Proof of (20):

First, we start deriving the convolution needed in the proof.

$$e^{-ax} \otimes e^{-bx} = \int_0^x e^{-a(x-u)} e^{-bu} du = e^{-ax} \frac{e^{(a-b)x}-1}{a-b}$$
$$= \frac{e^{-bx}-e^{-ax}}{a-b}$$

For two hop (k = 2)

$$P_{1+2} = P_1 \otimes P_2 = \beta_1 \beta_2 \int_0^x e^{-(\beta_1-\beta_2)t} e^{-\beta_2 x} dt$$
$$= \frac{\beta_1 \beta 2}{\beta_2-\beta_1} \left(e^{-(\beta_1 x - \beta_2 x)} \right) = \beta_1 \beta 2 \left[\frac{e^{-\beta_1 x}}{\beta_2-\beta_1} + \frac{e^{-\beta_2 x}}{\beta_1-\beta_2} \right]$$
$$= -C_1^{(2)} P_1(x) - C_2^{(2)} P_2(x)$$

By using induction for $k \geq 3$ we can get

$$P_{k-1} = \sum_{i=1}^{k_l-1} C_i^{k_l-1} . P_i(x)$$

$$P_k = P_{1+2+\ldots k-1} \otimes P_k =$$

$$\left[\prod_{i=1}^{k-1} \beta_i \right] \sum_{j=1}^{k_l-1} \frac{e^{\beta_j x}}{\prod_{i=1}^{k-1} (\beta_k - \beta_j)} \otimes P_k$$
$$= \sum_{i=1}^{k_l-1} C_i^{k_l-1} \left(\frac{\beta_k}{\beta_k-\beta_i} P_i(x) + \frac{\beta_i}{\beta_i-\beta_k} P_k(x) \right)$$

If we consider $C_i^{k_l} = C_i^{k_l-1} . \frac{\beta_i}{\beta_i-\beta_k}$, we get

$$P_k = \sum_{i=1}^{k_l-1} C_i^{k_l} . P_i(x) + \sum_{i=1}^{k_l-1} C_i^{k_l-1} . \frac{\beta_i}{\beta_i-\beta_k} P_k(x)$$

For the second term, we have

$$\sum_{i=1}^{k_l-1} C_i^{k_l-1} \frac{\beta_i}{\beta_i-\beta_k} = \sum_{i=1}^{k_l-1} \frac{\beta_i}{\beta_i-\beta_k}$$
$$\cdot \left(\prod_{j=1,\, j \neq i}^{k-1} \frac{\beta_j}{\beta_j-\beta_i} \right)$$
$$= \prod_{j=1}^{k-1} \beta_j . \sum_{i=1}^{k_l-1} \prod_{j=1,\, j \neq i}^{k-1} \frac{1}{\beta_j-\beta_i}$$
$$= \prod_{j=1}^{k-1} \frac{\beta_j}{\beta_j-\beta_k} = C_k^{k_l}$$

Therefore, we have

$$f_k(x) = P_k = \sum_{i=1}^{k_l-1} C_i^{k_l} P_i(x) + C_k^{k_l} P_k(x)$$
$$= \sum_{i=1}^{k_l} C_i^{k_l} P_i(x)$$

CDF:

$$F_k(x) = P(T_d < T) = \int_0^T f_k(x) dx =$$

$$\sum_{i=1}^{k_l} C_i^{k_l} \int_0^T P_i(x) dx = \sum_{i=1}^{k_l} C_i^{k_l} . (1 - e^{-\beta_i T})$$

CCDF:

$$= P(T_d > T) = \int_T^\infty f_k(x) dx = \sum_{i=1}^{k_l} C_i^{k_l} . e^{-\beta_i T}$$