Collective Human Mobility Pattern from Taxi Trips in Urban Area

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Supporting Information

S3 Moment Generation Function of α

In this section we discuss α as a random variable following a normalized binomial distribution described by Eq. (10) and Eq. (11). Then the moment generating function of α is

$$\begin{aligned}
&M(g)\\
=&\langle e^{g\times\alpha}\rangle\\
=&\sum_{\alpha\langle TN\rangle=0}^{pn} e^{g\times\alpha}\binom{pn}{\alpha\langle TN\rangle} r^{\alpha\langle TN\rangle}(1-r)^{pn-\alpha\langle TN\rangle}\\
=&\sum_{\alpha\langle TN\rangle=0}^{pn} \binom{pn}{\alpha\langle TN\rangle}(r\times e^{\frac{g}{\langle TN\rangle}})^{\alpha\langle TN\rangle}(1-r)^{pn-\alpha\langle TN\rangle}\\
=&(r\times e^{\frac{g}{\langle TN\rangle}}+(1-r))^{pn}\\
=&(r\times e^{\frac{g}{pn\times r}}+(1-r))^{pn}
\end{aligned} \tag{1}$$

and consequently, the first, second and third moments are as follows.

$$\begin{aligned}
&\mu_1\\
=&M'(g)|_{g=0}\\
=&(r+(1-r))^{pn-1}\\
=&1
\end{aligned} \tag{2}$$

$$\begin{aligned}
&\mu_2\\
=&M''(g)|_{g=0}\\
=&(pn-1)\frac{1}{pn}+\frac{1}{\langle TN\rangle}\\
\approx&\frac{1}{\langle TN\rangle}
\end{aligned} \tag{3}$$

$$
\begin{aligned}
&\mu_3 \\
=&M'''(g)|_{g=0} \\
=&\frac{pn-1}{pn}[(pn-2)r\frac{1}{\langle TN\rangle}+\frac{2}{\langle TN\rangle}] \\
&+\frac{1}{\langle TN\rangle}[(pn-1)r\frac{1}{\langle TN\rangle}+\frac{1}{\langle TN\rangle}] \\
\approx&1+\frac{2}{\langle TN\rangle}+\frac{1}{\langle TN\rangle}(1+\frac{1}{\langle TN\rangle}) \\
=&1+\frac{3}{\langle TN\rangle}+\frac{1}{(\langle TN\rangle)^2}
\end{aligned} \tag{4}
$$

Typically pn is quite large, so that $pn \approx pn-1 \approx pn-2$, and the approximations above are valid.

References