

## Fuzzy Approximate Model for Distributed Thermal Solar Collectors Control

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**Abstract:** This paper deals with the problem of controlling concentrated solar collectors where the objective consists of making the outlet temperature of the collector tracking a desired reference. The performance of the novel approximate model based on fuzzy theory, which has been introduced by the authors in [1], is evaluated comparing to other methods in the literature. The proposed approximation is a low order state representation derived from the physical distributed model. It reproduces the temperature transfer dynamics through the collectors accurately and allows the simplification of the control design. Simulation results show interesting performance of the proposed controller.

**Keywords:** solar energy, concentrate solar collectors, fuzzy transform theory, nonlinear control.

### INTRODUCTION

In the last few decades, an increasing interest is given to renewable energy in order to reduce reliability on fossil energy. Particularly, characterized by the high productivity at industrial scale, thermal solar power production attracted many researchers to optimize the production of solar collectors. The concentrated parabolic troughs are the most used to convert solar light into heat for high scale production. They use their parabolic surfaces to concentrate the solar irradiance to the pipe located at the focal line of the parabola in order to heat the fluid flowing through it. Then, considering the energy balance of the heat transfer along the pipe, the physical model of the collectors is given by a distributed representation (partial differential equation) [2-5].

The mathematical model of the solar collector makes its control problem not trivial because of the distributed nature of the equation. Therefore, many simplified models have been introduced to facilitate the controller design [2-7]. Usually, the proposed simplification is either a spatial semi-discretization or a lumped parameter approximation. However, both techniques present limitations such as loss of information or high dimension of the resulting system.

Consequently, to overcome the encountered difficulties, a fuzzy approximate model has been proposed in [1] to reproduce the temperature behavior along the collector pipe. Fuzzy transformation has been applied to the distributed model to derive a state space representation. The resulting model is a low dimensional set of ordinary differential equations that accurately approximates the collector's dynamics. Indeed, fuzzy theory has been widely used in the last years in order to deal with complex nonlinear systems with approximate models that simplify the system analysis [8-9].

The control goal aims at making the outlet oil temperature to track a set reference by tuning the oil flow rate through the collector pipe. Thus, based on the proposed fuzzy approximate model, a control law has been synthesized resorting to Lyapunov Control functions. Convergence of the closed loop has been proven by simulation runs with interesting performance.

### CONCENTRATED SOLAR COLLECTOR MODELING

Parabolic solar collectors are spatially distributed industrial systems. They exploit the thermal properties of the fluid flowing along the collector tube to produce thermal energy. Indeed, the receiver fluid is heated by the concentrated sunlight to feed an industrial process with the generated heat. Based on the energy balance, several models of the solar collectors have been introduced in the literature [3-4]. A simple mathematical description that has been widely used to represent the heat transfer along the receiver tube (for  $x \in [0, L]$ ), has the following form:

$$\begin{cases} \frac{\partial T(x,t)}{\partial t} + u(t) \frac{\partial T(x,t)}{\partial x} = f(t), \\ T(0,t) = T_{in}(t), \\ T(L,t) = T_{out}(t), \end{cases} \quad (1)$$

where  $T(x,t)$  is the fluid temperature,  $T_{in}(t)$  and  $T_{out}(t)$  are the inlet and the outlet oil temperatures,  $f(t)$  is the source term and  $u(t)$  is the control input.  $x$  denotes the space and  $t$  the time.  $u(t) = \frac{q(t)}{S}$  is the input control where  $q(t)$  is the fluid flow rate, which is the tuned variable.  $f(t) = \eta I(t)$  is the source term such that  $\eta$  is a parameter depending on the fluid and the mirrors characteristics and  $I(t)$  is the value of the solar irradiance.

### NONLINEAR FUZZY APPROXIMATOR

To apply the fuzzy transformation, the space along the tube has been subdivided into  $m$  fuzzy sets  $D_i$  characterized by their membership values  $\xi_i$ , for  $i = \{1, \dots, m\}$ :

$$D_i = \{(x, \xi_i(x)) \mid x \text{ in } [0, L]\}, \quad (2)$$

The membership functions have been chosen to be Gaussian in order to have a smooth, continuous and infinitely differentiable solution. Then, the inference rules of the fuzzy transformation are given by:

$$R^i: \text{ IF } x \text{ is } D_i \quad (3)$$

$$\text{ THEN } T(x, t) = \alpha_i(t),$$

for  $i = \{1, \dots, m\}$  and  $x \in [0, L]$  to end up with the fuzzy output written in the form:

$$T(x, t) = \sum_{i=1}^m \alpha_i(t) \xi_i(x) \quad (4)$$

$$= \underline{\xi}^T(x) \underline{\alpha}(t)$$

with

$$\underline{\alpha}(t) = [\alpha_1(t), \dots, \alpha_m(t)]^T, \quad \underline{\xi}(x) = [\xi_1(x), \dots, \xi_m(x)]^T$$

Considering  $p$  equidistant points on the domain  $[0, L]$  for which the substitution of the proposition (4) in (1) yields:

$$H \underline{\dot{\alpha}} + H_x \underline{\alpha} u(t) = F(t). \quad (5)$$

Therefore, the approximate dynamical model for the solar plant can be expressed as:

$$\begin{cases} \underline{\dot{\alpha}} = A \underline{\alpha}(t) u(t) + B \\ y(t) = C \underline{\alpha}(t) \end{cases} \quad (6)$$

where:

$$A = -(H^T H)^{-1} H^T H_x, \quad B = (H^T H)^{-1} H^T F(t),$$

$$C = [\xi_1(L) \dots \xi_m(L)] \text{ and } y(t) = T(L, t);$$

Such that:

$$H = \begin{bmatrix} \xi_1(0) & \dots & \xi_m(0) \\ \xi_1(\Delta x) & \dots & \xi_m(\Delta x) \\ \vdots & & \vdots \\ \xi_1(j) & \dots & \xi_m(j) \\ \vdots & & \vdots \\ \xi_1(L) & \dots & \xi_m(L) \end{bmatrix}_{p \times m}, \quad H_x = \begin{bmatrix} 0 & 0 & 0 \\ \xi_1^x(\Delta x) & \dots & \xi_m^x(\Delta x) \\ \vdots & & \vdots \\ \xi_1^x(j) & \dots & \xi_m^x(j) \\ \vdots & & \vdots \\ \xi_1^x(L) & \dots & \xi_m^x(L) \end{bmatrix}_{p \times m},$$

$$F(t) = f(t) \begin{bmatrix} 0 \\ \mathbf{1}_{(p-1) \times 1} \end{bmatrix} \text{ and } \xi_i^x = \frac{\partial \xi_i(x)}{\partial x},$$

for  $j = \{\Delta x, 2\Delta x, \dots, (p-1)\Delta x, L\}$  with  $\Delta x = \frac{L}{p-1}$ .

It is worth to point out that the system order is equal to the number of fuzzy sets  $m < p$ . For more details, the reader should refer to [1].

### LYAPUNOV NONLINEAR CONTROL

Assuming that the solar plant behavior is accurately approximated by the fuzzy model, the state representation of equation (6) has been considered for the control design. The control objective in the distributed solar field is to find the suitable  $u(t)$  that allows tracking the desired reference for the outlet temperature. Based on the Lyapunov control theory, the following control law has been designed in order to stabilize the tracking error  $e$  [1].

$$u(t) = \frac{(K e - C B)}{C A \alpha} \quad (7)$$

where  $e \triangleq T_r - T_{out}$  represents the difference between the reference  $T_r$  and the measured outlet oil temperature. The control design has been established based on the energy Lyapunov function defined by:

$$V : R \rightarrow R$$

$$V = \frac{1}{2} e^2 \quad (8)$$

where  $K$  is a positive parameter.

## NUMERICAL RESULTS

### Model validation

The first simulation test has been conducted to validate the proposed fuzzy model. Actually, the resulting temperature profile of the fuzzy model has been compared to the semi-discrete solution in different positions along the pipe. The simulations have been run using the ACUREX field Plataforma Solar de Almeria parameters [4] under evolving profiles for the external disturbances, the inlet temperature and the solar irradiance (see Fig. 1).

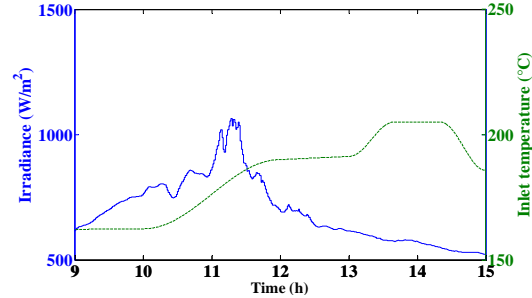


Fig. 1. Working conditions profiles.

In addition, the fluid flow has been injected with different velocities along time according to Fig. 2. For this simulation test, the fuzzy universe has been subdivided into 8 fuzzy sets ( $m = 8$ ) where the matrices  $H$  and  $H_x$  have been constituted based on a spatial grid of 500 nodes ( $p = 500$ ) to end up with a state representation of dimension equal to 8. The semi discrete solution has been also computed based on the same grid ( $p = 500$ ).

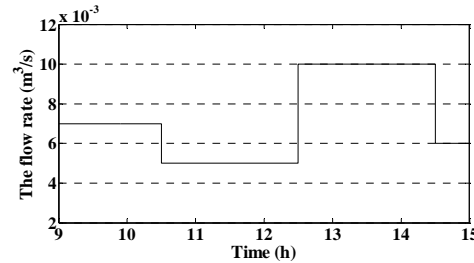


Fig. 2. Control input for model validation

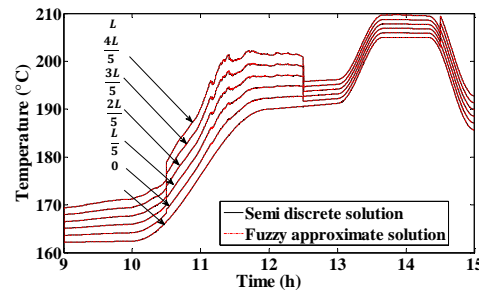


Fig. 3. Temperature evolution in different positions

Fig.3 presents the behavior of the two approximations. It is clear that the resulting fuzzy state-space model is able to approximate the partial

differential equation accurately. Indeed, the proposed fuzzy solution could reproduce the heat transfer behavior along the receiver tube with an error converging to zero. This conclusion can be justified by the profile of the difference between the fuzzy model and the semi-discrete one, which is presented in Fig. 4.

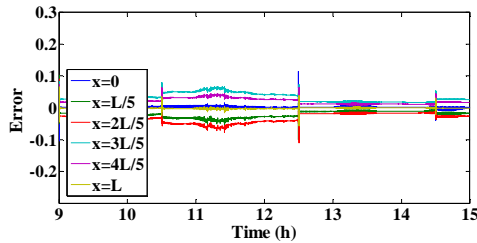


Fig. 4. Difference between the semi-discrete solution and the fuzzy solution.

Indeed, the fuzzy approximate solution proves its accuracy in approximating the behavior of the distributed solar collector with a low dimension representation. To evaluate the efficiency of this lumped parameter approximation, simulation tests have been run using less fuzzy sets and smaller grids. The errors mean are compared in Table. 1.

Table 1. The mean of the difference between the two models under different configurations.

x	Number of fuzzy sets m=4		
	Number of nodes in the grid (p)		
	100	250	500
0	0.0016	0.0016	0.0016
L/5	0.0022	0.0077	0.0095
2L/5	-0.0789	-0.0725	-0.0704
3L/5	-0.0140	0.0699	0.0708
4L/5	-0.0140	-0.0103	-0.0090
L	-0.0136	-0.0044	-0.0013
x	Number of fuzzy sets m=6		
	Number of nodes in the grid (p)		
	100	250	500
0	0.0017	0.0017	0.0017
L/5	-0.0589	-0.0518	-0.0494
2L/5	0.0519	0.0545	0.0554
3L/5	-0.0644	-0.0574	-0.0551
4L/5	0.0465	0.0489	0.0497
L	-0.0142	-0.0047	-0.0015
x	Number of fuzzy sets m=8		
	Number of nodes in the grid (p)		
	100	250	500
0	0.0016	0.0017	0.0017
L/5	-0.0278	-0.0224	-0.0205
2L/5	-0.0408	-0.0342	-0.0320
3L/5	0.0282	0.0313	0.0323
4L/5	0.0152	0.0194	0.0208
L	-0.0142	-0.0047	-0.0015

Moreover, it can be concluded that the resulting fuzzy approximate model presents acceptable approximation errors with smaller grids and fewer fuzzy sets. The heat transfer profile along the tube has been reproduced with an error mean lower than 0.1 with 4 fuzzy sets and a computational grid of 100

points. Consequently, the fuzzy approximate model presents interesting features for the control design thanks to its low dimensional state representation.

### Lyapunov control performance evaluation

The controller performance has been evaluated using the same system parameters under the working conditions of Fig. 5. The sampling time for the control has been considered equal to 36 seconds as it is in the Acurex Field according to [4]. The closed loop results are presented in Fig. 6 where the evolution of the measured outlet is compared to the set temperature reference. It is shown that the proposed Lyapunov control based on the fuzzy model ensures effectively the reference tracking with an error converging to zero. This can be verified by the error profile presented in Fig. 7.

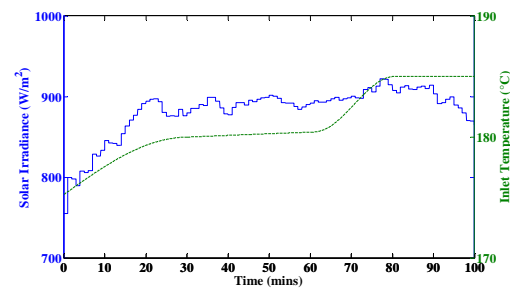


Fig. 5. External disturbances profiles

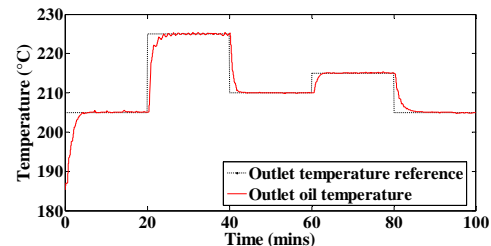


Fig. 6. Evolution of the outlet temperature tracking the reference

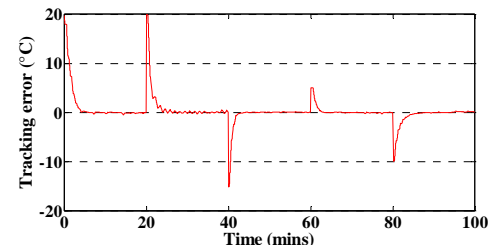


Fig. 7. Tracking error

Fig. 8 presents the generated control input to achieve the control goal. Effectively, the changes with respect to the variations in the temperature reference can be clearly noticed. The flow rate is decreased when an increasing step reference is injected and it is raised for a decreasing set temperature.

The response time has been estimated to be less than ten minutes which is very competitive comparing to others control methods presented in the literature. Therefore, the response time of the

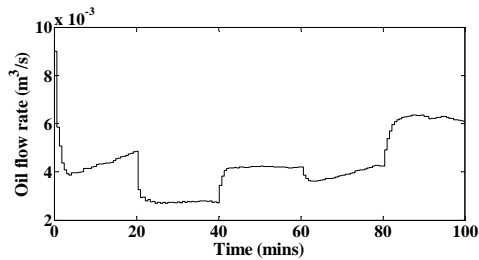


Fig. 8. Generated control input for the reference tracking

presented controller has been qualitatively compared to some control techniques that have been designed for the Acurex Field as well. The following table resumes the obtained results.

Table 2. Performance comparison.

Reference	Controller	Range of time response $T_r$ (in mins)
[6]	Fixed PI	$55 < T_r < 60$
[6]	Incremental Fuzzy PI	$35 < T_r < 45$
[10]	Fuzzy logic controller	$10 < T_r < 20$
[3]	Energy-based control	$10 < T_r < 20$
[11]	Variable Sampling Adaptive Control	$10 < T_r < 20$
[12]	adaptive PI	$25 < T_r < 35$
[12]	Adaptive Generalized Predictive Control	$10 < T_r < 15$
[13]	Adaptive neural output regulation	$5 < T_r < 10$
The presented work	Lyapunov control based fuzzy model	$T_r < 5$

Table 2. confirms the good performance of the Fuzzy model-based Lyapunov control. In addition to this, the implementation simplicity of the control law makes it computationally not demanding and very suitable for real time working stations.

## CONCLUSION

In this work, we have presented an approximate model based on fuzzy transform theory. The introduced model consists of a lumped parameter approximation defined as a set of ordinary differential equations. The latter is characterized by its low dimension compared to other models in the literature which presents an interesting advantage. Then, it is a simple accurate approximation for the concentrated solar collectors without simplifying hypothesis. Moreover, the proposed model simplifies the control design to manage the heat transfer along the distributed solar plant. Indeed, a Lyapunov control has been designed based on the fuzzy resulting approximation. The controller has presented promising performance under varying external disturbances. The authors are intending to validate experimentally the proposed model and to compare

its approximation under real working conditions with other simplified representations such as the lumped parameter bilinear representations and/or the linear transfer functions of low and high dimensions.

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