Unconventional scaling of the anomalous Hall effect accompanying electron localization correction in the dirty regime

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I. INTRODUCTION

The anomalous Hall effect (AHE) in ferromagnetic conductors, though first discovered by Hall in 1881, has received renewed interest in recent years due to its close connection with spin transport phenomena1 and the controversial mechanisms.2 It is now generally accepted that there are three mechanisms responsible for AHE. The first two involve the extrinsic mechanisms, namely skew scattering and side jump, both of which result from the asymmetric impurity scattering caused by the spin-orbit interaction and yield scaling relations between the anomalous Hall resistivity ρAH (Hall conductivity σAH) and longitudinal resistivity ρxx (conductivity σxx) as ρAH ∝ ρxx (σAH ∝ σxx) and ρAH ∝ ρ2xx (σAH ∼ const.), respectively.1,3 The other is the intrinsic mechanism, which arises from the transverse velocity of the Bloch electrons induced by the spin-orbit interaction together with interband mixing, and also gives ρAH ∝ ρ2xx (σAH ∼ const.).1 Nowadays, this intrinsic AHE has been reinterpreted in terms of the Berry curvature of the occupied Bloch states.2,6

Recently, a unified theory of AHE for multiband ferromagnetic metals with diluted impurities has been proposed and later verified by experimental results from a variety of itinerant ferromagnets.5,12 It predicts three distinct scaling regimes in the AHE that are functions of conductivity. In the clean regime (σxx > 104 S/cm), the skew scattering mechanism dominates, resulting in σAH ∝ σxx. With decreasing conductivity to the intermediate disorder regime (σxx ∼ 104–106 S/cm), the intrinsic contribution becomes dominant, yielding σAH ∼ const. In the dirty regime with high disorder (σxx < 104 S/cm), the intrinsic contribution is strongly decayed, resulting in a scaling relation expressed as σAH ∝ σγ xx with γ ∼ 1.6, irrespective of the different mechanisms responsible for the electron transport. It should be emphasized that, in the dirty regime, the physics of AHE can be enriched by the quantum effects of the Coulomb interaction (CI) and disorder-induced electron localization (EL),2,13–15 which the unified theory does not take into account.7,8 On this open issue, previous experimental studies mostly focused on the quantum corrections to σAH through altering the temperature of films with given thicknesses.16–19 The evolution of the scaling exponent γ, particularly in the presence of the quantum correction to σAH, remains poorly known.2 Interestingly, it has been recently reported that the low-temperature AHE scaling in CNi3 films exhibits a peculiar σAH ∝ σγ 1/2 xx relationship near the Mott-Anderson critical region, and CI was suggested to play a crucial role in the unconventional scaling exponent.20 In this paper we seek to elucidate how EL modulates the scaling exponent of σAH ∝ σγ xx in homogenous polycrystalline FePt films, which exhibit ideal features of negligible CI and well-defined quantum correction to σAH from EL. Our findings clarify the role of EL in affecting the AHE scaling and point to an experimental approach that can verify the role that EL plays.

II. EXPERIMENT

We deposited SiO2(5 nm)/FePt(1.2–100 nm)/SiO2(10 nm) sandwich films at room temperature in a multisource magnetron sputtering system controlled by a computer. The nominal thickness of the individual layers was calculated from sputtering time and rates, 0.062 nm/s for FePt and 0.076 nm/s for SiO2, determined by x-ray reflectivity on the single layer films of about 20 nm. Up to 18 uniform films were fabricated in a single deposition run. Thus the layer thickness uncertainty among different samples is much smaller than 0.1 nm. Thermally oxidized Si substrates adopted have an oxide layer of about 450 nm. In order to prevent FePt films from any interface/surface contamination and oxidation, all FePt films were deposited on a 5 nm SiO2 buffer layer and covered by a 10 nm SiO2 capping layer in situ. The equiatomic FePt alloy layers formed a homogenously continuous structure, which was confirmed by transmission electron microscopy observations.21 The x-ray diffraction pattern revealed the face-centered cubic polycrystalline structure of the FePt films (chemically disordered). Transport measurements were carried out on a Quantum Design PPMS-14H at 300–2 K. In studying Hall effect, the offset signal was removed, and ρAH was

PHYSICAL REVIEW B 87, 094405 (2013)
obtained as the zero field extrapolation of the high field \( \rho_{xx}(H) \). The magnetization of all samples measured by MPMS-XL yielded the bulk value of FePt and changes very little in the low temperature range. A detailed description of the film’s growth, structural characterization, and sample preparation for the transport measurements can be found in our previous paper.\(^{23}\)

### III. RESULTS AND DISCUSSION

The inset in Fig. 1(a) shows the dependence of thickness on sheet resistance \( R_{xx} \), measured at 300 K. The \( R_{xx} \) increased by about four orders of magnitude from 8.57 \( \Omega \) to 17.6 k\( \Omega \), when the film thickness \( \langle t_{FePt} \rangle \) was reduced from 100 to 1.2 nm. The corresponding \( \rho_{xx} \) also monotonically increased and fell in the range of 86–2100 \( \mu \Omega \) cm. The normalized sheet resistance \( R_{xx}(T)/R_{xx}(300 \, \text{K}) \) as a function of temperature \( T \) is presented in Fig. 1(a). In relation to this figure, we highlight that the FePt films that were 20 nm or thicker exhibited typical metallic behaviors.\(^{22,23}\)

An appreciable upturn in \( R_{xx} \sim T \) is evident in very thin films \( \langle t_{FePt} \rangle \lesssim 3 \, \text{nm} \). Quantitative analysis of the insulating resistance results shows that a linear logarithmic temperature dependence of \( R_{xx} \) emerges at 1.6 \( \lesssim \langle t_{FePt} \rangle \lesssim 3 \, \text{nm} \). When \( \langle t_{FePt} \rangle \) was further decreased to 1.4–1.2 nm, the \( R_{xx} \) varied faster than the ln \( T \) law in the low-temperature region. Actually it was appropriate to fit the \( R_{xx} \) of the 1.2 nm FePt film with a variable range hopping type of conduction,\(^{24}\) which will be detailed later.

As mentioned above, when the FePt thickness is reduced to 3–1.6 nm, \( R_{xx} \) changes linearly with ln \( T \) at low temperatures. This phenomenon can be ascribed to the two-dimensional (2D) weak EL and/or the 2D CI.\(^{13}\) To clarify if CI has a quantum correction to \( R_{xx} \), or sheet conductance \( G_{xx}/R_{xx} \) in the weakly localized region, we analyzed the low-temperature conductivity quantitatively. In the 2D case, \( G_{xx} \) can be expressed as

\[
G_{xx}(T) = G_0 + \rho \frac{e^2}{\pi h} \ln \left( \frac{T}{T'_0} \right) + (1 - F) \frac{e^2}{\pi h} \ln \left( \frac{T}{T'} \right),
\]

(1)

where the first term on the right-side of the equation stands for the Drude conductance, the second term is the contribution of weak EL, and the third term is the CI correction.\(^{13,14}\)

In Eq (1), the parameter \( F \) is a measure of the screening with \( 0 \leq F \leq 1 \), and \( p \) is the temperature exponent of the inelastic scattering length \( l_{\text{in}} \sim T^{-p/2} \). The value of \( p \) is governed by inelastic relaxation mechanisms: \( p = 1 \) for the electron-electron scattering, whereas \( p = 2, 3, \) or 4, depending on the material and the temperature, for the electron-phonon scattering (Ref. 13 and references therein). Figure 1(b) shows the \( G_{xx} \) obtained in a 1.8 nm FePt film as a function of ln \( T \). The perfect linear behavior for the data in the 2–20 K range on a logarithmic scale verifies the validity of the 2D assumption.\(^{13,14}\)

The fitting to the linear part of the \( G_{xx} \sim \ln T \) curve yields a slope of 1.2 \( \times 10^{-5} \, \text{s} \), i.e., the value of \( e^2/\pi h \). This leads to \( p + (1 - F) = 1 \), namely either \( p = F = 0 \) or \( p = F = 1 \). Considering that the AHE conductivity of the thin films receives a quantum correction from the weak EL, which will be shown later, the case of \( p = 0 \) and \( 1 - F = 1 \) can be ruled out. We thus have \( p = 1 \) and \( 1 - F = 0 \). This suggests that the CI correction to the Drude conductance is negligible. We have also fitted the data of other films with 1.6 \( \lesssim \langle t_{FePt} \rangle \lesssim 3 \, \text{nm} \) and found that all the fitted slopes of \( G_{xx} \) vs ln \( T \) converge nicely to the common value \( e^2/\pi h \). Therefore, the absence of the CI correction to the Drude conductance is universal in weakly localized FePt thin films.

In the 2D strongly localized case, electrons hop between localized states and the \( G_{xx} \) is given by

\[
G_{xx}(T) = C \exp[-(T''/T')^\beta],
\]

(2)

where the exponent \( \beta \) depends whether the hopping process is influenced by CI.\(^{24–26}\) \( \beta = 1/3 \) corresponds to Mott’s variable range hopping law, whereas \( \beta = 1/2 \) indicates the formation of a Coulomb gap resulting from strong electron correlation.\(^{26}\) Figure 1(c) displays the ln \( G_{xx} \) of the 1.2 nm FePt film as a function of \( T^{-1/3} \) along with the fitting line using Eq (2) with \( \beta = 1/3 \) at 2–20 K \( (C = 0.065 \, \text{mS}, T'' = 0.372 \, \text{K}) \).

The excellent fitting result corroborates the assumption that \( \beta = 1/3 \) in Eq. (2). It is worth noting that Mott’s variable range hopping can exist up to 150 K, which might be due to the phonon assisted hopping.\(^{27}\) Even so, we have still tried to
fit the data of 2–20 K using $\beta = 1/2$. The difference between the experimental points and the fitting result is not significant at 20 K, but the discrepancy becomes obvious above 50 K. It seems difficult to distinguish the minor difference in power laws applying $\beta = 1/3$ and $\beta = 1/2$ over the limited temperature range of 2–20 K, which suggests that CI cannot be totally ruled out in the strongly localized 1.2 nm FePt film. Anyway, CI correction to $G_{xx}$ can be considered to be insignificant at least in the weakly localized region in FePt films. Parenthetically, the perfect-fit exponent $\beta$ is 1/5 and 1/4 for 1.4 and 1.3 nm FePt films, respectively, reflecting the crossover from the weak EL limit to the strong EL in the metal-insulator transition.28

We now turn to the EL correction to anomalous Hall conductance $G_{AH}$ by changing temperature of the individual thin FePt films. In the weakly localized region, anomalous Hall resistance $R_{AH}$ at low temperatures (2–20 K) varies linearly with $\ln T$, similar to its $R_{xx}$ or $G_{xx}$ vs $T$ behavior. Following Bergmann and Ye’s notation,16 normalized relative changes $\Delta R_{xx}^{N}$, $\Delta R_{AH}^{N}$, $\Delta G_{AH}^{N}$ have been used to represent weak EL corrections to $R_{xx}$, $R_{AH}$, and $G_{AH}$ [$G_{AH} = R_{AH}/(R_{xx}^{2} + R_{CH}^{2})$], respectively, and are expressed as

$$\Delta R_{xx}^{N} = \left( \frac{\pi h}{e^{2}} \right) \frac{\delta R_{xx}}{R_{0}} = -A_{R} \ln \left( \frac{T}{T_{0}} \right), \quad (3)$$

$$\Delta R_{RH}^{N} = \left( \frac{\pi h}{e^{2}} \right) \frac{\delta R_{RH}}{R_{AH}} = -A_{RH} \ln \left( \frac{T}{T_{0}} \right), \quad (4)$$

for $|\delta R_{xx}| \ll R_{0}$ and $R_{AH}(T) \ll R_{xx}(T)$, which are true for all our films. $\Delta G_{AH}^{N}$ could be deduced as

$$\Delta G_{AH}^{N} = \left( \frac{\pi h}{e^{2}} \right) \frac{\delta G_{AH}}{G_{AH}} = (2A_{R} - A_{AH}) \ln \left( \frac{T}{T_{0}} \right), \quad (5)$$

where $R_{0}$ is the sheet resistance at reference temperature $T_{0}$.18 Using $T_{0} = 2$ K and $R_{0} = R_{xx}(2$ K), for a 1.8 nm FePt film, the normalized relative changes could fit well linearly with $\ln T$ below 20 K, as shown in Fig. 2(a). The fitting gives the prefactors as $A_{R} = 0.944$, $A_{RH} = 1.108$, and $2A_{R} - A_{AH} = 0.781$. In Fig. 2(b) we summarize the prefactors for weakly localized films (1.6 $\leq t_{FePt} \leq$ 3 nm). Note that $A_{R}$ is 0.92 $\pm$ 0.04 ($\sim$1), in excellent agreement with previous results in 2D weakly localized films with homogenous structures.16,18 Most interesting, the prefactor of $\Delta G_{AH}^{N}$, namely $2A_{R} - A_{AH}$, is far from zero in the weakly localized region, indicating that the weak EL correction to $G_{AH}$ is nonzero. We also note that the value of $2A_{R} - A_{AH}$ increases with increasing disorder strength. Due to the fact that CI correction to $G_{AH}$ vanishes for both the skew scattering and the side jump and that weak EL correction to $G_{AH}$ is zero for side jump but nonzero for skew scattering,29–31 the monotonic increase in the $2A_{R} - A_{AH}$ with reducing $t_{FePt}$ from 3 to 1.6 nm can be attributed to the increased weight of the skew scattering in the AHE as observed in homogeneous Fe films.18 It is worth noting that for the weakly localized 12–6 nm FePt films $\Delta G_{AH}^{N} \approx 0$, namely, the quantum correction to $G_{AH}$ is negligible.

As for the strongly localized film ($t_{FePt} = 1.2$ nm), similar to its $G_{xx}$ or $R_{xx}$ vs $T$ behavior [Fig. 1(c)], a hopping form $\ln R_{AH} \sim T^{-\beta}$ with $\beta = 1/3$ at low temperatures was observed as shown in the inset of Fig. 3. The normalized relative changes $\Delta R_{xx}^{N}$, $\Delta R_{RH}^{N}$, and $\Delta G_{AH}^{N}$, with respect to their reference values at 2 K, are displayed in Fig. 3. Note that $\Delta R_{xx}^{N}$, $\Delta R_{RH}^{N}$, or $\Delta G_{AH}^{N}$ does not have a nice linear relationship with $\ln T$ even below 20 K. Most remarkably, the $\Delta G_{AH}^{N}$ value quickly deviates from zero as the temperature increases from the reference one (2 K). For the 1.4 and 1.3 nm thick FePt films, nonzero $\Delta G_{AH}^{N}$ was also observed at temperatures other than 2 K. It is evident that there is a significant quantum correction to $G_{AH}$ in the strong localization region, although it is difficult to evaluate the CI and EL corrections to $G_{AH}$ separately in the hopping conduction regime.

With well-confirmed quantum corrections to the AHE in the individual thin FePt films, the scaling behavior of AHE at low
temperatures is further examined by varying the thickness of the film (accordingly altering the disorder strength). To gain a complete view of the thickness and temperature dependences of $\rho_{AH}$ ($R_{AH} \cdot t$), Fig. 4(a) displays the dependence of $\rho_{AH}$ on $\ln T$ for some representative samples. The metallic films with thicknesses between 20 and 100 nm always show a decrease in the $\rho_{AH}$ with decreasing temperature, but the $\rho_{AH}$ in weakly and strongly localized films displays $\rho_{AH} \sim \ln T$ and $\ln(\rho_{AH}) \sim T^{-\beta}$ tendencies at low temperatures [also see Fig. 2(a) and the inset of Fig. 3]. Figure 4(b) shows the $\rho_{AH}$ vs $\rho_{xx}$ plot at 5 and 2 K for films of different thicknesses. $\rho_{AH}$ increases with the rise in $\rho_{xx}$ as $t_{FePt}$ decreases from 100 to 3 nm, mainly caused by the surface scattering, but when $t_{FePt}$ further decreases, $\rho_{AH}$ becomes saturated while $\rho_{xx}$ continues to rise. As shown in Fig. 4(b), the $\rho_{AH}$ maintains an almost constant value of $\sim 4.4 \mu\Omega$ cm in an appreciably broad $\rho_{xx}$ region (100-300 $\mu\Omega$ cm), corresponding to 1.2 \leq t_{FePt} \leq 3$ nm. This disorder-independent $\rho_{AH}$ introduces a significant deviation from the unified theory. Figure 4(c) shows $\sigma_{AH}(G_{AH}/t)$ as a function of $\sigma_{xx}$ at 2 K. For films with metallic behaviors (20 \leq t_{FePt} \leq 100 nm) but a high residual resistivity, $\sigma_{xx}$ is about $1.58 \times 10^{4}$ S/cm and $\sigma_{AH} \approx 780$ S/cm, regardless of the thickness of the film. The value of $\sigma_{AH}$ is close to the theoretical prediction of the intrinsic contribution of the order of $e^2/\hbar a \sim 10^5$ S/cm (with a lattice constant of $a \approx 4 \text{Å}$) in the 3D intermediate disorder regime. We hence argue that the AHE in metallic FePt films is dominated by the resonant intrinsic Berry-phase contribution. For films with thicknesses in the range of 12–6 nm, the magnitude of $\sigma_{xx}$ ($1.46 - 1.25 \times 10^{3}$ S/cm) seems to drop into the critical regime between intermediate and high disorder, and the $\sigma_{AH}$ gradually decreases from 700 to 590 S/cm. As $\sigma_{xx}$ passes through this crossover and goes into the dirty regime ($\sigma_{xx} \approx 0.91 \times 10^4$ S/cm) at $t_{FePt} \leq 3$ nm, where $G_{AH}$ (or $\sigma_{AH}$) is well demonstrated to receive a quantum correction, the $\sigma_{AH}$ scales as $\sigma_{AH} \sim \sigma_{xx}^{-\gamma}$ with $\gamma = 2$. This is in striking contrast with the universal scaling exponent $\gamma \approx 1.6$. It should be pointed out that the scaling exponent $\gamma = 2$ has been observed in ultrathin CNi$_3$ films with 2D strong EL near the threshold of the metal-insulator transition ($k_F l \sim 4.00 - 0.74$), where the disorder-enhanced CI was taken as the key to modulating the scaling exponent of AHE. In our FePt films, the scaling relation $\sigma_{AH} \sim \sigma_{xx}^{-\gamma}$ holds in a wide dirty regime ranging from weak to strong localization ($k_F l \sim 7.09 - 0.95$ in Ref. 28). Moreover, the CI is negligible in the weakly localized regime of FePt films as demonstrated in the electron transport characterization, which excludes the possibility of CI modulating the scaling exponent of AHE. Most important, the emergence of an unconventional scaling exponent $\gamma = 2$, starting at $t_{FePt} = 3$ nm, coincides with the onset of the quantum correction to AHE by changing the temperature, strongly suggesting that the electron localization correction modulates the AHE scaling exponent at low temperatures.

IV. CONCLUSION

In conclusion, we have found an unconventional low-temperature AHE scaling ($\sigma_{AH} \propto \sigma_{xx}^{-\gamma}$) in polycrystalline FePt films in the dirty regime, which is accompanied by a quantum correction to $\sigma_{AH}$ in both weak and strong EL regions. Since the electron transport characterization reveals a negligible CI in localized films, the unique scaling relation is very likely due to the emergence of the EL correction to AHE rather than CI. Detailed theoretical studies are needed to understand how AHE is influenced by electron localization.

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Based on the 2D localization theory, the disorder parameter $k_F l$ can be calculated from the $R_{xx}$, i.e., $k_F l = (h/e^2)/R_{xx}$, where $k_F$ is the Fermi wave number, $l$ is the elastic mean free path, and $h/e^2$ is the quantum resistance [see Ref. 13 and D. C. Licciardello and D. J. Thouless, Phys. Rev. Lett. 35, 1475 (1975)]. At 2 K, we obtained the values of $k_F l$ = 70.9, 10.0, 2.72, and 0.95 for our films with 3, 1.6, 1.3, and 1.2 nm thicknesses, respectively. This result suggests that the system disorder increases with decreasing thickness and approaches the Ioffe-Regel limit ($k_F l \sim 1$) at 1.2 nm causing the thickness-driven metal-insulator transition.