Higher Order Modes Excitation of Micro Cantilever Beams

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ABSTRACT

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Nizar R. Jaber

In this study, we present analytical and experimental investigation of electrically actuated micro cantilever based resonators. These devices are fabricated using polyimide and coated with chrome and gold layers from both sides. The cantilevers are highly curled up due to stress gradient, which is a common imperfection in surface micro machining. Using a laser Doppler vibrometer, we applied a noise signal to experimentally find the first four resonance frequencies. Then, using a data acquisition card, we swept the excitation frequency around the first four natural modes of vibrations. Theoretically, we derived a reduced order model using the Galerkin method to simulate the dynamics of the system. Extensive numerical analysis and computations were performed. The numerical analysis was able to provide good matching with experimental values of the resonance frequencies. Also, we proved the ability to excite higher order modes using partial electrodes with shapes that resemble the shape of the mode of interest. Such micro-resonators are shown to be promising for applications in mass and gas sensing.
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LIST OF SYMBOLS

$L$ Length of micro beam ($\mu m$)

$b$ Width of the micro beam ($\mu m$)

$d$ Air gap thickness ($\mu m$)

$m$ Mass of the micro beam ($kg$)

$\rho$ Density of micro beam ($kg/m^3$)

$w$ Deflection of micro beam ($\mu m$)

$E$ Young’s modulus ($GPa$)

$\nu$ Poisson’s ratio

$A$ Cross sectional area of micro beam ($\mu m)^2$

$c_{non}$ Non-dimensional viscous damping coefficient ($N.s/m$)

$c_{com}$ Compressible viscous damping coefficient ($N.s/m$)

$c_{inc}$ Incompressible viscous damping coefficient ($N.s/m$)

$k$ Structural stiffness of micro beam ($N/m$)

$\varepsilon_0$ Permittivity in free space ($F/m$)

$V_{DC}$, dc voltage

$V_{AC}$, ac voltage

$F_e$ Electrostatic force developed per unit length

$\omega_{non,n}$ First natural frequency (rad/s)

$\Omega$ Frequency of ac signal (rad/s)

$K_n$ Knudsen number
\( \sigma \) Squeeze number

\( \mu_{eff} \) Effective dynamic viscosity \((Pa.s)\)

\( P \) Vacuum chamber pressure \((Pa)\)

\( \lambda \) Mean free path of gas molecule \((\mu m)\)

\( \phi_n \) The \( n^{th} \) mode shape
Chapter 1

Introduction

MEMS or Micro-electromechanical systems are devices and technologies that have been derived from the microelectronics industry. Researchers investigated ways to create Micro-electromechanical devices using microelectronic fabrication method. MEMS devices have salient features such as the smaller sizes, the ability to work in harsh environments and power efficiency [1, 2].

In particular, MEMS resonators such as micro-plates and micro-beams are the main building block of many MEMS sensors and actuators that are used in variety of applications, such as toxic gas sensors [3], mass and biological sensors [4,5], temperature sensors[6], force and acceleration sensors [7], and earthquake detectors [8]. The demand to develop sensors with high sensitivity and low power consumption is driving the development of cantilever based resonators [3, 4]. MEMS resonators are excited using different type of forces such as electromagnetic [9, 10], thermal [11] and electrostatic [8]. Electrostatic actuation is the most commonly used method because of its simplicity and availability [12]. An efficient approach to improve the oscillation of electrostatically actuated resonator is to use parametric excitation [13] and secondary resonance [14]. Moreover, Alsaleem et al [15] proved
experimentally and analytically that MEMS resonator can be stabilized using a delayed feedback controller.

In this thesis, we presented a static and dynamic simulation of electrostatically excited cantilever beam near its higher order modes of vibrations. Moreover, we compared the experimental result with the theoretical. MEMS resonators excited near its higher order modes are proposed for mass and gas detection in micro scale. Exciting the resonators near its higher order modes improve the sensitivity and the quality factor of the mass sensor. Jen et al [10] defined the sensitivity $S_n$ and the quality factor $Q_n$ of a resonant cantilever as

$$S_n = \frac{\omega'_n - \omega_n}{m} \approx \frac{-\omega_n}{2m_{\text{eff}}}.$$  \hspace{1cm} (1.1)

$$Q_n = \frac{3\pi b h}{\omega_n (256\mu)^{-1}}.$$ \hspace{1cm} (1.2)

where $m$ is the cantilever mass, $m_{\text{eff}}$ is the $n^{th}$ mode effective mass of the cantilever, $\omega_n$ is the resonance frequency of the cantilever, $\omega'_n$ is the final resonance frequency after detecting a mass, $b$ is the cantilever width, $h$ is the cantilever thickness and $\mu$ is the air viscosity. As we notice from Eq. (1.1) and Eq. (1.2) the sensitivity and the quality factor are directly proportional to the excited mode number. High quality factor implies a sharper and stable resonance peak.
1.1 Literature Review

Many researchers focused on modeling cantilever beam and determining its response to harmonic loads. Researchers developed models that predict the microstructure response to electrostatic force near its first mode, super harmonic and sub harmonic resonance frequencies accurately.

Younis et al [16] developed an accurate and effective model to study the behavior of electrostatically actuated micro structure. Their model accounts for dynamical load, coupling between electrical and mechanical forces. Moreover, the validated their result with the finite element and experimental solutions available in literature. Also they were able to predict the static pull in voltage accurately by solving algebraic equation.

Alsaleem and Younis [17] they investigated the dynamical pull in clamped-clamped and micro cantilever. They showed experimentally that despite the structural difference between the devices, they have similar escape behavior near their primary resonance. Also, they studied the effect of damping, sweeping type and the resonance type on the dynamical pull in. In addition, they correlated the experimental pull in bands to the escape band with large difference near the pull in.

Alsaleem et al [18] studied the nonlinear resonance of electrostatically actuated resonators. They studied the nonlinear dynamics near the sub-harmonic, primary and super harmonic resonance experimentally and analytically. Also, they modeled
the electrostatic force and the squeeze film damping using a nonlinear spring-mass-damper model. They showed experimentally jumps, hysteresis and dynamical pull in near the primary resonance. Moreover, they showed the Dover-cliff curves which help designers to study the stability and reliability of their resonators.

Nayfeh et al [19] studied transient and the steady state dynamics of electrostatically actuated micro beam near the secondary resonance excitation. They modeled the dynamics using a reduced-order model and studied its stability using shooting technique. Also, they showed analytically the effect of varying the DC and AC voltages near the super harmonic and sub harmonic resonance on the frequency response curve. Moreover, they showed dynamical pull in near the primary resonance occurs at a combined AC and DC voltages much lower than the static pull in voltage. They proposed this resonator actuated near their subharmonic resonance to design a band pass RF filter.

Abdel-Rahman et al [20] presented nonlinear model of electrostatically actuated microstructure that accounts for mid-plane stretching and the electrostatic force. Their simulation results agreed with experimental data available in literature. Moreover, they showed the importance of including the mid-plane stretching and geometric nonlinearities to accurately calculate the static stability limit. Also, they found that increasing the axial force, increases the natural frequency and the pull in voltage. On the other side, they noticed that increasing the mid-plane stretching increases the pull in voltage and decreases the natural frequency of the micro structure.
Oukad et al [21] studied the nonlinear dynamics of electrostatically actuated clamped-clamped arches. They modeled it using Galerkin method and calculated the natural frequencies and mode shapes for different values of DC and AC voltages. Moreover, they used the perturbation method to calculate the vibration of the microstructure and the non-linear resonance frequency as function of the AC amplitude, DC load and the initial rise of the arch. Also they showed that the frequency response curve is of softening type.

Several works have focused on exciting higher order modes of microstructure using different excitation method for various applications. The promising advantages of using higher order modes drove the developments of resonators excited near its higher order modes.

Dohn et al [4] designed and fabricated a resonant cantilever based mass sensor that has sensitivity of $5 fg/Hz$. Also, they proved experimentally that increasing the mode number increase the quality factor. In addition, they observed experimentally the effect of the point-mass position on the sensor sensitivity and using the fourth mode the sensitivity increases by a factor of 300. Moreover, they proposed that the position of the detected mass can be calculated by measuring different modes of vibration.
Jen et al. [10] developed a resonant cantilever for detection in air environment. They integrated a piezoresistive bridge for sensing and aluminum loop for Lorentz-force actuation. They excited the cantilever near its first and second natural frequencies using a sinusoidal electrical current generated using a network analyzer from Agilent. A small magnetic bulk placed on the sensor package to generate a magnetic field for excitation. The aluminum coil configuration depends on the excited mode: first mode, the driving force placed near the cantilever tip and for the second mode, the driving force is placed near the tip and the middle of the cantilever. These locations match the first and second mode maximum deflection, respectively. The piezoresistive sensing bridge is placed near the fixed side of the cantilever since it
has the maximum bending surface stress. Figure 1.1 shows the location of the aluminum coils and Lorentz force directions. Using the second configuration near the second mode resonance, they were able to improve the resolution by 5.9 times compared with the first configuration near the first resonance. In addition to that, the quality factor has improved from 195 to 857.

Nayfeh et al [22] developed a mathematical model of electrostatically actuated micro cantilever and derived the static and dynamic solutions. Also, they studied the stability of the cantilever by combining the FDM discretization with Floquet theory. Their results showed potential application in micro scale gas sensing where a detected a mass shift, the natural frequency of the cantilever, changes the location of stable equilibrium and causes dynamical pull in.
Subhashini et al [3] fabricated a $SiO_2$ micro cantilever coated with a selective material to detect $CO$ and $CO_2$. This material adsorbs the gases and changes the cantilever resonant frequency. They optimized the cantilever parameters to obtain an optimum response. To measure the cantilever vibration, they fabricated it between two electrodes as shown in Figure 1.2. One of these electrodes will act as source and the other one will act as drain. As the cantilever vibrates, it makes contact with these electrodes. The generated signal will be measured using a buzzer circuit. When the cantilever adsorbs a point-mass, its natural frequency decreases. This change in frequency is correlated to the amount of mass detected.

Figure 1.2: micro cantilever vibration. Part (a) shows the vibration before exposure to the gas, Part (b) shows the vibration after exposure to gas [3].
Chow et al [23] demonstrated the ability to excite higher order modes of microstructure using square wave. As shown in Figure 1.3, using a square wave swept from 0-333 KHz, they excited the first five modes within 0-1 MHz. However, using a sinusoidal signal swept from 0-333 KHz only three modes were excited.

![Figure 1.3: Response and excitation signals under 0-333 KHz. Sweep.](image)

**Figure 1.3:** Response and excitation signals under 0-333 KHz. Sweep. (a)-(d) show the spectrum of sine wave for excitation, dynamic response of sine wave, show the spectrum of square wave for excitation, dynamic response of square wave [23].

Alsaleem et al [24] investigated experimentally and theoretically the dynamic pull in of electrostatically actuated microstructure near the first and second mode of vibration. Also, they identified the dynamic pull in range and showed the effect of the initial condition, the ac amplitude, the ac frequency and the sweeping type on the frequency response of the resonator. In addition, they constructed the bifurcation diagram to operate the resonator away from the pull in bands.
Hossain *et al* [25] investigated the dynamic response of micro cantilever submerged in viscous media where they validated the finite element result with the corresponding experiments as shown in Figure 1.4. Also, they found that the

![Graph showing the resonance frequency and quality factor vs. frequency in air](image)

Figure 1.4: Finite element and experimental investigation of a cantilever in air [25].

resonance frequency and the quality factor are inversely related to the viscosity of the fluid. In addition to that, they repeated the experiments when the cantilever was submerged with different depth in the viscous media.
Chapter 2

Design and Modeling

In this chapter, we will define our micro cantilever parameters, describe materials used in fabrication and present the lower electrode configuration that is used to efficiently excite higher order modes. Moreover, the cantilever partial differential equation (PDE) of motion is presented. These PDE’s solved by reducing them into a set of coupled ordinary differential equations (ODE’s) in time. Then these ODEs are integrated numerically.

2.1 Parameters

The cantilever’s parameter include the cantilever length $l$, the polyimide layer thickness $h$, the polyimide and metal layer widths $b$, the chrome layer thickness $h_{ch}$, the gold layer thickness $h_{Au}$ and the amorphous silicon layer thickness which later will define the air gap $d$ after release. In our design, we fabricated micro structures with different length that range from 100 $\mu m$ to 700 $\mu m$ to study the effect of beam length on the curvature and resonance frequency. The beam width $b$ was fixed at 50 $\mu m$ due to dry etching recommendation. The metal width was 10 $\mu m$ narrower
because of the in house process rules. The total cantilever thickness was fixed around 4 μm. The amorphous silicon layer thickness is around 3 μm.

### 2.2 materials

For our micro cantilever we selected the material based on their availability in KAUST’s clean room. Moreover, the clean room staff has developed and optimized accurate recipes to deposit and etch these materials. So, we selected polyimide as main structural material coated with Gold and Chrome layer from bottom. This metal layer will form the upper electrode of our micro cantilever. Another Chrome and Gold layer is sputtered on the substrate to form the lower electrode. The Chrome used as adhesion promoter between the Gold and Polyimide. Moreover, we selected the Amorphous Silicon as sacrificial layer because it can be etched using dry etchant such as $XeF_2$. Table 2.1 summarizes the material properties.

<table>
<thead>
<tr>
<th>Property</th>
<th>Chrome (Cr)</th>
<th>Gold (Au)</th>
<th>Polyimide (PI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus of elasticity ($E$) in GPa</td>
<td>279</td>
<td>79</td>
<td>8.5</td>
</tr>
<tr>
<td>Poisson's ratio ($\nu$)</td>
<td>0.21</td>
<td>0.44</td>
<td>0.34</td>
</tr>
<tr>
<td>Density ($\rho$) in $Kg/m^3$</td>
<td>7190</td>
<td>19300</td>
<td>1400</td>
</tr>
</tbody>
</table>
2.3 Lower electrode configuration

Figure 2.1: lower electrode configuration.
Our optimum goal is to excite the higher modes by changing the lower electrode configuration such that it resembles the excited mode shape of the cantilever. As we can see in Figure 2.1, we used full electrode to excite first mode, half electrode to excite the second and third mode. To excite the forth mode, we used two one-third electrodes one near the cantilever’s fixed anchor and one near the tip.

2.4 Problem Formulation

\[
E I \frac{d^4 w}{dx^4} + \rho A \frac{d^2 w}{dt^2} + c \frac{dw}{dt} = F_e, \tag{2.1}
\]

where: \( w(x, t) \) is the beam deflection, which depends on variable on space \( x \) and time \( t \), \( I \) is the moment of Inertia of the micro cantilever, \( \rho \) is the density, \( A \) is the
cross sectional area of the cantilever, $E$ is the modulus of elasticity, $c$ is the viscous damping coefficient and $F_e$ is the electrical excitation force per unit length given by:

$$F_e = \frac{\epsilon_o (V_{DC} + V_{AC} \cos(\Omega t)) b}{2(d - w)^2}, \tag{2.2}$$

where $b$ is the beam width, $d$ is the air gap between the two electrodes, $\epsilon_o$ is the air permittivity, $V_{DC}$ is the DC polarization voltage, $V_{AC}$ is the harmonic voltage amplitude and $\Omega$ is the forcing frequency. The boundary and initial conditions for the cantilever equation of motion are given as below:

$$\frac{\partial^3 w(l,t)}{\partial x^3} = 0, \quad \frac{\partial^2 w(l,t)}{\partial x^2} = 0, \quad \frac{\partial w(0,t)}{\partial x} = 0, \quad w(0,t) = 0. \tag{2.3}$$

$$\frac{\partial w(x,0)}{\partial t} = 0, \quad w(x,0) = 0. \tag{2.4}$$

Also, in Eq. (2.1) we set the damping and forcing terms equal to zero and assuming $w(x,t) = \phi_i(x)e^{iot}$ we get

$$\phi''''_i(x) - \omega^2 \phi_i = 0. \tag{2.5}$$

### 2.4.1 Normalization

Normalization has many advantages in simplifying the analysis and the numerical computations such as rescaling all the quantities into the same order of magnitude and analyzing the system behavior regardless of the units.
So, in Eq. (2.1) \( x \) is replaced by \( \hat{x}l \), \( w \) is replaced by \( \hat{w}d \) and \( t \) with \( \hat{t}T \) where

\[
T = \sqrt{\frac{\rho Al^4}{EI}},
\]

(2.6)

after substituting and simplifying. Eq. (2.1) becomes:

\[
\frac{\partial^4 \hat{w}}{\partial \hat{x}^4} + \frac{\partial^2 \hat{w}}{\partial \hat{t}^2} + c_{non} \frac{\partial \hat{w}}{\partial \hat{t}} = \frac{\alpha_2 (V_{DC} + V_{AC} \cos(\Omega t))^2}{(1 - \hat{w})^2},
\]

(2.7)

where:

\[
c_{non} = \frac{cl^4}{EIT'}, \quad \alpha_2 = \frac{\epsilon_0 bl^4}{2EI'd}.
\]

### 2.4.2 Galerkin Method

The Galerkin method is used to reduce the partial differential equation into set of second order ordinary differential equations. The beam deflection can be approximated as:

\[
w(x, t) = \sum_{i=1}^{n} \phi_i(x) \ u_i(t)
\]

(2.8)

where: \( \phi_i(x) \) is the \( i^{th} \) cantilever beam mode shape, \( u_i(t) \) is the \( i^{th} \) time-varying function and \( n \) is the number of the coupled ordinary differential equations used in the approximation.
\[ \phi_i(x) = \]
\[ \cosh(\sqrt{\omega_{non,i}x}) + \cos(\sqrt{\omega_{non,i}x}) - \sigma_i \sinh(\sqrt{\omega_{non,i}x}) - \sigma_i \sin(\sqrt{\omega_{non,i}x}), \quad (2.9) \]

The values of \( \omega_{non,i} \) and \( \sigma_i \) is extracted from the following table for the first four mode shapes:

Table 2-2: The first four non-dimensional natural frequencies and mode shapes of a cantilever [10].

<table>
<thead>
<tr>
<th>( i )</th>
<th>( \omega_{non,i} )</th>
<th>( \sigma_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.51602</td>
<td>0.7341</td>
</tr>
<tr>
<td>2</td>
<td>22.0345</td>
<td>1.0185</td>
</tr>
<tr>
<td>3</td>
<td>61.6972</td>
<td>0.9992</td>
</tr>
<tr>
<td>4</td>
<td>120.902</td>
<td>1.0</td>
</tr>
</tbody>
</table>

The cantilever beam mode shapes satisfies the orthonormality conditions which:

\[ \int_0^1 \phi_i(x)\phi_j(x) \, dx = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases} \quad (2.10) \]

To derive the reduced order model, we apply The Galerkin method on the normalized partial deferential equation. We substitute Eq. (2.8) into Eq. (2.7) and then simplify using Eq. (2.5) and Eq. (2.10) we get:

\[ \sum_{i=1}^{n} \phi_i''''(\tilde{x}) \, u_i(\tilde{t}) + \sum_{i=1}^{n} \phi_i(\tilde{x}) \, \ddot{u}_i(\tilde{t}) + c_{non} \sum_{i=1}^{n} \phi_i(\tilde{x}) \, \dot{u}_i(\tilde{t}) = \alpha_2 (V_{DC} + V_{AC} \cos(\Omega t))^2 \frac{1}{(1 - \sum_{i=1}^{n} \phi_i(\tilde{x}) \, u_i(\tilde{t}))^2}. \quad (2.11) \]
Next, we multiply Eq. (2.11) by \((1 - \sum_{i=1}^{n} \phi_i(\hat{x}) u_i(\hat{t})) \) and the mode shape function \(\phi_j(x)\) then integrate from 0 to 1. The resulted equation is simplified using Eq. (2.5) and Eq. (2.10) to get:

\[
\int_{0}^{1} \phi_j(\hat{x}) \left(1 - \sum_{i=1}^{n} \phi_m(\hat{x}) u_m(\hat{t}) \right)^2 \sum_{i=1}^{n} \phi_l(\hat{x}) \omega^2_{\text{non},i} u_i(\hat{t}) \, dx 
\]

\[
+ \int_{0}^{1} \phi_j(\hat{x}) \left(1 - \sum_{i=1}^{n} \phi_m(\hat{x}) u_m(\hat{t}) \right)^2 \sum_{i=1}^{n} \phi_l(\hat{x}) \ddot{u}_i(\hat{t}) \, dx 
\]

\[
+ c_{\text{non}} \int_{0}^{1} \phi_j(\hat{x}) \left(1 - \sum_{i=1}^{n} \phi_m(\hat{x}) u_m(\hat{t}) \right)^2 \sum_{i=1}^{n} \phi_l(\hat{x}) \dot{u}_i(\hat{t}) \, dx 
\]

\[
= \alpha_2 (V_{DC} + V_{AC} \cos(\Omega t)) \int_{0}^{1} \phi_j(\hat{x}) \, dx. \tag{2.12}
\]

We solve Eq. (2.12) using long time integration method until steady state is reached. The numerical integration is carried on using Mathematica where we used the first four fundamental mode shapes \((n=4)\) to approximate the solution. The result for different values of voltages and different range of frequencies are discussed in Chapter 3.

### 2.5 Pull in

The cantilever beam is actuated using electrostatic force. This force pulls the cantilever down through an electrostatic attraction force; a mechanical restoring
force is created in the opposite direction. Equilibrium is reached at certain values of voltages and beam deflection. If the magnitude of the applied voltage increases beyond a certain limit called the pull in voltage the upper electrode moves towards the lower electrode till they come in contact.

In order to get the static response of the cantilever, we set all time derivatives in Eq. (2.12) equal to zero and replacing the time varying functions with constant $C$, hence we get:

\[
\int_0^1 \phi_j(\hat{x}) \left( 1 - \sum_{i=1}^{n} \phi_m(\hat{x}) C_m \right) \sum_{i=1}^{n} \phi_i(\hat{x}) \omega_{non,i}^2 C_m \, dx
\]

\[
= \alpha_2 V_{DC}^2 \int_0^1 \phi_j(\hat{x}) \, dx.
\]  

(2.13)

Using one mode of vibration and substituting for $\omega_{non,1}$, we get:

\[
\omega_{non,1}^2 C_1 \int_0^1 \phi_1(\hat{x}) (1 - \phi_1(\hat{x}) C_1)^2 \phi_1(\hat{x}) \, dx = \alpha_2 V_{DC}^2.
\]  

(2.14)

Eq. (2.14) is solved numerically using Matlab and Mathematica. This equation has three solutions when the applied voltage is below the pull in voltage: stable physical, unstable physical and unphysical solution. Figure 2.3 shows the case when the applied voltage is below the pull in voltage for a cantilever beam of dimensions $500 \, \mu m \times 50 \, \mu m \times 3 \, \mu m$ and air gap of $3 \, \mu m$. 
Figure 2.4 shows the case when the applied voltage equals to the pull in voltage. In this case, we have two solutions one physical and the other one is nonphysical.

Also, at this point, the beam deflection equals to one third of the air gap.
Figure 2.5 shows the last case when the applied voltage is greater than pull in. In this case the upper electrode hits the lower electrode.

2.6 Squeeze film damping

Squeeze film damping is an energy dissipating mechanism of the moving microstructures and the electrodes separated by a few microns. When the upper electrode vibrates, the pressure of the air trapped between the two electrodes will increase and hence opposes the motion. In this section, a model of the squeeze film damping is presented. This model is based on Blech et al [26] to calculate the
damping coefficient $c$. The calculation is done on cantilever beam of dimensions $500 \mu m \times 50 \mu m \times 3 \mu m$ and the air gap $d = 3 \mu m$.

Here, we define a set of parameters related to our viscous damping calculation. The air mean free path $\lambda$ defined as: The distance traveled by a gas molecule between two successive collisions. $\lambda$ is calculated at any pressure $P$ with equation

$$\lambda = \frac{\lambda_o \times P_o}{P},$$

(2.15)

where $\lambda_o = 0.064 \mu m$ is the air mean air free path at the atmospheric pressure $P_o = 101.3 \text{ KPa}$. Figure 2.7 shows the change in $\lambda$ as a function of the air pressure on a Log-Log scale plot. As expected, the air mean free path is inversely related to the pressure.

![Air mean free path](image)

Figure 2.7: the change in air mean free path for different values of pressure.
Knudsen number \( (K_n) \) is a dimensionless number that relates the air mean free path to the air gap depth between the two electrodes. The gas flow is divided into four regimes based on Knudsen number \([10]\): Continuum flow when \(K_n < 0.01\), slip flow when \(0.01 < K_n < 0.1\), transitional flow when \(0.1 < K_n < 10\) and free molecular flow when \(K_n > 10\). For each one of these regimes there is a governing equation for the gas flow that accounts for the boundary conditions. Younis \([27]\) used the continuum approach to model the squeeze film damping which is based on Reynolds equation:

\[
\frac{\partial}{\partial x} \left( H^3 P \frac{\partial P}{\partial x} \right) + \frac{\partial}{\partial y} \left( H^3 P \frac{\partial P}{\partial y} \right) = 12\mu \left( H \frac{\partial P}{\partial t} + P \frac{\partial H}{\partial t} \right) \tag{2.16}
\]

where \(H\) is the distance between the two plates, \(P\) is the air pressure, \(t\) is the time and \(\mu\) is the fluid viscosity.

Younis \([27]\) solved Eq. (2.16) for the compressible and the incompressible gas models. He used the effective viscosity correction factor Eq. (2.17) calculated by Veijola et al \([28]\) to account for the change in viscosity.

\[
\mu_{eff} = \frac{\mu}{1 + 9.638 K_n^{1.159}} \tag{2.17}
\]

The solution of Eq. (2.16) of the compressible case gives the following analytical expression for the compressible damping coefficient and stiffness:

\[
k = \frac{64\sigma^2 \rho A}{\pi^8 d} \sum_{m, n \text{ odd}} \frac{1}{(mn)^2 \left( m^2 + \beta^2 n^2 \right)^2 + \frac{\sigma^2}{\pi^4}} \tag{2.18}
\]
\[ c_{com} = \frac{64\sigma PA}{\pi^6 \omega d} \sum_{m,n \text{ odd}} \frac{m^2 + \beta^2 n^2}{(mn)^2 \left\{ (m^2 + \beta^2 n^2 + \frac{\sigma^2}{\pi^4}) \right\}^2} \]  

(2.19)

where \( \beta = b/l, A = bl \) and \( \sigma \) is the squeeze number.

The squeeze number \( \sigma \) – calculated in Eq. (2.20) – is used to measure the compressibility of the gas where a high squeeze number means compressible fluid and a low squeeze number means incompressible fluid.

\[ \sigma = \frac{12A \omega \mu}{P d^2}, \]  

(2.20)

where \( A \) is the plate surface area, \( \omega \) is the excitation frequency.

The series in Eq. (2.18) and Eq. (2.19) converge fast. Hence, one term is sufficient to give good results [10]. As shown in Figure 2.8, for low values of \( \sigma \) the spring force effect is negligible while for large values a spring force added to Eq. (2.16) to act in parallel with microstructure stiffness. The cut off squeeze number \( \sigma_c \) defined as the point at which the non-dimensional spring and damping forces are equal. Blech [28] approximated the cut off squeeze number \( \sigma_c \) as

\[ \sigma_c = \pi^2 (1 + \beta^2)^2. \]  

(2.21)
The solution of Eq. (2.16) for the incompressible case gives the following analytical expression for the damping coefficient of a cantilever plate of length \( l \) and width \( b \):

\[
c_{inc} = \frac{\mu l b^3}{d^3} f \left( \frac{b}{l} \right),
\]

(2.22)

Where the value of the function \( f \left( \frac{b}{l} \right) \) can be calculated from table 2-3.

Table 2-3: Values of the function \( f \) for a rectangular plate [10].

<table>
<thead>
<tr>
<th>( \frac{b}{l} )</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f \left( \frac{b}{l} \right) )</td>
<td>0.94</td>
<td>0.87</td>
<td>0.81</td>
<td>0.75</td>
<td>0.69</td>
<td>0.63</td>
<td>0.57</td>
<td>0.52</td>
<td>0.47</td>
<td>0.42</td>
</tr>
</tbody>
</table>

Figure 2.8: Non-dimensional damping and spring force versus squeeze number.
Figure 2.9 shows the squeeze number calculated near the first four natural modes of vibration for different values of pressure. The squeeze number is always below the cut-off and the fluid is incompressible near the first resonance. On the other hand, near the higher order modes and at low pressure values the squeeze number is higher than cut-off and the fluid is compressible.

Based on the pressure and the actuation frequency the value of the damping coefficient can be calculated for a particular cantilever dimensions. This value then substituted in Eq. (2.12) after normalizing it.

![Squeeze number graph](image)

**Figure 2.9:** The squeeze number near the first four fundamental resonances at different values of pressure.
2.7 Equivalent section method

Since the width of our micro beams is five times greater than its depth, the modulus of elasticity $E$ is replaced by $E/(1 - \nu)^2$ where $\nu$ is the Poisson’s ratio. Also, our micro cantilever is composed of different materials with different thicknesses as shown in Figure 2.10. Thus, we need to find the effective flexural of rigidity to substitute back into Eq. (2.12).

Because the micro cantilever’s cross section is double-symmetric as shown in Figure 2.10, the neutral axis is located at mid-height. Therefore, the effective flexural of rigidity is calculated as follows

$$E 1_{\text{equivalent}} = E'_{\text{Cr}1_{\text{Cr}}} + E'_{\text{Au}1_{\text{Au}}} + E'_{\text{PI}1_{\text{PI}}}$$

(2.23)
where

\[ E'_{Cr} = \frac{E_{Cr}}{(1 - \nu)^2} \]  \hspace{1cm} (2.24)

\[ E'_{Au} = \frac{E_{Au}}{(1 - \nu)^2} \]  \hspace{1cm} (2.25)

\[ E'_{PI} = \frac{E_{PI}}{(1 - \nu)^2} \]  \hspace{1cm} (2.26)

\[ I_{PI} = \frac{1}{12} b h_{PI}^3 \]  \hspace{1cm} (2.27)

\[ I_{Cr} = \frac{1}{12} b ((2h_{cr} + h_{PI})^3 - (h_{PI})^3) \]  \hspace{1cm} (2.28)

\[ I_{Au} = \frac{1}{12} b ((2h_{cr} + 2h_{Au} + h_{PI})^3 - (2h_{cr} + h_{PI})^3) \]  \hspace{1cm} (2.29)

where \( b \) is the beam width, \( h_{cr} \) is the chrome layer thickness, \( h_{PI} \) is the polyimide layer thickness and \( h_{Au} \) is the gold layer thickness. After calculating all of the previous parameters and substituting in Eq. (2.23) we get the effective flexural rigidity that is used to solve Eq. (2.13).
Chapter 3

Simulation Results

In this chapter the solution of Eq. (2.13) is presented for a cantilever beam of dimensions 500$\mu m \times 50\mu m \times 3\mu m$, under different values of excitation voltage and frequency. Also, we compare the result under different values of chamber pressure and for different lower electrode configuration.

3.1 Static results

Figure 3.1: beam tip deflection subjected to electrostatic force.

Figure 3.1 represents the beams tip deflection versus $\alpha_2 V_{DC}^2$ as we can see once the beam deflection exceeds 40% of the gap; the cantilever will accelerate towards the lower electrode.
3.2 Frequency response curve at 10 pa pressure for the full electrode case.

The following figures present the simulation of Eq. (2.12) for different combinations of DC and AC excitation near the first four resonance modes. Although, the static pull in voltage of this particular cantilever is 5.6 V, near resonance, the dynamical pull in occurs at voltages much lower than that.

In Figures 3.2, 3.3 and 3.4, we generate the frequency response curve near the first, second, third and fourth resonance frequency. We notice softening behavior for the first three modes and hardening behavior near the fourth mode. Softening moves the natural frequency to lower values due to electrostatic nonlinearities that reduce the effective stiffness of the micro structure. On the other hand, hardening shifts the resonance frequency to higher values because of geometric nonlinearities [27]. Moreover, we notice dynamical pull in near resonance occurs at voltage load much lower than the static pull in and as we increase the AC voltage a pull-in band is formed. The dynamical pull-in is a major instability issues in MEMS resonators because it collapses the upper electrode into the lower electrode and cause electrical problems.
Figure 3.2: Displacement frequency response curve near the first resonance at the cantilever tip $V_{dc} = 2V$.

Figure 3.3: Displacement frequency response curve near the second resonance at the cantilever tip $V_{dc} = 2V$. 
Figure 3.4: Displacement frequency response curve near the third resonance at the cantilever tip $V_{dc} = 2\, V$.

Figure 3.5: Displacement frequency response curve near the fourth resonance at the cantilever tip, $V_{dc} = 3\, V$ and $V_{ac} = 1\, V$. 
3.3 Different lower electrode configuration

The following figures show the simulation result of Eq. (2.13) for different lower electrode configuration. To account for the lower electrode different configuration we multiply Eq. (2.13) with a unit step function that has the same shape as the lower electrode. For example, if we are using half electrode, we multiply right-hand side of Eq. (2.13) with \( u(x) - u(x - 0.5) \).

Figure 3.6: Displacement frequency response curve near the first resonance at the cantilever tip at \( V_{dc} = 2 \, V \) and \( V_{ac} = 0.5 \, V \) for different lower electrode configuration.

In Figure 3.6, as expected, near the first resonance, the full electrode has the highest amplitude compared with other configurations. Figures 3.7 and 3.8 show the simulation results near the second and third resonance. We notice that the half electrode configuration has the highest amplitude compared with other configurations. Moreover, near the second resonance, the one-third configuration
has the lowest amplitude because its shape opposes the second mode shape of the cantilever.

Figures 3.9 and 3.10 show the displacement and velocity frequency response curve near the fourth resonance. Although, the partial electrodes effect is not clear on the displacement curve, on the velocity curve, we see the one-third configuration has the highest amplitude compared with other two configurations.

Figure 3.7: Displacement frequency response curve near the second resonance at the cantilever tip, $V_{dc}=2$ V and $V_{ac}=0.5$ V, for different lower electrode configuration.
Figure 3.8: Displacement frequency response curve near the third resonance at the cantilever tip, $V_{dc}=2\,\text{V}$ and $V_{ac}=0.5\,\text{V}$, for different lower electrode configuration.

Figure 3.9: Displacement frequency response curve near the fourth resonance at the cantilever tip, $V_{dc}=2\,\text{V}$ and $V_{ac}=0.5\,\text{V}$, for different lower electrode configuration.
3.4 Frequency response curve under different pressure values

Figure 3.11 shows the effect of increasing the chamber pressure near the first resonance of the cantilever. As we discussed previously in section 2.6, the pressure is directly proportional to the coefficient of viscous damping. As shown in Figure 3.11, the frequency response curve has a higher amplitude at 10 Pa than at 100 Pa.
Chapter 4

Fabrication Process

In this chapter, we explain the In house fabrication process, the tools that are used in masks and microstructure fabrication and the fabrication sequence from the silicon wafer up to the actual functional micro structures.

4.1 In-house process

We fabricate our microstructures using the In-house process devolved by Erensto [29]. The basic process consists of set of rules and six physical layers. The In-house rules define the allowed configuration and features based on manufacturing limitation such as minimum resolution of lithography system, alignment between the different layers and etching requirements. The physical layers that define the fabricated devices are:

1- A thermally grown silicon dioxide layer of 500 nm thickness. This layer provides electrical isolation from the wafer and good adhesion properties.

2- The first metal layer of 50 nm Chrome and 250 nm gold. This layer provides the ability to create ground planes and connection to the external actuation source.

3- A 3 μm amorphous silicon sacrificial layer. This layer defines the air gap and will be removed during the release step.
4- The second metal layer of 50 nm Chrome and 250 nm gold. This layer provides the routing of the electrical signal on the micro structures and the creation of the upper electrode.

5- Polyimide is the main structural material. It has desirable film properties such as low stress, low moisture uptake and high ductility.

6- The third metal layer of 50 nm Chrome and 250 nm gold. This layer protects the structural layer during the polyimide etching.

4.2 Materials.

In this section, we describe the material properties and thicknesses used in the In-house fabrication process.

1- Chrome and gold layer used as an electrical ground of thicknesses 50 nm and 250 nm, respectively. The Chrome is used to enhance the adhesion between the gold and polyimide.

2- Amorphous silicon layer of 3 \( \mu m \) thickness used as sacrificial layer. This layer defines the air gap and is etched completely during the release step.

3- Polyimide layer of 4 \( \mu m \) thicknesses is used as a structural layer.
4.3 Masks and Microstructure Fabrication

A mask is basically a glass plate with pattern that is to be printed on the wafer. The pattern is defined by an opaque layer of Chrome or Molybdenum Silicide. This layer blocks the light from reaching the photoresist during exposure. In our masks, we draw the pattern using Tanner L-edit to export the drawing to machine compatible format. These masks have alignment marks placed in the middle of the mask and spaced out to align subsequent layers.

1- Metal-0

As shown in figure 4.1, the Metal-0 mask is used to pattern the first Chrome and Gold physical layer. In this layer, the external connection to our micro structure is connected.

Figure 4.1: Metal-0 Mask.
2- Anchor

As shown in figure 4.2, the anchor layer defines etching hole in the amorphous silicon layer to connect the two metal layers.

![Anchor Mask](image)

Figure 4.2: Anchor Mask.

3- Dimple

As shown in figure 4.3, dimples are depressions in the amorphous silicon layer to generate bumps on the undersurface of the structural layer to minimize stiction.

![Dimple Mask](image)

Figure 4.3: Dimple Mask.
4- Metal-1

As shown in figure 4.4, the Metal-1 mask used to pattern the second chrome and gold layer. This layer is attached to the bottom of the microstructure and defines the upper electrode pattern.

5- Structural

As shown in figure 4.5, the structural mask is used to pattern and to define the structural layer features.
4.3 Fabrication Process

After collecting the masks of micro structures, we wash them using Acetone and Isopropanol Alcohol (IPA), we dry these masks by Nitrogen to remove all contaminants. Then we cover the wafer with 500 nm thermally grown isolation layer of Silicon Dioxide (SiO$_2$). Figure 4.6 presents a sectional view of the final microstructure.

![Figure 4.6: Sectional view of the microstructure.](image)

The following steps summarize the fabrication process of the microstructure.

4.3.1 Metal-0

a. A 4 µm photoresist (EIC 3027) layer is spun onto the substrate according to the standard recipe shown in table 4.1. Then it is baked for 60 sec at 100°C as shown in figure 4.7a.

b. Using the mask aligner machine, the photoresist is exposed through the Metal-0 Mask after aligning it with the wafer. The parameters that used in this tool are:
Table 4.1: standard recipe for the photo resist

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Photoresist Thickness</td>
<td>4 (\mu m)</td>
</tr>
<tr>
<td>Mask Thickness</td>
<td>2.5 mm</td>
</tr>
<tr>
<td>Wafer Thickness</td>
<td>0.52 mm</td>
</tr>
<tr>
<td>Wafer Size</td>
<td>4 in</td>
</tr>
<tr>
<td>Contact Type</td>
<td>Hard contact</td>
</tr>
<tr>
<td>Exposure Doss</td>
<td>200 mJ/cm(^2)</td>
</tr>
<tr>
<td>Separation Distance</td>
<td>60 – 80 (\mu m)</td>
</tr>
</tbody>
</table>

c. The wafer is developed using AZ 726 developer for 1 minute. Then rinsed with distilled water as shown in Figure 4.7b.
d. Using the Plasma Ion Etching (PIE) tool, \(O_2\) descum for 20 sec is applied on the wafer to remove all the contaminants that is remained after developing.
e. Using the sputter deposition tool, 50 nm of Chrome and 300 nm of Gold layer is sputtered where the required sputtering time for the Chrome and Gold is 200 sec and 352 sec, respectively to get the desired thickness. As shown in Figure 4.7c.
f. In the ultrasonic acetone bath, the Chrome and Gold layer is lifted off for 15 min. Then, the wafer rinsed with IPA and dried with nitrogen gas. As shown in Figure 4.7d.
4.3.2 Amorphous silicon

Amorphous silicon is a 3 μm sacrificial layer that is deposited using physical chemical vapor deposition all over the wafer. As shown in figure 4.8.
4.3.3 Anchor

a. Again, a 4 µm photoresist layer is spun onto the substrate according to the standard recipe of (EIC3027) to produce Anchors on the sacrificial layer. As shown in Figure 4.9a.

b. The photoresist is exposed through the Anchor Mask after aligning it with the wafer.

c. The wafer is developed using AZ 726 developer for 1 minute before rinsing it by distilled water.

d. Using the plasma Ion Etching (PIE) tool, O2 descum for 20 sec and then SF₆ etch for 12 min were applied on the wafer. As shown in Figure 4.9b.

f. After Acetone and IPA rinsing (to remove the photoresist), we checked the anchors feature using the profilo-meter and noticed that the anchor's depth is 3.1 µm which is close to what should we get based on the RIE etching rate (250nm/min * 12min =3 µm). As shown in Figure 4.9c.
4.3.4 Dimple

a. Again, a 4 µm photoresist layer is spun onto the substrate according to the standard recipe of (EIC3027) to produce dimples on the sacrificial layer. As shown in Figure 4.10a.

b. The photoresist is exposed through the Anchor Mask after aligning it with the wafer.

c. The wafer is developed using AZ 726 developer for 1 minute before rinsing it by distilled water.

d. Using the plasma Ion Etching (PIE) tool, O2 descum for 20 sec and then $SF_6$ etch for 4 min were applied on the wafer. As shown in Figure 4.10b.
e. After Acetone and IPA rinsing (to remove the photoresist), we checked the dimple feature using the profilo-meter and noticed that the dimple’s depth is 1.1 $\mu$m which is close to what should we get based on the etching rate (250nm/min * 4min = 1 $\mu$m).

![Figure 4.10: Dimple fabrication.](image)

### 4.3.5 Metal-1

a. A 4 $\mu$m photoresist (ECI 3027) layer is spun onto the substrate according to the standard recipe (1750 rpm for 30 sec). Then it is baked for 60 sec at 100°C. As shown in Figure 4.11a.
b. Using the mask aligner machine, the photoresist is exposed through the Metal-1 Mask after aligning it with the wafer.

c. The wafer is developed using AZ 726 developer for 1 minute. Then, it is rinsed with distilled water.

d. Using the Plasma Ion Etching (PIE) tool, O₂ descum for 20 sec is applied on the wafer to remove all the contaminants that is remained after developing. As shown in Figure 4.11b.

e. Using the sputter deposition tool, 50 nm of Chrome and 300 nm of Gold layer is sputtered. The required sputtering time for the chrome and gold is 200 sec and 352 sec, respectively to get the desired thicknesses.

f. In the ultrasonic acetone bath, the chrome and gold layer is lifted off for 15 min. Then the wafer rinsed with IPA and dried with nitrogen gas. As shown in Figure 4.11c.
4.3.5 Structural

a. A dehydration bake is done at 120°C for 300 sec on the wafer.

b. The wafer is immersed in an adhesion promoter solution that consists of 1 L of distilled and 1 mm³ of VM-651 for 40 sec to enhance the Polyimide to stick properly on the wafer.

c. A 4 µm polyimide (HD-2611) structural layer is spun using a recipe (extracted from the below figure 4.12) that consists of the following two steps in order to get 4µm structural layer.

![Spin speed curve](image)

Figure 4.12: Spin speed curve [30].

1. 500 rpm for 5 sec

2. 4500 rpm for 45 sec

d. Then the wafer is post baked at 90°C for 90 sec and then at 150°C for 90 sec.
e. After that the structural layer is cured by rising the temperature from 150 °C to 350°C in 50 min then the temperature fixed at 350°C for 30 min. Figure 4.13 shows the final cured structural layer.

4.3.6 Polyimide etching

- A 4 μm photoresist layer is spun onto the substrate. As shown in Figure 4.14a.
- The photoresist is exposed through the structural mask after aligning it with the wafer.
- The wafer is developed using AZ 726 developer for one minute before rinsing it by distilled water. As shown in Figure 4.14b.
- Using the sputter deposition tool, 50 nm of Chrome and 300 nm of Gold layer is sputtered where the required sputtering time for the Chrome and
Gold is 200 sec and 352 sec, respectively to get the desired thicknesses. As shown in Figure 4.14c.

e. In the ultrasonic acetone bath, the chrome and gold layer is lifted off for 15 min before rinsing the wafer with IPA and drying it with N₂. This metal layer protects our structures during Polyimide etching.

f. Using the plasma Ion Etching (PIE) tool, the wafer has been etched by CF₄ & O₂ at flow rates of 10 SCCM and 40 SCCM, respectively. The gases are applied for 18 min on 2 stages each one of 6 min long. The final etched structure is shown in figure 4.14.d.

Figure 4.14: Polyimide etching.
4.3.9 XeF$_2$ Release

After completing the etching process, each chip is released separately. The sacrificial layer is etched using the $XeF_2$ gas. Where the number of cycles is 25, the etching time is 20 seconds for each cycle and the $XeF_2$ Pressure is 2 Torr.

Figure 4.15: Final released structure.
Chapter 5

Testing and Characterization

In this chapter, we aim to verify experimentally the ability to excite higher order modes of microstructure using partial and full electrode actuation and to validate the model presented in Chapter 3.

5.1 Experimental Setup

In this section, we describe the experimental set-up used for testing and characterization. The setup for static characterization consists of a wire bonded chip placed under the micro surface analyzer. For the dynamic characterization, the set up consists of oscilloscope, amplifier, a vacuum chamber connected to vacuum pump and a micro system analyzer to measure the microstructure displacement and velocity. The schematic setup is shown in Figure 5.1.
Figure 5.1: The schematic setup for dynamic characterization.

5.2 Experimental Procedure

Figure 5.2: Microstructures placed under the micro surface analyzer.
For static characterization, we placed the microstructures under the microscope and then focused the microscope. After that, the vertical scanning range and exposure time were defined. Finally, the topography of the microstructure is generated and shown on the PC screen, as shown in Figure 5.2.

For dynamic characterization, we placed the wire-bonded chip inside the vacuum chamber. The pressure inside the vacuum chamber is kept around 20 mtorr during the experiments. A noise signal with different ranges of voltages and frequencies is applied on the microstructure. This signal is generated using the built-in function generator of the microsystem analyzer.

The microstructure movement is measured using the laser-Doppler vibrometer. The laser beam is focused onto the microstructure using the microscope and the x-y positioning stage. The measurement is based on interferometry, where the laser beam is split into two beams: one is focused on the moving structure and the other one is focused on a reference target. The difference between the two beams in phase and distance traveled is translated into displacement or velocity of the microstructure as a function of the frequency [31].

After getting the estimate value of the resonant natural frequencies, we actuated the microstructure using a sweep sinusoidal signal generated from the National Instrument data acquisition card (DAQ). The frequency of the signal is swept around
the natural frequencies of the microstructure to get the frequency response curve of the microstructure.

5.3 Experimental Result

5.3.1 Topography Characterization

We start our experimental investigation with static topography of the microstructure. We aim to find the microstructure curvature and define the air gap depth.

This experiment is conducted on cantilever beams of dimensions $350 \mu m \times 50 \mu m \times 3 \mu m$. After placing the cantilever under the micro system analyzer and defining the required parameters, the combined cantilever thickness and capacitor gap is around $40 \mu m$ which are higher than the design value $6 \mu m$. The curvature

Figure 5.3: Cantilever beam topography.
shown in Figure 5.3 is due to thermal stresses induced during fabrication where the layers deposited at extremely different temperatures.

5.3.2 Dynamic Characterization

Our dynamical characterization is divided into two parts: The first one is to experimentally find the resonant frequencies of the cantilever. The other part is to get the frequency response curve by sweeping a sinusoidal signal around the resonant frequency of the cantilever.

5.3.2.1 Random excitation

Using the built-in function generator of the micro system analyzer (MSA), a noise signal is generated to excite our microstructures. The vibrations are measured using the laser Doppler vibrometer. The measurements is taken near the fixed end

Figure 5.4: Wire bonded chip inside the vacuum chamber and under the micro system analyzer.
because it has better reflectivity and the high curvature near the cantilevers tip scatters the laser signal away from detector. The measurements are conducted on a cantilever beam of dimensions $600 \, \mu m \times 50 \, \mu m \times 3 \, \mu m$ with full and one-third lower electrode configuration. We applied a 15 DC and 10 AC $V$ on both the full electrode and the partial electrode cantilevers with a measurement frequency range up to $40 \, kHz$. For both cantilevers, we excited the first and second mode but the results of the partial electrode are sharper and clearer as shown in Figures 5.5 and 5.6.
Figure 5.5: Frequency response curve of a cantilever with full electrode up to 40 KHz.

Figure 5.6: Frequency response curve of a cantilever with one third electrode up to 40 KHz.
Then, we increased the measurement frequency range and applied a 15 DC and 10 AC V. Using the one-third electrodes, we were able to excite up to the eighth mode of vibration. The response amplitude near the second, third and fourth resonance is higher than the first resonance as shown in Figure 5.7. This is because the one-third electrode shape resembles these mode shapes and the measurement is taken near the fixed side of the cantilever. As shown in Figure 2.1 the beam deflection near the fixed end of the first mode is much smaller than the other modes.

Figure 5.7: Frequency response curve of a cantilever with one third electrode up to one MHz.
On the other hand, using the full electrode, only the first three modes were excited even though the result was not clear and sharp as the partial electrode ones as shown in Figure 5.8.

![Figure 5.8: Frequency response curve of a cantilever with one third electrode up to 400 KHz.](image)

Table 5-1: Experimental and simulation values of the first four modes of vibration.

<table>
<thead>
<tr>
<th>Mode</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{\text{exp}}$ (KHz)</td>
<td>5.8</td>
<td>37</td>
<td>100.5</td>
<td>191.2</td>
</tr>
<tr>
<td>$f_{\text{sim}}$ (KHz)</td>
<td>6</td>
<td>37.61</td>
<td>105.31</td>
<td>206.37</td>
</tr>
</tbody>
</table>

As shown in Table 5-1, the experimental values of the resonance frequency matched our simulation result with an error less than 8%.
5.3.2.2 Frequency sweep

After fixing the chip inside the vacuum chamber, the pressure was reduced down to 20 mTorr using the vacuum pump. We conducted the measurement on a cantilever with one-third electrode configuration as shown in Figure 5.11 and dimensions 600 μm × 50 μm × 3 μm.

A sinusoidal signal sweep was conducted around the first four modes of vibration with different DC and AC voltages. Figure 5.12 shows the result of sweeping a sinusoidal signal from 5 KHz to 7 KHz with 15 DC V and 10 AC V.

Figure 5.11: Partial electrodes length equal to one-third of the total beam length.

Figure 5.12: Frequency response curve of a cantilever near the first resonance frequency with one-third electrode actuation. $V_{dc} = 15 V$. 
Figure 5.13: Frequency response curve of a cantilever near the second resonance frequency with one-third electrode actuation. $V_{dc} = 15\ V$.

Figure 5.14: Frequency response curve of a cantilever near the third resonance frequency with one-third electrode actuation. $V_{dc} = 15\ V$.

Figure 5.13 shows the result of sweeping a sinusoidal signal around the second mode. The frequency range of the sweep signal is 34.5 $KHz$ – 36.5 $KHz$ with 15 DC $V$ and 10 AC $V$. Figure 5.14 shows the result of sweeping a sinusoidal signal around the third mode. The frequency range of the swept signal is 99$KHz$ – 101$KHz$. with 15 DC volts and AC voltages as shown.
Figure 5.15 shows the result of sweeping a sinusoidal signal around the fourth mode. The frequency range of the swept signal was 194.5 $kHz$ – 197 $kHz$ with 15 DC volts and AC voltages as shown.

Due to the cantilever high curvature, the measurement was taken near the fixed end of the cantilever. This affected the frequency response curve amplitudes and sharpness as shown in the previous figures.
Chapter 6

Conclusions

In this thesis, we presented a mathematical model of electrostatically actuated micro structure. We modeled it to account for the partial electrode configuration. Using Galerkin method, we reduced the order of the cantilever partial differential equation into set of coupled ordinary differential equations in time that has been solved using Mathematica up to the fourth mode of vibration. Displacement and velocity frequency response curve showed the dynamical pull-in ranges of the polyimide cantilever. Using the MSA, we were able to measure the air gap thickness near the cantilever tip. The gap thickness was ten times greater than the design value. Moreover, using the (DAQ), we swept the frequency around the first four modes of vibration. Due to high curvature, the measurement is taken near the fixed end of the cantilever. Also, we noticed dynamic pull in snaps the cantilever into the substrate and cause electrical problems. Moreover, we experimentally proved the ability to excite higher order modes using partial electrodes. The experimental numerical values of resonant frequency quantitatively matched our theoretical calculation as shown in table 6.1 for 600 $\mu$m long cantilever.
REFERENCES


