Decentralized Control of Transmission Rates in Energy-Critical Wireless Networks

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***Abstract*— In this paper, we discuss the decentralized optimization of delay and energy consumption in a multi-hop wireless network. The goal is to minimize the energy consumption of energy-critical nodes and the overall packet transmission delay of the network. The transmission rates of energy-critical nodes are adjustable according to the local information of nodes, i.e., the length of packets queued. The multi-hop network is modeled as a queueing network. We prove that the system performance is monotone w.r.t. (with respect to) the transmission rate, thus the "bang-bang" control is an optimal control. We also prove that there exists a threshold type control policy which is optimal. We propose a decentralized algorithm to control transmission rates of these energy-critical nodes. Some simulation experiments are conducted to demonstrate the effectiveness of our approach.**

I. Introduction

Energy saving is one of the most critical challenges for modern wireless networks since most of the mobile end equipments are powered by battery. On the other hand, low latency of transmission is another important requirement for many real-time applications of network. High transmission rate can reduce the end-to-end network delay with the cost of consuming extra power. Therefore, it is fundamental to ask how to schedule the packet transmission to optimize the tradeoff of transmission delay and energy consumption of the entire network [3], [17]. Without loss of generality, we study this problem where the network is hybrid. That is, the network consists of two types of communication nodes, one has adequate energy supply and the other has limited one. We call it *partial energy-critical* wireless network.

This partial energy-critical wireless network exists in practice. For example, the hybrid ad-hoc network deployed in deserts may have part nodes with infrastructure and power supply. Other nodes neither have infrastructure nor power supply [2]. Another example is that a heterogeneous wireless sensor network may have sensor nodes whose remaining energy is relatively low [12]. The goal is to optimize the energy consumption of such nodes in order to extend their life times without largely increasing the data delay of the entire network.

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The optimization problem about the transmission power and delay in wireless networks has been studied in the literature. Some of the studies discuss the optimization of energy consumption with the delay constraint [6], [10], [22]. As a comparison, some other studies propose to minimize the delay under a constraint of power consumption level [8], [21]. However, all of the studies above consider only one single node as the optimization target. In a multi-hop wireless networks, communication nodes are interconnected and the dynamics of nodes have mutual influences on each other. Therefore, it is necessary to consider the mutual influences of nodes during the power and delay optimization [13], [16]. On the other hand, many studies use a centralized control mode where the central controller has to know the full information of the entire network [6], [16]. Decentralized control has its advantages on scalability and implementability [1], [11]. How to develop a decentralized optimization approach under a network scenario is an important topic in the research field of wireless networks [18]. Moreover, there are many other works studying the rate control and scheduling problem in wireless networks from other viewpoints, such as the joint consideration of congestion control and resource scheduling problems [7], [14]. But these studies do not take the energy efficiency as their first optimization target in a multi-node and multi-hop communication scenario.

In this paper, we study the decentralized optimization problem as a tradeoff of energy and delay in a multi-hop wireless network. We aim to develop a decentralized algorithm which is adopted by every energy-critical node and achieves an efficient optimization of energy consumption and data delay of the entire network. Each node adjusts its own transmission rate by observing its local information, i.e., the buffer status. With analyzing and learning the historical information about the node actions and the system rewards, nodes will improve their scheduling strategies to obtain a better energy consumption and data delay of the entire network.

We use a queueing network model, Jackson network, to formulate our energy-critical multi-hop wireless network. Our problem is formulated as a Markov decision process (MDP). We utilize the special structure of product-form solution of Jackson network and develop an iterative optimization algorithm which is distributedly executed by each communication node. This algorithm does not require the whole information of the system and can be implemented with an online manner. Since the decisions of nodes have mutual influence on each other, our problem has similarity of game theory. We show that all of these analysis can be induced from a key equation called performance difference equation [4], which describes how is the change of system performance when system parameters have changes.

The remainder of the paper is organized as follows. In section 2, we give a detailed description of our problem and formulate it as a mathematical model. In section 3, we utilize the special structure of Jackson network to obtain the performance difference equation. We discuss some optimality properties of this problem and propose a decentralized control algorithm. In section 4, we conduct some simulation experiments to demonstrate the effectiveness of our approach. Finally, we conclude the paper with section 5.

II. Problem Description and Model Formulation

Consider a partial energy-critical wireless network. The network consists of M communication nodes. The number of energy-critical nodes is denoted as M_1 and the number of energy-noncritical nodes is $M - M_1$. Without loss of generality, we assume that first M_1 indexed nodes are energy-critical. That is, server i is energy-critical and server j is not energy-critical, $i = 1, 2, \cdots, M_1$, $j = M_1 + 1, \cdots, M$. We assume that the data packet to be transmitted is independently generated at each node. The data generation process is assumed as a Poisson process and its rate at node i is denoted as λ_i, $i = 1, 2, \cdots, M$. The data packet size is exponentially distributed with a unit mean. Since the network has multi-hop transmission routes, the generated packet will be transmitted among nodes according to routing protocols. When a packet arrives at node i, this packet will be forwarded to adjacent node j with routing probability q_{ij}, $i, j = 1, 2, \cdots, M$. On the other hand, a packet may arrives at its destination node i with probability q_{i0} and disappears from the network, $i = 1, 2, \cdots, M$. Obviously, $\sum_{j=0}^{M} q_{ij} = 1$ for all i.

We assume that the wireless channel scheduling is highly synchronized by certain schemes, such as TDMA or OFDMA. Thus, we do not consider the media contention or interference among adjacent nodes. When a node is busy in transmitting packets, newly arriving packet will be queued in buffer. Since the buffer size of nodes is limited, we simply assume that the capacity of the entire network has an upper limit denoted as N. That is, when the total number of packets in the entire network reaches N, any newly generated packets will be rejected by the network instantly. As the packet size is exponentially distributed, the transmission time of data packet is also exponentially distributed. The transmission rates of energy-noncritical nodes are fixed and denoted as μ_i, $i = M_1 + 1, \cdots, M$. The transmission rates of energy-critical nodes are varied according to the number of queued packets in the buffer. We call it *load-dependent* transmission rates and denote it as μ_{i,n_i}, where n_i is the number of data packets at node i (including the packet being transmitted), $i = 1, 2, \cdots, M$, $n_i = 0, 1, \cdots, N$. Obviously, $\mu_{i,0} = 0$.

Energy-critical nodes can adjust their transmission rates within $[0, \mu_i^{max}]$, $i = 1, 2, \cdots, M_1$. It is known that the energy consumption of a wireless communication node heavily depends on its transmission rate. In this paper, we assume that the transmission power is linear to the transmission rate, which is true for many situations. To keep a certain transmission rate μ_{i,n_i}, a node has an energy consumption rate $b\mu_{i,n_i}$ per unit time, where b is a coefficient. In order to save energy, nodes may adopt a lower transmission rate. However, low transmission rates will decrease the throughput of the network and increase the transmission delay. We assume that each node has to pay a cost rate c per unit time for each packet which is waiting in the buffer. Our target is to find the optimal transmission rates of energy-critical nodes, which minimizes the average system cost including the energy consumption of energy-critical nodes and the waiting cost of all the packets in the entire network.

Under proper simplifications, we use a Jackson network model to formulate this problem. We use a virtual node 0 to model the external data generation. The service rate of node 0 is $\mu_0 := \sum_{i=1}^{M} \lambda_i$. The routing probabilities from node 0 to other nodes are $q_{0i} := \lambda_i / \mu_0$, $i = 1, 2, \cdots, M$. The queue length of node 0 is defined as $n_0 := N - \sum_{i=1}^{M} n_i$. Therefore, the wireless network can be modeled as a closed Jackson network with $M + 1$ servers and N customers. The system state is denoted as $\boldsymbol{n} = (n_0, n_1, \cdots, n_M)$. The state space is denoted as $\mathcal{S} = \{\text{all } \boldsymbol{n} : \sum_{i=0}^{M} n_i = N\}$. The cost function f is a column vector and its element is defined as follows.

$$f(\boldsymbol{n}) = c \sum_{i=1}^{M} n_i + b \sum_{i=1}^{M_1} \mathbf{1}_{n_i > 0} \mu_{i,n_i}, \tag{1}$$

where $\mathbf{1}_{n_i > 0}$ is an indicator function which is defined as $\mathbf{1}_{n_i > 0} = 1$ when $n_i > 0$, otherwise $\mathbf{1}_{n_i > 0} = 0$. The first term of the cost function (1) is the cost rate of data delay for all the packets in the network and the second term is the cost rate of energy consumption of all the energy-critical nodes.

Jackson network is a continuous time Markov process. The state transition rate matrix of the system is denoted as B and its element $B(\boldsymbol{n}, \boldsymbol{n}')$ is determined as follows. For all $\boldsymbol{n} \in \mathcal{S}$, it has $B(\boldsymbol{n}, \boldsymbol{n}) = -\sum_{i=0}^{M} \mathbf{1}_{n_i > 0} \mu_{i,n_i}$; $B(\boldsymbol{n}, \boldsymbol{n}_{-i+j}) = \mathbf{1}_{n_i > 0} \mu_{i,n_i} q_{ij}$, where $\boldsymbol{n}_{-i+j}$ is the neighboring state of $\boldsymbol{n}$ and defined as $\boldsymbol{n}_{-i+j} := (n_0, \cdots, n_i - 1, \cdots, n_j + 1, \cdots, n_M)$ for $n_i > 0$, $i, j = 0, 1, \cdots, M$; $B(\boldsymbol{n}, :) = 0$ for all the other states. Note that for the energy-noncritical nodes and node 0, their service rates μ_{i,n_i} are fixed as μ_i, $i = 0, M_1 + 1, \cdots, M$ for all n_i.

The system steady state distribution is denoted as a row vector π and its element $\pi(\boldsymbol{n})$ is the probability that the steady system stays at state $\boldsymbol{n}$. The average cost of system is $\eta = \pi f$. From the structure of B, different μ_{i,n_i} will induce different B. We define the system policy as a set of transmission rates $\mathcal{L} := \{\mu_{i,n_i}, \text{ for all } i = 1, \cdots, M_1, n_i = 1, \cdots, N\}$. The policy space is defined as $\Psi := \{\text{all } \mathcal{L}\}$. Different policy $\mathcal{L}$ has different π, different f, and different η. Our target is to find the optimal $\mathcal{L}^*$ which minimizes the system cost η.

$$\mathcal{L}^* = \arg\min_{\mathcal{L} \in \Psi} \{\eta^{\mathcal{L}}\} = \arg\min_{\mathcal{L} \in \Psi} \{\pi^{\mathcal{L}} f^{\mathcal{L}}\}, \tag{2}$$

where $\eta^{\mathcal{L}}$, $\pi^{\mathcal{L}}$, and $f^{\mathcal{L}}$ are the system average cost, steady state distribution, and cost function, when the system adopts the policy $\mathcal{L}$, respectively.

III. Optimization Method and Optimality Structure

First, based on the product-form solution, we introduce a special property of conditional probability in Jackson networks as follows. This special property can help solve the violation of state independent actions.

For a particular server i, if we change μ_{i,n_i} to μ'_{i,n_i} and fix other servers' service rates, the conditional probability $\pi(\boldsymbol{n}|n_i)$ has the following property [20]

$$\pi(\boldsymbol{n}|n_i) = \pi'(\boldsymbol{n}|n_i), \text{when } \mu_{i,n_i} \text{ changes for a particular } i. \tag{3}$$

The above property means that the conditional probability $\pi(\boldsymbol{n}|n_i)$ remains unchanged when only server i's service rate changes. Next, we further utilize this property to analyze the system performance of our formulated problem.

A. Difference Equation and Optimality Structure

First we give a brief overview on the theory of performance difference equation, which can be viewed as the extension of perturbation analysis theory [4], [5], [19]. Performance difference equation can describe the performance change of Markov systems when system parameters or policies change. We apply this theory to analyze the performance of our system when the transmission scheduling policy changes.

Consider a continuous-time Markov process $\{X_t, t \geq 0\}$, where X_t is the system state at time t. When the policy of Markov system changes from $\mathcal{L}$ to $\mathcal{L}'$, the corresponding state transition rate matrix will change from B to B' and the cost function will change from f to f'. We have the following difference equation to measure the change of system long-run average performance [5].

$$\eta' - \eta = \pi'[(B' - B)g + (f' - f)], \tag{4}$$

where the column vector g is called *performance potential* and its associated element $g(u)$, $u \in \mathcal{S}$, is defined as below.

$$g(u) = \lim_{T\to\infty} E\left\{ \int_0^T [f(X_t) - \eta]dt \Bigg|_{X_0=u} \right\}. \tag{5}$$

From (5), we find that $g(u)$ can be obtained from the current system and it has no relation to the perturbed system with new policy.

As we describe in Section II, our system to be optimized is a Markov process. We apply the difference equation (4) to analyze this problem. When transmission rates of an energy-critical node i change from μ_{i,n_i} to μ'_{i,n_i}, $n_i = 1, 2, \cdots, N$, the transition rate matrix B and the cost function (1) will change to B' and f'. With the structure of matrix B, (4) can be rewritten as follows.

$$\begin{aligned}
\eta' - \eta &= \sum_{n_i=1}^{N} \sum_{\boldsymbol{n}\in\mathcal{S}_{n_i}} \pi'(\boldsymbol{n}) \Bigg\{ (\mu'_{i,n_i} - \mu_{i,n_i}) \sum_{j=0}^{M} q_{ij} \\
&\quad [g(\boldsymbol{n}_{-i+j}) - g(\boldsymbol{n})] + b(\mu'_{i,n_i} - \mu_{i,n_i}) \Bigg\} \\
&= \sum_{n_i=1}^{N} \pi'(n_i) \sum_{\boldsymbol{n}\in\mathcal{S}_{n_i}} \pi'(\boldsymbol{n}|n_i)(\mu'_{i,n_i} - \mu_{i,n_i}) \\
&\quad \left\{ \sum_{j=0}^{M} q_{ij}[g(\boldsymbol{n}_{-i+j}) - g(\boldsymbol{n})] + b \right\}.
\end{aligned} \tag{6}$$

Since we only change the transmission rates of node i and keep other nodes' rates unchanged, we can apply the property (3) to (6) and obtain

$$\begin{aligned}
\eta' - \eta &= \sum_{n_i=1}^{N} \pi'(n_i)(\mu'_{i,n_i} - \mu_{i,n_i}) \sum_{\boldsymbol{n}\in\mathcal{S}_{n_i}} \pi(\boldsymbol{n}|n_i) \\
&\quad \left\{ \sum_{j=0}^{M} q_{ij}[g(\boldsymbol{n}_{-i+j}) - g(\boldsymbol{n})] + b \right\}.
\end{aligned} \tag{7}$$

We further define an aggregated parameter $\alpha(i, n_i)$ as follows.

$$\alpha(i, n_i) = \sum_{\boldsymbol{n}\in\mathcal{S}_{n_i}} \pi(\boldsymbol{n}|n_i) \left\{ \sum_{j=0}^{M} q_{ij}[g(\boldsymbol{n}_{-i+j}) - g(\boldsymbol{n})] + b \right\}. \tag{8}$$

Difference equation (7) can be rewritten as below.

$$\eta' - \eta = \sum_{n_i=1}^{N} \pi'(n_i)(\mu'_{i,n_i} - \mu_{i,n_i})\alpha(i, n_i). \tag{9}$$

From the definition (8), $\alpha(i, n_i)$ is uniquely determined by the current system and it has no relation to the perturbed system with μ'_{i,n_i}'s. Therefore, we can calculate or estimate the value of $\alpha(i, n_i)$ based on the system sample path of current parameters or policy. With (9), we observe that $\pi'(n_i)$ is always positive although we do not know its exact value. Therefore, if $\alpha(i, n_i) < 0$, we choose $\mu'_{i,n_i} > \mu_{i,n_i}$ which induces $\eta' < \eta$; otherwise if $\alpha(i, n_i) > 0$, we choose $\mu'_{i,n_i} < \mu_{i,n_i}$ which induces $\eta' < \eta$. We can find a better setting of parameters or policy with (9) and the value of $\alpha(i, n_i)$. Repeating this process, we can continuously improve the system performance.

With (9), we obtain the following theorem that describes the property of system performance.

Theorem 1: The system performance η is monotone w.r.t. the transmission rate μ_{i,n_i}, $i = 1, \cdots, M_1$, $n_i = 1, \cdots, N$.

Proof: Suppose that a particular transmission rate changes from μ_{i,n_i} to μ'_{i,n_i} while other transmission rates do not change. The difference equation (9) can be rewritten as below.

$$\eta' - \eta = \pi'(n_i)(\mu'_{i,n_i} - \mu_{i,n_i})\alpha(i, n_i). \tag{10}$$

On the other hand, we reversely suppose that this transmission rate changes from μ'_{i,n_i} to μ_{i,n_i} and other transmission rates remain unchanged. Similarly, we have the following difference equation.

$$\eta - \eta' = \pi(n_i)(\mu_{i,n_i} - \mu'_{i,n_i})\alpha'(i, n_i). \quad (11)$$

Comparing (10) and (11), we have

$$\frac{\alpha'(i, n_i)}{\alpha(i, n_i)} = \frac{\pi'(n_i)}{\pi(n_i)} > 0, \quad (12)$$

since $\pi'(n_i)$ and $\pi(n_i)$ are always positive. Therefore, the sign of $\alpha(i, n_i)$ is fixed if we change a particular transmission rate μ_{i,n_i}. From (10), we can obtain the following derivative equation by letting $\mu'_{i,n_i} \to \mu_{i,n_i}$

$$\frac{d\eta}{d\mu_{i,n_i}} = \pi(n_i)\alpha(i, n_i). \quad (13)$$

Since the sign of $\alpha(i, n_i)$ is fixed and $\pi(n_i)$ is always positive, the sign of derivative $\frac{d\eta}{d\mu_{i,n_i}}$ is also fixed and η is monotone w.r.t. μ_{i,n_i}. ■

With Theorem 1, we derive the following corollary.

Corollary 1: The optimal transmission rate μ^*_{i,n_i} is either 0 or μ_i^{max}, $i = 1, \cdots, M_1$, $n_i = 1, \cdots, N$.

Proof: This is a direct conclusion from Theorem 1. The proof is neglected for simplicity. ■

With Corollary 1, we see that "bang-bang" control is optimal for this problem. We only need to search the optimal value from 0 and μ_i^{max} for every μ_{i,n_i}, without considering the intermediate values between 0 and μ_i^{max}. This greatly reduces the search complexity. Furthermore, we prove that the optimal policy for transmission rates has a threshold form, which is described by the following theorem.

Theorem 2: The optimal transmission rates have a threshold form. There exists a threshold θ_i for each energy-critical node i, $i = 1, \cdots, M_1$, which has $\mu^*_{i,n_i} = 0$, if $n_i < \theta_i$; $\mu^*_{i,n_i} = \mu_i^{max}$, if $n_i \geq \theta_i$, where $\theta_i \in \{1, \cdots, N+1\}$.

Proof: With Theorem 1, we know that the optimal transmission rate can be either 0 or μ_i^{max}. We assume that $\mu^*_{i,k} = 0$ for a particular node i and $n_i = k$. We observe that when the queue length of node i reaches k, the transmission rate of node i becomes 0 and its queue length n_i will always grow up. Since this problem is a Markov process, this phenomena means that the states of $\bigcup_{n_i \leq k} \mathcal{S}_{n_i}$ are *transient*. Because the transient states have no effect on the long-run average performance of system, the optimal transmission rates at these states can be chosen as 0. That is, $\mu^*_{i,n_i} = 0$ when $n_i \leq k$. Therefore, we have proved that if there exists $\mu^*_{i,k} = 0$, then $\mu^*_{i,n_i} = 0$ for all $n_i \leq k$. It is straightforward that the optimal transmission rates obey a threshold form. ■

Theorem 2 can further simplify the optimization process. The original scheduling policy is $\mathcal{L} := \{\mu_{i,n_i}, i = 1, \cdots, M_1, n_i = 1, \cdots, N\}$ with $\mu_{i,n_i} \in [0, \mu_i^{max}]$ and it has a huge search space. With Theorem 2, the policy becomes $\mathcal{L} := \{\theta_i, i = 1, \cdots, M_1\}$ with $\theta_i \in \{1, \cdots, N+1\}$ and the policy search space is greatly reduced to $(N+1)^{M_1}$. Therefore, we only focus on the threshold type policy. Note that this threshold type policy is not a standard one for MDP theory, because it violates the assumption of state independent action. We cannot use the classical approach of MDP to solve this problem. However, we can use the theory of difference equation to solve it. Below, we discuss when the threshold type policy has changes, what is the difference of system performance.

Suppose that the threshold of node i changes from θ_i to θ'_i. Without loss of generality, we assume that $\theta_i < \theta'_i$. This change of threshold means that the transmission rates change from $\mu_{i,n_i} = \mu_i^{max}$ to $\mu'_{i,n_i} = 0$, for $n_i = \theta_i, \cdots, \theta'_i - 1$. Therefore, the difference equation (9) can be rewritten as follows.

$$\eta' - \eta = \sum_{n_i=\theta_i}^{\theta'_i - 1} \pi'(n_i)(0 - \mu_i^{max})\alpha(i, n_i). \quad (14)$$

The system with threshold θ'_i has two types of states, transient states $\mathcal{S}_{n_i < \theta'_i - 1}$ and positive recurrent states $\mathcal{S}_{n_i \geq \theta'_i - 1}$. Therefore, the marginal probability $\pi'(n_i)$ has the following property, $\pi'(n_i) = 0$, for $n_i = 0, \cdots, \theta'_i - 2$; $\pi'(n_i) > 0$, for $n_i = \theta'_i - 1, \cdots, N$. The difference equation (14) becomes

$$\boxed{\eta' - \eta = -\pi'(\theta'_i - 1)\mu_i^{max}\alpha(i, \theta'_i - 1),} \quad (15)$$

where the threshold of a particular server i changes from θ_i to θ'_i, $\theta_i < \theta'_i$, and $\theta_i, \theta'_i = 1, 2, \cdots, N+1$.

Comparing difference equations (9), (14), and (15), we observe that (15) is the simplest. When the threshold of a server i is increased from θ_i to θ'_i, the performance difference only involves three terms, $\pi'(\theta'_i - 1)$ which is always positive, μ_i^{max} which is a known parameter, and $\alpha(i, \theta'_i - 1)$ which can be calculated or estimated based on the sample paths of the current system. There is no need to consider the terms $\pi'(n_i)$ and $\alpha(i, n_i)$, $n_i = \theta_i, \cdots, \theta'_i - 2$, which are shown in (14). Since $\pi'(\theta'_i - 1)$ is always positive, we do not calculate or estimate its value. After we obtain the value of $\alpha(i, \theta'_i - 1)$ with calculation or estimation, we can judge the performance of the new system as follows. If $\alpha(i, \theta'_i - 1)$ is positive, $\eta' < \eta$ and the system performance gets improved. Otherwise, the new system is worse than the current system and we have to try other thresholds θ'_i. This is the basis for us to develop the iterative optimization algorithm.

B. Distributed Optimization Algorithm

In a networking scenario, distributed control is a desired mode for many scheduling algorithms. A central controller is not convenient for an autonomous network consisting of many individual nodes, because a central controller usually has high requirements on time synchronization and frequent communication between controller and nodes. Distributed control distributes the task of control and decision to each node and all of the nodes cooperate together to achieve a good performance of the entire system.

As we have obtained a concise difference equation (15) for an energy-critical node, we further discuss how to develop an optimization algorithm which can be distributed among all the energy-critical nodes. The difference equation (15) clearly quantifies how the system performance will change when the threshold-type scheduling policy of a node changes.

Based on (15), we develop the following algorithm to find the optimal scheduling policy for a particular node.

Algorithm 1 Optimization algorithm for the threshold of an energy-critical node i.

Initialization

- Choose the initial threshold as $\theta^{(0)} = 1$, set $\Theta = \{2, \cdots, N+1\}$ and $k = 0$.

Evaluation

- For the current policy with threshold $\theta^{(k)}$, calculate or estimate the aggregated performance potential $\alpha(i, n_i)$, $n_i = \theta^{(k)}, \cdots, N$.

Reduction

- If $\alpha(i, j) \leq 0$, remove the element $j+1$ from the set Θ, where $j = \theta^{(k)}, \cdots, N$.

Stopping Rule

- If $\Theta = \varnothing$, set $\theta^* = \theta^{(k)}$ and stop the algorithm;
- Otherwise, choose the new threshold as $\theta^{(k+1)} = \min\{\Theta\}$ and remove $\theta^{(k+1)}$ from the set Θ, set $k := k+1$ and go to step 2.

In step 3, the thresholds with worse system performance are identified by (15) and removed from the feasible search space Θ. Every element of the refined set Θ has dominant performance over that of the current threshold $\theta^{(k)}$. In step 4, we choose the minimal element of Θ as the new threshold. Thus, the system average cost (cost for energy consumption and data delay) will be reduced through the update in step 4. The system performance will be strictly improved after each iteration. Since the size of search space Θ is finite (the maximal size of space is N), Algorithm 1 will converge to the optimal threshold after a finite number of iterations.

With Algorithm 1, we can find the optimal threshold for one particular energy-critical node i. However, the original problem (2) is to find the optimal scheduling policy for all the energy-critical nodes i, $i = 1, \cdots, M_1$. We have to consider how to distribute Algorithm 1 to all the energy-critical nodes. Below, we use a rotation procedure to apply Algorithm 1 to every energy-critical node.

Algorithm 2 Round-Robin rotation procedure to distribute Algorithm 1 on every node.

Initialization

- Choose the initial policy as $\theta_i^{*(0)} = 1$, for all $i = 1, \cdots, M_1$. Set $k = 0$.

Rotation Optimization

- Apply Algorithm 1 to optimize the threshold of node i in order $i = 1, \cdots, M_1$.
- For a particular node i, set the threshold of other nodes j as: $\theta_j^{*(k+1)}$ for $j = 1, \cdots, i-1$ and $\theta_j^{*(k)}$ for $j = i+1, \cdots, M_1$.
- Run Algorithm 1 for node i and obtain its optimal threshold as $\theta_i^{*(k+1)}$.

Stopping Rule

- If $\theta_i^{*(k+1)} = \theta_i^{*(k)}$, for all $i = 1, \cdots, M_1$, stop the algorithm and output the current thresholds $\theta_i^{*(k)}$, $i = 1, \cdots, M_1$, as the final results;
- Otherwise, set $k := k+1$ and go to step 2.

Algorithm 2 describes a rotation procedure to distribute Algorithm 1 among all the energy-critical nodes. It applies Algorithm 1 to every node. The performance of the total network will be continuously improved with every iteration. When the rotation procedure stops, the system performance cannot be improved under the current optimization framework.

IV. Numerical Simulation

In this section, we conduct numerical experiments to testify the performance of our optimization algorithm. To illustrate the main idea of our approach, we adopt a simple network with a small number of nodes.

Suppose that the network consists of 3 nodes. Node 1 and 2 are energy-critical, while node 3 is energy-noncritical. That is, $M = 3$ and $M_1 = 2$. The total capacity of the network is $N = 6$ (e.g., $N = 6$ means that the capacity of the network is 6k packets and we use 1k packets as a basic unit). The data generation rates at nodes are $\lambda_1 = 15$, $\lambda_2 = 20$, $\lambda_3 = 15$, respectively. The routing probabilities of nodes are $q_{10} = 0.3$, $q_{12} = 0.5$, $q_{13} = 0.2$, $q_{20} = 0.3$, $q_{21} = 0.3$, $q_{23} = 0.4$, $q_{30} = 0.2$, $q_{31} = 0.4$, $q_{32} = 0.4$. Since the external data generation process can be treated as a virtual node 0, the transmission rate of node 0 is $\mu_0 = \lambda_1 + \lambda_2 + \lambda_3 = 50$ and $q_{01} = 0.3$, $q_{02} = 0.4$, $q_{03} = 0.3$. The transmission rate of node 3 is fixed as $\mu_3 = 100$. The transmission rates of node i are adjustable within $[0, \mu_i^{max}]$ and $\mu_i^{max} = 100$, $i = 1, 2$. The coefficients in cost function (1) are $b = 0.2$ and $c = 10$.

We apply Algorithm 2 to optimize the thresholds of energy-critical nodes. During the first round of optimization, Algorithm 2 selects node 1 as the target node and uses Algorithm 1 for the optimization process. The aggregated performance potentials $\alpha(1, n_1)$, $n_1 = 1, 2, \cdots, 7$, can be obtained under the current system setting. We find that the signs of $\alpha(1, n_1)$ are all negative. Therefore, all the candidate thresholds for $\theta_1 = 2, 3, \cdots, 7$ are removed according to step 3 of Algorithm 1. Repeating this process, we obtain the optimal solution $\theta_1^* = 1, \theta_2^* = 1$ as the final output of Algorithm 2.

To verify the effectiveness of our algorithm, we enumerate the system performance under all of the possible thresholds, $\theta_1, \theta_2 = 1, 2, \cdots, 7$. The curve plane of system performance under different threshold settings is illustrated by Fig. 1. We find that $(\theta_1, \theta_2) = (1, 1)$ is truly the optimal threshold. Considering the edge of the curve plane, e.g., the left side of edge curve depicting η w.r.t. θ_2 when θ_1 is fixed as 1, we find that $\theta_2 = 1$ is truly optimal, which demonstrates the effectiveness of Algorithm 1.

It is worth pointing out that our Algorithm 2 can only find the local optimum, not the global optimum. The above exper-

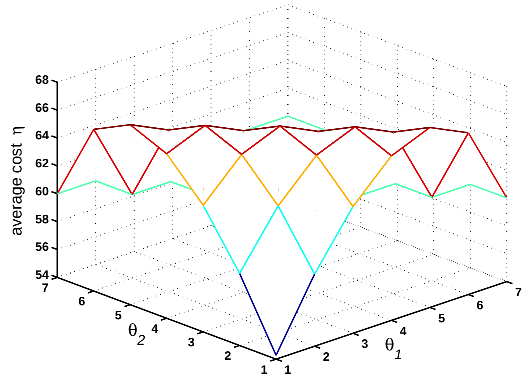

Fig. 1. The curve plane of system performance w.r.t. thresholds

iment is a special example, where the output of Algorithm 2 is exactly the global optimum.

From the above experiments, we find that Algorithm 2 has a good searching capability to find the optimal solution. Based on the difference equation (15), the algorithm can efficiently utilize the performance sensitivity information from the system sample path. We only need to run the system under the current threshold and observe the system sample path. Based on the sample path, we can estimate or calculate $\alpha(i, n_i)$'s and directly find the better solutions which are indicated by the signs of the corresponding $\alpha(i, n_i)$'s. Here we obtain another advantage of our approach. Since the *sign of an estimate* is much robuster than the *exact value of an estimate*, our estimation or calculation for $\alpha(i, n_i)$ does not require a very high accuracy. Our approach has a robust performance for the optimization.

V. Conclusion

In this paper, we present a decentralized optimization framework for the energy and delay optimization problem of energy-critical nodes in wireless networks. We formulate this problem as an MDP and prove that the optimal scheduling policy has a threshold form. This optimality lets the algorithm focus on only searching the optimal thresholds. This greatly reduces the optimization complexity. Based on the equation of performance difference under any different threshold policies, we develop a decentralized algorithm to optimize every node's scheduling policy. This decentralized algorithm is rotatingly executed by every node according to their local information. We further implement this algorithm with an online manner based on the system sample path. Simulation results demonstrate the effectiveness of our approach. In the future, we will further study how to improve the estimation efficiency and accuracy of our approach.

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