

A Suboptimal Scheme for Multi-User Scheduling in Gaussian Broadcast Channels

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Abstract

This work proposes a suboptimal multi-user scheduling scheme for Gaussian broadcast channels which improves upon the classical single user selection, while considerably reducing complexity as compared to the optimal superposition coding with successful interference cancellation. The proposed scheme combines the two users with the maximum weighted instantaneous rate using superposition coding. The instantaneous rate and power allocation are derived in closed-form, while the long term rate of each user is derived in integral form for all channel distributions. Numerical results are then provided to characterize the prospected gains of the proposed scheme.

Index Terms

Broadcast channels, block-fading, multi-user diversity, resource allocation, superposition coding.

I. INTRODUCTION

It is now well known that next generation wireless systems will be required to support high data rates [1]. Hence, recent years have seen considerable work being put into improving the spectral efficiency of wireless communication systems. Many new and innovative techniques, such as cooperation [2], cognitive radio [3] etc, have been proposed to support higher data rates. In addition to new techniques, known optimal techniques, which were too complex to implement, have also been re-visited. One such technique is optimal superposition coding (SC) with successive interference cancellation (SIC) which is the capacity

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achieving scheme for the Gaussian broadcast channel [4]. Even though SC with SIC is the optimal scheme, it is difficult to implement in practice due to the complexity it entails. In SC with SIC, the rate and power allocation depend on the marginal utility function of each user which needs to be calculated for a range of values of the interference levels as elaborated in Section III. Moreover, depending on the channel quality of each user, the user needs to know the response of other users with worse channel conditions which contributes to the overhead and consumes precious resources. Furthermore, the number of users scheduled at a time depends upon the channel condition and varies with time. If there are a significant amount of users, the complexity and feedback load might reduce spectral efficiency. Hence, to reduce complexity and overhead, optimal single user selection (SU), in which only a single user which has the maximum weighted rate is selected was considered such as in [5]. Selecting a single user reduces complexity considerably while still exploiting multi-user diversity. SU becomes the optimal scheme when the objective is to maximize the sum rate and is suboptimal when the weights of the users are different.

Reference [6] compared the performance of SU and SC with SIC for three operating points, namely hard fairness, proportional fairness and fixed weights. Reference [6] showed that SC with SIC can achieve a 10% or higher gain than SU, so it may be feasible in some scenarios. Other works which considered the performance of SC with SIC include [7]–[10]. The work in [7] only limited itself to only two users, while the works in [8], [9] did not consider optimal rate and power allocation. The work in [10] studied SC with SIC for the special case of the proportional fair scheduler. Moreover, [10] did not utilize the optimal weights for the proportional fair scheduler.

Motivated by the work in [6], this work proposes a suboptimal scheduling scheme for Gaussian broadcast channels which improves upon the classical SU and still has significantly less complexity than SC with SIC. In the proposed scheme, only the two ‘best’ users are first selected according to their respective weights and instantaneous capacity and then are combined using SC. Now as there are only two users for SC, the rate and power allocation are found in closed-form. Moreover, the source only needs to broadcast the channel response of one user to the other user. Hence, significantly reducing overhead. Numerical results show that the proposed scheme can provide considerable gain over classical SU and can be a viable alternative to SC with SIC in cases where simplicity is desired.

The rest of the letter is organized as follows. Section II describes the system and channel model. The proposed scheme is discussed in Section III. The rate and power allocation is derived and then the

long term rate of each user is characterized. Numerical results are provided in Section IV to analyze the performance and complexity of the proposed scheme. Finally, Section V concludes the paper.

II. SYSTEM MODEL

Consider the classical broadcast channel in which a source (S) communicates with M users or destinations (D). Each user has its own rate requirements which are specified by its weight, μ_i . Also, each terminal is assumed to be equipped with a single antenna. The channel is modeled as block-fading. Hence, the time-frequency grid is partitioned into blocks. The channel is assumed to stay constant over a channel block (CB) and change independently and randomly from one channel block to another. This is a standard assumption in most works on multi-user scheduling and can be achieved by setting the duration of the channel block significantly less than the coherence time and the setting the subcarrier bandwidth significantly less than the coherence bandwidth which is the case in practice. Without loss of generality, it is assumed that each CB has the same duration and bandwidth. The users are also assumed to undergo independent fading which is the case when the users are sufficiently far apart.

We assume a constant power of \bar{P} per CB. It is also assumed, without loss of generality, that the noise variance of all user are equal and are given by σ^2 . The source is assumed to have full CSI¹ of all the links to the users and makes scheduling decisions in each CB according to the instantaneous channel conditions as explained in the next section.

III. PROPOSED SCHEDULING SCHEME

A. User Selection and Resource Allocation

In the proposed scheme, the users are first sorted, in the k th CB, according to the criteria

$$\zeta_i[k] = \mu_i R_i^c[k] \quad i = 1, 2, \dots, M, \quad (1)$$

where $R_i^c[k]$ is the instantaneous capacity of the i th user in the the k th CB given by

$$R_i^c[k] = \log_2 \left(1 + \frac{\bar{P}\gamma_i[k]}{\sigma^2} \right) \quad (2)$$

¹It is noted that the source only requires knowledge of the amplitude of the channel gain.

$$\mathbb{A}_i[k] \equiv \left\{ z \in \left[0, \frac{\bar{P}}{\sigma^2} \right] : u_i(k, z) > u_j(k, z) \quad \forall j \neq i, \{i, j\} \in \{m, n\} \right\} \quad (9)$$

and $\gamma_i[k]$ is the channel power gain of the i th user in the k th CB. Then, the two users with the two maximum ζ_i are selected. Thus, in each CB, the first user is selected as

$$m = \arg \max \zeta_i \quad \forall i = 1, 2, \dots, M, \quad (3)$$

and the second user is selected as

$$n = \arg \max \zeta_i \quad \forall i \neq m. \quad (4)$$

After selection, both the selected users are then combined using superposition coding. In superposition coding, a marginal utility function is defined for all the competing users and then power and rate are allocated to each user in accordance with their respective utility function. The marginal utility function of the i th user is defined as [4]

$$u_i(k, z) = \frac{\mu_i}{\frac{1}{\gamma_i[k]} + z} - \lambda[k] \quad i = m, n, \quad 0 \leq z \leq \frac{\bar{P}}{N_0}, \quad (5)$$

where z denotes the interference level,

$$\lambda[k] = \max \lambda_i[k] \quad i = m, n, \quad (6)$$

where

$$\lambda_i[k] = \frac{\mu_i}{\frac{1}{\gamma_i[k]} + \frac{\bar{P}}{\sigma^2}} \quad i = m, n. \quad (7)$$

The rate and power are then allocated to each user as

$$R_i[k] = \frac{1}{\ln 2} \int_{\mathbb{A}_i[k]} \frac{1}{\frac{1}{\gamma_i[k]} + z} dz, \quad P_i[k] = \sigma^2 \int_{\mathbb{A}_i[k]} dz, \quad (8)$$

where the period $\mathbb{A}_i[k]$ is defined as in (9). It is noted here that if $\mathbb{A}_i[k]$ is an empty set then the i th user is not selected in the k th CB. In general, it is difficult to obtain closed-form solutions for the optimal power and rate allocation. Hence, the utility functions for all the users need to be found and from there the optimal rate and power for each user are calculated using (8). This leads to significant computations

and hence, the considerable complexity of SC.

For the proposed scheme, as there only two users, the rate and power allocation can be obtained in closed-form. Due to their being two users, the case where both the users are scheduled is when the intersection point of their marginal utility functions lies in the interval $[0, \frac{\bar{P}}{\sigma^2}]$. Equating the utility functions and simplifying yields the intersection point in each CB, denoted by $z^*[k]$ as

$$z^*[k] = \frac{\frac{\mu_m}{\gamma_n[k]} - \frac{\mu_n}{\gamma_m[k]}}{\mu_n - \mu_m} \quad \mu_n \neq \mu_m. \quad (10)$$

If $\mu_m = \mu_n$, then the user with higher power gain, $\gamma[k]$, is selected in the k th CB. Now using (8) and (10), the rate of the i th user, $i \in \{m, n\}$, in the k th CB can be obtained as

$$R_i[k] = \begin{cases} 0 & z^*[k] \leq 0 \text{ or } z^*[k] \geq \frac{\bar{P}}{\sigma^2}, \lambda_j[k] > \lambda_i[k] \\ \log_2 \left(1 + \frac{\bar{P}\gamma_i[k]}{\sigma^2} \right) & z^*[k] \leq 0 \text{ or } z^*[k] \geq \frac{\bar{P}}{\sigma^2}, \lambda_i[k] > \lambda_j[k] \\ \log_2 \left(1 + \frac{\mu_m \gamma_m[k] - \mu_n \gamma_n[k]}{\gamma_m[k](\mu_n - \mu_m)} \right) & 0 < z^*[k] < \frac{\bar{P}}{\sigma^2}, \lambda_j[k] > \lambda_i[k] \\ \log_2 \left(\frac{(\sigma^2 + \bar{P}\gamma_i[k])\gamma_j[k](\mu_n - \mu_m)}{\sigma^2 \mu_i (\gamma_m[k] - \gamma_n[k])} \right) & 0 < z^*[k] < \frac{\bar{P}}{\sigma^2}, \lambda_i[k] > \lambda_j[k]. \end{cases} \quad (11)$$

Similarly using (8) and (10), the power allocated to the i th user in the k th CB, $P_i[k]$, can be obtained as

$$P_i[k] = \begin{cases} 0 & z^*[k] \leq 0 \text{ or } z^*[k] \geq \frac{\bar{P}}{\sigma^2}, \lambda_j[k] > \lambda_i[k] \\ \bar{P} & z^*[k] \leq 0 \text{ or } z^*[k] \geq \frac{\bar{P}}{\sigma^2}, \lambda_i[k] > \lambda_j[k] \\ \sigma^2 \frac{\frac{\mu_m}{\gamma_n[k]} - \frac{\mu_n}{\gamma_m[k]}}{\mu_n - \mu_m} & 0 < z^*[k] < \frac{\bar{P}}{\sigma^2}, \lambda_j[k] > \lambda_i[k] \\ \bar{P} - \sigma^2 \frac{\frac{\mu_m}{\gamma_n[k]} - \frac{\mu_n}{\gamma_m[k]}}{\mu_n - \mu_m} & 0 < z^*[k] < \frac{\bar{P}}{\sigma^2}, \lambda_i[k] > \lambda_j[k]. \end{cases} \quad (12)$$

B. Long Term Achievable Rate

The long term achievable rate of the i th user is given by

$$\bar{R}_i = \frac{1}{\ln 2} \int_0^{\frac{\bar{P}}{\sigma^2}} \int_0^\infty \frac{1}{\frac{1}{\gamma} + z} f_{\gamma_i}(\gamma) \quad (13)$$

$$\text{Prob}(i\text{th user is selected} | \gamma_i = \gamma, z) d\gamma dz,$$

where $f_{\gamma_i}(\cdot)$ is the probability density function (pdf) of the channel power gain of the i th user. The i th user is selected when its ζ_i is among the two maximum ζ_i s and then its utility function is the maximum

in any part of the region $0 \leq z \leq \frac{\bar{P}}{\sigma^2}$. Note that, a user's ζ_i being either first or second are mutually exclusive events. Also, noting that all the users experience independent fading, the probability of i th user selection can be found as

$$\begin{aligned} \text{Prob}(i\text{th user is selected}|\gamma_i = \gamma) &= \sum_{l \neq i} G_l(\gamma) \prod_{j \neq i, l} F_j \left(\left(\left(1 + \frac{\bar{P}\gamma}{\sigma^2} \right)^{\frac{\mu_j}{\mu_l}} - 1 \right) \frac{\sigma^2}{\bar{P}} \right) + \\ &\sum_{l \neq i} \int_0^{\min\left(\frac{\sigma^2}{\bar{P}} \left(\left(1 + \frac{\bar{P}\gamma}{\sigma^2} \right)^{\frac{\mu_l}{\mu_l}} - 1 \right), \left[\frac{\mu_l \gamma}{\mu_l + (\mu_l - \mu_i) \gamma z} \right]^* \right)} \prod_{m \neq i, l} F_m \left(\left(\left(1 + \frac{\bar{P}y}{\sigma^2} \right)^{\frac{\mu_l}{\mu_m}} - 1 \right) \frac{\sigma^2}{\bar{P}} \right) f_{\gamma_l}(y) dy, \end{aligned} \quad (14)$$

where

$$G_l(\gamma) = \left[F_l \left(\left[\frac{\mu_l \gamma}{\mu_l + (\mu_l - \mu_i) \gamma z} \right]^* \right) - F_l \left(\left(\left(1 + \frac{\bar{P}\gamma}{\sigma^2} \right)^{\frac{\mu_l}{\mu_l}} - 1 \right) \frac{\sigma^2}{\bar{P}} \right) \right]^+, \quad (15)$$

$F_j(\cdot)$ is the cumulative distribution function (cdf) of the channel power gain of the j th user, $[x]^+ = \max(0, x)$ and using the notation in [11]

$$[a]^* = \begin{cases} \infty & a < 0 \\ a & \text{otherwise.} \end{cases} \quad (16)$$

The first term in (14) is the probability that the i th user has the second largest ζ_i and i th user's utility function is greater than the utility function of the user with the maximum ζ_i . The second term is the probability the i th user has the largest ζ_i . Note that if a user has the maximum ζ_i , its marginal utility function will also be maximum over a region as the ζ_i is the integral of the marginal utility function over the complete range of z . A more detailed derivation from (14) is given in the appendix.

IV. NUMERICAL RESULTS

We now present numerical results to compare the performance of the proposed scheme to SU and SC with SIC. We use the same model to generate the users' average power gain as in [6, Section III]. We also utilize Rayleigh fading to model the fading process of all the users. Hence, the pdf and the cdf of the fading process are given by

$$\begin{aligned} f_\gamma(\gamma) &= \frac{1}{\bar{\gamma}} \exp\left(-\frac{\gamma}{\bar{\gamma}}\right) \quad \text{and} \\ F_\gamma(\gamma) &= 1 - \exp\left(-\frac{\gamma}{\bar{\gamma}}\right) \quad \gamma \geq 0, \end{aligned} \quad (17)$$

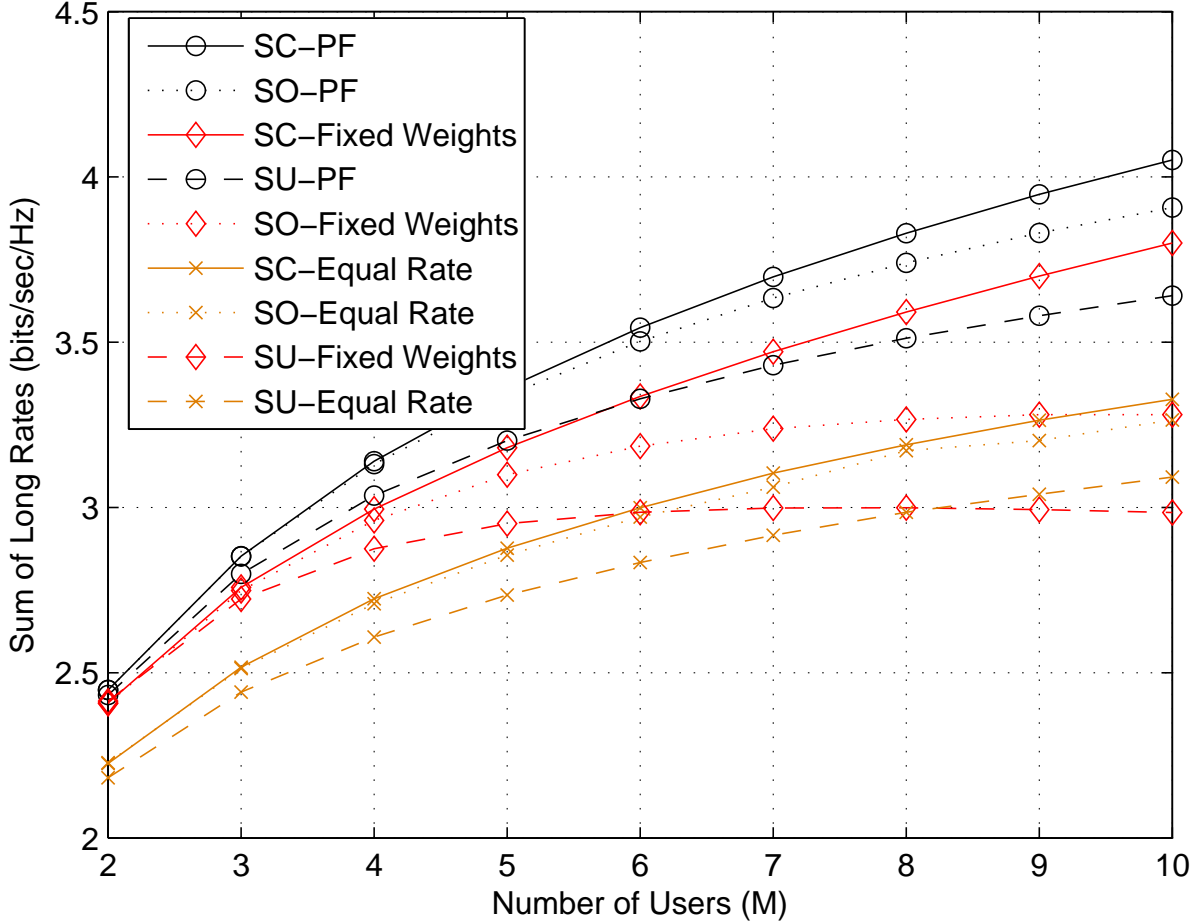


Fig. 1: Comparison of SC, SO and SU in term of sum of long term throughputs of all the users.

respectively, where $\bar{\gamma}$ is the average power gain. The channel power gain and all the the other simulation parameters are the same as in [6]. We compare the schemes for three specific schedulers, namely hard fairness, proportional fairness and fixed weights. These are the same as in [6]. However, the algorithms given in [6] for calculating the user weights for proportional fairness and hard fairness need to be slightly modified. Here, R_i needs to be calculated by the procedure outlined in Section III. The weight update equation remains the same as in Algorithm 1 in [6]. All the simulation parameters are the same as in [6]. In the following we denote the proposed suboptimal scheme as SO.

Fig. 1 shows the sum of the long term rates for all three operating points for the three schemes of SC, SO and SU. It can clearly be seen that SO provides good gain over SU, particularly as the number of users increases. Moreover, the difference between the SC and SO is not large, particularly for PF and fixed weights. A similar behaviour is seen in Fig. 2 where the gains of SO and SC over SU are plotted.

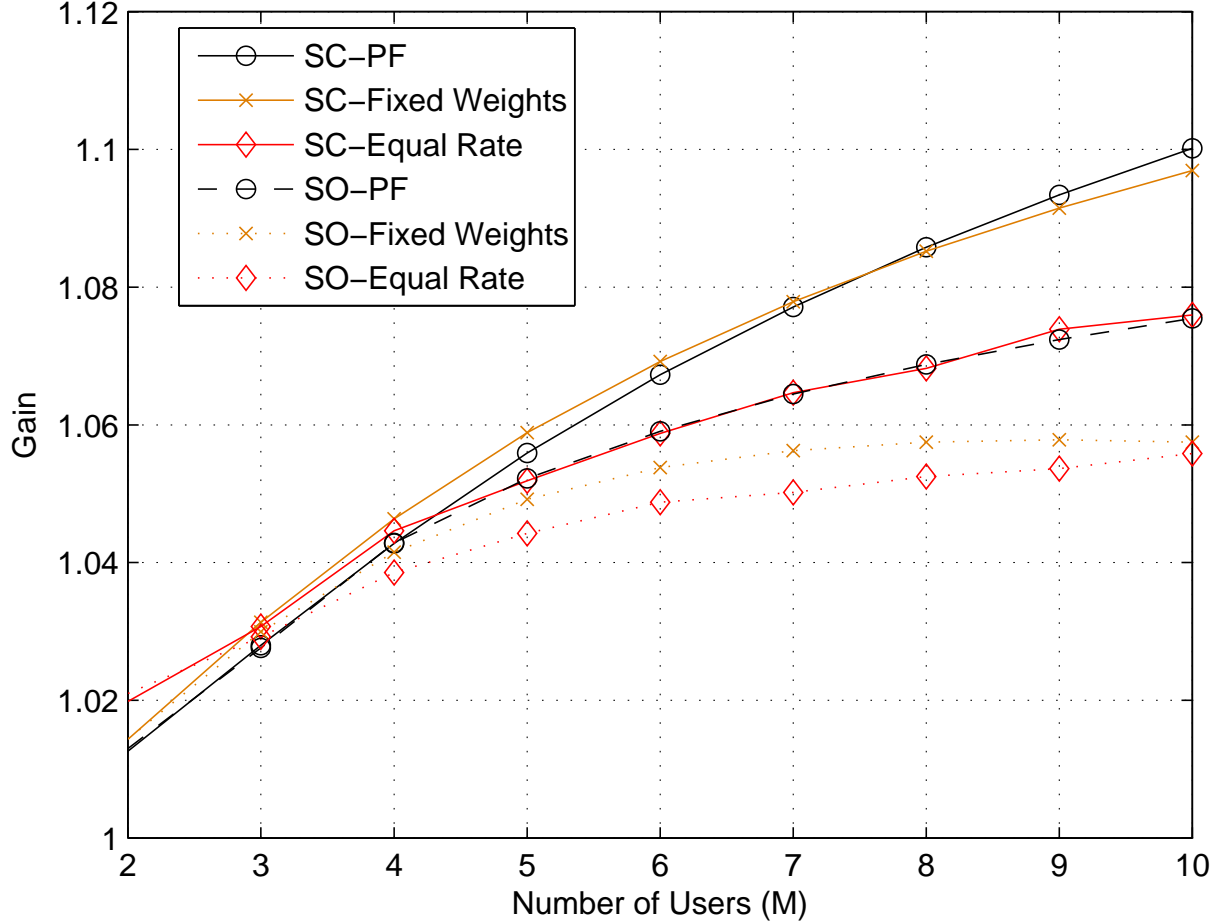


Fig. 2: Gain in performance of SC and SO over SU as a function of number of users, M .

While SO does not provide the same gain as SC, as expected, it can still provide more than 5% gain over SU. The benefit of these gains becomes clear below when we discuss the complexity of these schemes.

The complexity for these schemes comes from the feedback of channel state information (CSI) and the computational complexity of implementing the algorithm. For all three schemes, the scheduler needs to know the channel gains of all the links. Thus, this amount of feedback is common for all schemes. However, for SU, the scheduler does not need to inform the selected user of any channel gains which is not the case for both SC and SO. Assuming that Q bits are used to represent a channel gain, then SO requires Q bits to feedback the response of the low priority user to the high priority user when two users are scheduled and zero bits when only when user is scheduled. Conservatively, SO can be said to utilize Q bits of feedback. Similarly for SC, feedback bits depend on the number of users. If N users are scheduled, then the number of feedback bits $\frac{N(N-1)}{2}Q$. Thus, SO significantly reduces feedback, particularly when N

is large. Now coming to the computational complexity, SU just requires comparison between M users to select the best one and then 3 multiplications and an addition and an logarithm operation to allocate the rate and power. SO, on the other hand, requires a maximum of comparison among M users to select the best two and then 12 multiplications and 8 additions and 2 logarithm operations to find the optimal power and rate allocation. In the case of SC, it depends on the resolution of z , where $z \in \left[0, \frac{\bar{P}}{\sigma^2}\right]$. Assuming z is resolved into P points, then SC requires $2MP$ multiplications and MP additions to find the utility functions and then P comparisons among the M users to identify $A_i \forall i$. Then additional computation will be required depending on the algorithm utilized to calculate the integrals. However, even from the computations in calculating the marginal utility functions, it can be seen that as the number of users increases, the computational complexity grows significantly as compared to SO. Hence, SO can be useful in scenarios where a little more resource consumption can be sacrificed for increase in performance.

V. CONCLUSIONS

This paper has proposed and analyzed a suboptimal scheduling scheme which significantly reduces feedback overhead and computational complexity as compared to the optimal SC with SIC while still providing good gain over SU. The scheduling policy has been described and the optimal rate and power allocation have been found in closed-form. Additionally, the long term average rate of each user was also derived in integral form which can be calculated numerically. The proposed scheme has been numerically compared to SC with SIC and SU in terms of complexity and long term rate performance. It has been shown that the proposed scheme can provide good gain in return for a small increase in feedback overhead and computational complexity.

APPENDIX

Recall that the long term rate of the i th user is given by

$$\bar{R}_i = \int_0^{\frac{\bar{P}}{\sigma^2}} \int_0^\infty \frac{1}{\frac{1}{\gamma} + z} f_{\gamma_i}(\gamma) \text{Prob}(i\text{th user is selected} | \gamma_i = \gamma, z) d\gamma dz. \quad (18)$$

We need to find the probability that the i th user is scheduled. In this regard, we recognize two mutually exclusive cases:

- 1) i th user has the second highest $\mu_j R_j \quad j = 1, 2, \dots, M$.

2) i th user has the highest $\mu_j R_j$ $j = 1, 2, \dots, M$.

We find the probability in each of this case and then as these events are mutually exclusive, add them to obtain the probability that the i th user is scheduled.

A. i th user has the second highest $\mu_j R_j$ $j = 1, 2, \dots, M$

Denoting the highest user with the index l , we have

$$\mu_l \log_2 \left(1 + \frac{P\gamma_l[k]}{\sigma^2} \right) > \mu_i \log_2 \left(1 + \frac{P\gamma_i[k]}{\sigma^2} \right) \quad (19)$$

From (19), we obtain the relation

$$\gamma_l > \frac{\sigma^2}{P} \left(\left(1 + \frac{P\gamma_i}{\sigma^2} \right)^{\frac{\mu_i}{\mu_l}} - 1 \right). \quad (20)$$

Similarly, we have

$$\gamma_j < \frac{\sigma^2}{P} \left(\left(1 + \frac{P\gamma_i}{\sigma^2} \right)^{\frac{\mu_i}{\mu_j}} - 1 \right) \quad j \neq l, i. \quad (21)$$

Additionally, for the i th user's utility function should be greater than the utility function of the l th user.

So

$$\frac{1}{\frac{1}{\gamma_i[k]} + z} - \lambda[k] > u_l(k, z) = \frac{1}{\frac{1}{\gamma_l[k]} + z} - \lambda[k]. \quad (22)$$

From (22), we obtain

$$\gamma_l < \frac{\mu_i \gamma_i}{\mu_l + (\mu_i - \mu_l) \gamma_i z}. \quad (23)$$

Note that the above is always true when $\mu_l + (\mu_i - \mu_l) \gamma_i z < 0$. So from (20) and (23), we have

$$\frac{\sigma^2}{P} \left(\left(1 + \frac{P\gamma_i}{\sigma^2} \right)^{\frac{\mu_i}{\mu_l}} - 1 \right) < \gamma_l < \frac{\mu_i \gamma_i}{\mu_l + (\mu_i - \mu_l) \gamma_i z} \quad (24)$$

Hence, combining (21) and (24), we obtain the probability of the i th user being scheduled when it has the second highest $\mu_j R_j$ as

$$\sum_{l \neq i} G_l(\gamma) \prod_{j \neq i, l} F_j \left(\left(\left(1 + \frac{\bar{P}\gamma}{\sigma^2} \right)^{\frac{\mu_i}{\mu_j}} - 1 \right) \frac{\sigma^2}{\bar{P}} \right), \quad (25)$$

where

$$G_l(\gamma) = \begin{cases} 1 - F_l \left(\left(\left(1 + \frac{\bar{P}\gamma}{\sigma^2} \right)^{\frac{\mu_i}{\mu_l}} - 1 \right) \frac{\sigma^2}{\bar{P}} \right) & x < 0 \\ 0 & \left(\left(1 + \frac{\bar{P}\gamma}{\sigma^2} \right)^{\frac{\mu_i}{\mu_j}} - 1 \right) \frac{\sigma^2}{\bar{P}} > \frac{\mu_i\gamma}{x} > 0 \\ F_l \left(\frac{\mu_i\gamma}{\mu_l + (\mu_l - \mu_i)\gamma z} \right) - F_l \left(\left(\left(1 + \frac{\bar{P}\gamma}{\sigma^2} \right)^{\frac{\mu_i}{\mu_l}} - 1 \right) \frac{\sigma^2}{\bar{P}} \right) & \text{otherwise,} \end{cases} \quad (26)$$

$x = \mu_l + (\mu_i - \mu_l)\gamma_i z$ and the summation is due to the fact that any user having the highest $\mu_j R_j$ is a mutually exclusive event. The three cases of G_l can be obtained from (24). If we have $\mu_l + (\mu_i - \mu_l)\gamma_i z < 0$, then the right inequality of (24) is always true and we get the first case. The second case is the case when the inequality is not true. The last case is when $\mu_l + (\mu_i - \mu_l)\gamma_i z > 0$ and (24) holds.

B. i th user has the highest $\mu_j R_j$ $j = 1, 2, \dots, M$

In this case, denote the second highest user as l . Thus we have three conditions in this case

$$\gamma_l < \frac{\sigma^2}{\bar{P}} \left(\left(1 + \frac{P\gamma_i}{\sigma^2} \right)^{\frac{\mu_i}{\mu_l}} - 1 \right), \quad (27)$$

$$\gamma_l < \frac{\mu_i\gamma_i}{\mu_l + (\mu_i - \mu_l)\gamma_i z}, \quad (28)$$

$$\gamma_m < \frac{\sigma^2}{\bar{P}} \left(\left(1 + \frac{P\gamma_l}{\sigma^2} \right)^{\frac{\mu_l}{\mu_m}} - 1 \right) \quad m \neq l, i. \quad (29)$$

Hence, combining these condition gives the probability of selecting the i th user as

$$\sum_{l \neq i} \int_0^{\min \left(\frac{\sigma^2}{\bar{P}} \left(\left(1 + \frac{P\gamma_i}{\sigma^2} \right)^{\frac{\mu_i}{\mu_l}} - 1 \right), \left[\frac{\mu_i\gamma_i}{\mu_l + (\mu_i - \mu_l)\gamma_i z} \right]^+ \right)} \prod_{m \neq i, l} F_m \left(\left(\left(1 + \frac{\bar{P}y}{\sigma^2} \right)^{\frac{\mu_l}{\mu_m}} - 1 \right) \frac{\sigma^2}{\bar{P}} \right) f_{\gamma_l}(y) dy, \quad (30)$$

where

$$[x]^* = \begin{cases} \infty & x < 0 \\ x & \text{otherwise.} \end{cases} \quad (31)$$

C. Probability of i th user being scheduled

Thus, adding the two probabilities we have

$$\begin{aligned} \text{Prob}(i\text{th user is selected}|\gamma_i = \gamma, z) &= \sum_{l \neq i} G_l(\gamma) \prod_{j \neq i, l} F_j \left(\left(\left(1 + \frac{\bar{P}\gamma}{\sigma^2} \right)^{\frac{\mu_j}{\mu_l}} - 1 \right) \frac{\sigma^2}{\bar{P}} \right) + \\ &\sum_{l \neq i} \int_0^{\min\left(\frac{\sigma^2}{\bar{P}} \left(\left(1 + \frac{P\gamma_i}{\sigma^2} \right)^{\frac{\mu_i}{\mu_l}} - 1 \right), \left[\frac{\mu_i \gamma_i}{\mu_l + (\mu_i - \mu_l) \gamma_i z} \right]^+ \right)} \prod_{m \neq i, l} F_m \left(\left(\left(1 + \frac{\bar{P}y}{\sigma^2} \right)^{\frac{\mu_l}{\mu_m}} - 1 \right) \frac{\sigma^2}{\bar{P}} \right) f_{\gamma_l}(y) dy \end{aligned} \quad (32)$$

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