isosceles triangle and Yoffe-0.1 models, respectively.
Figure 2.3: (a) Reference (top row) and inverted rupture models using an evolutionary algorithm assuming different source time functions: (b) isosceles triangle, (c) Yoffe with acceleration time of 0.1 s, (d) Yoffe with acceleration time of 0.3 s. Arrows in the left column indicate slip direction and amplitude. Contour lines in the right column mark the rupture time distribution (contour-interval: 1 s).
Figure 2.4: Amplitude differences between the reference and inverted model parameters (peak slip-rate, rise time, and slip amplitude). The black contour-line limits the area of the fault used in the error analysis.

Table 2.3: Spatial cross-correlation coefficient between the reference and each inverted model.

<table>
<thead>
<tr>
<th>Model</th>
<th>Peak slip-rate</th>
<th>Slip</th>
<th>Rise time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle</td>
<td>0.71</td>
<td>0.74</td>
<td>0.60</td>
</tr>
<tr>
<td>Yoffe-0.1</td>
<td>0.83</td>
<td>0.84</td>
<td>0.76</td>
</tr>
<tr>
<td>Yoffe-0.3</td>
<td>0.72</td>
<td>0.73</td>
<td>0.80</td>
</tr>
</tbody>
</table>

* Yoffe source time function with acceleration time of 0.1 s
† Yoffe source time function with acceleration time of 0.3 s

In Figure 2.4, we investigate the discrepancies between reference and inverted rupture models in terms of amplitude differences of source parameters at individual point on the fault. For this error analysis, we do not consider the area of zero slip in the reference model (Figure 2.3a), the region into which the dynamic rupture did not propagate. In Figure 2.4, this area is marked with a black contour line. Figure 2.4 shows that the residuals are not entirely random, but follow certain trends. The peak slip-rate residuals for the isosceles
triangle and Yoffe-0.3 show similar pattern, with overpredicting around the hypocenter and underpredicting away from the hypocenter. However, the peak slip-rate residual for Yoffe-0.1 shows a different pattern, with higher slip rates over almost the entire rupture area. The discrepancies of peak slip rate near the hypocenter are related to the spontaneous nucleation of the dynamic rupture simulation, which is difficult to resolve by inversion. The medians of the ratio of reference to inverted peak slip rate over the entire rupture area are 1.23, 1.03, and 0.66 for the isosceles triangle, Yoffe-0.3, and Yoffe-0.1, respectively.

To unambiguously define rise time for the three different STFs used, we use the time to accumulate 5% - 95% of the total slip in this error analysis. The rise time residual appears to be overestimated only near the edge of the rupture and underestimated over most of the fault. This originates from the almost negligible seismic radiation in the tail of the dynamic STF, which is difficult to constrain using a simple analytical STF. Finally, for the slip amplitude, the residuals for the three STFs show similar shape, with large residuals along the edge of the area of zero slip. The medians of the ratio of reference to inverted slip are 1.21, 0.98 and 1.30 for the isosceles triangle, Yoffe-0.1 and Yoffe-0.3, respectively.

Next, we compare the STF of the dynamic reference model and the inverted solutions using the three different STFs at eight selected points (Figure 2.5). When peak slip-rate is low (less than 2 m/s) in the reference model, the imaged slip-rate pulse occurs later for all STFs, with a delay of 0.5 to 2 s. Conversely, in cases when peak slip-rate is greater than 2 m/s, the peaks of the three STFs occur at around the same time as in the reference model. We attribute this observation to the difficulty of resolving the temporal rupture pattern, demonstrating the non-uniqueness of the problem, as these rupture times are all acceptable values when examining the posterior PDFs at these points on the fault. Part of this uncertainty originates from the trade-off between neighboring points of the fault. As including spatial smoothing between neighboring points, with a smoothing parameter $\alpha^2$ of 20, generally reduces this time difference of the peaks, particularly for Yoffe-0.1 close to the hypocenter and at some points for triangle STF (see Appendix A).

Figure 2.6 presents the level of fits of the synthetics obtained from the best fitting models with the data at a selection of 12 stations. At all stations, the predictions fit the data well,
Figure 2.5: (a) Reference rupture model with eight selected points on the fault. (b) STF at eight points on the fault, for the reference model (black), and from the inversion results, using as STF the isosceles triangle (red), Yoffe-0.1 (blue), and Yoffe-0.3 (green).
with an average variance reduction of 93.9, 94.2, and 94.1% for the triangle, Yoffe-0.1, and Yoffe-0.3, respectively. To check the dependence of these best models on the initial state of the random model parameters, we run the evolutionary algorithm using different initial states, and find that the high slip and slip-rate patches around the hypocenter are maintained for the generated rupture models.

Figure 2.7 shows posterior PDFs for peak slip-rate, rise time, rupture time and the calculated slip, at two points of the fault (point four and five in Figure 2.5) located at 5 km and 12 km from the hypocenter, for the three different STFs. They reveal that the characteristics of the uncertainty vary from point to point, and they generally do not follow a Gaussian distribution due to the non-linearity of the kinematic source imaging problem. The peak slip-rate PDFs close to the hypocenter (4 in Figure 2.5), for example, show distributions that are skewed toward higher values for the three STFs, with a larger uncertainty for Yoffe-0.3. Farther from the hypocenter, on the other hand, the PDFs become broader, following nearly uniform distributions. Therefore, the PDFs at those two points suggest that the accuracy of source parameters decreases with increasing distance from the hypocenter.

Another interesting aspect of these PDFs is that the rupture for the triangle propagates faster than for the Yoffe (see PDFs of rupture time, Figure 2.7). This feature of rupture behavior originates from the sensitivity of ground motion to peak slip-rate. In this case, to obtain the correct timing for the peak slip-rate, the rupture time needs to be earlier due to the symmetric form of this STF. The slip-rate functions on points 3 and 5 (Figure 2.5) nicely illustrate this process. At these specific points, the rupture times are artificially earlier by about 0.8 s (or about 25%) compared with the Yoffe function, and the target model. This incorrect timing has the impact of overestimating the rupture speed, which is one of the key parameters needed to better understand earthquake source physics.

Figure 2.8 and Figure 2.9 present the maximum, mean, and median posterior models for the triangle and Yoffe-0.1, respectively. These statistical estimates are extracted from the posterior PDFs. At each node, we find the kinematic source parameters according to the maximum, mean, and median independently of the other node. We then assemble the composite to obtain the model estimates in Figure 2.8 and Figure 2.9. These figures show
Figure 2.6: Level of fits produced by the best model for isosceles triangle (thin black line), Yoffe-0.1 (light grey), and Yoffe-0.3 (dark grey) with the synthetic data (thick black line). The maximum velocity for the synthetic record (cm/s) for each waveform is shown below each black trace, the variance reduction for each modeled seismogram is given at the end of the traces.
Figure 2.7: The marginal posterior density for peak slip rate, rise time, rupture onset time, and slip at two points on the fault located (a) 5 km from the hypocenter and (b) 12 km from the hypocenter (indicated as points 4 and 5, respectively Figure 2.5. We assume as STF the isosceles triangle (black), Yoffe-0.1 (light gray), and Yoffe-0.3 (dark gray).

that all models capture the main pattern of the rupture, characterized by high slip and slip-rate patches around the hypocenter (compare to Figure 2.3a). However, the roughness of the models and the extension of the patches change between the various estimates. In addition, the rupture time PDFs are skewed in the case of the triangle STF solutions, such that the inferred rupture times for the maximum extracted from the posterior become earlier than for the mean/median, implying faster rupture propagation.

Table 2.4: Spatial cross-correlation coefficient between the reference and the model estimates.

<table>
<thead>
<tr>
<th>Model</th>
<th>Peak slip-rate</th>
<th>Slip</th>
<th>Rise time</th>
<th>Peak slip-rate</th>
<th>Slip</th>
<th>Rise time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum</td>
<td>0.72</td>
<td>0.61</td>
<td>0.55</td>
<td>0.88</td>
<td>0.78</td>
<td>0.80</td>
</tr>
<tr>
<td>Mean</td>
<td>0.77</td>
<td>0.67</td>
<td>0.76</td>
<td>0.85</td>
<td>0.77</td>
<td>0.85</td>
</tr>
<tr>
<td>Median</td>
<td>0.77</td>
<td>0.64</td>
<td>0.73</td>
<td>0.86</td>
<td>0.77</td>
<td>0.84</td>
</tr>
</tbody>
</table>

* Yoffe source time function with acceleration time of 0.1 s
In addition, we statistically compare the estimated models with the reference model computing the 2D spatial cross correlation between source parameters \cite{Mai2007}. As listed in Table 2.4, we find that for triangle STF, the maximum posterior model has the lowest cross-correlation for the peak slip-rate, slip, and rise time (0.72, 0.61, and 0.55, respectively). On the other hand, the mean and median models have comparable cross-correlation values. They both have a correlation of 0.77 for the peak slip-rate, 0.64 and 0.67 for the slip, and 0.73 and 0.76 for the rise time and have higher cross-correlation value compared to the maximum model. For Yoffe-0.1 on the other hand, the three model estimates (maximum, mean, and median) have similar ability to represent the reference model (see Table 2.4), and with larger values than for the triangle STF. Therefore, the Yoffe STF is a better representation of the dynamic rupture behavior. As we assess the level of fits of the synthetics with respect to the synthetic data, we find that among these estimates, the median posterior for the triangle STF explains the data best, with a variance reduction of 93% compared to the maximum and mean which respectively give 66% and 87%. The seismograms of the maximum posterior shows differences in timing, due to the fast rupture. For the Yoffe function (Figure 2.9) on the other hand, the overall pattern of the source models are similar and the fits vary from 89% to 92%. The levels of fit is consistent with the cross-correlation comparison, as the higher the variance reduction, the higher the cross-correlation coefficient. Therefore, both analyses show that the Yoffe STF more accurately captures the dynamic rupture behavior.

Figure 2.10 displays the marginal distribution of the seismic moment generated from the posterior source models and considering the three different STFs. The maximum values of the seismic moment marginals for isosceles triangle, Yoffe-0.1, and Yoffe-0.3 are about $1.1 \times 10^{19}$ Nm, $1.3 \times 10^{19}$ Nm, and $1.6 \times 10^{19}$ Nm, respectively. The triangle STF best fits the seismic moment of the reference model, while the Yoffe-0.1 and Yoffe-0.3 overestimate the seismic moment by about 18% and 40%.

In terms of 2D marginal distributions, Figure 2.11 presents the joint PDFs of rise time and rupture time for the isosceles triangle and Yoffe STF at the same eight points as in Figure 2.5. In general, the farther away from the hypocenter the node is located, the stronger
the trade-off between parameters. This can be understood from the fact that the problem is less ill-posed near the hypocenter as rupture is constrained to be close to the hypocenter at the beginning of the earthquake. Therefore, the inversion converges easier toward the true model. Later in the rupture, the increasing complexity of parameters combination makes things more complicated and decrease the accuracy of the source parameters.
Figure 2.8: (a) Maximum, (b) mean, and (c) median estimates extracted from the posterior for an isosceles triangle STF. (d) Levels of fit produced by these three estimates, shown at eight stations for the maximum (green), mean (blue), and median (red). Synthetic data are shown in black lines.
Figure 2.9: (a) Maximum, (b) mean, and (c) median estimates extracted from the posterior for Yoffe-0.1. (d) The levels of fit produced by these three estimates, shown at eight stations for the maximum (green), mean (blue), and median (red). Synthetic data are shown in black lines.
Figure 2.10: Marginal posterior PDFs for the seismic moment assuming as STFs the isosceles triangle (black), Yoffe with Tacc of 0.1 s (light grey), and Yoffe with Tacc of 0.3 s (dark grey). The black square marks the seismic moment of the reference solution.

A possible explanation the behavior of the inversion in this test case can be given using the isochrone concept (Bernard & Madariaga, 1984; Spudich & Frazer, 1984). The isochrone consists of all points on the fault from which the radiated seismic energy arrives at a specific station at the same time. The radiated energy is significant in areas with large slip, high isochrone velocity, and/or a strong change in isochrone velocity. Therefore, large slip near the hypocenter would radiate more strongly than the same amount of slip away from the hypocenter (Schmedes & Archuleta, 2008). In our test case, the energy is released mainly from the area around the hypocenter, where the slip and the isochrone velocity are highest. Moreover, the isochrones around the hypocenter are in general quite different from station to station. Hence, the source parameters in this area can be well constrained. In contrast, away from the hypocenter, the isochrones become shorter with roughly the same spacing (velocity), and they do not change significantly from station to station (Schmedes & Archuleta, 2008). These lead to a wider range of plausible values for the source parameters and hence to a decrease in the accuracy when inferring these source parameters, in particular when slip is low farther away from the hypocenter, as in this example.
Figure 2.11: 2D marginal distributions between rise time and rupture time on the eight points shown in Figure 5 for (a) an isosceles triangle STF and (b) the Yoffe-0.1 STF.
We also observe an interesting feature of the 2D marginals for the triangle STF (Figure 2.11a). The rupture time and rise time are negatively correlated; that is, larger values of rupture time are expected for smaller rise times. Therefore, by constraining the rupture time for a triangle STF, the data also constrain the rise time. To the best of our knowledge, this implicit constraint of temporal parameters for triangle STF has never been described before. This originates from the fact that the ground motions are most sensitive to the time of the peak slip velocity. This feature was also reported in Goto & Sawada (2010), and by Oglesby & Mai (2012) who analyzed slip-rates and waveforms from rupture dynamics and related ground motion. This negative correlation between temporal parameters for the triangle STF still appears when considering the spatial correlation between neighboring points. In contrast, for the Yoffe function this trade-off is not observed because of the antisymmetric shape of this function.

2.4.2 Crustal Model Variability

In this section we analyze the impact of 1D crustal structure variability on the rupture model uncertainty. In Figure 2.12, we show the posterior PDFs of the source parameters at two points of the fault generated using a single Earth model and the two types of crustal model uncertainties (Figure 2.2), with uncertainties in wavespeeds only and with variability in both wavespeeds and layer depths. A significant change appears in the resolution of the parameters, including that velocity uncertainty broadens the posterior PDFs with some shift in peak location. This is consistent with the findings of Graves & Wald (2001), which show how the use of the incorrect Earth model reduces the resolution of the source image. Our results quantify this loss of resolution (blurring effect) in terms of changes in the PDFs of kinematic parameters. Near the hypocenter, the posterior PDFs for the slip and peak slip rate becomes broader but maintains the peak value. The PDFs for the rise time, on the other hand, is nearly uniform. For the rupture time, the distribution appears to be shifted; the rupture tends to propagate faster if the Earth model uncertainties are included. Because the arrival time of each phase of the seismogram is related to both travel time and rupture time, the distortion of the travel time due to the changes in Earth structure
Figure 2.12: Posterior probability density function of slip rate, rise time, rupture onset time, and slip at two points on the fault located, (a) 5 km from the hypocenter and (b) 12 km from the hypocenter. Results are shown without crustal structure variability (gray), with variations in wavespeed (black dashed line), and with simultaneous variations in wavespeed and layer depth (black line).

ultimately affects the estimated rupture time. In addition, because most of the variations in shear-wave velocity occur in the third layer (where the fault is located) and are mostly smaller than the reference wavespeed, the resulting wave propagation is slower. Therefore, to compensate for this slower shear-wavespeed effect, the rupture velocity needs to be higher leading to earlier rupture arrival. On the other hand, away from the hypocenter, the PDFs of the resulting source parameters are mostly uniform, hence we cannot constrain the source parameters over this area of the fault.

Figure 2.13 displays the spatial variations of source parameters in terms of the posterior median for the two crustal model uncertainties. The result shows that despite the uncertainties in the Earth model, the inversion still retrieves the high slip and slip-rate features near the hypocenter, with values comparable to the reference model (Figure 2.3a). However, the patch is more extended towards the surface for both types of crustal model uncertainty. Additionally, slip on the fault boundary, which generally is not well constrained, appears to be larger than in previous results. Comparing the cross-correlation coefficient for the median model with and without Earth structure variability (see Table 2.5), we find that
Figure 2.13: Median extracted from the posterior PDFs for Yoffe-0.1 STF and including variability in the 1D crustal structure. (a) Wavespeed variability only; (b) wavespeed and layer depth variabilities.

the correlation becomes lower, by 20% and 10% for the peak slip-rate and slip, respectively, as we incorporate the crustal variability.

Comparing the results for the two crustal model variabilities, we do not find significant discrepancies for the posterior PDFs and the extracted source parameters (with a comparable value of correlation, as displayed in the Table 2.5), mainly because the major differences between the two crustal model variabilities appears at depth greater than 15 km. Only a small fraction of the rupture is located in this region, and thus it has only a small effect on the inferred source models.

Table 2.5: Spatial cross-correlation coefficient between the reference and the median posterior models for Yoffe-0.1, including 1D crustal structure variability.

<table>
<thead>
<tr>
<th>Case</th>
<th>Peak slip-rate</th>
<th>Slip</th>
<th>Rise time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without uncertainty</td>
<td>0.86</td>
<td>0.77</td>
<td>0.84</td>
</tr>
<tr>
<td>Wave speed only</td>
<td>0.68</td>
<td>0.70</td>
<td>0.85</td>
</tr>
<tr>
<td>Wave speed and layer depth</td>
<td>0.69</td>
<td>0.70</td>
<td>0.85</td>
</tr>
</tbody>
</table>
2.5 Discussion

In this study, we apply a Bayesian inference approach to obtain a kinematic rupture model from synthetic near fault ground motion, generated from a spontaneous dynamic rupture calculation. The principal advantage of this technique compared to using only an optimization technique, is its ability to not only solve the non-linear inversion, but to also provide a quantitative assessment of the uncertainty. Instead of obtaining one single optimal model, it yields an ensemble of plausible models represented in terms of posterior PDF. We can then assess the resolution of rupture models by analyzing the posterior distributions. Broad posterior distributions mean for example that the source parameters are not well constrained. Additionally, to learn more about the ensemble of models, we can extract statistical information regarding the posterior distribution at each point on the fault independently.

In Figure 2.8, for example, we present three models, maximum, mean, and median of the posterior PDFs from isosceles triangle. These estimates are not always well representative of the full solution space as they are just representations of a statistical model. They are calculated by combining the estimated values from the PDFs at each individual point of the fault. Hence, the combination of these selected source parameters does not in general represent a single rupture model that has been tested during the inversion stage and found to fit the data. Instead, these are independently generated rupture models that may not fit the data as well as any of the inverted solutions. We may expect these three estimates to be similar, reflecting the center of the distribution, only if the posterior distributions are Gaussian. However, if the distribution is broad, skewed, or contains several peaks, the maximum posterior is not preferable, as the peak is not aligned with the bulk of the distribution. For distributions that are not Gaussian, the mean and median do not well represent the central tendency of the distribution, which explains the high values on the boundary of the fault. In this part of the fault, the source parameters are not well resolved, and posterior distributions are nearly uniform. These characteristics of the extracted statistical models from the posterior PDFs are also reported in [Minson et al. 2013].

We also test the effects on the inversion when using different STFs, an isosceles triangle
and two regularized Yoffe functions with different acceleration times. The fits to the data of the synthetics produced by these rupture models give a variance reduction around 94%, lower than reported by [Shao & Ji (2012)], who reported a variance reduction of about 99% the initial blind test of the source inversion validation exercise. The reason for this difference is that the true model in our case is based on a dynamic rupture simulation, whose temporal properties are difficult to recover in detail by a kinematic source inversion [Konca et al. 2013]; the initial blind test featured a simple kinematic rupture as reference rupture model. We also find that the use of an isosceles triangle causes two main issues. First, it generates an artificial rupture acceleration, which has important implications for earthquake physics, as it would overestimate the rupture speed. Second, the use of an isosceles triangle generates artificial linear correlation between rupture time and rise time, by constraining the rupture time for a triangle STF, the data also constrain the rise time. The former problem could be remedied by a multitime window inversion, whereas it is not clear at present how the rise time/rupture time correlation would manifest itself in this case.

Finally, away from the hypocenter, the distribution for all kinematic source parameters becomes nearly uniform (which means the accuracy of the parameter decreases) as the point is located away from the hypocenter. Investigating the rupture process of the 2000 Tottori earthquake, [Monelli et al. (2009)] also found that the source parameters are well resolved around the high slip patch located above the hypocenter and poorly resolved elsewhere. These two observations suggest the source parameter resolution is related to the amount of seismic energy released at different parts of the fault and to the position of these strongly radiating regions with respect to the hypocenter and the observational network. The stronger the seismic radiation is, the better resolved the source parameters should be at this specific area of the fault. The radiated energy from the slip patches close to the hypocenter should be large [Schmedes & Archuleta, 2008], hence the source parameters in this area will be more accurately resolvable than the same slip patch located far away from the hypocenter. The type of data being used also controls the resolution of the rupture patch, as various data sets are sensitive to different aspects of the rupture process.

One crucial element to achieve a robust rupture model is adopting a realistic Earth
model. In this study, the Greens function was first calculated using a single 1D Earth model assuming perfectly known Earth structure. We then include the variability in Earth structure as part of the modeling error in the Bayesian inference, to mimic the realistic case in which we do not know the Earth structure precisely. This allows us to capture the uncertainty in wave propagation that in turn affects the resolution of the kinematic model. Our result (Figure 2.12) shows that the use of inaccurate Earth structure significantly reduces the resolution of the rupture model parameters, consistent with the study of Graves & Wald (2001). This approach to include Earth structure variability allows the capture of uncertainty of the Green’s functions Yagi & Fukahata (2011). However, these are still approximate representations of the Earth structure. Therefore, an extensive analysis of the rupture model uncertainty adopting a 3D Earth model that includes the site effects and topographic scattering deserves an extensive exploration. This will particularly help in improving the rupture model resolution, which has important implications for near-fault ground-motion prediction and seismic-hazard assessment.

Another factor that controls the source inversion result is the fault parameterization. In this study we adopt a planar fault of 36 km by 16 km, discretized based on the choice of maximum frequency. This particular choice is reasonable for the synthetic case considered. However, for real earthquakes, different modelers may use different fault geometry, which are likely to affect the result. For some cases, too-simplistic fault geometry might not be well representative of the local tectonics. On the other hand, using very complex fault geometry may induce other complications for the source parameter resolution due to the increase of free parameters and rupture complexity at fault segment boundaries. It is therefore worthwhile to extend this Bayesian approach by including uncertainty in the fault parameterization, as part of the modeling error. This choice of fault parameterization is even more challenging for an earthquake early warning purpose, as we may not have any prior information about the fault geometry. Therefore, the source geometry needs to be solved simultaneously with the rupture models. As an example, Minson et al. (2013) recently use Bayesian inference to find the optimal fault geometry and the distribution of possible slip models for that geometry, and then apply the approach to the 2011 Tohoku-Oki and the
2003 Tokachi-Oki earthquakes.

2.6 Conclusions

Kinematic source models are obtained considering the full non linearity of the inversion problem. We use a Bayesian technique to obtain the posterior probability function of each source parameter on a regular grid representing the fault. We find that a symmetric triangle STF compensates missing dynamic constraints by adding an artificial correlation between rise time and rupture time. Incorporating spatial smoothing reduces the skewness of the posterior distribution, as this constrain reduces the trade-off of the parameters at neighboring nodes. To avoid biasing the result due to the assumption on an Earth model, we propose incorporating Earth model variability in the inference of source parameters. Our results indicate that including the uncertainty in Earth structure broadens the PDFs of the source parameters, and may shift the location of the peak rupture time. Rupture time is the parameter that is most strongly affected both by variations of the STF and crustal structure.

To further explore the effect of Earth structure variability, one should (1) consider the implication of the choice of the inverted data frequency range on source model uncertainty, and (2) analyze the use of 3D Earth structure, which includes site effects and topographic scattering. Additionally, despite our tests of different acceleration time $T_{\text{acc}}$ for the regularized Yoffe STF, an inappropriate choice on $T_{\text{acc}}$ may create artifacts in the source models (Tinti et al., 2009). Therefore, the inference and effects of this parameter needs further exploration. Additional work on exploring the uncertainty due to assumption of fault plane complexity and fault parameterization is also warranted, as in general they are chosen differently by different research teams. Several studies for example propose different ways of choosing grid spacing (Bernard et al., 1996 Emolo & Zollo, 2005 Page et al., 2009). A natural extension of our work would consider a transdimensional inversion (Bodin et al., 2012) in which the data itself are used to define the model parameterization and hence dictate the overall uncertainties.
Chapter 3

Estimating the rupture process of the 2009 L’Aquila earthquake

The work within this chapter is a paper in preparation:

Abstract

Published finite-fault rupture models for the Mw 6.3 L’Aquila earthquake reveal considerable intra-event variability. One potential source of this variability arises from the non-unique choice of crustal structure. This earthquake occurred in an area of complex geology, including a small sedimentary basin and pronounced topography. Therefore, the use of a one-dimensional crustal structure may be insufficient to infer the earthquake rupture process accurately.

One way to understand this variability is to examine the effect of crustal structures in the inferred model. In this study, we use both one- and three-dimensional Earth structures in the inversion. Bayesian inference is applied to assess the characteristics of the space-time rupture evolution quantitatively. Two cases are considered, a synthetic and a real case of the 2009 L’Aquila earthquake. We find that inaccurate crustal structure degrades the data misfit. However, no significant changes are observed in the source parameters and the corresponding uncertainty. The rupture model of the 2009 L’Aquila earthquake also shows slow rupture in the strike direction as other released models. Comparing the inferred model with the afterslip models, we find that afterslip region tends to occur adjacent to large coseismic slip areas, where the fault zone was loaded during the earthquake.

3.1 Introduction

The devastating Mw 6.3 L’Aquila earthquake occurred on 2009 April 6 at 01 : 32 UTC in Abruzzo, one of the most seismically vulnerable regions in Italy. This area is dominated by extensional stress regime related to subduction of Adria microplate, and Africa-Eurasia collision, which accommodates 2.5-3 mm/yr of NE-SW extension [D’Agostino et al., 2008]. Hence, small-to-moderate earthquakes are frequent in the region. Historical records indicate that severe earthquakes occurred in 1349, 1461, and 1703, which affected the area around L’Aquila [Guidoboni et al., 2012].

The 2009 L’Aquila earthquake caused more than 300 fatalities and rendered about 67 thousand people homeless. The hypocenter of this earthquake is located at 42.35°N,
13.38°E, and a depth of 9.0 km (see http://www.ingv.it/ National Institute of Geophysics and Volcanology). The corresponding seismic moment tensor solution indicates an NW-striking normal fault \cite{Pondrelli2010}. A number of studies investigated the main fault (location and geometry) responsible for this earthquake. \cite{Chiarabba2009} found that the aftershocks coincided with the Paganica fault trace and estimated a rupture length of 15-18 km, dipping 45° to the SW between 2 and 10 km depth. \cite{Walters2009}, on the other hand, used InSAR data and found rupture length of 12 km and 19 km for uniform and non-uniform slip, respectively. They also suggest that the rupture happened along the Paganica normal fault, with strike angle of 140°-145° and dip of 45°-55° toward SW. Geological observations also indicated surface fractures and cracks along the Paganica fault, where the ground surface exhibited dip-slip displacements up to 10 cm \cite{Emergeo-Working-Group2009, Falcucci2009, Boncio2010}.

Several studies have reported the rupture process of the 2009 L’Aquila earthquake using various datasets and techniques. \cite{Atzori2009} and \cite{Walters2009} used geodetic data and inferred one single 0.6-0.9 m of slip located at a depth of 7 km. \cite{Cirella2009} employed a two-stage nonlinear technique considering both strong motion and GPS data. They inferred two slip patches, one of which is small located up-dip of the hypocenter, and another larger and deeper (7-11 km) situated southeastwards. Despite the moderate size of the earthquake, \cite{D’Amico2010} used teleseismic data and a back-projection method to study the source. They identified two slip patches with a rupture propagating toward the south, and then up-dip toward the east. Applying the multiple finite-extent inversion methods with strong motion data, \cite{Gallovic2012} inferred three major slip patches: two located up-dip the hypocenter and the third one much deeper in the southeast. \cite{Gallovic2015} used ground motion and GPS data, and found a large-slip asperity southeast of the hypocenter.

In these source studies, uncertainty analyses are not typically included. Even though, nonlinear ill-posed problem such as finite fault source inversions warrant explicit assessment of the associated uncertainty. This observation is quite apparent from the spatial variability in the inferred rupture parameters of the 2009 L’Aquila earthquake. One potential source
of this variability arises from the non-unique choice of the crustal structure. This event occurred in an area of complex geology, including a small sedimentary basin and pronounced topography. Therefore, use of a one-dimensional description of crustal structure may be insufficient to infer the earthquake rupture process accurately. A number of studies attempt to incorporate this complex crustal structure: Cirella et al. (2009) used two different regional structures, one at stations where significant site effects are observed, and another one at the remaining stations. Trasatti et al. (2011) and Gallović et al. (2015) considered the effects of topography and crustal heterogeneities to retrieve the rupture process. The present study complements these efforts by mapping the rupture model uncertainty. We adopt a Bayesian inversion to estimate the posterior distribution function of kinematic source parameters and to identify the well-resolved part of the fault.

This paper addresses the effects of crustal structure variability in the rupture process. We consider synthetic and real cases for the 2009 L’Aquila earthquake. We particularly focus on the uncertainties related to one- and three-dimensional Earth structures that are used to compute Green’s functions. In doing so, we evaluate the influence and advantages of using more realistic crustal structure in resolving the rupture model parameters.

3.2 Data and Model Parameterization

The 2009 l’Aquila earthquake was recorded by a number of seismic networks and the data are available at \url{http://itaca.mi.ingv.it/}. In this study, we consider three-component velocity waveforms from 12 seismic stations (see Figure 3.1). Four of the stations (EX9, E12, E14, and E16) are hypothetical ones used for the sensitivity test purposes.
Figure 3.1: Source-station geometry. The data consists of 8 seismic stations (blue triangles) and 4 synthetic stations (black triangles). The yellow star denotes the epicenter, the black line marks the surface projection of the 50°-dipping fault.

These stations are located within 50 km epicentral distance, at different elevations (varying from 568 to 1072 m) and have distinct soil properties (soft soil or rock sites). They were chosen to highlight the crustal structure and topographic effects on the finite-fault inversion.

3.2.1 Green’s Function

We are aiming to solve for the source parameters, using recorded data and information on the crustal structure response. In finite-fault inversion, realistic crustal structure is required since the amplitude and phase of the seismic waves will be perturbed by the presence of basins, strong lateral heterogeneities or steep topography. Therefore, oversimplification of crustal structure could potentially affect the estimation of the source parameters.

To highlight the impact of the crustal structure on the inferred source parameters,
Figure 3.2: Cross sections of the shear velocity structure between points A and B in Figure 3.1: (top) one dimensional; (bottom) three-dimensional (Gallović et al., 2015).

we compute Green’s functions and subsequently the synthetic seismograms using one- and three-dimensional velocity structures for the L’Aquila area (Figure 3.2). The one-dimensional structure was proposed by Ameri et al. (2012) and consists of a layered medium in which velocity changes only with depth. The three-dimensional structure, on the other hand, was proposed by Di Stefano et al. (2011) and includes both topography and crustal structure complexity. These two structures have also been used in the study of Gallović et al. (2015). The response of the medium was generated using a finite-difference wave propagation code (WPP; Nilsson et al., 2007). Then, we extracted the synthetics through its convolution with the local source-time function. We filtered both generated synthetics and observed waveforms using the same filter, a second order Butterworth bandpass in the frequency range of 0.01 Hz - 0.5 Hz.
Table 3.1: Search ranges (model space) for slip, rise time, and rupture speed, and rake.

<table>
<thead>
<tr>
<th>Case</th>
<th>Inner nodes</th>
<th>Nodes at boundary</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slip (cm)</td>
<td>0 - 400</td>
<td>0 - 200</td>
</tr>
<tr>
<td>Rupture speed (km/s)</td>
<td>0.8 - 4.8</td>
<td>0.8 - 4.8</td>
</tr>
<tr>
<td>Rise time (s)</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Rake angle (°)</td>
<td>-90</td>
<td>-90</td>
</tr>
</tbody>
</table>

3.2.2 Fault Parameterization

We assume a planar fault with fixed length and width of 20 and 15 km. The fault is dipping 50° toward southwest, with a strike of 150°. The fault is discretized into rectangular elements of 1 km x 1 km. For each cell, we assume constant values of the source parameters. Since the grid size is smaller than the shortest wavelength corresponding to the chosen frequency, we incorporate spatial correlation between nodes. We consider a Brune source-time function at each point of the fault, and invert for slip and rupture onset time. In total, we infer at 300 nodes two source parameters (slip and rupture time). Table 3.1 shows the parameter ranges used in the prior.

3.3 Bayesian Inference

In Bayesian formulation, the inference is not limited to a single model, but rather it provides all the potential models that represent the data. These possible solutions are defined probabilistically in terms of probability density function. The Bayesian inversion is based on Bayes theorem, in which the posterior probability (\( \pi(\theta|d) \)) is obtained by updating the prior information (expressed in terms of probability, \( \pi(\theta) \)) of the source parameters \( \theta \) based on the data information \( d \). It is expressed as follows:

\[
\pi(\theta|d) \propto f(d|\theta) \pi(\theta) = \int f(d|\theta) \pi(\theta) d\theta
\]  

(3.1)

where \( \pi(d) \) denotes the evidence, or marginal likelihood, which is independent of the model \( \theta \), and can be considered as a normalization constant [Sen & Stoffa 1995]. \( f(d|\theta) \) is the
likelihood that quantifies the probability of how well the models represent the data. For a Gaussian error, it is expressed as follows:

\[
    f(d|\theta) = \frac{1}{\sqrt{(2\pi)^n|C_d|}} \exp\left\{ -\frac{1}{2}(g(\theta) - d)^T C_d^{-1} (g(\theta) - d) \right\}
\]

where \( n \) denotes the size of data vector \( d \), \( C_d \) is the data covariance matrix, \( g(\theta) \) is the forward modeling, and \( T \) denote the matrix transpose.

The prior was chosen as a uniform distributions for each parameter \( \theta \), and expressed as follows:

\[
    \pi(\theta) = \prod_\alpha (\theta_{\text{max}}^\alpha - \theta_{\text{min}}^\alpha)^{-1}
\]

where \( \theta_{\text{min}}^\alpha \) and \( \theta_{\text{max}}^\alpha \) are its minimum and maximum values.

The models are sampled with Markov Chain Monte Carlo (MCMC) based on Metropolis algorithm (Metropolis et al., 1953). This algorithm consists of generating a random walk of points distributed according to a required probability distribution. From an initial state, \( \theta \), draw a new proposal state, \( \theta' \), of the Markov chain from a proposal distribution \( q(\theta'|\theta) \), a Gaussian centered at the current state \( \theta \). The proposed state is either accepted or rejected based on the acceptance probability,

\[
    \alpha(\theta, \theta') = \min \left[ 1, \frac{f(d|\theta')}{f(d|\theta)} \right]
\]

This acceptance probability can be interpreted as follows, draw a uniform random number \( u \in [0, 1] \); if \( \alpha(\theta, \theta') > u \) accept the candidate sample, \( \theta = \theta' \); if \( \alpha(\theta, \theta') < u \), reject the candidate sample (keep the current state). We then thin each chain to retrieve independent samples.

### 3.4 Data covariance matrix

Equations 3.1 and 3.2 show that the form of the posterior is highly related to the nature of the data covariance matrix, which represents the data uncertainty. In earthquake seis-
mology, the characteristics of data noise are not entirely known. It includes instrumental, observational errors as well as theoretical errors. To address this issue, Monelli et al. (2009) use an empirical likelihood function, in which they first adopt optimization (evolutionary algorithm; Beyer, 2001) technique to retrieve the level of fit for an optimal model. Then, they use this information to define correlation function assigned to each model. Bodin et al. (2012) and Dettmer et al. (2012), on the other hand, use Bayesian in hierarchical formulation (Gelman et al., 1995). In this formulation, instead of fixing the variance of observational error, the characteristics of data noise are considered as unknown parameters. In this study, we address this lack of knowledge of data uncertainties through the empirical likelihood function of Monelli et al. (2009).

3.5 Modeling Result

First, we present the results using a synthetic case based on the source-station geometry of the 2009 L’Aquila earthquake (Figure 3.1). The synthetic data were obtained using the target rupture model in Figure 3.3 and the three-dimensional velocity structure (Figure 3.2). Then, we consider a real case using recorded waveforms. For the two cases, we assume that the rupture occurs over a rectangular fault of about 20 km x 15 km. To highlight the effect of different crustal structures, we infer the earthquake source parameters using both one- and three-dimensional precomputed Green’s functions.

Using Metropolis algorithm, we produce 600,000 samples for each parameter at each node. To minimize the initial values effect in the posterior inference, we discard the first 5000 iterations (burn-in period) of the Markov chain samples. We also thin the generated chain by extracting only the uncorrelated samples to ensure that the samples are independent. The autocorrelation are computed using Geyer initial monotone sequence estimator (Geyer, 1992). The remaining thinned samples are then used to approximate the posterior probability density function.

To facilitate the comparison of the uncertainties related to the choice of crustal structure in the inferred source parameters, we measure the shape of the posterior PDFs in terms
of skewness and Kurtosis values (see Figures 3.4). The skewness identifies the symmetrical shape of the distribution. Skewness of 0 indicates that the distribution is symmetric, whereas positive and negative values mean that most of the values tend to cluster toward the small and large values, respectively. The Kurtosis value, on the other hand, evaluates the sharpness of the distribution. Kurtosis of 1.8 indicates that the distribution is nearly flat (Uniform distribution). As this value increases, the distribution becomes sharper; Kurtosis of 3 means that the distribution is normal; and greater than 3 means sharp distribution.

Figure 3.3: (a) Target rupture model, with the inverted ones using (b) one-dimensional and (c) three-dimensional structures.
Figure 3.4: Shape measurements of various distribution: (a) Skewness and (b) kurtosis values.

3.5.1 Synthetic Case

Figure 3.3 shows the inferred rupture models applying the evolutionary algorithm (Beyer, 2001) and using seismic waveforms from 12 stations (see black and blue triangles in Figure 3.1). We adopt two different crustal structures: (a) Correct structure referring to the three-dimensional velocity structure used for generating the data; (b) incorrect structure consisting of the one-dimensional structure in Figure 3.2 (top figure). The overall slip patterns are found to be identical for the two crustal structures, with maximum slip around 145 cm. The major slip patches are located between 10 and 18 km along strike, similar as that for the target model. However, the inferred slip patches are more prolonged. For the rupture times, on the other hand, the inferred models are able to capture the sudden changes in the rupture speed at about 2 km from the hypocenter in the strike direction. Near the edge of the fault, the inferred rupture time tends to be overestimated. This overestimation is due to decrease in the accuracy of the source parameters away from the hypocenter (Konca et al, 2013; Razafindrakoto & Mai 2014). The seismic moments of the inferred models
using the correct and incorrect structure are $2.3 \times 10^{18}$Nm and $2.2 \times 10^{18}$Nm, respectively, compared to that of $2.5 \times 10^{18}$Nm for the target model.

To examine the resolution of inverted rupture models with respect to the target, we examine the amplitude differences between the target and inverted models (Figure 3.5). The differences are not entirely random. Overall, they follow the same pattern for the correct and incorrect structure, with slip overestimation near the edge of the fault and very close (1 km) to the hypocenter. The slip overestimation near the hypocenter could be related to restricted resolution due to the used frequency range. The small scale features above the hypocenter are thus too small to be captured. The residuals for the rupture time, on the other hand, show that the rupture time tends to be overestimated along the fault boundaries.

Figure 3.5: Amplitude difference between target and inferred rupture models.
The evolution of the model fitness from the starting model until the convergence (Figure 3.6) clearly shows that after convergence, the misfit values (normalized L2 norm) for the correct structure is smaller than that for the incorrect structure. We express the quality of the seismogram fits in terms of variance reduction (Cohee & Beroza [1994]) that can attain any value between 0 to 100% (best fit). Figure 3.7 presents the seismogram-fit of the inferred rupture model with respect to the synthetic data at each station. It reveals that the fits using the correct structure are, in general, better than that with incorrect structure. The inferred rupture models explain the data with overall variance reductions of 93% and 88% for the correct and incorrect structure, respectively.

Figure 3.8 depicts the characteristics of the inferred source parameters posterior PDFs at different points of the fault. In low slip areas (less than 50 cm), the distributions are skewed positively. This is due to the constraints imposed for the slip and temporal parameters; they have to be positive. We also find that posterior distributions are sharp near the hypocenter and the surface. As the point is located away from the hypocenter, the corresponding PDF becomes wider. For the rupture time, the posterior PDFs have kurtosis values of 3 or greater (sharp distribution) around the hypocenter and the high slip patch areas. The remaining locations are characterized by broad posterior distributions.
Figure 3.7: Levels of fit produced by the best model for the correct (blue) and incorrect crustal structure (red) with the synthetic data (black line). The maximum velocity for the synthetic record (cm/s) for each waveform is shown below each black trace, and the variance reduction for each modeled seismogram is given at the end of the traces. Waveforms are in the frequency range 0.01 to 0.5 Hz.
Figure 3.8: Shape measurements of the inferred posterior PDFs for (a) slip and (b) rupture time.

To examine how the choice of station distributions changes the inferred model. We perform additional inversions using 7 stations (blue triangles in Figure 3.1) considering the same crustal structures. Then, we compare the performance of the inverted slip distribution for different station-networks as well as crustal structures. Interestingly, we find that although smaller number of stations are used, the location of the major slip patch is still captured. However, the slip patch is dispersed over a wider area (Figure 3.9).

To quantify the (dis)similarity between the inverted models, we use a gray-scale metric and multidimensional scaling (MDS) method (Razafindrakoto et al., 2015). Figure 3.10 displays the derived MDS point-cloud for slip models with different structures and station-networks. The summary of the findings is shown in Table 3.2. The configuration reveals that slip model generated using both small number of stations and inaccurate structure (Model 5) falls into a ‘poor’ similarity level (greater than 40% dissimilarity with respect to the target model). The remaining models have between 20 and 40% dissimilarities (‘fair’). We also find that Model 3 (12 stations; incorrect structure) and Model 4 (7 stations; correct structure) are very similar to each other (less than 5% dissimilarity). Therefore, in this specific case, the structure inaccuracy or lack of data coverage have similar effect on the
inferred slip models. Both extend the major slip patch. By examining the alignment of the model along each axis, we find that the two principle sources of variability correspond to the shape/compactness of the patches (dimension 1) and their location (dimension 2).

![Figure 3.9](image)

**Figure 3.9:** (a) Target and inferred rupture model using 7 seismic stations, and considering (b) correct and (c) incorrect crustal structure.
Figure 3.10: MDS configuration of dissimilarity between inferred slip models using the gray-scale metric, centered at the reference model (Model 1). Circles limit different levels in the similarity scale, excellent (dark grey), good (grey), fair (light grey), and poor (outside the circles). The proportion of the dissimilarity explained by each configuration is quantified in the bottom right of each figure.

Table 3.2: Similarity of the inverted models with respect to the target model.

<table>
<thead>
<tr>
<th>Model</th>
<th>Number of stations</th>
<th>Structure</th>
<th>Dissimilarity (%)</th>
<th>Category</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>12</td>
<td>Correct</td>
<td>20 – 40</td>
<td>Fair</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td>Incorrect</td>
<td>20 – 40</td>
<td>Fair</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
<td>Correct</td>
<td>20 – 40</td>
<td>Fair</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
<td>Incorrect</td>
<td>&gt; 40</td>
<td>Poor</td>
</tr>
</tbody>
</table>

3.5.2 2009 L’Aquila Earthquake Model

For the real case of the 2009 L’Aquila earthquake, the optimal rupture models for both one- and three-dimensional structure (Figure 3.11) show two major slip patches, one located near the surface, up-dip the hypocenter. The second slip patch is located at about 6 km from the hypocenter, between 10 and 14 km depth, extending about 5 km, and with a maximum
of 160 cm. In terms of rupture time, it appears that the rupture propagates faster along dip than along strike (see contours on rupture time panel, Figure 3.11). Signature of super-shear were suspected during the nucleation of this event (Ellsworth & Chiaraluce 2009; Mello et al. 2010).

The inferred slip models yield seismic moments of $3.2 \times 10^{18}$ Nm and $3.6 \times 10^{18}$ Nm for the one- and three-dimensional structures, respectively. Similarly to the synthetic case, we find that the use of unrealistic crustal structure degrades the data fitting. The optimal models for one- and three-dimensional structures produce an overall variance reduction of 71 % and 73 %, respectively.

To validate our inferred model, we compare in Figure 3.12 the inverted rupture models (coseismic slip) and published post-seismic slips (Cheloni et al., 2014; Gualandi et al., 2014). The post-seismic slip consists of cumulative 307 days after-slip magnitude. The comparison shows that the main after-slip tends to occur adjacent to areas with large coseismic slip. This originates from the fact that the large slip area was locally driven by high stress drops. Therefore, high slip region became areas of low stress after the earthquake, whereas, the low slip regions, adjacent to high slip patches, underwent stress increase after the earthquake (negative static stress drop).

In addition, we examine the inferred source parameter posterior PDFs (See Appendix B). Figure 3.13 displays the characteristics of the posterior PDFs in terms of skewness and kurtosis values. Only small differences are noticeable between the resulting rupture model uncertainty from the two parameterizations of crustal-structure. They show sharp slip posterior distribution near the hypocenter. As the point is located away from the hypocenter, the distribution becomes broad. The posterior PDFs of the rupture time, on the other hand shows are sharp around the shallow slip patch up-dip the hypocenter. At the remaining points, the rupture time posterior PDFs are broad.
Figure 3.11: inverted rupture models using real data for (a) 1D and (b) 3D structure. (c) The corresponding seismogram fits for the 1D (blue) and 3D (red) structures with the observed data (black line). The maximum velocity for the synthetic record (cm/s) for each waveform is shown below each black trace.
Figure 3.12: Inverted rupture models using real data for 1D and 3D structure. The estimated post-seismic slip (Gualandi et al., 2014) is represented by the grey contour lines.

Figure 3.13: Shape measurements of the inferred posterior PDFs for (a) slip and (b) rupture time.
3.6 Discussion and Conclusions

In this study, we examine inferred rupture models for a synthetic case and the real of the 2009 L’Aquila earthquake based on one- and three-dimensional crustal structures. The change of crustal structure complexity has a minor effect on the earthquake source parameters. A potential explanation of this minor influence of the crustal structure complexity is due to low upper frequency. The major structural complexity effect on the waveform could already filtered out. Therefore, we could extend this study considering higher upper frequency. In this case, the calculation of the Green’s functions would be more computationally expensive.

For the real case, there are discrepancies in the shape as well as the extension of the slip patch near the surface for one- and three-dimensional structure. This is consistent with the results proposed by Ji et al. (2002) and Piatanesi et al. (2007), who found for 1999 Hector Mine and 2000 Tottori that only the near-surface slip estimate is strongly affected by the velocity structure variability. The deep feature of the rupture models are maintained despite the inaccuracy of the crustal structure. Degradation of the data fitting is also observed as we consider an inaccurate crustal structure, particularly for stations away from the earthquake source (Figure 3.14).

For the synthetic case, we find that there is high slip patch around the hypocenter, that is not present in the target model. It could be related to the contribution of the small scale slip patches above the hypocenter. These patches are too small to be resolved for the used frequency range (0.01 to 0.5 Hz). The inversion thus re-distributes them to locations that contribute to the same isochrone. It is also important to note that the data cannot resolve a cell size finer than the shortest wavelength corresponding to the highest frequency considered. In this study, the corresponding wavelength is about 5 km considering the upper crustal layers of the chosen crustal structure.

For both synthetic and real cases, the resulting posterior PDFs show that the accuracy of the source parameters decreases as the point is located away from the hypocenter. Another interesting aspect of these PDFs is that the posterior of rupture time appears to be sharp near the high slip patch areas. These observations suggest that ground motions have
different sensitivity to various source parameters.

For the real case of the 2009 L’Aquila earthquake, we observe slow rupture propagation along strike, as the rupture front reaches the high slip patch in depth. The second asperity, which is about 8 km away from the hypocenter, failed at about 4.5-5 s. This slow rupture could be explained by the fault complexity, such as presence of topography/roughness on the fault. This slow rupture propagation was also reported by previously published kinematic models (e.g., Cirella et al. 2009; Yano et al. 2014). They suggest that this is due to spatial heterogeneity of fault dynamic properties. Other studies also suggest the existence of fault segmentation, in which the rupture propagation slows down as it encounters the boundaries of asperities. As we compare our resulting rupture model with previously reported after-slip, we find consistencies with the concept of velocity-strengthening friction. After-slip areas tend to occur adjacent to large coseismic slip areas, where the fault zone was loaded during the earthquake.

3.6.1 Role of station weighting

The waveform fits in Figure 3.6 show that the stations with higher amplitudes, which are also the closest stations, have much larger variance reduction (VR > 85 %) compared to stations farther away from the fault. To better understand this feature, we analyze the spatial pattern of the variance reduction (Figure 3.14), as well as its variability with respect to the waveform amplitude (Figure 3.15). The variance reduction appears to increase exponentially with waveform amplitude. Therefore, the kinematic source inference is strongly influenced by the stations with larger amplitude close to the fault.
Figure 3.14: Spatial pattern of the variance reduction.
Figure 3.15: Variance reduction for different maximum velocity waveform amplitude.

To analyze the contribution of various stations in constraining the kinematic slip models, an additional inversion is performed using different weighting factors. The weighting factor has been chosen based on the waveform amplitude. The resulting rupture models show a decrease of the slip near the surface, as we down-weight the large amplitude stations (Figure 3.16). Therefore, the closest stations control the amplitude of the shallow feature. It is important to note that the feature near the surface is not significant (with slip amplitude less than 50 cm) for published models from geodetic data (e.g., Walters et al., 2009; Trasatti et al., 2011; Gualandi et al., 2014). Furthermore, Gallović et al. (2015) obtained similar results as the geodetic data inversion for strong smoothing factor (0.05 and 0.10 m). With smoothing factor of 0.01 m, they found slip patches near the surface, as other inference from strong motion data (e.g., Cirella et al., 2012). However, they argued that these shallow patches are artifacts due to both smoothing and the use of imperfect Green’s functions. It is important to note that Gallović et al. (2015) apply different weighting factor for different stations (1 for stations close to the fault and 3 for stations farther away from the fault).
Figure 3.16: Inverted rupture models using real data for 3D structure and (a) decrease (b) increase the weight of the stations close to the fault.

3.6.2 Influence of likelihood formulation on inferred rupture models

So far, the likelihood function used in the Bayesian inference consists of the empirical likelihood function which depends on the fitness of an optimal model from optimization technique. This problem set up may bias the inferred models, as the optimal model could vary depending on the chosen optimization algorithm. We hence performed additional inference using the standard Gaussian likelihood function (see equation 3.2). Despite the modification in the likelihood formulation, the overall spatial pattern of the shape measurement remain similar (Figure 3.17). We also find that minor variability appears on the shape of the inferred posterior PDFs when using 1D and 3D crustal structures. However, using the new likelihood function, the posterior distributions near the hypocenter become less skewed.
and with kurtosis around 3, compared with those from empirical likelihood (kurtosis around 5). This feature is expected because as we use a Gaussian likelihood function, the posterior distribution would be Gaussian-like distribution. Figure 3.18 illustrates the posterior of the slip at different points on the fault.

**Figure 3.17:** Shape measurements of the inferred posterior PDFs using new likelihood for (a) slip and (b) rupture time.

**Figure 3.18:** Example of the posterior PDFs inferred with and without optimal model (a) (x=6 km,y=5 km); (b) (x=15 km,y=6 km); (c) (x=10 km,y=8 km); (d) (x=6 km,y=15 km). Note the different y-axis scaling.
Chapter 4
Quantifying Variability in Earthquake Rupture Models using Multi Dimensional Scaling

The work within this chapter is related to the publication:

Abstract

Finite-fault earthquake source inversion is an ill-posed inverse problem leading to non-unique solutions. In addition, various fault parameterizations and input data may have been used by different researchers for the same earthquake. Such variability leads to large intra-event variability in the inferred rupture models. One way to understand this problem is to develop robust metrics to quantify model variability. We propose a Multi Dimensional Scaling (MDS) approach to compare rupture models quantitatively. We consider normalized squared and gray-scale metrics that reflect the variability in the location, intensity and geometry of the source parameters. We test the approach on two-dimensional random fields generated using a von Kármán autocorrelation function and varying its spectral parameters. The spread of points in the MDS solution indicates different levels of model variability. We observe that the normalized squared metric is insensitive to variability of spectral parameters, whereas the gray-scale metric is sensitive to small-scale changes in geometry. From this benchmark, we formulate a similarity scale to rank the rupture models. As case studies, we examine inverted models from the Source Inversion Validation (SIV) exercise and published models of the 2011 Mw 9.0 Tohoku earthquake, allowing us to test our approach for a case with a known reference model and one with an unknown true solution. The normalized squared and gray-scale metrics are respectively sensitive to the overall intensity and the extension of the three classes of slip (very large, large, and low). Additionally, we observe that a three-dimensional MDS configuration is preferable for models with large variability. We also find that the models for the Tohoku earthquake derived from tsunami data and their corresponding predictions cluster with a systematic deviation from other models. We demonstrate the stability of the MDS point-cloud using a number of realizations and jackknife tests, for both the random field and the case studies.

4.1 Introduction

Spatial and temporal analysis of rupture across a fault area is a useful tool for understanding the complexity of earthquake sources and the influence of such complexity on seismic
and tsunami hazard assessment. Since the 1980s, finite fault models have been developed to characterize the kinematics of the earthquake rupture process. These models are increasingly generated in an almost routine fashion and used in subsequent seismological research. For the purpose of earthquake early warning, Minson et al. (2014) even proposed real-time inversions for slip models of finite faults. Soon after an earthquake occurs, source studies are now able to provide corresponding rupture models based on different datasets (e.g., seismic waveform, GPS and/or InSAR data), based on alternative assumptions in the problem setup, and utilizing different inversion algorithms such as the multi-time window approach (e.g., Olson & Apsel 1982; Hartzell & Heaton 1983) or non-linear inversion with a predefined, analytical source-time function (e.g., Cotton & Campillo 1995; Liu & Archuleta 2004; Tinti et al. 2005b). Resulting rupture models often differ widely, although they typically all fit the data well. Kinematic source models of the 2011 Mw 9.0 Tohoku earthquake are prime examples of this variability.

The 2011 Tohoku event occurred off the Pacific coast of northeastern Honshu, Japan. Data from such well-recorded earthquake have been used in numerous source studies to capture the rupture process. These studies were based on various datasets including seismic data (e.g., Shao et al. 2011; Hayes 2011; Lay et al. 2011), geodetic data (e.g., Feng & Jónsson 2012), tsunami data (e.g., Fujii et al. 2011; Satake et al. 2013), or a combination of different types of datasets (e.g., Yue & Lay 2013; Simons et al. 2011). Most of the proposed rupture models suggest that the largest slip (over 50 m) was near the trench, although there are significant discrepancies regarding the spatial pattern of the rupture process. Such discrepancies eventually affect the evaluation of seismic and tsunami hazard (Goda et al. 2014). Without a rigorous way to compare slip models quantitatively, it is therefore difficult to assess their common and stable features, as well as the limits of their resolution.

Statistical techniques have been applied to characterize and quantify the complexity of rupture models (e.g., Somerville et al. 1999; Mai & Beroza 2002; Lavallée et al. 2006). Significant efforts have also been expended on assessing the uncertainties in finite-fault source inversions. Beresnev (2003) discussed the levels of uncertainties in source inversion.
To account for uncertainties, Piatanesi et al. (2007), for instance, performed a statistical analysis of large sets of inferred source models, while others used Bayesian techniques (e.g., Monelli & Mai 2008; Minson et al. 2013; Razafindrakoto & Mai 2014). In this context, the Source Inversion Validation (SIV; Page et al. 2011; Mai et al. 2007) project seeks to understand and quantify the variability in slip-model inversions. There have also been efforts to characterize the differences and similarities between rupture models. Shao & Ji (2012), for instance, used residual analysis and spatial cross correlation to quantify spatial heterogeneity. These classical quantifications, however, cannot classify and rank rupture models.

In this study, we adopt an embedding method based on multidimensional scaling (MDS) to compare rupture models quantitatively. This approach embeds the dissimilarities between all pairs of slip models in low-dimensional Euclidean space. Although the MDS technique has been widely applied in the medical, biological, and social sciences, very few studies have utilized this approach in earthquake seismology. Of these few, Dzwinel et al. (2005) used MDS to investigate earthquake patterns, whereas Yuen et al. (2009) used it in earthquake forecasting.

This paper develops a set of benchmarks and metrics that can help to assess and rank rupture models quantitatively. In doing so, we evaluate how different datasets and techniques constrain a rupture model. Our analysis is done in two steps. First, we test the performance of MDS on two-dimensional random fields generated using a von Kármán autocorrelation function, parameterized with different correlation lengths and/or Hurst parameters. In the second step, we conduct case studies on six inverted slip models from an SIV exercise and on 21 published slip models of the 2011 Mw 9.0 Tohoku earthquake. The SIV models were obtained using an identical dataset and essentially identical source geometry but with different inversion techniques, and they can be compared with a known reference solution. On the other hand, the models for the Tohoku earthquake were obtained using different inversion techniques, source parameterizations, and datasets, and they include some variations in the assumed fault geometry as well. In this case, a known reference solution does not exist.
4.2 Embedding Method

In this section, we present the embedding method based on MDS for comparing a set of 2D random fields. The essence of this technique is to reduce the spatial variability of random fields, and their corresponding differences, to points in a lower dimensional space. A key step of this technique is selecting appropriate metrics that are sensitive to various spatial properties of the random field.

4.2.1 Metrics

Dissimilarity can be loosely defined as a quantitative measure of how close two sets of variables (random fields, denoted $A$, $B$, $C$) are. A dissimilarity metric needs to fulfill three requirements, consisting of reflectivity ($d(A, B) = 0$ if and only if $A = B$), symmetry ($d(A, B) = d(B, A)$), and triangle inequality ($d(A, C) \leq d(A, B) + d(B, C)$). In this study, we consider normalized squared and gray-scale metrics owing to their sensitivity to the variability in features’ locations, intensities, and shapes. These two metrics detect complementary information. The normalized squared metric captures the magnitude of differences between two objects through point-by-point differences regardless of the position or intensity. It is computationally efficient and therefore commonly used for comparing images. The gray-scale metric, on the other hand, is more complicated. It requires transformation of the image to different intensity levels. It is computed based on the distance from grid points to sets of features with various intensities, instead of point-to-point distances used in the normalized squared metric. Hence, the gray-scale metric tends to detect differences in shape, as well as features with similar intensities. Despite its complexity compared with the normalized squared metric, the gray-scale metric has the advantage of gaining additional information on the spatial variability of 2D random fields. Wilson et al. (1997) presented particular examples for which these two metrics are able to identify specific features based on a ‘letters’ image. They found that the gray-scale metric is sensitive to differences such as removing the dot on the letter ‘i’, while the normalized squared metric is more sensitive to image translation.
For two random fields (or slip models) $A$ and $B$, the normalized squared metric is defined as the square of the difference of the two random fields divided by the mean of their individual squared values. It is expressed as a percentage (Kragh & Christie, 2002):

$$d_1(A, B) = 100 \frac{\sum_x [A(x) - B(x)]^2}{(\sum_x [A(x)]^2 + \sum_x [B(x)]^2)^{1/2}} = 200 \frac{\sum_x [A(x) - B(x)]^2}{\sum_x [A(x)]^2 + \sum_x [B(x)]^2}, \quad (4.1)$$

where $x$ denotes the grid-points on the rupture surface. This metric consists of point-by-point matching.

The gray-scale metric (Wilson et al., 1997), on the other hand, is an extension of the binary Baddeley metric (Baddeley, 1992) and is defined, for $1 \leq p \leq \infty$, as

$$d_2(A, B) = \left\{ \frac{1}{NG} \sum_x \sum_g |\Delta[(x, g), \Gamma_A] - \Delta[(x, g), \Gamma_B]|^p \right\}^{1/p}, \quad (4.2)$$

where $G$ presents the number of chosen gray levels, $g$, $N$ is the number of elements, $(x, g)$ is a point in set $S(\text{rupture surface}) \times G(\text{gray level})$, $\Gamma_A$ and $\Gamma_B$ respectively denote the subgraphs of random fields $A$ and $B$, which give a set representation of the fields in different gray-scale levels, and $\Delta[(x, g), \Gamma_A]$ is a distance function that represents the shortest distance between a point $(x, g) \in S \times G$ and the subgraph of $A$.

In this study, we choose the intensity levels following Mai et al. (2005) who defined the slip heterogeneity based on a large set of finite-fault rupture models. They found that earthquake rupture tends to start close to a region they defined as a large-slip area ($\frac{1}{3}U_{max} < U < \frac{2}{3}U_{max}$, where $U$ is the slip value and $U_{max}$ is the maximum slip). It then needs to encounter a very-large-slip area ($U \geq \frac{2}{3}U_{max}$) within half of the rupture length to grow into a large earthquake. These characteristics of a rupture are found to be consistent with the energy balance in the dynamic rupture process. Hence, based on these findings, we consider three color levels consisting of very-large-slip ($U \geq \frac{2}{3}U_{max}$), large-slip ($\frac{1}{3}U_{max} < U < \frac{2}{3}U_{max}$), and moderate to low-slip ($U \leq \frac{1}{3}U_{max}$) areas.

The metric $d_1$ is expressed as a percentage and its value is not limited within the range of 0% (best similarity) to 100%. The theoretical maximum is 200%, in which one of the
random fields contains only zeros. Hence, this metric can attain any value between 0% and 200%. However, $d_2$ is within the range of $(0, \infty)$ with 0 indicating the best similarity. To scale the two metrics similarly, we convert $d_2$ into a percentage as follows:

$$d_2(A, B) = 200 \frac{\left\{ \sum |\Delta[(x, g), \Gamma_A] - \Delta[(x, g), \Gamma_B]|^p \right\}^{1/p}}{\left\{ \sum |\Delta[(x, g), \Gamma_A]|^p \right\}^{1/p} + \left\{ \sum |\Delta[(x, g), \Gamma_B]|^p \right\}^{1/p}} .$$ (4.3)

### 4.2.2 Classical Multidimensional Scaling

In classical MDS, the purpose is to generate an $m$-dimensional configuration of $n$ points in Euclidean space based on the (dis)similarity of objects under investigation (2D fields in our case). Accordingly, we can then visualize and examine point configurations in a lower-dimensional representation that best preserves the distances (dissimilarities). The general procedure is as follows (Borg & Groenen, 2005):

1. Start with a matrix of metric distances, $D$, with elements $d_{ij}$ containing pairwise-computed dissimilarity values between all random fields (see equations 4.1 and 4.3).

2. Construct a matrix, $B$, from double centering matrix $D$, which consists of subtracting the row and column means of a matrix from its elements and adding the grand mean,

$$b_{ij} = -\frac{1}{2} [d_{ij}^2 - d_i^2 - d_j^2 + d_{ij}] .$$ (4.4)

The double centering is particularly important to make sure that the matrix is symmetric. It can also be obtained as follows:

$$B = -\frac{1}{2} HDH ,$$ (4.5)

where $H = I - \frac{1}{n} 11'$, $I$ represents the identity matrix, and $1$ is a vector with the value of unity in each of its cells.

3. Apply Singular Value Decomposition (SVD) to the symmetric matrix, $B$, by eigen-decomposition of $B$ into $VAV^T$, where $V$ is a matrix containing the eigenvectors
of $B$ and $\Lambda$ is a diagonal matrix whose diagonal elements, $[\lambda_1, ..., \lambda_n]$, represent the corresponding eigenvalues.

4. The coordinates of $n$ points in $m$-dimensional Euclidean space are then given by

$$y_{ij} = V_{ij} \lambda_j^{\frac{1}{2}} \quad i = 1, ..., n; \quad j = 1, ..., m$$

The $Y$ coordinates are constructed such that each column sums to zero (i.e., the origin of configuration $Y$ coincides with the centroid). The construction is invariant under rotations and reflections. If a reference model exists or can be defined, this can be used to define the origin. In this case, the point coordinates become

$$Y_s = (I - 1w)Y = P_wY$$

in which $Y_s$ are the new coordinates and $w$ controls the position of the origin. For instance, $w = [0, ..., 0, 1, 0, ..., 0]$ defines the origin at the position of 1. Likewise, selecting the centroid as the origin can be obtained using $w = [1/n, ..., 1/n]$. Therefore, instead of finding the SVD of $B$, it is computed for the matrix, $B_s$, defined as

$$B_s = P_wBP'_w$$

The dimension of the space of the derived coordinates ($Y$ or $Y_s$) is chosen by selecting the first $m$ eigenvalues. Then, the spatial patterns can be analyzed using the resulting point-clouds centered either at the centroid (mean model) or at any chosen reference model in $m$-dimensional space. The separation of points with respect to each other maps their percentage dissimilarity, because of the chosen normalization in equations 4.1 and 4.3. The point-cloud therefore clusters similar random fields based on any chosen metrics. The eigenvalues typically help in determining the number of dimensions, $m$, that are necessary to represent the dissimilarity matrix accurately. The sum of eigenvalues, $\lambda_j$, is the total variance in the dissimilarity matrix. Hence, individual eigenvalues expressed as a proportion of the sum of the eigenvalues yield the proportion of variance explained by each axis. The purpose is to select enough dimensions to appropriately capture the data. However, for
practical reasons, $m$ is typically restricted to $m = 2$ or $m = 3$. It would be possible to use $m = 4$ by making a 3D plot and coloring the points according to their percentage value. [Hair et al. (2010)] suggest a proportion of 60% as the minimum acceptable level accounted for by the approximated representation. We note that the actual physical meaning of each dimension, $m$, in an MDS configuration needs to be assessed based on the physical field under consideration and the chosen dissimilarity metric.

4.3 Comparison of 2D Random Fields

To test the performance of the MDS technique for comparing rupture models, we use 2D random fields generated using the von Kármán autocorrelation function [Goff & Jordan (1988)] belonging to the Matérn family of correlation functions [Guttorp & Gneiting (2006)] and defined as

$$C(r) = \frac{G(r)}{G(0)}$$

in which $G(r) = r^H K_H(r)$ is the covariance, $r$ is the distance, $H$ is the Hurst exponent, $K_H$ is the modified Bessel function of the second kind of order, $H$. The choice of this autocorrelation function is motivated by its flexibility, as it includes a range of autocorrelation functions (e.g., the exponential and Gaussian functions). Therefore, it is well suited for a variety of applications including the earthquake rupture process. It is also a commonly used class of autocorrelation functions in the spatial statistics community. In addition, by analyzing 44 published finite-source rupture models of 24 different earthquakes, [Mai & Beroza (2002)] found that the von Kármán autocorrelation function best describes the spatial characteristics of the rupture models.

An autocorrelation function can be converted into its power spectral density in the wave number domain and vice versa through Fourier transforms. Then, the corresponding spectral representation can be written as
\[ P(k) = \frac{C^2}{(1 + k^2)^{H+1}} \]  

(4.9)

where \( k \) denotes the wavenumber and \( C \) is the correlation length for an isotropic random field. Seed values are used to control the phase spectrum of the randomized von Kármán autocorrelation function, and inverse Fourier transform is applied to obtain the random field distribution in the spatial domain. The seed value therefore determines the spatial locations of high- and low-slip values as well as the large-scale characteristics of the random field. On the other hand, the small-scale details of the generated random fields are sensitive to the choice of \( H \) and \( C \). Small values of \( H \), for instance, generate highly heterogeneous 2D fields, while values of \( H \) close to unity result in smoother distributions. \( H \) is typically within the range \( 0 < H < 1 \), although in some examples we consider a value up to 1.5 for the purpose of extended sensitivity tests. The correlation length, \( C \), on the other hand, scales with the source dimension. Figure D.1 presents random field realizations for various correlation lengths and Hurst parameters.

We examine the MDS configuration and its sensitivity to different parameterizations for six random fields generated on a predefined plane, including: (a) variable \( H \), (b) variable \( C \) (4 to 19 km in steps of 3 km), (c) variable \( H \) and \( C \), and (d) variable \( H \), \( C \), and seed-number. The motivation of this test is to define and benchmark similarity scales using the normalized squared metric, \( d_1 \) and the gray-scale metric, \( d_2 \) (see equations 4.1 and 4.3). We define four categories or levels of similarity between the 2D random fields: ‘excellent’, ‘good’, ‘fair’, and ‘poor’, adopting quantitative comparisons that are similar to those developed by Kristeková et al. (2009) for seismic signals. In Table 4.1 we define the values of \( d_1 \) and \( d_2 \) for each category. ‘Excellent’, for instance, indicates that two random fields have a less than 5% dissimilarity, while ‘poor’ corresponds to a greater than 40% dissimilarity.
### Table 4.1: Similarity scale.

<table>
<thead>
<tr>
<th>Case</th>
<th>Excellent</th>
<th>Good</th>
<th>Fair</th>
<th>Poor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_1^*$ (%)</td>
<td>$&lt; 5$</td>
<td>$5 - 20$</td>
<td>$20 - 40$</td>
<td>$&gt; 40$</td>
</tr>
<tr>
<td>$d_2^\dagger$ (%)</td>
<td>$&lt; 5$</td>
<td>$5 - 20$</td>
<td>$20 - 40$</td>
<td>$&gt; 40$</td>
</tr>
</tbody>
</table>

* $d_1$, normalized squared metric
† $d_2$, gray-scale metric

#### 4.3.1 MDS Configuration

Figs. 4.1 and 4.2 show the MDS configuration using the normalized squared and gray-scale metrics, respectively. For the normalized squared metric, each 2D representation accounts for more than 90% of the variations. Hence, the choice of dimension $m = 2$ is sufficient to capture the dissimilarity between random fields. The point-cloud follows a similar pattern when we vary only the Hurst parameter or correlation length. It aligns with the axis corresponding to the largest eigenvalues (Dimension 1) and is centered at the mean model (Model 7). Figure 4.1c presents the case for varying correlation length. Among the six random fields, Fields 1 and 6 are most dissimilar to each other, but yet with a dissimilarity ($d_1$) of about 3%. These two random fields consist of the minimum ($C = 4$ km) and maximum ($C = 19$ km) spectral parameters, respectively. They also are most dissimilar with respect to the mean model, with $d_1$ of 1.5%. However, Model 3 with correlation length $C = 10$ km is closest to the mean model. Figure 4.1c also shows that for larger correlation lengths, the variability between two random fields with neighboring $C$-values decreases.

As we simultaneously change the two spectral parameters (correlation length and Hurst parameter), the six points no longer lie on a line. This can be understood from the fact that, by adding an additional source of variability, we increase the dimension of the dissimilarity representation. We also find that the points are located inside a circle centered at the mean model and with a radius of 5% if identical seed values are used and areas of high (low) values of the field occur at the same spatial location (Figures 4.1c and 4.1a). In this case, we find that the dissimilarity of each random field with respect to the mean model is less than 5%. This value falls into the ‘excellent’ similarity level (see Table 4.1). However, in Figure 4.1b,
Figure 4.1: MDS configuration of dissimilarity between six random fields using the normalized squared metric. (a) Variable Hurst parameter or correlation length; (b) variable Hurst parameter and correlation length; (c) variable Hurst parameter and correlation length with the features assumed to appear anywhere on the rectangular plane. Note the different axis scaling. Random field 7 corresponds to the mean model. The proportion of the dissimilarity explained by each configuration is listed in the bottom right of each figure.

we no longer constrain the seed value. The slip patch for the six random fields can thus be anywhere on the rectangular plane. This results in a widely spread point distribution that occupies a larger circle of radius 20%. We define this value of $d_1$ (see Table 4.1) as the limit between similarity level ‘Good’ ($d_1 = 5 - 20\%$) and ‘Fair’ ($d_1 = 20 - 40\%$). For the normalized squared metric, the MDS configuration is strongly influenced by the patch locations, while the spread of the points shows different similarity levels. Dispersed points correspond to random fields with low similarity, whereas grouped points represent models with high similarity.

In the Appendix C (Figure C.1c), the Spatial Prediction Comparison Test (SPCT) results of Zhang et al. (2015a) using the squared-error (SE) loss function are presented. We find consistency on the model ranking from MSD and SPCT.

For the gray-scale metric (Figure 4.2), the MDS configuration displays more distinct clusters, with larger variability along the two dimensions compared with the normalized
Figure 4.2: MDS configuration of dissimilarity between six random fields using the gray-scale metric. (a) Variable Hurst parameter or correlation length; (b) variable Hurst parameter and correlation length; (c) variable Hurst parameter and correlation length with the features assumed to appear anywhere on the rectangular plane. The proportion of the dissimilarity explained by each configuration is listed in the bottom right of each figure.

squared metric. However, the overall point configurations are similar. Figure 4.2a displays the MDS representation for random fields with different Hurst parameters or correlation lengths. In this case, the point configuration is characterized by a quadratic function (parabola). The axes of symmetry pass through the centroid, with the vertex at random field 3, which is closest to the mean model. The gray-scale metric is not a point-by-point distance metric; therefore the central tendency of the models is not represented by the mean model. Instead, the model with the averaged position of the structure/feature in all intensity levels defines the central tendency. The figure also shows, as in the case of the normalized squared metric, that random fields 1 and 6 are the most dissimilar with respect to each other, with dissimilarity, $d_2$, of about 50%. Additionally, we find that random fields 4, 5, and 6 are very close to each other with less than 5% dissimilarity for both the normalized squared metric $d_1$, and the gray-scale metric, $d_2$. These random fields thus share common features in terms of the intensity and location of the regions of high (low) values. These values of $d_1$ and $d_2$ fall into the category ‘excellent’. Similarly to the normalized
squared metric, we find in Figure 4.2c that if we allow the slip patch to occur anywhere in
the rectangular plane, the points are dispersed farther from the centroid. Random fields 1,
3, and 4, for instance, are dissimilar with respect to the centroid by more than 40%. This
means that the high-slip patches in these three random fields are located away from the
mean patch location.

4.3.2 Sensitivity Analysis

To capture the sensitivity of the point configurations, we generate 1000 realizations of
random fields following again the four parameterizations (the variable Hurst parameter or
correlation length, the variable correlation length and Hurst parameter, and finally the
variable Hurst parameter, correlation length, and the seed-number controlling the patch
location). We find that the pattern of point configuration follows the same pattern as for a
single realization when varying only the spectral parameters.

For the normalized squared metric, Figure 4.3 displays the distribution of points cor-
responding to the variability of the relative point location for 1000 replications. Although
the variability for six random fields with different Hurst parameters (Figure 4.3b) is slightly
larger than for the random field with different correlation lengths (Figure 4.3a), the distribu-
tions of the point configurations are similar for these two parameterizations. The variability
of the point-cloud is mainly along a line, parallel to the axis of the largest eigenvalue. The
variability along the second dimension (second largest eigenvalue) is very small (less than
0.5%). We also notice overlapping point-clouds, implying that these models are very simi-
lar. Figure 4.3c shows the case where both the correlation length, $C$, and Hurst parameter,
$H$, are different in the six random fields. We increase $C$ from 4 to 19 km in steps of 3 km
and decrease $H$ as follows [1.5; 1.3; 1.0; 0.8; 0.5; 0.3]. The variability of the point locations
for these six random fields shows a circular shape, meaning that they are equally sensitive
to the two dimensions. The variability is highest for random field 1 ($H = 1.5; C = 4$ km).
The smallest uncertainty appears for random field 5 ($H = 0.5; C = 16$ km), which is closest
to the mean model. These realizations also reveal that the metric, $d_1$, is more sensitive to
the variability of the Hurst parameter compared to that of the correlation length.
Figure 4.3: Sensitivity of the MDS configuration of dissimilarity between six random fields, each with 1000 realizations and using the normalized squared metric. (a) Variable Hurst parameter; (b) variable correlation length; (c) variable Hurst parameter and correlation length. The bottom panels show zoomed versions of the top panels.
Figure 4.4: Sensitivity of the MDS configuration of dissimilarity between six random fields, each with 1000 realizations and using the gray-scale metric. (a) Variable Hurst parameter; (b) variable correlation length; (c) variable both Hurst parameter and correlation length.

The point configuration of the 1000 realizations of the gray-scale metric (Figure 4.4) also reveals an identical configuration to that of only one realization (Figure 4.2c). The point-cloud for random fields with different Hurst parameters (Figure 4.4b) and correlation lengths (Figure 4.4a) follows a similar pattern. They both consist of a parabolic trend with the vertex around random field 3. The dissimilarities along dimensions 1 and 2 correspond to smoothness and patch extension, respectively. As we vary both the Hurst parameter and correlation length (Figure 4.4c), the point-cloud becomes circular.

When we assume that the slip patch can occur anywhere in the fault, the point-clouds do not follow any pattern in both the normalized squared and gray-scale metrics (Figure 4.5). They spread randomly over the 2D space because, in this case, the mean model for each realization can be very different as can be the dissimilarity between each point and with respect to their corresponding centroid model. This result shows that both metrics are more sensitive to slip patch location variability and less sensitive to the spectral parameters.

These sensitivity tests highlight the stability of the MDS configuration and its low sensitivity to spectral parameters (the Hurst parameter and correlation length) compared
Figure 4.5: Sensitivity of the MDS configuration of dissimilarity between six random fields, each with 1000 realizations for the variable Hurst parameter, correlation length, and assuming that the features appear anywhere on the rectangular plane. (a) Using the normalized squared metric; the bars indicate the x and y range of 5, as used in Figure 4.3. (b) Using the gray-scale metric.

to the slip location variability. These findings are also consistent with the results of Zhang et al. (2015a) who used statistical hypothesis testing on the same data. In addition, the MDS technique can be used to rigorously rank and assess the similarity between any 2D geophysical models and eventually to rank them. In the following, we apply this technique to compare earthquake rupture models.

4.4 Case Studies

In this section, we compare inverted slip models from the SIV exercise (accessible at http://equake-rc.info/SIV/) and for the 2011 Mw 9.0 Tohoku earthquake, respectively. These models provide cases with and without a reference model. Figure 4.6 depicts the reference model and six inverted slip models from the SIV exercise. These models have approximately the same fault parameterization with lengths of 30 – 35 km and widths of 15 – 20 km. However, the slip models have been generated using various grid sizes and inversion techniques. For the Tohoku earthquake, Table 4.2 lists 21 models obtained from the source model database (http://equake-rc.info/srcmod/; Mai & Thingbaijam 2014). These slip models were derived using different inversion techniques and different datasets, and
they have different fault parameterizations (e.g., single/multiple segments, fault/subfault size).

![Selection of rupture models from the SIV exercise. (a) Reference model and (b-g) six inverted slip models.](image-url)

**Figure 4.6:** Selection of rupture models from the SIV exercise. (a) Reference model and (b-g) six inverted slip models.
Table 4.2: 2011 Tohoku earthquake rupture models used in this study.

<table>
<thead>
<tr>
<th>Data</th>
<th>Length (km)</th>
<th>Width (km)</th>
<th>Maximum slip (m)</th>
<th>Seismic moment (Nm)</th>
<th>Numbering</th>
<th>Author</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teleseismic</td>
<td>500</td>
<td>200</td>
<td>41.18</td>
<td>5.01 $10^{22}$</td>
<td>1</td>
<td>Shao et al. (2011)</td>
</tr>
<tr>
<td>Teleseismic</td>
<td>500</td>
<td>200</td>
<td>60.11</td>
<td>4.84 $10^{22}$</td>
<td>2</td>
<td>Shao et al. (2011)</td>
</tr>
<tr>
<td>Teleseismic</td>
<td>475</td>
<td>200</td>
<td>56.76</td>
<td>5.01 $10^{22}$</td>
<td>3</td>
<td>Shao et al. (2011)</td>
</tr>
<tr>
<td>Teleseismic</td>
<td>475</td>
<td>200</td>
<td>62.04</td>
<td>5.01 $10^{22}$</td>
<td>4</td>
<td>Shao et al. (2011)</td>
</tr>
<tr>
<td>Teleseismic</td>
<td>625</td>
<td>260</td>
<td>34.13</td>
<td>4.22 $10^{22}$</td>
<td>8</td>
<td>Hayes (2011)</td>
</tr>
<tr>
<td>Teleseismic</td>
<td>380</td>
<td>200</td>
<td>57.10</td>
<td>3.55 $10^{22}$</td>
<td>9</td>
<td>Lay et al. (2011)</td>
</tr>
<tr>
<td>Teleseismic</td>
<td>500</td>
<td>200</td>
<td>51.32</td>
<td>5.75 $10^{22}$</td>
<td>10</td>
<td>Yagi &amp; Fukahata (2011)</td>
</tr>
<tr>
<td>Teleseismic</td>
<td>525</td>
<td>240</td>
<td>31.09</td>
<td>3.55 $10^{22}$</td>
<td>12</td>
<td>Wei and Sladen (2011)</td>
</tr>
<tr>
<td>Teleseismic</td>
<td>445</td>
<td>240</td>
<td>30.72</td>
<td>3.55 $10^{22}$</td>
<td>21</td>
<td>Ide et al. (2011)</td>
</tr>
<tr>
<td>Teleseismic+GPS</td>
<td>625</td>
<td>280</td>
<td>30.95</td>
<td>6.00 $10^{22}$</td>
<td>13</td>
<td>Wei et al. (2011)</td>
</tr>
<tr>
<td>Teleseismic+GPS</td>
<td>600</td>
<td>210</td>
<td>41.02</td>
<td>3.55 $10^{22}$</td>
<td>5</td>
<td>Ammon et al. (2011)</td>
</tr>
<tr>
<td>Teleseismic+Tsunami</td>
<td>340</td>
<td>200</td>
<td>67.06</td>
<td>3.55 $10^{22}$</td>
<td>11</td>
<td>Yamazaki et al. (2011)</td>
</tr>
<tr>
<td>Tsunami</td>
<td>500</td>
<td>200</td>
<td>48.57</td>
<td>3.55 $10^{22}$</td>
<td>6</td>
<td>Fujii et al. (2011)</td>
</tr>
<tr>
<td>Tsunami</td>
<td>500</td>
<td>200</td>
<td>39.11</td>
<td>3.55 $10^{22}$</td>
<td>7</td>
<td>Fujii et al. (2011)</td>
</tr>
<tr>
<td>Tsunami</td>
<td>550</td>
<td>200</td>
<td>35.09</td>
<td>3.55 $10^{22}$</td>
<td>18</td>
<td>Satake et al. (2013)</td>
</tr>
<tr>
<td>Tsunami</td>
<td>550</td>
<td>200</td>
<td>38.14</td>
<td>3.55 $10^{22}$</td>
<td>19</td>
<td>Satake et al. (2013)</td>
</tr>
<tr>
<td>Tsunami</td>
<td>550</td>
<td>200</td>
<td>44.85</td>
<td>3.55 $10^{22}$</td>
<td>20</td>
<td>Satake et al. (2013)</td>
</tr>
<tr>
<td>Tsunami+GPS</td>
<td>450</td>
<td>200</td>
<td>44.37</td>
<td>3.55 $10^{22}$</td>
<td>15</td>
<td>Gusman et al. (2012)</td>
</tr>
<tr>
<td>Tsunami+GPS</td>
<td>450</td>
<td>200</td>
<td>42.56</td>
<td>3.55 $10^{22}$</td>
<td>16</td>
<td>Gusman et al. (2012)</td>
</tr>
<tr>
<td>GPS+GM</td>
<td>525</td>
<td>260</td>
<td>48.31</td>
<td>5.50 $10^{22}$</td>
<td>14</td>
<td>Wei et al. (2012)</td>
</tr>
<tr>
<td>GPS+Teleseismic+Tsunami</td>
<td>420</td>
<td>240</td>
<td>75.72</td>
<td>5.92 $10^{22}$</td>
<td>17</td>
<td>Yue &amp; Lay (2013)</td>
</tr>
</tbody>
</table>


As a preliminary analysis, we examine the variability of slip models based on a single physical value such as maximum slip and centroid location. Table 4.3 compares the seismic moment, maximum slip, and centroid location of the slip models from the SIV exercise. The seismic moments of Models 3 and 4 are closest to the seismic moment of the reference model. On the other hand, the maximum slips for Models 7 and 3 are closest to the reference model, while those for Models 2 and 4 are the farthest with maximum slips of 0.95 and 1.02 m, respectively. In addition, we compute a first-order estimate of the centroid location, $C(c_i, c_j, c_k)$, defined as follows:

$$
\begin{align*}
  c_i &= \frac{\sum_{x=1}^{n} U_i^x i_x}{\sum_{x=1}^{n} U_i^x} ; \\
  c_j &= \frac{\sum_{x=1}^{n} U_j^x j_x}{\sum_{x=1}^{n} U_j^x} ; \\
  c_k &= \frac{\sum_{x=1}^{n} U_k^x k_x}{\sum_{x=1}^{n} U_k^x}, 
\end{align*}
$$

(4.10)

where $U_i^x$, $U_j^x$, and $U_k^x$ correspond to the projection of the slip at point $x$ of the fault along the $i$-, $j$-, and $k$-axis, and $n$ represents the total number of points. This estimation is in line with McGuire (2004), who considered the temporal centroid. For the SIV slip models, we estimate this centroid location along the strike and dip directions (see Table 4.3). We find that the centroid location of Model 3 and the reference model are close, 0.24 km away from each other. The ranking with respect to the reference model in terms of the centroid location is as follows: Models 3, 2, 7, 1, 4, and 6. According to this first-order analysis, the best model is Model 3 in terms of centroid location and seismic moment, whereas it is Model 7 in terms of maximum slip. The worst model is Model 6 in terms of centroid location and seismic moment, whereas it is Model 2 in terms of maximum slip.

Table 4.3: Physical description of the different slip models from the SIV exercise.

<table>
<thead>
<tr>
<th>Case</th>
<th>Maximum slip (m)</th>
<th>Centroid location (km)</th>
<th>Seismic moment (Nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Along strike</td>
<td>Along dip</td>
<td></td>
</tr>
<tr>
<td>Model 1</td>
<td>1.29</td>
<td>18.5377</td>
<td>10.7413</td>
</tr>
<tr>
<td>Model 2</td>
<td>0.91</td>
<td>18.5950</td>
<td>9.89439</td>
</tr>
<tr>
<td>Model 3</td>
<td>1.54</td>
<td>19.2123</td>
<td>10.3017</td>
</tr>
<tr>
<td>Model 4</td>
<td>1.02</td>
<td>18.4979</td>
<td>9.21181</td>
</tr>
<tr>
<td>Model 5*</td>
<td>1.85</td>
<td>19.3622</td>
<td>10.1033</td>
</tr>
<tr>
<td>Model 6</td>
<td>2.35</td>
<td>17.6148</td>
<td>9.18940</td>
</tr>
<tr>
<td>Model 7</td>
<td>2.05</td>
<td>18.5438</td>
<td>10.1822</td>
</tr>
</tbody>
</table>
For the Tohoku earthquake, about half of the slip models have identical seismic moments \((M_0 = 3.55 \times 10^{22} \text{ Nm})\), and most of these models are from tsunami data (aside from Models 5, 9, and 21; see Table 4.2). The seismic moments of the remaining models vary from \(4.22 \times 10^{22}\) to \(6.00 \times 10^{22} \text{ Nm}\), with Model 13 having the largest seismic moment. In terms of maximum slip, the minimum and maximum values are observed for Models 21 and 17, respectively. The variability is large, with a mean of 46.7 m and a standard deviation of 12.6 m. We also compute the centroid location for the 21 proposed slip models for the Tohoku earthquake along latitude, longitude, and depth (see Figure 4.7). The result shows that the centroid locations are in the depth range of 14 – 30 km. Models 9, 19 and 20 have the shallowest depths, whereas Models 8, 17, and 21 have the deepest locations. About half of the models, including Models 3, 4, 6, 7, 10, 11, 15, 16, and 18, are clustered around latitude 38°N, longitude 143.1°E, and depth 17 km. The second cluster consists of Models 1, 2, and 14 that are located to the southeast of the first group of models (latitude 37.9°N, longitude 142.8°E). The third cluster includes models 12 and 13 located at about the same latitude and longitude as the second cluster, although the centroid location is much deeper. The remaining models do not belong to any group, as each corresponding location is isolated. According to this analysis, models 13, 17, and 21 have extreme values in most of the cases.

These comparisons provide preliminary insight into the rupture model variability. However, they are limited, as they do not allow for comparison of spatial distributions of slip. Thus, we use the MDS technique to compare the spatial variability of the slip models.

To facilitate the slip model comparison, we reconfigure the models such that they all have the same grid spacing. We adopt a grid spacing of 1 km and 5 km for the SIV and Tohoku models, respectively. Since the fault geometries for the SIV models are approximately identical, we do not require any additional model transformations. However, for the Tohoku slip models, we additionally adopt a single-plane representation and consider an average strike, dip, and depth of the top-edge of the fault from the surface (htop).

Before applying the MDS, we also need to calculate the dissimilarity between each pair of models using equations 4.1 and 4.3 for the normalized squared metric and the gray-scale metric, respectively. For the normalized squared metric, we directly use the reconfigured
models. However, for the gray-scale metric, we transform the reconfigured slip models such that they consist of only three areas (very large, large, and low slip).

### 4.4.1 Source Inversion Validation Models

Figure 4.8 displays the point-cloud obtained from application of MDS to the slip models for the SIV exercise, in which Model 5 corresponds to the reference model. We examine two cases in which the point-clouds are centered at the centroid of the model ensemble and at the reference model. For each case, we use both the normalized squared and gray-scale metrics. All configurations reveal that Models 2 and 4 are very similar to each other (less than 5\% dissimilarity) and cluster in one group, as their slip patches occupy roughly the same area. Model 6, on the other hand, contains inconsistent high slip patches. It is hence isolated from the other slip models.

Figure 4.8 also reveals some discrepancies, for instance, in the nearest neighbor models. The three nearest neighbors in the normalized squared metric (Figure 4.8a) to Model 5 are

---

**Figure 4.7:** Centroid slip locations for 21 rupture models of the 2011 Mw 9 Tohoku earthquake.
Models 3, 4, and 2. However, when we center the configuration at the reference (Figure 4.8c), Models 3, 2, and 7 are closest to the center. This is expected because there are multiple sources of spatial pattern dissimilarity (intensity, various patch locations, patch extension, shape, and so forth) for the SIV slip models. Therefore, the choice of a two-dimensional representation may no longer be sufficient to represent the model dissimilarity fully. For the normalized squared metric, the dissimilarity is less than 20% with respect to the centroid for all models. Models 3 and 5 are closest to the centroid with \( d_1 \) less than 5%. However, in comparison with the reference, Models 3, 7, and 2 have less than 20% dissimilarity, \( d_1 \). These models fall into the category ‘good’ in the similarity scale. The remaining models belong to the ‘fair’ similarity category, for which the dissimilarity is between 20 and 40%. According to this analysis, Model 3 is the best solution to the target/reference Model 5 in the inversion exercise.

The large variability of the point-clouds for the two cases is not surprising when we use the gray-scale metric. The first case, in which the point-cloud is centered at the central tendency of the model ensemble, reveals three main clusters, Models 7 and 5, Models 3 and 1, Models 2 and 4, and one individual model, Model 6. These clusters consist of slip models that share the same intensity of slip at the same location. All these models have between 20 and 40% dissimilarity with respect to the centroid, aside from Model 3, which has less than 20% dissimilarity. For the second case, in which the point-cloud is centered at the reference model (Figure 4.8d), Models 3 and 7 are ‘fairly similar’ (\( d_2 \) between 20 and 40%) to the reference model in terms of location and extension of the three areas of slip (very large, large, and low slip). The other models underestimate the feature of the reference model, and hence have a gray-scale metric, \( d_2 \), that is greater than 40% (‘poor’) compared to the reference model.

Each of the two-dimensional representations of the slip model variability for the SIV exercise explains about 70% of the full dissimilarity. This value is considered acceptable as suggested by [Hair et al. 2010]. However, this level might change depending on the context. Hence, to examine possible contributions to the dissimilarity measure when considering high dimensions, we analyze the three-dimensional representation of the point-clouds (Fig-
Figure 4.8: MDS configuration of dissimilarity between slip models in the SIV exercise. Centered at the central tendency of the model ensemble using (a) the normalized squared metric and (b) the gray-scale metric. (c) and (d) configuration with respect to reference model (model 5). Circles limit different levels in the similarity scale, excellent (dark grey), good (grey), fair (light grey), and poor (outside the circles). The proportion of the dissimilarity explained by each configuration is quantified in the bottom right of each figure.
When we include the third dimension, the MDS representations explain about 90% and 80% of the full dissimilarity for the normalized squared and gray-scale metric, respectively. This representation therefore provides a more complete point distribution. By comparing the configurations centered at the centroid and reference model in 2D and 3D, we obtain more consistent results in the 3D point distribution, particularly for the nearest neighbors of each point. The three nearest neighbors to Model 5 of the normalized squared metric are Models 3, 2, and 7 for both configurations centered at the centroid and the reference. Clearly, the third dimension captures a rather detailed aspect of the spatial variability, compared to the first and second dimensions. In the normalized squared metric centered at the centroid, the third dimension consists of patch extensions. This analysis also reveals that Model 3 is the best model.

We find that the percentage of dissimilarity when the gray-scale metric is used appears larger than that for the normalized squared metric, because of their sensitivity to different properties of the slip models. The gray-scale metric is more sensitive to small-scale spatial variability. According to the three-dimensional representations in Figure 4.9 (normalized squared metric) Model 3 is the best solution to the reference model (Model 5), with between 5 and 20% dissimilarity. This falls into the category ‘good’. The rest of the models are ‘fair’, with between 20 and 40% dissimilarity. The rank of these models is consistent with the findings of Zhang et al. (2015a) for squared loss functions. For the gray-scale metric, Models 3 and 7 are closest to the reference model with between 20 and 40% dissimilarity. The rest of the models have greater than 40% dissimilarity. We also find that Models 2, 3, and 4, which have similar spatial patterns, belong to a single cluster. This analysis illustrates the importance of a high-dimensional representation to fully capture the large variability among the models. It also reveals the ability (strength) of each metric.

### 4.4.2 2011 Tohoku Earthquake Models

We examine two cases of the 2011 Tohoku earthquake to compare slip models because the fault geometry of the 21 models varies significantly across the models (see Figure 4.10). We consider (a) the smallest common rupture area for all models and (b) the largest fault area
Figure 4.9: 3D MDS configuration of dissimilarity between slip models in the SIV exercise. Centered at the central tendency of the model ensemble using (a) the normalized squared metric and (b) the gray-scale metric. (c) and (d) configuration with respect to reference model (Model 5).
that contains all models. These two cases capture the variability in terms of fault geometry and slip distribution, respectively. It is also important to note that for real earthquakes, no reference model exists. The comparison is therefore done based on a centroid model, which is adopted as the reference although it does not represent the best model.

Figs. 4.11 and 4.12 display the MDS configuration of the 21 Tohoku earthquake slip models using a two-dimensional configuration. For the smallest common rupture plane, similarity appears in the overall distribution of the point-clouds of both the normalized squared and gray-scale metrics. We find that Models 15 and 16 are closest to the centroid with dissimilarity metrics, $d_1$ and $d_2$, less than 5%. Model 17, on the other hand is farthest from the center of the configuration, with greater than 40% dissimilarity for both metrics. More than half of the slip models fall into the category ‘good’. Table 4.4 summarizes the similarity of each model with respect to the centroid model for these two metrics.

It is important to note that we do not know in advance the meaning of each dimension in terms of the physical parameter of the fields. In fact, it can be interpreted as a physical parameter that appears to order the models in the configuration. For the smallest common rupture plane (Figs. 4.11 and 4.12), Models 17 and 12 are quite dissimilar on dimension 1, but rather similar on dimension 2. Hence, dimension 1 relates to the overall intensity (or magnitude) level of the slip. Dimension 2, on the other hand, corresponds to the extent of the slip patches. We also find that most of the models are aligned along the diagonal, aside from Models 1, 2, 5, and 17 which are separated. This diagonal alignment clearly shows that the variability of the slip models encompasses both the intensity and the slip patch extensions.

In the Appendix (see Figure D.7), we present the 3D MDS configuration. The third dimension contributes about 9% and 20% of the slip model variability in the gray-scale and normalized squared metrics, respectively. For the gray-scale metric, the alignment of Models 9 and 3, as well as 6 and 13 shows that the third dimension is related to the variability of the slip patch extension along the strike direction. However, the contribution of this component is small. For the normalized squared metric, the alignment of Models 1, 2, 19, and 7 suggests that this dimension indicates the compactness of the slip patches. It also
Figure 4.10: Twenty slip models for the 2011 Tohoku earthquake (excluding Model 16 that has a similar pattern to Model 15). The original fault geometry is indicated with red lines. The red dashed lines outline single plane representations of the corresponding model. The smallest and largest areas used in the slip model comparison are denoted by black lines and black dashed lines, respectively.
can be interpreted as the change in the patch locations along the dip direction. Figure D.7 particularly illustrates that the 3D-visualization becomes more challenging as we have more models. Additionally, with the eigenvalues ordered from the largest to the smallest, the first dimension contributes most to the dissimilarity, followed by the second dimension and so forth. The physical interpretation of these dimensions becomes more difficult (non-unique) as the relative contribution of the dimension becomes insignificant. We then restrict our analysis to two-dimensional representation for the following cases.

**Table 4.4**: Tohoku slip model similarity compared to mean model (smallest common area).

<table>
<thead>
<tr>
<th>Case</th>
<th>Excellent</th>
<th>Good</th>
<th>Fair</th>
<th>Poor</th>
</tr>
</thead>
<tbody>
<tr>
<td>(d_1^*)</td>
<td>15,16,20</td>
<td>3,4,6,8,9,10,11,13,14,18,19</td>
<td>1,2,5,7,12,21</td>
<td>17</td>
</tr>
<tr>
<td>(d_2^\dagger)</td>
<td>15,16</td>
<td>3,4,6,7,8,9,10,11,13,14,18,19,20,21</td>
<td>1,2,5,12</td>
<td>17</td>
</tr>
</tbody>
</table>

* \(d_1\), normalized squared metric
† \(d_2\), gray-scale metric

To validate our result from the 2011 Tohoku slip models, we compared the MDS results with the results obtained from the SPCT ([Hering & Genton](2011)). SPCT is a statistical test that consists of comparing loss functions between competing forecasts. It was developed for general spatial fields and applied to wind speed ([Hering & Genton](2011)), precipitation fields ([Gilleland](2013)), and earthquake slip models ([Zhang et al.](2015a)). These studies extensively describe the technique. In Figure 4.13, we present the SPCT result using the squared-error (SE) loss function when considering the mean model (see appendix; Figure [D.2]) as a reference model. For further details, we refer to [Zhang et al.](2015a) who note that negative values (blue) indicate that the case named in the corresponding row is the better model in terms of SE loss functions, and the location with letter ‘a’ indicates that the corresponding two models differ significantly from each other at the 5% confidence level.

Figure 4.13 shows that Model 17 differs significantly from all other models at the 5% level. This model is thus the most dissimilar to the mean model. This model is in the category ‘poor’ for MDS. Models 1, 2, 5, 7, and 12, on the other hand, are significantly
Figure 4.11: MDS point-cloud considering the smallest and largest common area of the 2011 Tohoku slip models and using the normalized squared and gray-scale metrics. The proportion of the dissimilarity explained by each configuration is specified in the bottom right of each figure.
Figure 4.12: Zoomed version of Figure 4.11. Note the different axis scaling.
different at the 5% level from more than four models. These models fall into ‘fair’ category in the MDS. The rest of the models are either ‘good’, ‘excellent’, or along the boundary between ‘fair’ and ‘good’. They are significantly different from fewer than three models. Both techniques also reveal that Models 15 and 16 best represent the mean model (see Figure D.2 in the Appendix). This validation test shows that the MDS results are consistent with the SPCT method.

**Figure 4.13:** Mean loss differentials for the squared loss function with the hypothesis test results from the spatial prediction comparison test. Locations with letter ‘a’ indicate that the corresponding two models differ significantly from each other at the 5% level. Negative values (blue) indicate that the case named in the corresponding row is the better model.

The patterns of the MDS configurations for the largest common rupture plane (Figs 4.11
and 4.12) are not similar between the normalized squared metric and gray-scale metric. The metrics have different sensitivities. The variability among the slip models with the normalized squared metric, for instance, is large. The points appear to be more scattered and occupy a wider area (greater than 40%). This illustrates the large variability in the geometry, particularly the strike. Despite the scattered points, we still observe that Models 6, 7, 18, and 19 are clustered. These models are all obtained by inverting the tsunami data. As we unify the fault geometry (strike, dip, and htop), the slip models become more similar. With the gray-scale metric, all models appear similar, with less than 20% dissimilarity with respect to the centroid. This is due to numerous zero-valued grid points over the enlarged area of the slip models (see white area between red and black dashed line in Figure 4.10). We also examine the alignment of the model along each axis and find that for the normalized squared metrics, the two principle sources of variability correspond to the compactness of the patch (dimension 1) and the intensity (dimension 2). With the gray-scale metrics, on the other hand, these two dimensions correspond to the slip patch extension and the shape of the features. The summary of classification of the 21 Tohoku slip models with respect to the centroid considering the largest common rupture area is shown in Table 4.5.

Table 4.5: Tohoku slip model similarity compared to mean model (largest common area).

<table>
<thead>
<tr>
<th>Case</th>
<th>Excellent</th>
<th>Good</th>
<th>Fair</th>
<th>Poor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_1^*$</td>
<td>13</td>
<td>3,6,14</td>
<td>4,5,7,8,9,10,11,12,15,16,18,19,20,21</td>
<td>1,2,17</td>
</tr>
<tr>
<td>$d_2^+$</td>
<td>3,4,6,13,15,16</td>
<td>1,2,5,7,8,9,10,11,12,14,17,18,19,20,21</td>
<td>1,2,17</td>
<td></td>
</tr>
</tbody>
</table>

$^*$ $d_1$, normalized squared metric  
$^+$ $d_2$, gray-scale metric

**MDS Sensitivity**

To assess the stability and accuracy of the MDS solution, we use a jackknife approach, which is a resampling method that is applied to statistical inference. It is widely used due to its simplicity and efficiency. In particular, we use the delete-one jackknife (Efron 1982) that works as follows:

1. Omit one slip model and generate the dissimilarity matrix using the remaining slip
models.

2. Carry out MDS.

3. Apply the similarity transformation (Gower & Dijksterhuis 2004) with respect to the original model.

We repeat this procedure for each slip model. Ultimately, we have \( n - 1 \) jackknifed coordinates for each slip model, in which \( n \) is the number of slip models. Figure 4.14 depicts the variability of the point-cloud considering the largest common area of the Tohoku slip models and using the normalized squared metric. We chose this metric because it shows the largest dissimilarities among models. The jackknifed point-cloud shows similar pattern as in the original configuration. Only Model 17, which is far away from the average model, has a significant effect in changing the MDS configuration. We also find that the point-clouds of very similar models such as Models 15 and 16 as well as 6 and 7 overlap.

**Figure 4.14:** A jackknifed MDS point-cloud for the largest common area of the Tohoku slip models using the normalized squared metric.
Predicted Displacement Comparison

The MDS technique has been applied to some slip models of the Tohoku earthquake that required processing in order to map them onto the same physical space. An alternative approach is to compute an independent and hence unbiased physical predictor quantity from each model and to use that predictor for the MDS analysis. In this section, we use MDS to compare the three components of seafloor displacement computed from the 21 slip models. The displacements are computed using Okada (1985) and are presented in the Appendix (Figs D.3, D.4, and D.5). They all predict displacements over 20 m and 8 m on the horizontal and vertical components, respectively. However, significant variability appears in their spatial patterns that will affect the prediction of tsunami properties (Tappin et al., 2014; Goda et al., 2014).

Figure 4.15 displays the MDS point-cloud of the predicted seafloor displacements using the normalized squared metric. We observe that dissimilarities among the horizontal components are less pronounced than those among the vertical. For the EW component (Figure 4.15a), for instance, the dissimilarities of the predicted displacements with respect to the mean displacement are all less than 40%, aside from the predicted displacements from Models 9, 19, and 20, which have greater than 40% dissimilarity. For the NS component (Figure 4.15b), the dissimilarity is less than 40% for all models, aside from the displacement corresponding to Models 9, 11, 13, 19, and 21. However dissimilarities among the vertical displacements (Figure 4.15c) are greater than 40% for most of the models. Hence, more variability appears in the vertical component of the seafloor displacements that in fact largely govern the tsunami generation process. Hence, we conjecture that the predictive ability to match the tsunami observations from all these models is very different. We also identify clusters of points for the vertical component, such as points 5, 6, 7, 15, 16, 18, 19, 20, and 21; points 3, 10, 13, and 14; and points 1, 2, and 17. The remaining models are isolated. The first cluster consists mostly of predictions from slip models generated from tsunami data. Figs. D.5 and 4.15c also suggest that the first and second axes correspond to variability in the location and in the significance of the uplifted area, respectively.

We observe that the computed seafloor displacements from Model 9 appear to be isolated
Figure 4.15: MDS configurations of dissimilarity between predicted seafloor displacement from the 21 Tohoku slip models in Figure 4.10. (a) EW component; (b) NS component; and (c) Vertical component (see Appendix, Figs D.3-D.5).

for all components. This model has a compact and shallow slip patch. It is not very different from the other models in terms of the normalized squared metric. However, the 3D cloud points of the slip dissimilarity for the gray-scale metric (see Appendix, Figure D.7) show that this model is isolated and has between 20 and 40% dissimilarity (‘fair’) compared to the central tendency of the model. The corresponding EW seafloor displacement is low (around 10 m), and the NS seafloor displacement is large (around 20 m), compared to the predictions near the trench from other models.

4.5 Discussion

In this study, we apply MDS to investigate the characteristics of the dissimilarity between slip models. This technique augments standard residual analysis through its ability to cluster the models with common intensity and patch geometry. This ability eventually allows for ranking the models based on a reference or central tendency of the model ensemble. Additionally, we investigate the effect of various physical assumptions and datasets. For example, Figure 4.11 shows that slip models from tsunami data form a single cluster. This suggests that the inverted slip models from tsunami data vary systematically from the
mean, which consists of underestimation of slip values compared to the mean model. An alternative explanation is that the differences are due to the fault geometry, as they use an average strike of 192° to 194°, compared to those of 198° to 202° for slip models obtained from seismic and/or geodetic data.

We also examine the MDS results based on gray-scale and normalized squared metrics. These metrics are sensitive to different spatial characteristics of the slip. The normalized squared metric is more sensitive to the overall intensity, whereas the gray-scale metric particularly detects the patch geometry and the location of the areas of large and low slip. Therefore, models that are very similar using both metrics share common aspects in terms of patch intensity and geometry. Models with different smoothness and correlation lengths, for instance, show that they are very similar in terms of the normalized squared metric, with less than 5% dissimilarity (Figure 4.1c). However, the variability using the gray-scale metric is larger (Figure 4.2), since the change in smoothness or correlation length particularly affects the geometry of the patches.

In this study, we compared only the slip values. However, the proposed approach can be easily extended and applied to examining the variability of the full spatio-temporal parameters of the rupture process. Also, the results we obtained are based on two metrics relevant to capturing the spatial variability of the slip. However, additional metrics are proposed that could be utilized and tested to quantify the dissimilarity of the slip models. Hence, this study could be extended to examine alternative metrics such as correlation and warping loss functions [Gilleland 2013], which could potentially contribute to additional information on the source of the discrepancies. We could additionally mix and weight the different metrics, although the MDS point-cloud would be more complex as it would be a mixture of different properties of the spatial pattern.

There is no rigorous statistical method to evaluate the quality and reliability of a representation produced by MDS. Here, we assess the accuracy based on the variability of the configuration using a number of realizations. For the case studies, we obtain the variability from a jackknife test. An MDS configuration also provides a richer interpretation compared to a simple residual analysis, as it reveals classes of slip models sharing common features
such as slip patch intensity, shape, and extension.

4.5.1 Insights for Source Inversion

As an application of MDS to a real earthquake, we compared the slip models from the 2011 Tohoku earthquake. We could, however, consider this analysis as a benchmark to assess the accuracy of any earthquake for which multiple finite-fault source models have been published. We can also use this tool to investigate possible systematic classification depending on earthquake magnitudes and tectonic regimes. Additionally, the sensitivity of datasets in resolving the rupture process can be extensively explored using this comparison tool.

We find that the gray-scale metric complements the normalized squared metric for comparing slip models. The gray-scale metric thus appears to be a useful Euclidean norm in kinematic source inversion to statistically discriminate between various proposed solutions. To the best of our knowledge, the use of this metric to examine the resolution of inverted slip models has never been described before.

4.5.2 Accuracy and Limitation

We considered an approximate representation using a dimension-reduction technique with two or three dimensions. However, two- or three-dimensional representations may be insufficient to represent the full dissimilarity between slip models. In Figure 4.1c, for instance, the source of the variability comes from one source, the Hurst parameter, $H$, or the correlation length, $C$. Therefore, even one dimension suffices to represent the dissimilarity between the random fields. In this case, the interpretation of the dimension axes is straightforward: it is the $C$ and $H$ values. As we increase the complexity and the source of variability in the models, the dimensionality of the dissimilarity becomes larger. Therefore, the 2D representation is just an approximation to facilitate the visualization and the interpretation of the different classes, although it still constitutes a limited representation of the dissimilarity. A comparison of the slip models for the SIV exercise illustrates this limitation and shows
the importance of higher dimensional-representations. Therefore, it is necessary to assess percentages of dissimilarity accounted for by each dimension. For the SIV case, for instance, the two-dimensional representation accounts for 79% of the full dissimilarity. As we incorporate the third dimension, they account for 95%. The physical meaning of each dimension also changes depending on the main source of variability among the model ensemble. The factors involve patch location, intensity, dimension, extension, shape, or their alternative combinations.

One limitation of our technique is that we do not have a reference model for real earthquake. We make comparisons with respect to the central tendency of the models. However, this mean model does not, in general, represent the best model in the sense of, for instance, its capability to fit observation data. Consequently, this approach does not rely on the best and optimal model, but it helps to identify those models that share common features and hence may be able to achieve similar predictions of data or physical quantities.

4.6 Conclusions

The objective of our study is to quantify both differences and similarities between rupture models. We find that the MDS technique efficiently identifies slip models sharing common spatial pattern characteristics in terms of the slip patch intensity and geometry. We also propose a similarity scale for the rupture models based on this technique. The scale allows for ranking the models with respect to the centroid or reference model. We find that the generated MDS point-clouds change depending on the choice of metric. The normalized squared metric is insensitive to the spectral parameters of the random field but very sensitive to the slip patch locations. The gray-scale metric, on the other hand, is sensitive to the patch geometry and hence to small-scale variability in the model. The case studies also reveal the importance of higher-dimensional representations for rupture models with large sources of variability. A natural extension of this work would be to assess the accuracy of inter-event rupture models and consider both spatial and temporal patterns.
Chapter 5

Concluding Remarks

This dissertation addressed several key issues and yet opened questions in earthquake seismology, such as the inconsistency of the kinematic source models published by different authors for a particular earthquake. These problems have implications not only for the earthquake source community, but also for subsequent seismological research requiring finite-fault models as input data. For a better understanding of these issues, the uncertainty of kinematic source inversion was studied.

A Bayesian approach was utilized to analyze the effects of source-time function (STF) and crustal structure in finite-fault source inversion for a synthetic test case with a known solution. This technique mapped the solution space of the earthquake source parameters in terms of posterior PDFs. One of the advantages of the Bayesian inference is its ability to extract potential trade-off between sample parameters. To highlight the effect of source-time functions, three functions were used - an isosceles triangle and two asymmetric Yoffe functions with different acceleration times. All the inferred models from these functions could reproduce the essential high-slip and high-slip-rate features of the target model. However, details of later rise time and slip-rate were difficult to resolve, because of weak seismic radiation in the latter part of the target model. The Yoffe function was found to be preferable since the isosceles triangle function caused an artificial acceleration of the rupture propagation and generated artificial correlations between temporal parameters.

To explore the effects of inaccurate Earth structure on kinematic source models, two tests based on variations in wave speed as well as in wave speed and layer depth were per-
formed. These two tests show little to no differences among each other, both significantly broadening posterior PDFs and causing faster rupture propagation. Therefore, the choice of crustal structure and STF could accelerate the rupture time leading to an overestimation of the rupture speed, which is a critical parameter for dynamic rupture modeling. As an application to a real earthquake and complementary analyses, the effects of crustal structure complexity in the rupture process of the 2009 L’Aquila earthquake were examined. Particular emphasis was placed on assessing the rupture model uncertainty related to one- and three-dimensional Earth structures. The variation in crustal structure complexity had a minor effect on the inferred earthquake source parameters. Nevertheless, a degradation of the data fitting was observed when the one-dimensional crustal structure was used, particularly for stations located away from the earthquake source. The resolution of the earthquake source parameters also decreased for points located away from the hypocenter. Moreover, rupture time appeared to be less accurate in low slip areas. These analyzes on the uncertainty of finite-source models have significant implications for the earthquake physics and the advanced seismic hazard analysis for earthquake risk mitigation. For instance, the hypothesized correlations (or anti-correlations) among source parameters (e.g., Schmedes et al., 2010; Guatteri et al., 2004) could result from only partially correct assumptions on the rupture behavior.

Finally, this dissertation presented the ability of multidimensional scaling approach for quantifying slip model variability. This method was rarely used in earthquake seismology. First, the method was tested on random fields generated from a von Kármán autocorrelation function. Then, two case studies were examined, first, the Source Inversion Validation exercise with six inverted slip models, and second, the 2011 Mw 9.0 Tohoku earthquake with more than 20 published slip models. Given the considerable intra-event variability in the inferred rupture models, it is useful to classify published slip models and quantify their variability (similarities and differences). This analysis is of particular importance in revealing classes of slip models sharing common spatial patterns. This, in turn, allows defining a ranking scheme for the inverted rupture models. These comparison tests for slip models serve as benchmarks for further investigation on the model variability. The comparison tool
could be extensively used to investigate possible systematic classification - depending on
earthquake magnitudes and tectonic regimes - as well as further exploring the sensitivity of
datasets used for the inversion. As an example, for the 2011 Tohoku earthquake, slip mod-
els generated from tsunami data and their corresponding seafloor displacements clustered
systematically compared to those from seismic and/or geodetic data, indicating a potential
bias in the tsunami-data inversions.

5.1 Future perspectives

The work presented in this dissertation is one step in understanding better the uncertainties
in kinematic source inversion. However, there is still room for lots of improvement in terms
of the inversion technique. There are also associated studies that could be pursued to answer
important open questions in earthquake science. These include a better understanding of
the ground motion complexity and the physical process of the earthquake rupture.

5.1.1 Implication for future research

In the present work, the inference is focused on kinematic representations of the rupture
process. The next step is to analyze how the uncertainties affecting kinematic parameters
are mapped into the estimation of dynamic parameters, which are critical for a physical un-
derstanding of the rupture initiation, propagation, and arrest. From the inferred kinematic
model, we could consider/extract either representative models (mean, median and maxi-
mum) of the posterior distributions or the full PDFs, from which then the spatio-temporal
evolution of the stress is estimated using the Staggered-Grid-Split-Node method (SGSN;
Dalguer & Day, 2007). Then, based on the calculated stress-slip curves, the dynamic pa-
rameters (strength excess, dynamic stress drop, and slip-weakening distance) distributions
on the fault with the corresponding uncertainties could be retrieved. Such proposed study
helps in refining scaling relations of dynamic source parameters (e.g., Causse et al., 2014).

The inferred source model ensemble that was used to approximate the posterior PDF
could also be used to provide a probabilistic constraint for shaking-scenario analysis and
Coulomb stress change. First, each of the inferred models could be incorporated into scenario simulation to capture the ground motion prediction adequately. Such simulation is rather computationally demanding. An alternative approach would be to include the finite-fault PDFs as a prior information for the strong motion simulation. This would give a better assessment of the earthquake and tsunami hazard uncertainties for a specific area.

Furthermore, the improved understanding of the earthquake source uncertainty helps to shed light on the examination of the aftershock occurrence. Woessner et al. (2012) used, for instance, the inferred finite-fault candidates to assess the reliability of Coulomb failure stress change.

5.1.2 Technical and methodological improvements

Most inversion techniques suffer from the restrictive assumptions imposed in the inversion. These include the predefined STF, simplification in the fault parameterization (planar fault), datasets (ground motion), as well as the chosen frequency range of the data. These restrictions are considered based on previous studies or data resolution power/limitations, and help in decreasing the total number of parameters. However, they potentially affect the resolutions of the source parameters.

The inversions were performed using a predefined STF at each point of the fault. This problem setup was found to bias the estimation of dynamic parameters constrained by the kinematic models (e.g., Piatanesi et al., 2004). The assumed STF also limits the characteristic of the rupture process to allow each point on the fault to only rupture once. However, potential re-rupturing of fault patches during the same event was reported/discussed based on finite-fault earthquake source inversion (e.g., Lee et al., 2011; Ide et al., 2011), numerical simulation (e.g., Noda & Lapusta, 2013; Gabriel et al., 2012), as well as laboratory models (e.g., Nielsen et al., 2010). Therefore, a natural extension of this study would be to remove the constraints on the shape of STF, and let the inversion resolves the nature of the function (e.g., Fan et al., 2014; Galović et al., 2015). This would allow for complex rupture patterns and re-rupturing of the points on the fault.

Source modelers also limit the inversion using low frequency of the observed waveforms,
although the earthquake radiates waves over a wide frequency range. Another improvement is then to perform the inversion considering higher frequency. This would increase the resolution of the kinematic source model and highlight the effect of crustal structure complexity and topography further. However, the challenge would be to compute/obtain reliable Green’s functions at high frequencies.

Beside STF and crustal structure, we also need to examine the role of fault geometry complexity and discretization in constraining the rupture history. Fault geometry is typically fixed based on the spatial distribution of the aftershocks, and the effect of its uncertainty in the inferred rupture process are rarely investigated. This is particularly worth to analyze not only to examine its importance in the inference but more importantly to understand the physical process between the seismogenic zone (area where the earthquake source studies are usually performed) and the root of the fault (area known as creeping). Moreover, the presented inversion used fixed and simple fault parameterization. More elaborate parameterization of the fault could be considered, in which irregular grid and complex faulting system are allowed. Furthermore, the Bayesian inference presented in this dissertation could be extended in the following directions.

1. Address the lack of knowledge of the data uncertainty characteristics using Hierarchical Bayes formulation (Gelman et al., 1995) that consider the data noise parameters as hyper-parameters. In this case, we let the data itself constrain the nature of the data uncertainty. Hence, both source parameters and the data noise parameters are inferred simultaneously.

2. Incorporate the parameterization uncertainty through trans-dimensional Bayesian (Dettmer et al., 2014) that is not typically included in the earthquake source inversion. This technique would reduce the number of free parameters, yet resolve in a better way the area with a significant slip.

3. Use of empirical Green’s function, instead of computed Green’s function from a particular structure that already has some uncertainties. This proposition would reduce the bias of the inferred model on the choice of crustal structure.
APPENDICES

A Effect of spatial smoothing in uncertainty analysis

To minimize the trade-off between rupture parameters at neighboring points, we include the spatial smoothing using an eight-neighbors Laplacian filter $D$,

$$D = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

and respecting the distance between patches and the boundary.

Figure A.1 shows that including spatial smoothing between neighboring points, with a smoothing parameter $\alpha^2$ of 20, generally reduces this time difference between the peaks of the slip-rate pulse for the reference model and the inversion solutions, particularly for Yoffe-0.1 close to the hypocenter. However, for triangle source time function, the improvement is only apparent for a few points on the fault. This suggest that the trade-off between neighboring points is not the only source of uncertainty in source parameters. To analyze this effect on the parameter uncertainties in more detail, we constrain the inversion following the penalized likelihood (Eq. 2.10) in which the smoothing factor is chosen deterministically. We present in Figure A.2 the kinematic source models generated from an evolutionary
algorithm (Beyer, 2001) applying different smoothing factors $\alpha$ (Eq. 2.10). For $\alpha^2 = 0.1$ and $\alpha^2 = 20$, we identify high slip-rate and slip around the hypocenter, which is not observed for $\alpha^2 = 200$ due to oversmoothing of the parameters. The variability of misfits to the seismic data with respect to the model roughness allows us to deterministically choose the preferred model. We choose a smoothing factor $\alpha^2 = 20$, based on the curvature of the trade-off function. Therefore, we use this factor in Figure A.1. Sekiguchi et al. (2000) also discuss a way to optimally choose this smoothing value. In Figure A.3, we also show the resulting posterior PDFs at two selected points on the fault (point four and five in Figure 2.5) for different smoothing factors. The area around the hypocenter shows more constrained slip and slip-rate by incorporating the spatial smoothing. We find that the PDFs become less skewed as this constraint reduces the trade-off between neighboring points that partially leads to this skewness.

Figure A.1: Source time functions on the same eight points as in Figure 2.5 for the reference model (dashed line) and different inversion solutions: (a) using triangle slip-rate function and including spatial smoothing; (b) using Yoffe-01 source time function, and including spatial smoothing.
Figure A.2: (a) Trade-off curve. Kinematic source model for three different smoothing factors ($\alpha^2$): (b) $\alpha^2 = 0.1$, (c) $\alpha^2 = 20$, and (d) $\alpha^2 = 200$. Arrows in the left column (peak slip rate) indicate slip direction and amplitude. Contour lines in the right column (final slip) mark the rupture time distribution (contour interval: 1 s).
Figure A.3: Posterior PDF for peak slip rate, rise time, rupture onset time, and slip at two points on the fault: (a) located at 5 km from the hypocenter and (b) 12 km from the hypocenter, assuming a Yoffe source time function with acceleration time (Tacc) of 0.1 s and considering different smoothing factors (blue, no smoothing; green, $\alpha^2 = 0.1$; magenta, $\alpha^2 = 20$; red, $\alpha^2 = 200$). The black square marks the reference model, given by the known benchmark model.
B Posterior Probability Density Function for the 2009 L’Aquila Earthquake

The following figures show the inferred posterior probability density functions of the slip and rupture time, considering the recorded waveforms of the 2009 L’Aquila earthquake and using 1D and 3D crustal structures.
Figure B.1: Posterior probability density function of slip at the inner nodes of the fault plane using 1D crustal structure. Color represents the maximum estimates of the posterior.
Figure B.2: Posterior probability density function of slip at the inner nodes of the fault plane using 3D crustal structure. Color represents the maximum estimates of the posterior.
Figure B.3: Posterior probability density function of rupture time at the inner nodes of the fault plane using 1D crustal structure. Color represents the maximum estimates of the posterior.
Figure B.4: Posterior probability density function of rupture time at the inner nodes of the fault plane using 3D crustal structure. Color represents the maximum estimates of the posterior.
C Comparing earthquake slip models with the spatial prediction comparison test

This Appendix is related to the publication:


SUMMARY: Earthquake rupture models inferred from inversions of geophysical and/or geodetic data exhibit remarkable variability due to uncertainties in modelling assumptions, the use of different inversion algorithms, or variations in data selection and data processing. A robust statistical comparison of different rupture models obtained for a single earthquake is needed to quantify the intra-event variability, both for benchmark exercises and for real earthquakes. The same approach may be useful to characterize (dis-)similarities in events that are typically grouped into a common class of events (e.g. moderate-size crustal strike-slip earthquakes or tsunamigenic large subduction earthquakes). For this purpose, we examine the performance of the spatial prediction comparison test (SPCT), a statistical test developed to compare spatial (random) fields by means of a chosen loss function that describes an error relation between a 2-D field (model) and a reference model. We implement and calibrate the SPCT approach for a suite of synthetic 2-D slip distributions, generated as spatial random fields with various characteristics, and then apply the method to results of a benchmark inversion exercise with known solution. We find the SPCT to be sensitive to different spatial correlations lengths, and different heterogeneity levels of the
slip distributions. The SPCT approach proves to be a simple and effective tool for ranking the slip models with respect to a reference model.

**Figure C.1:** Mean loss differentials for the squared loss function with the hypothesis test results from the spatial prediction comparison test. Locations with letter ‘a’ indicate that the corresponding two models differ significantly from each other at the 5% level. Negative values (blue) indicate that the case named in the corresponding row is a better model. (a) Variable Hurst parameter $H$; (b) variable Hurst parameter and correlation length $C$; and (c) variable $H$, $C$ and seed number.
D  Random Field Realization,
Predicted Seafloor Displacement
and 3D MDS Configuration

Realizations of random fields

Figure D.1 shows realizations of random fields using different parameterizations. At the
top of the figure, we plot the mean of the random field ensemble. The generated random
fields locate the slip patches at the same position (same seed value) while varying the Hurst
parameter, $H$, and the correlation length, $C$. 
Figure D.1: Random field example of different Hurst parameters, $H$, and correlation lengths, $C$. 
Mean model and predicted seafloor displacement

The following figures show the mean model and the predicted displacements from the 21 inverted slip models for the 2011 Tohoku earthquake. Figure D.2 presents the mean model. Figures D.3, D.4, and D.5 on the other hand, illustrate the computed EW, NS, and vertical seafloor displacement, respectively.

We also show in Figures D.6 and D.7 the 3D MDS configuration for the smallest common rupture plane of the 2011 Tohoku slip models using normalized squared and gray-scale metrics.

**Figure D.2:** Mean model of the 21 inverted source models for the 2011 Tohoku earthquake.
Figure D.3: East-West horizontal seafloor displacement from 20 inverted slip models for the 2011 Tohoku earthquake (excluding the prediction from Model 16 that has a similar pattern as the one from Model 15). The green dashed line indicates the Japan trench subduction zone, while the gray curve indicates the coast line of central and northern Honshu, Japan.
Figure D.4: North-South horizontal seafloor displacement from 20 inverted slip models for the 2011 Tohoku earthquake (excluding the prediction from Model 16 that has a similar pattern as the one from Model 15). The green dashed line indicates the Japan trench subduction zone, while the gray curve indicates the coast line of central and northern Honshu, Japan.
Figure D.5: Vertical seafloor displacement from 20 inverted slip models for the 2011 Tohoku earthquake (excluding the prediction from Model 16 that has a similar pattern as the one from Model 15). The green dashed line indicates the Japan trench subduction zone, while the gray curve indicates the coast line of central and northern Honshu, Japan.
3D MDS configuration of the Tohoku slip models

Figure D.6: 3D MDS point-cloud considering the smallest common area of the 2011 Tohoku slip models and using the normalized squared metric.
Figure D.7: 3D MDS point-cloud considering the smallest common area of the 2011 Tohoku slip models and using the gray-scale metric.
E Publications Based on This Dissertation


REFERENCES


Efron, B., 1982. The jackknife, the bootstrap, and other resampling plans, Society for Industrial and Applied Mathematics.


Hartzell, S. & Heaton, T. H., 1983. Inversion of strong ground motion and teleseismic wave-
form data for the fault rupture history of the 1979 Imperial Valley, California, earthquake,

Hartzell, S., Stewart, G. S., & Mendoza, C., 1991. Comparison of L1 and L2 norms in a tele-
seismic waveform inversion for the slip history of the Loma Prieta, California, earthquake,

Investigation of rupture velocity, risetime, and high-frequency radiation, *J. Geophys. 

of finite-fault slip inversions: Application to the 2004 Parkfield, California, earthquake,

Hayes, G., 2011. Rapid source characterization of the 03-11-2011 Mw 9.0 off the Pacific 

414–425.

Hisada, Y., 2000. A theoretical omega-square model considering the spatial variation on 


Ide, S., Baltay, A., & Beroza, G. C., 2011. Shallow dynamic overshoot and energetic deep 

Iida, M., Miyatake, T., & Shimazaki, K., 1990. Relationship between strong motion array 
**80**(6A), 1533–1552.

Imperatori, W. & Mai, P. M., 2012. Broad-band near-field ground motion simulations in 

California, earthquake, part i: Wavelet domain inversion theory and resolution analysis,


