Power Allocation Strategies for Fixed-Gain Half-Duplex Amplify-and-Forward Relaying in Nakagami-m Fading

Ammar Zafar, *Student Member, IEEE*, Redha M. Radaydeh, *Senior Member, IEEE*, Yunfei Chen, *Senior Member, IEEE*, and Mohamed-Slim Alouini, *Fellow, IEEE*

Abstract

In this paper, we study power allocation strategies for a fixed-gain amplify-and-forward relay network employing multiple relays. We consider two optimization problems for the relay network: 1) maximizing the end-to-end signal-to-noise ratio (SNR) and 2) minimizing the total power consumption while maintaining the end-to-end SNR over a threshold value. We investigate these two problems for two relaying protocols of all-participate (AP) relaying and selective relaying and two cases of feedback to the relays, full and limited. We show that the SNR maximization problem is concave and the power minimization problem is convex for all protocols and feedback cases considered. We obtain closed-form expressions for the two problems in the case of full feedback and solve the problems through convex programming for limited feedback. Numerical results show the benefit of having full feedback at the relays for both optimization problems. However, they also show that feedback overhead can be reduced by having only limited feedback to the relays with only a small degradation in performance.

Index Terms

Amplify-and-forward, energy-efficiency, fixed-gain, full feedback, half-duplex, limited feedback, Nakagami-m fading, optimal power allocation.

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A. Zafar and M.-S. Alouini are with the Computer, Electrical and Mathematical Sciences & Engineering (CEMSE) Division, King Abdullah University of Science and Technology (KAUST), Thuwal, Makkah Province, Saudi Arabia. 23955-6900 (E-mail: {ammar.zafar, slim.alouini}@kaust.edu.sa).

Y. Chen is with the School of Engineering, University of Warwick, Coventry, UK. CV4 7AL (E-mail: yunfei.chen@warwick.ac.uk).

R. M. Radaydeh is now with the Electrical and Software Engineering Department, Alfaisal University, Riyadh, Saudi Arabia. 11533 (E-mail: rradaydeh@alfaisal.edu).

I. INTRODUCTION

The classical three-terminal relay channel has been around since the 1960s when it was first introduced by Van Der Meulen [1], [2]. Cover and Gamal then characterized the capacity of a three-terminal relay channel where a relay assists a source node in communicating with a destination node [3]. Since then precious little work was carried out on relays until a decade ago. The rapid progress in wireless communications technology, the increasing popularity of tetherless connectivity and satisfying the high quality-of-service (QoS) requirements have rekindled interest in relays [4], [5]. Recent years have seen a dearth of work being carried out to study the performance of relay-assisted systems [6]–[13]. It has been shown that relays can provide spatial diversity [14]–[16], increase capacity [17]–[19], conserve power [20], [21] and enhance coverage [22]–[24].

Power is a precious resource. It was reported in [25] that information and communication technology (ICT) utilizes more than 3% of the total electrical energy consumed worldwide and this percentage is expected to increase with time. Furthermore, in wireless communications, vendors and operators are searching for energy-efficient algorithms and devices to cut down on energy and operating costs [26]. In addition, mobile devices need to conserve energy as they have limited battery. Therefore, it is of paramount importance to use the available power as efficiently as possible. In a cooperative relay network, this corresponds to allocating power efficiently among the source and relay nodes to improve performance. Moreover, it is also essential to reduce overhead in the system which comes from feedback to the relays in the form of CSI, transmit power and other information. This usually requires the controller [1] to feedback information to each relay. This causes great overhead and consumes precious system resources such as bandwidth and time. Thus, it is crucial to come up with power allocation strategies which can work with limited feedback to the relay to minimize the overhead [27]. However, this reduced overhead comes at the cost of decrease in performance as there is less knowledge to work with and exploit. Therefore, there is a performance-overhead trade-off. It is important to have insight into this trade-off, so that optimal decisions can be taken to improve system performance under different scenarios.

Power allocation for cooperative network with a single fixed-gain AF relay was studied in [28]–[32]. In [28], the authors proposed power allocation schemes to maximize the sum and product of the average SNR of the source-destination and average SNR of the relay-destination link for a fixed gain relay-assisted

[1]The place where system decisions are taken. For instance it can be the destination.

source-destination pair. Moreover, the schemes required only knowledge of the channel statistics. For the same model as in [28], [29] proposed optimal and near-optimal power allocation algorithms to maximize the end-to-end SNR under slowly varying channel conditions. In [30], power allocation to minimize an upper bound on the symbol error rate (SER) for M-ary phase shift keying (MPSK) was derived for a source communicating with destination through a single fixed-gain AF relay. It was shown in [30] that the power allocation method provided better performance when the relay was near the destination. References [31] and [32] proposed power allocation strategies to minimize the bit error rate (BER) of a communication system employing a single fixed-gain AF relay. In [31], an upper bound on the pairwise error probability (PEP) is minimized for Rayleigh fading channels by efficiently allocating power between the source and relay nodes for different transmission protocols. The authors in [32] expanded upon the work in [31] by using the same PEP bound as in [31] and considering BER minimization for two single relay scenarios 1) moving relay 2) access point relay. For the moving relay case, the links were modeled as cascaded Nakagami random variables and for the access point case, the links were modeled as Nakagami random variables. For both cases, the upper bound on the PEP could not be obtained in closed-form in general. Closed-form bounds were obtained in the high SNR regime for the case when the relay is very close to the destination. These bounds involved the hypergeometric U-function [33, Eq. 9.211.4] which is difficult to implement. Hence, [32] suggested to numerically solve the problem for a large number of SNRs and Nakagami parameters and store the results in a look up table to be used for practical purpose.

Power allocation for networks with multiple AF relays was studied in [34]–[36]. The authors in [34] proposed a power allocation scheme to minimize an upper bound on the outage probability in the high SNR regime for Rayleigh fading channels assuming channel knowledge at the relays. Reference [34] reduced the $N + 1$ dimension problem, where N denotes the number of relays, to a one dimension convex optimization problem which could be efficiently solved. Power allocation to minimize the outage probability of a network consisting of multiple AF relay was also studied in [35] for the two cases of full CSI and knowledge of channel statistics only. For the case of full CSI, closed-form expressions were obtained for the relay power, however, due to the complexity of the problem, the source power could not be found in closed-form nor through an efficient algorithm. Hence, an exhaustive search is required to find the optimal source power and from it, the optimal relay powers can be obtained. For the case of knowledge of channel statistics only, as it was difficult to find the outage probability in closed-form, upper and lower bounds for the outage probability were derived for the high SNR regime for Rayleigh fading channels. Then power allocation was performed to minimize both bounds simultaneously. Again as the problem was too complex, only closed-form expressions for the relay powers were derived and the optimal source power had to found using an exhaustive search.

In this work, we consider a source-destination pair which communicates with the help of m fixed-gain AF relays. In addition to the m dual-hop links, the source and destination are also connected through a direct link[2]. The relays are assumed to have only one antenna and work in half-duplex mode. To avoid interference, all the relays are assumed to operate on orthogonal channels[3]. Moreover, coherent detection is assumed. Hence, the destination has CSI of all the links which are modeled as Nakagami-m random variables with arbitrary Nakagami parameter. To overcome the difficulty of obtaining a closed-form solution or efficient algorithm to calculate the source and relay powers jointly as was the case in [35], a relay gain model is used which depends on neither the instantaneous source power nor the instantaneous source-relay channel. Different from [35], this allows us to to show that the joint optimization problem is convex for the source as well as the relay powers. For this system, we consider two relay participation schemes of AP relaying in which all the m relays forward the signal to the destination and selective relaying in which only the selected [4] relay forwards the signal received from the source to the destination. For both schemes, we consider two optimization problems of optimal power allocation (OPA) and energy-efficiency. We refer to these as dual problems. OPA refers to the problem of allocating power to the source and the relays to maximize the end-to-end SNR under a total power constraint on the system. In the dual problem of energy-efficiency, the total power consumed is minimized while keeping the end-to-end SNR above a certain threshold. For both problems, as the destination has full CSI, the power allocation is performed at the destination and the power of each relay is then fed back to it[5]. In this work, two cases of feedback to the relays are considered

1) Full feedback: In this case, power allocation is performed for each set of channel realization and fed back to the relays and the source. Thus, this is the optimal scenario and serves as a benchmark for

[2]The results in this paper include the case of no direct link as a special scenario by replacing the fading gain of the source-destination link with 0.

[3]They can be orthogonal in time, frequency, space, or code.

[4]How selection takes place is discussed in Section III where we study selective relaying in detail.

[5]This is similar to multiple user systems with opportunistic scheduling where full CSI is required at a central controller to schedule the users [37], [38]. The destination then feeds back the scheduling decision and the optimal transmit powers to the scheduled users. Moreover a similar assumption is made in [35].

practical schemes. However, the drawback of this case is the great overhead due to feedback for each set of channel realizations. Note that the solution to this case is the same as the case when power allocation is performed at the source and the relays and they each have full CSI[6]. It is shown that all the problems for full feedback can be efficiently solved in closed-form, including closed-form expression for the source power which was not obtained in previous related works.

2) Limited feedback: In this case, power allocation is performed using only the channel statistics. Hence, now the destination only needs to feedback the powers once before system startup. This results in degradation of performance. However, it reduces the feedback overhead considerably which is an important issue for cooperative relaying systems [27], [31], [32], [34], [35] The solution in this instance is the same as that when power allocation is performed at relays with knowledge of only channel statistics. This problem is solved assuming Nakagami-m fading with arbitrary Nakagami parameter which is different from previous works, such as [34], [35] considering multiple relays. Moreover, the exact average end-to-end SNR is used as the selection metric. The concavity and convexity of the maximizing SNR and minimizing total power consumption is first established. Hence, the problems are then solved using convex optimization techniques. In addition, if the destination prefers simplicity and cannot solve a $m+1$ dimension convex optimization problem, then an upper bound is used for the average end-to-end SNR for integer Nakagami parameter which leads to simple closed-form solutions.

We study the two dual optimization problems for the two relay participation schemes under both cases of feedback given above. In [36], we studied the two dual problems for a similar system AP setup[7], not selective relaying. Moreover, in [36], we assumed that power allocation was performed at the relays which had only partial CSI of the links, not at the destination with full CSI. Such a system gives similar results to the full feedback case for AP relaying considered here. Hence, for full feedback with AP relaying in this work, we only give the results and a perfunctory discussion without derivation. The interested reader is referred to [36] for more details. Thus, the contribution of this work are the novel power allocation strategies for the different optimization metrics (i.e. maximizing end-to-end SNR and minimizing total power consumption), amount of feedback (i.e. full and limited), Nakagami-m fading with

[6]Such an assumption was made in [35] and [39].

[7]We made a slight mistake for the implementation of the energy-efficiency problem solution in [36] which makes the results presented in [36] a little worse than they actually should be. This case will be discussed in Section II-C1

arbitrary Nakagami parameter and relaying strategy (i.e AP and selective). These novel power allocation strategies not only optimize the relay powers but the source power as well. Furthermore, the power allocation strategies work on the exact performance metric, i.e. instantaneous end-to-end SNR and average end-to-end SNR, and not on bounds except in the case where the destination prefers simplicity.

As well, we choose the end-to-end SNR as the performance metric for the following reasons. For the case of full feedback, the end-to-end SNR is the optimal criteria from an outage probability and ergodic capacity point of view, and it has to be used. For the case of limited feedback, it is not the optimal criteria anymore and the outage probability and the ergodic capacity should be used to find the optimal solution instead. However, it is very difficult to obtain these two metrics in a form suitable for evaluation, if not impossible. Most of the existing works which consider power allocation for the minimization of outage probability use bounds for Rayleigh fading channels to provide suboptimal solutions in the high SNR regime often around 20 dB, while in practical systems, the SNR may lie in the range of 5-10 dB because it is intractable to find the closed-form expressions of outage and capacity, even bounds for Nakagami-m fading at medium SNRs for multiple relays, as considered in this work. For example, the outage probability of dual hop transmissions with a single fixed-gain relay over arbitrary Nakagami-m fading channels was derived in [40, Eq. (20)]. However, [40] could not find the ergodic capacity in closed-form, and from [40], power allocation to minimize the outage probability seems intractable even for the case of one relay as it contains multiple Gamma functions [33, Eq. (8.310.1)], generalized hypergeometric functions [33, Eq. (9.14.1)] and the Beta function [33, Sec. (8.380)], not to mention multiple relays. Similarly for works that use the SER [32], the suboptimal PEP bound can be obtained in closed-form only for high SNR where only one relay close to the destination, and yet the result still contains the hypergeometric-U function which is difficult to implement. From the SER derived in [41], the power allocation seems intractable even for a single relay with no direct link. Furthermore, there have not been any significant works on the maximization of the ergodic capacity with knowledge of only channel statistics. In summary, all existing works that consider the optimal criteria such as outage probability, BER and ergodic capacity in fact provide suboptimal solutions for limited simple cases. It is beyond the scope of this work to derive closed-form expressions for these optimal metrics to find the optimal solution for these criteria. Instead, we relax the metric used to the end-to-end SNR, which is also an important metric in wireless communications, to find the optimal solution for this criteria. As all the other performance metrics depend on the end-to-end SNR, generally, maximizing the average end-to-end SNR increases the instantaneous SNR on average which in turns means better outage performance and ergodic capacity.

The remainder of the paper is organized as follows. In Section II, we consider AP relaying. Section III focuses on selective relaying. Numerical results are presented in Section IV. Finally, Section V summarizes the main results of the paper.

II. All-Participate Scheme

A. System Model

Consider a cooperative system with a source node, a destination node, and m relays. Each relay is assumed to be equipped with only one antenna and operates in half-duplex mode. The source and the relays transmit on orthogonal channels. Without loss of generality, we assume time division multiple access (TDMA). The transmission takes place in two phases. In the first phase, the source broadcasts information to the relays and the destination. The signals at the ith relay and the destination are given by $y_{si} = \sqrt{E_s} h_{si} s + n_{si}$ and $y_{sd} = \sqrt{E_s} h_{sd} s + n_{sd}$, respectively, where E_s is the source energy, s is the transmitted signal with unit energy, h_{si} and h_{sd} are the instantaneous channel gains from the source to the ith relay and destination, respectively, modeled as independent Nakagami-m faded, n_{si} and n_{id} are the complex Gaussian noise with mean zero and variances σ_{si}^2 and σ_{sd}^2, respectively. In the second phase, the relays, after amplification, forward the signal to the destination. The received signal at the destination from the ith relay is $y_{id} = \sqrt{a_i E_s E_i} h_{si} h_{id} s + n_i$, where a_i is the ith relay gain, E_i is the ith relay energy, h_{id} is the instantaneous channel gain from the ith relay to the destination again modeled as Nakagami-m faded, n_i is the equivalent noise with $n_i \sim CN(0, \sigma_i^2)$, and $\sigma_i^2 = a_i E_i |h_{id}|^2 \sigma_{si}^2 + \sigma_{id}^2$. One can write the $m+1$ received signals at the destination in matrix form

$$\mathbf{y} = \mathbf{h} s + \mathbf{n}, \tag{1}$$

where $\mathbf{h} = \left[\sqrt{\frac{E_s}{\sigma_{sd}^2}} h_{sd} \ \sqrt{\frac{a_i E_s E_1}{a_i E_1 |h_{1d}|^2 \sigma_{s1}^2 + \sigma_{1d}^2}} h_{s1} h_{1d} \dots \sqrt{\frac{a_i E_t E_m}{a_i E_m |h_{md}|^2 \sigma_{sm}^2 + \sigma_{md}^2}} h_{sm} h_{md} \right]^T$, $\mathbf{n} \sim CN(\mathbf{0}, \mathbf{I})$, and $\mathbf{y} = \left[\frac{1}{\sigma_{sd}} y_{sd} \ \ \frac{1}{\sigma_1} y_{1d} \dots \frac{1}{\sigma_m} y_{md} \right]^T$. Throughout this paper, it is assumed that the destination has complete CSI of all the links. It is also assumed that all the links experience independent fading. Furthermore, the fading gain of each link changes independently from one time slot to the other.

B. Optimal Power Allocation

First, we consider the problem of OPA. In OPA, the end-to-end SNR is maximized under power constraints on the system. For OPA, we consider the two cases of full and limited feedback.

1) Full Feedback: In this subsection, we assume that the destination feeds back information to each relay and the source. Using (1), and assuming maximal-ratio-combining (MRC) at the destination, the end-to-end SNR of the system is given by

$$\gamma = E_s \left(\sum_{i=0}^{m} \alpha_i - \sum_{i=1}^{m} \frac{\alpha_i \zeta_i}{a_i E_i \beta_i + \zeta_i} \right), \tag{2}$$

where $\alpha_0 = \frac{|h_{sd}|^2}{\sigma_{sd}^2}$, $\alpha_i = \frac{|h_{si}|^2}{\sigma_{si}^2}$, $\beta_i = \frac{|h_{id}|^2}{\sigma_{id}^2}$, and $\zeta_i = \frac{1}{\sigma_{si}^2}$. In this work, we consider a total power constraint on the whole system and individual power constraints on all the nodes. Hence, the power allocation problem can be written as

$$\max_{E_s, E_i} \gamma, \quad \text{subject to} \quad E_s + \sum_{i=1}^{m} E_i \leq E_{tot}, \ 0 \leq E_s \leq E_s^{max}, \ 0 \leq E_i \leq E_i^{max}. \tag{3}$$

where E_{tot} is the total power constraint on the whole system, E_s^{max} is the peak power constraint at the source, and E_i^{max} is the peak power constraint on the ith relay. In general, γ is not a concave function of the source and relay powers as its Hessian is not negative semi-definite. However, as we show in Appendix A, the objective function in (3) is concave given the constraints on the system. Hence, the optimization problem in (3) can be solved using the Lagrange dual method [42]. Moreover, as the constraints are affine, Slater's condition [42] is satisfied, i.e. the duality gap between the primal and dual solutions is zero. Therefore, the solution obtained for the Lagrange dual problem is also the optimal solution of the problem in (3). Hence, solving the problem in (3) using the Lagrange dual method, with the help of [36], yields the solution

$$E_s = \left(\frac{\delta \left(\sum_{i=1}^{m} \sqrt{\frac{\alpha_i \zeta_i}{a_i \beta_i}} \right)^2}{\left(\sum_{i=0}^{m} \alpha_i - \delta \right)^2} \right)_0^{E_s^{max}} \tag{4}$$

$$E_j = \left(\frac{\left(\sum_{i=1}^{m} \sqrt{\frac{\alpha_i \zeta_i}{a_i \beta_i}} \right)}{\left(\sum_{i=0}^{m} \alpha_i - \delta \right)} \sqrt{\frac{\alpha_j \zeta_j}{a_j \beta_j}} - \frac{\zeta_j}{a_j \beta_j} \right)_0^{E_j^{max}}, \tag{5}$$

where

$$\delta = \sum_{i=0}^{m} \alpha_i - \left(\sum_{i=1}^{m} \sqrt{\frac{\alpha_i \zeta_i}{a_i \beta_i}} \right) \sqrt{\frac{\left(\sum_{i=0}^{m} \alpha_i \right)}{E_{tot} + \sum_{j=1}^{m} \frac{\zeta_j}{a_j \beta_j}}}. \tag{6}$$

It is noted here that as stated in the Introduction the above solution is different from the solutions obtained in [35] due to a different relay gain model which enables joint optimization of both source and relay powers. Moreover, as we jointly optimize source and relay power, the expression fo the relay powers is different from the one obtained in [35]. From equations (4) and (5), one can conclude that the OPA follows a water-filling solution. Hence, power is allocated in an iterative manner. However, unlike traditional water-filling algorithm, here the closed-form solution may change in an iteration according to the results in the previous iteration due to the fact that source and relay powers have different closed-form solutions unlike in works where only relay power allocation is performed and a generalized closed-form of the relay powers is obtained. Thus, if the source power exceeds its peak constraint, the expression for the relay powers changes accordingly. Also, if a relay power and not the source power exceeds its peak constraint, only the constraints and optimization variables are updated. If in the result of the current iteration the source power, E_s, satisfies its constraints and any relay power does not satisfy its individual constraint, then the analytical solution to the problem remains the same, however, the optimization variables and the constraint changes in the next iteration. Therefore, in this case, the optimal solution in the next iteration is given by

$$E_s = \left(\frac{\delta \left(\sum_{i \in \mathbb{X}} \sqrt{\frac{\alpha_i \zeta_i}{a_i \beta_i}} \right)^2}{\left(\sum_{i \notin \mathbb{Y}} \alpha_i - \delta - \sum_{i \in \mathbb{Z}} \frac{\alpha_i \zeta_i}{a_i E_i^{max} \beta_i + \zeta_i} \right)^2} \right)_0^{E_s^{max}} \tag{7}$$

$$E_j = \left(\frac{\left(\sum_{i \in \mathbb{X}} \sqrt{\frac{\alpha_i \zeta_i}{a_i \beta_i}} \right) \sqrt{\frac{\alpha_j \zeta_j}{a_j \beta_j}}}{\sum_{i \notin \mathbb{Y}} \alpha_i - \delta - \sum_{i \in \mathbb{Z}} \frac{\alpha_i \zeta_i}{a_i E_i^{max} \beta_i + \zeta_i}} - \frac{\zeta_j}{a_j \beta_j} \right)_0^{E_j^{max}} \tag{8}$$

and

$$\delta = \sum_{i \notin \mathbb{Y}} \alpha_i - \sum_{i \in \mathbb{Z}} \frac{\alpha_i \zeta_i}{a_i E_i^{max} \beta_i + \zeta_i} - \left(\sum_{i \in \mathbb{X}} \sqrt{\frac{\alpha_i \zeta_i}{a_i \beta_i}} \right) \sqrt{\frac{\sum_{i \notin \mathbb{Y}} \alpha_i - \sum_{i \in \mathbb{Z}} \frac{\alpha_i \zeta_i}{a_i E_i^{max} \beta_i + \zeta_i}}{E_{tot} - \sum_{i \in Z} E_i^{max} + \sum_{j \in \mathbb{X}} \frac{\zeta_j}{a_j \beta_j}}}, \tag{9}$$

where $\mathbb{X}$, $\mathbb{Y}$ and $\mathbb{Z}$ represent the sets of powers which satisfy the individual constraints and are less than zero and greater than the peak individual constraint, respectively. Now, if the source power comes out to be greater than E_s^{max} in the current iteration, then it is set at E_s^{max}. The updated optimal solution for the relay powers in the next iteration now becomes

$$E_j = \left(\sqrt{\frac{E_s^{max} \alpha_j \zeta_j}{\delta a_j \beta_j}} - \frac{\zeta_j}{a_j \beta_j} \right)_0^{E_j^{max}}, \tag{10}$$

where the Lagrangian multiplier can be obtained as

$$\delta = \frac{E_s^{max}\left(\sum_{j\in\mathbb{X}}\sqrt{\frac{\alpha_j\zeta_j}{a_j\beta_j}}\right)^2}{\left(E_{tot}-E_s^{max}-\sum_{i\in\mathbb{Z}}E_i^{max}+\sum_{j\in\mathbb{X}}\frac{\zeta_j}{a_j\beta_j}\right)^2}. \tag{11}$$

2) Limited Feedback: In Section II-B1, it was assumed that the destination fed back the transmit powers to each relay and the source for each channel realization. However, this greatly increases the overhead. The destination has to inform all relays of the power allocation through dedicated feedback channels. This consumes a considerable amount of resources. Moreover, the reverse link between the destination and the relays might be poor and communication might not be possible[8]. Therefore, it is desirable to be able to work with less feedback. Hence, destination calculates the powers using channel statistics and only feeds back the powers once before system startup . Thus, to perform power allocation, the end-to-end SNR needs to be averaged over all the links. As the channels are modeled as Nakagami-m distributed, α_i and β_i are both Gamma random variables and their probability density functions are given by

$$f_{\alpha_i}(x) = \frac{1}{\Gamma(k_{\alpha_i})\bar{\gamma}_{\alpha_i}^{k_{\alpha_i}}}x^{k_{\alpha_i}-1}e^{-\frac{x}{\bar{\gamma}_{\alpha_i}}} \quad \text{and} \quad f_{\beta_i}(x) = \frac{1}{\Gamma(k_{\beta_i})\bar{\gamma}_{\beta_i}^{k_{\beta_i}}}x^{k_{\beta_i}-1}e^{-\frac{x}{\bar{\gamma}_{\beta_i}}} \qquad x \geq 0, \tag{12}$$

respectively, where k_{α_i} and k_{β_i} are the shape parameters of the links, $\bar{\gamma}_{\alpha_i}$ and $\bar{\gamma}_{\beta_i}$ are the average SNRs of the links, and $\Gamma(.)$ is the Gamma function. As all the links are assumed to be independent, the average end-to-end SNR can be found by averaging (2) over the density functions in (12) giving

$$\begin{aligned}\bar{\gamma} = \int_0^\infty \cdots \int_0^\infty E_s\left(\sum_{i=0}^m x_i - \sum_{i=1}^m \frac{x_i\zeta_i}{a_iE_iy_i+\zeta_i}\right)\frac{1}{\Gamma(k_{\alpha_0})\bar{\gamma}_{\alpha_0}^{k_{\alpha_0}}}x^{k_{\alpha_0}-1}e^{-\frac{x}{\bar{\gamma}_{\alpha_0}}}\times \\ \left(\prod_{i=1}^m \frac{1}{\Gamma(k_{\alpha_i})\bar{\gamma}_{\alpha_i}^{k_{\alpha_i}}}x_i^{k_{\alpha_i}-1}e^{-\frac{x_i}{\bar{\gamma}_{\alpha_i}}}\frac{1}{\Gamma(k_{\beta_i})\bar{\gamma}_{\beta_i}^{k_{\beta_i}}}y_i^{k_{\beta_i}-1}e^{-\frac{y_i}{\bar{\gamma}_{\beta_i}}}\right)dx_0dx_1\dots dx_mdy_1\dots dy_m.\end{aligned} \tag{13}$$

This can be simplified to

$$\bar{\gamma} = E_s\sum_{i=0}^m k_{\alpha_i}\bar{\gamma}_{\alpha_i} - E_s\sum_{i=1}^m \frac{k_{\alpha_i}\bar{\gamma}_{\alpha_i}\zeta_i}{\Gamma(k_{\beta_i})\bar{\gamma}_{\beta_i}^{k_{\beta_i}}}\int_0^\infty \frac{1}{a_iE_iy_i+\zeta_i}y_i^{k_{\beta_i}-1}e^{-\frac{y_i}{\bar{\gamma}_{\beta_i}}}dy_i \tag{14}$$

[8]The results in this section are also applicable to the case where power allocation is done at the relays with knowledge of channel statistics only.

Solving the above with the help of [33, Eq. (3.383.10)] gives the average end-to-end SNR as

$$\bar{\gamma} = E_s \sum_{i=0}^{m} k_{\alpha_i} \bar{\gamma}_{\alpha_i} - E_s \sum_{i=1}^{m} \frac{k_{\alpha_i} \bar{\gamma}_{\alpha_i} \zeta_i^{k_{\beta_i}}}{a_i^{k_{\beta_i}} \bar{\gamma}_{\beta_i}^{k_{\beta_i}}} \frac{1}{E_i^{k_{\beta_i}}} e^{\frac{\zeta_i}{a_i E_i \bar{\gamma}_{\beta_i}}} \Gamma\left(1 - k_{\beta_i}, \frac{\zeta_i}{a_i E_i \bar{\gamma}_{\beta_i}}\right), \tag{15}$$

where $\Gamma(.,.)$ is the upper incomplete gamma function given by $\Gamma(s,z) = \int_z^{\infty} e^{-t} t^{s-1} dt$ [33, Eq. (8.350.2)]. The optimization problem is the same as in II-B1, however the objective function is now changed to $\bar{\gamma}$ instead of γ. We show that $\bar{\gamma}$ is a concave function of the optimization parameters on the domain of interest in Appendix B. Thus, the optimization problem is concave. However, it is difficult to find closed-form expressions for the optimal solution due to the complexity of the objective function. Fortunately, as the problem is concave, we can utilize well-known algorithms for convex optimization. So, the interior point algorithm can be used to find the optimal solution [42]. For the special case of Rayleigh fading, $\bar{\gamma}$ in (15) simplifies to

$$\bar{\gamma} = E_s \sum_{i=0}^{m} \bar{\gamma}_{\alpha_i} - E_s \sum_{i=1}^{m} \frac{\zeta_i \bar{\gamma}_{\alpha_i}}{a_i E_i \bar{\gamma}_{\beta_i}} e^{\frac{\zeta_i}{a_i E_i \bar{\gamma}_{\beta_i}}} E_1\left(\frac{\zeta_i}{a_i E_i \bar{\gamma}_{\beta_i}}\right), \tag{16}$$

where $E_1(.)$ is the exponential integral function of the first kind and is related to the exponential integral function as $E_1(x) = -E_i(-x)$ [43].

One last remark is that (15) contains the upper incomplete Gamma function which can be complex to implement. Moreover, the optimization problem is a numerical one with $m+1$ dimensions. Therefore, it is possible that the destination, if it is a simple node, does not have the capability to handle such computations. Furthermore, if power allocation is performed at the relay and the relay nodes are simple, then in this scenario to the relays will have difficulty in solving the $m+1$ dimension problem. Hence, a simple suboptimal solution is required in such scenarios. Using [44, Eq. (6.5.9)], $\bar{\gamma}$, for integer values of the Nakagami parameter, can be written as

$$\bar{\gamma} = E_s \sum_{i=0}^{m} k_{\alpha_i} \bar{\gamma}_{\alpha_i} - E_s \sum_{i=1}^{m} \frac{k_{\alpha_i} \bar{\gamma}_{\alpha_i} \zeta_i}{a_i E_i \bar{\gamma}_{\beta_i}} e^{\frac{\zeta_i}{a_i E_i \bar{\gamma}_{\beta_i}}} E_{k_{\beta_i}}\left(\frac{\zeta_i}{a_i E_i \bar{\gamma}_{\beta_i}}\right). \tag{17}$$

Now utilizing [44, Eq. (5.1.19)], an upper bound on $\bar{\gamma}$ can be found as

$$\bar{\gamma} \leq E_s \sum_{i=0}^{m} k_{\alpha_i} \bar{\gamma}_{\alpha_i} - E_s \sum_{i=1}^{m} \frac{k_{\alpha_i} \bar{\gamma}_{\alpha_i} \zeta_i}{a_i E_i \bar{\gamma}_{\beta_i} (k_{\beta_i} - 1) + \zeta_i}. \tag{18}$$

Thus, in the case of simple destination or relay nodes, the Nakagami parameter is first approximated to its closest integer and then the upper bound in (18) can be maximized which as we show now yields simple closed-form results. Note that (18) can be obtained from (2) by replacing α_i by $k_{\alpha_i}\bar{\gamma}_{\alpha_i}$ and β_i by $\bar{\gamma}_{\beta_i}(k_{\beta_i}-1)$. Hence, the closed-form solution to maximizing the upper bound in (18) can simply be obtained from the closed-form solution derived in Section II-B1 by making the given substitutions.

C. Energy-Efficiency

In Section II-B, we considered the problem of maximizing the end-to-end SNR under individual and total power constraints. In this section, we study the problem of energy-efficiency. The objective is to minimize the total power consumed, $E_{tot} = E_s + \sum_{i=1}^{m} E_i$, while keeping the instantaneous end-to-end SNR above a threshold, γ^{th}, and ensuring the source and relay powers do not exceed their respective individual constraints. Like Section II-B, we consider the two cases of full and limited feedback.

1) Full feedback: The optimization problem is given by

$$\min_{Es,Ei} \; E_{tot}, \;\; \text{subject to} \;\; \gamma \geq \gamma^{th}, \; 0 \leq E_s \leq E_s^{max}, \; 0 \leq E_i \leq E_i^{max}. \tag{19}$$

Now, we need to show that the optimization problem in (19) is convex and hence, the optimal solution can be found. The objective function and the individual constraints are convex functions. So, it is only left to show that γ is concave, meaning $\gamma^{th} - \gamma$ is convex, on the domain of the problem. Employing the same notation and procedure as in Appendix A, to prove concavity of γ, we need to show that $D(x, y) = (f(x) - f(y))(g(x) - g(y)) \leq 0$. If $f(x) > f(y)$, then $g(y) > g(x)$, to satisfy the constraint on γ and vice versa. Thus, $D(x, y) < 0$ and γ is concave on the domain. Moreover, γ is a monotonically increasing function of E_s and E_is, hence the optimal solution to (19) is achieved when $\gamma = \gamma^{th}$. As the other two constraints are affine and the objective function is convex, Slater's condition is satisfied. Therefore, the solution obtained using the Lagrange dual method will be optimal. Solving the problem (19) using the Lagrange dual method, with the help of [36], gives the optimal solution as

$$E_s = \left(\frac{\rho \left(\sum_{j=1}^{m} \sqrt{\frac{\alpha_j \zeta_j}{a_j \beta_j}} \right)^2}{(\rho \sum_{i=0}^{m} \alpha_i - 1)^2} \right)_0^{E_s^{max}} \tag{20}$$

$$E_j = \left(\frac{\rho \left(\sum_{i=1}^{m} \sqrt{\frac{\alpha_i \zeta_i}{a_i \beta_i}} \right)}{(\rho \sum_{i=0}^{m} \alpha_i - 1)} \sqrt{\frac{\alpha_j \zeta_j}{a_j \beta_j}} - \frac{\zeta_j}{a_j \beta_j} \right)_0^{E_i^{max}} , \tag{21}$$

where

$$\rho = \frac{\left(\sum_{j=1}^{m} \sqrt{\frac{\alpha_j \zeta_j}{a_j \beta_j}}\right)}{\sum_{i=0}^{m} \alpha_i \sqrt{\gamma^{th}}} + \frac{1}{\sum_{i=0}^{m} \alpha_i}. \tag{22}$$

From (20) and (21), one can see that the solution again follows a water-filling algorithm described in Section II-B1. Hence, the power allocation process is repeated until all the powers satisfy the constraint. However, the optimal solution changes depending on the initial power allocation. There are two cases like for the problem of OPA: 1) E_s lies between 0 ans E_s^{max}, 2) E_s is greater than E_s^{max}. Considering case 1 first, the optimal power allocation is given by

$$E_s = \left(\frac{\rho \left(\sum_{j\in\mathbb{X}} \sqrt{\frac{\alpha_j \zeta_j}{a_j \beta_j}}\right)^2}{\left(\rho \sum_{i\notin\mathbb{Y}} \alpha_i - \rho \sum_{i\in\mathbb{Z}} \frac{\alpha_i \zeta_i}{a_i E_i^{max} \beta_i + \zeta_i} - 1\right)^2} \right)_0^{E_s^{max}} \tag{23}$$

$$E_j = \left(\frac{\rho \left(\sum_{i\in\mathbb{X}} \sqrt{\frac{\alpha_i \zeta_i}{a_i \beta_i}}\right) \sqrt{\frac{\alpha_j \zeta_j}{a_j \beta_j}}}{\rho \sum_{i\notin\mathbb{Y}} \alpha_i - \rho \sum_{i\in\mathbb{Z}} \frac{\alpha_i \zeta_i}{a_i E_i^{max} \beta_i + \zeta_i} - 1} - \frac{\zeta_j}{a_j \beta_j} \right)_0^{E_i^{max}} \tag{24}$$

and

$$\rho = \frac{\sqrt{\frac{\left(\sum_{j\in\mathbb{X}} \sqrt{\frac{\alpha_j \zeta_j}{a_j \beta_j}}\right)^2}{\gamma^{th}} + 1}}{\sum_{i\notin\mathbb{Y}} \alpha_i - \sum_{i\in\mathbb{Z}} \frac{\alpha_i \zeta_i}{a_i E_i^{max} \beta_i + \zeta_i}}. \tag{25}$$

In the second case, $E_s = E_s^{max}$ and

$$E_j = \left(\sqrt{\frac{\rho \alpha_j \zeta_j E_s^{max}}{a_j \beta_j}} - \frac{\zeta_j}{a_j \beta_j} \right)_0^{E_j^{max}}, \tag{26}$$

$$\rho = \frac{E_s^{max} \left(\sum_{i\in\mathbb{X}} \sqrt{\frac{\alpha_i \zeta_i}{a_i \beta_i}}\right)^2}{\left(E_s^{max} \left(\sum_{i\notin\mathbb{Y}} \alpha_i - \sum_{i\in\mathbb{Z}} \frac{\alpha_i \zeta_i}{a_i E_i^{max} \beta_i + \zeta_i}\right) - \gamma^{th}\right)^2}. \tag{27}$$

As stated above, the power allocation in this section follows the same procedure as in Section II-B1. Hence, the power is allocated in an iterative manner. However, the channel conditions can be such that the solution obtained is where some powers are set at their maximum constraints and some at 0. A situation can arise in this instance where $\gamma > \gamma^{th}$ due to the powers greater than their constraint being treated first in the algorithm. Hence, there should be a check at the end of the algorithm and if $\gamma > \gamma^{th}$, then the whole water-filling process is repeated, however, this time the powers which came out to be zero in the first run of the complete procedure are not included in the algorithm from the start[9].

2) Limited feedback: Now, we consider the case of limited feedback. As was the case in Section II-B2, the end-to-end SNR now needs to be averaged over all the links. Therefore, in this instance, the energy-efficiency optimization problem is given by

$$\min_{Es,Ei} E_{tot}, \text{ subject to } \bar{\gamma} \geq \gamma^{th},\ 0 \leq E_s \leq E_s^{max},\ 0 \leq E_i \leq E_i^{max}, \tag{28}$$

where $\bar{\gamma}$ is given by (15). Using Appendix B and a similar argument as that in Section II-C1, it can be shown that the the optimization problem, (28), is convex. Hence, the Lagrange multiplier and other convex optimization algorithms can be applied to solve the problem. However, due to the complexity of the problem, it is very difficult to obtain closed-form expressions for the optimal solution. Therefore, we use the interior-point algorithm as utilized in Section II-B2 to obtain the optimal solution. Also, similar to Section II-B2, for simple nodes, the upper bound on $\bar{\gamma}$ can be used, the results of which can be obtained from the results in Section II-B2 by making the appropriate substitutions.

III. SELECTION SCHEME

In Section II, AP relaying was considered. However, AP relaying requires additional complexity at the destination to combine the relays. Also, as the relays transmit on orthogonal channels, it consumes a huge amount of system resources and decreases throughput. To ameliorate these drawbacks of AP relaying, selective relaying has been proposed in which only the "best" relay is selected to forward the signal from the source to the destination. The selection criteria depends on the objective. For OPA, the relay which maximizes the end-to-end SNR after power allocation is selected. For energy-efficiency, the relay which minimizes the consumed energy while fulfilling the constraint on the end-to-end SNR is selected. If no relay fulfills the constraint on the end-to-end SNR, then the relay which achieves the maximum end-to-end SNR is selected.

[9]This was our mistake in [36]. We did not account for these special cases

An important point to note is that while selective relaying is a special case of AP relaying and the solutions can be obtained in the same manner as in the AP case, it is better to re-formulate the problem as it reduces the number of computations and hence, conserves time and resources. For example, for OPA with full feedback, the new formulation requires only to find one variable and then both powers can be obtained from this variable with a simple multiplication. If this problem was solved using the methodology for AP, then three variables, source power, relay power and the Lagrange multiplier would need to be calculated which requires more computations. The computation saving is not significant for one channel realization, but becomes significant for large channel realizations. For the OPA limited feedback case, the convex optimization problem is converted from a two-dimensional problem to a one-dimensional problem which can be solved more efficiently. Therefore, re-formulating the problems has its benefits.

A. Optimal Power Allocation

1) Full feedback: For selective relaying, noting that the power is now only divided between the source and one relay, one can re-formulate the end-to-end SNR in (2), in the case that the ith relay is selected to transmit, as

$$\gamma_i = \eta_i E_{tot} \left(\alpha_0 + \alpha_i - \frac{\alpha_i \zeta_i}{a_i (1-\eta_i) E_{tot} \beta_i + \zeta_i} \right) \qquad i = 1, 2, \ldots, m, \tag{29}$$

where we have replaced $E_s = \eta_i E_{tot}$, $E_i = (1-\eta_i)E_{tot}$ and $0 < \eta_i \leq 1$. Ignoring the individual power constraints, the optimization problem becomes

$$\max_{\eta_i} \quad \eta_i E_{tot} \left(\alpha_0 + \alpha_i - \frac{\alpha_i \zeta_i}{a_i (1-\eta_i) E_{tot} \beta_i + \zeta_i} \right) \qquad i = 1, 2, \ldots, m. \tag{30}$$

The concavity of the objective function follows from the concavity of the problem for AP relaying. Taking the derivative of the objective function in (30) and equating it to zero yields the optimal solution

$$\eta_i = 1 - \frac{1}{a_i E_{tot} \beta_i} \left(\sqrt{\frac{(a_i E_{tot} \beta_i + \zeta_i) \alpha_i \zeta_i}{\alpha_0 + \alpha_i}} - \zeta_i \right), \tag{31}$$

where α_i and β_i are the links associated with the ith relay. If η_i is found to be such that one of the powers exceeds its individual constraints, then η_i is adjusted so that the power lies on its peak individual constraint. The case where both powers exceed their constraints is when $E_{tot} > E_s^{max} + E_i^{max}$. In this case, both the source and the selected relay transmit at their individual constraints. The algorithm for power allocation for selective relaying is

- Calculate η_i for all the relays using (31).
- Compute the resulting γ_i for each relay.
- Select the relay which has the maximum γ_i.

2) Limited feedback: As was the case for AP relaying, in the case of limited feedback, the end-to-end SNR has to be averaged over the channels before power allocation is performed. Hence, now the relay which gives best performance on average is now selected. The optimization problem now, with the help of (15), is

$$\max_{\eta_i} \quad (1-\eta_i)E_{tot}\left(k_{\alpha_0}\bar{\gamma}_{\alpha_0}+k_{\alpha_0}\bar{\gamma}_{\alpha_i}-\frac{k_{\alpha_i}\bar{\gamma}_{\alpha_i}\zeta_i^{k_{\beta_i}}}{a_i^{k_{\beta_i}}\bar{\gamma}_{\beta_i}^{k_{\beta_i}}}\frac{1}{\eta_i^{k_{\beta_i}}E_{tot}^{k_{\beta_i}}}e^{\frac{\zeta_i}{a_i\eta_i E_{tot}\bar{\gamma}_{\beta_i}}}\Gamma\left(1-k_{\beta_i},\frac{\zeta_i}{a_i\eta_i E_{tot}\bar{\gamma}_{\beta_i}}\right)\right) \tag{32}$$

where now $E_s=(1-\eta_i)E_{tot}$, $E_i=\eta_i E_{tot}$ and $0\leq\eta_i<1$ due to ease of analysis. The concavity of the problem follows directly from the concavity of the AP case. Taking the derivative of the objective function in (32) and equating it to zero gives

$$\begin{aligned}0=&-E_{tot}\left(k_{\alpha_0}\bar{\gamma}_{\alpha_0}+k_{\alpha_0}\bar{\gamma}_{\alpha_i}-\frac{k_{\alpha_i}\bar{\gamma}_{\alpha_i}\zeta_i^{k_{\beta_i}}}{a_i^{k_{\beta_i}}E_{tot}^{k_{\beta_i}}\bar{\gamma}_{\beta_i}^{k_{\beta_i}}}\frac{1}{\eta_i^{k_{\beta_i}}}e^{\frac{\zeta_i}{a_i\eta_i E_{tot}\bar{\gamma}_{\beta_i}}}\Gamma\left(1-k_{\beta_i},\frac{\zeta_i}{a_i\eta_i E_{tot}\bar{\gamma}_{\beta_i}}\right)\right)-\\&(1-\eta_i)\frac{k_{\alpha_i}\bar{\gamma}_{\alpha_i}\zeta_i^{k_{\beta_i}}}{a_i^{k_{\beta_i}}E_{tot}^{k_{\beta_i}-1}\bar{\gamma}_{\beta_i}^{k_{\beta_i}}}\left(-\frac{\zeta_i}{a_i E_{tot}\bar{\gamma}_{\beta_i}}\frac{1}{\eta_i^{k_{\beta_j}+2}}\Gamma\left(1-k_{\beta_i},\frac{\zeta_i}{a_i\eta_i E_{tot}\bar{\gamma}_{\beta_i}}\right)e^{\frac{\zeta_i}{a_i\eta_i E_{tot}\bar{\gamma}_{\beta_i}}}-\right.\\&\left.k_{\beta_i}\frac{1}{\eta_i^{k_{\beta_i}+1}}\Gamma\left(1-k_{\beta_i},\frac{\zeta_i}{a_i\eta_i E_{tot}\bar{\gamma}_{\beta_i}}\right)e^{\frac{\zeta_i}{a_i\eta_i E_{tot}\bar{\gamma}_{\beta_i}}}+\frac{\zeta_i^{-k_{\beta_i}+1}}{a_i^{-k_{\beta_i}+1}\eta_i^2E_{tot}^{-k_{\beta_i}+1}\bar{\gamma}_{\beta_i}^{-k_{\beta_i}+1}}\right).\end{aligned} \tag{33}$$

Equation (33) can be solved numerically using algorithms such as bisection, Newton's method etc to yield the optimal value of η_i. Similar to the full feedback case, η_i is found for all the relays and then the relay which maximizes the averaged end-to-end SNR is selected. For the special case of Rayleigh fading, η_i can be obtained from

$$\begin{aligned}0=&-E_{tot}\left(\bar{\gamma}_{\alpha_0}+\bar{\gamma}_{\alpha_i}-\frac{\zeta_i\bar{\gamma}_{\alpha_i}}{a_i\eta_i E_{tot}\bar{\gamma}_{\beta_i}}e^{\frac{\zeta_i}{a_i\eta_i E_{tot}\bar{\gamma}_{\beta_i}}}E_1\left(\frac{\zeta_i}{a_i\eta_i E_{tot}\bar{\gamma}_{\beta_i}}\right)\right)-(1-\eta)\frac{\zeta_i\bar{\gamma}_{\alpha_i}}{a_i\bar{\gamma}_{\beta_i}}\\&\left(-\frac{1}{\eta_i^2}e^{\frac{\zeta_i}{a_i\eta_i E_{tot}\bar{\gamma}_{\beta_i}}}E_1\left(\frac{\zeta_i}{a_i\eta_i E_{tot}\bar{\gamma}_{\beta_i}}\right)+\frac{1}{\eta_i}\left(\frac{1}{\eta_i}-E_1\left(\frac{\zeta_i}{a_i\eta_i E_{tot}\bar{\gamma}_{\beta_i}}\right)e^{\frac{\zeta_i}{a_i\eta_i E_{tot}\bar{\gamma}_{\beta_i}}}\frac{\zeta_i}{a_i\eta_i^2E_{tot}\bar{\gamma}_{\beta_i}}\right)\right).\end{aligned} \tag{34}$$

For the case of simple nodes, then the solution can be obtained from replacing α_i by $k_{\alpha_i}\bar{\gamma}_{\alpha_i}$ and β_i by $\bar{\gamma}_{\beta_i}(k_{\beta_i}-1)$ in (31).

B. Energy-Efficiency

1) Full CSI: In the case of full CSI, the energy-efficiency problem for the ith selected relay is

$$\min_{Es,Ei}\ E_{tot}, \quad \text{subject to} \quad \gamma_i \geq \gamma^{th},\ 0 \leq E_s \leq E_s^{max},\ 0 \leq E_i \leq E_i^{max}, \tag{35}$$

where

$$\gamma_i = E_s\left(\alpha_0 + \alpha_i - \frac{\alpha_i \zeta_i}{a_i E_i \beta_i + \zeta_i}\right). \tag{36}$$

The optimization problem, (35), is solved for all the relays and the relay which minimizes E_{tot} while fulfilling the constraint on γ_i is selected. If no relay fulfills the constraint on γ_i, then the relay which maximizes γ_i is selected.

Ignoring the individual constraints and taking advantage of the fact that at the optimal solution $\gamma = \gamma^{th}$, we can write

$$E_s = \frac{a_i E_i \beta_i \gamma^{th} + \zeta_i \gamma^{th}}{\alpha_0 \zeta_i + a_i E_i \alpha_0 \beta_i + a_i E_i \alpha_i \beta_i}. \tag{37}$$

Thus, ignoring the individual constraints, (35) can be re-written as

$$\min_{E_i} \qquad \frac{a_i E_i \beta_i \gamma^{th} + \zeta_i \gamma^{th}}{\alpha_0 \zeta_i + a_i E_i \alpha_0 \beta_i + a_i E_i \alpha_i \beta_i} + E_i. \tag{38}$$

Taking the derivative and equating it to 0 gives E_i as

$$E_i = \frac{\sqrt{a_i \alpha_i \beta_i \zeta_i \gamma^{th}} - \alpha_0 \zeta_i}{a_i \beta_i \left(\alpha_0 + \alpha_i\right)}. \tag{39}$$

Substituting (39) in (37) yields

$$E_s = \frac{\left(\sqrt{a_i \alpha_i \beta_i \zeta_i} + \zeta_i \alpha_0\right)\gamma^{th}}{\left(\alpha_0 + \alpha_i\right)\sqrt{a_i \alpha_i \beta_i \zeta_i \gamma^{th}}}. \tag{40}$$

Incorporating the individual constraints gives the water-filling solution

$$E_s = \left(\frac{\left(\sqrt{a_i \alpha_i \beta_i \zeta_i} + \zeta_i \alpha_0\right)\gamma^{th}}{\left(\alpha_0 + \alpha_i\right)\sqrt{a_i \alpha_i \beta_i \zeta_i \gamma^{th}}}\right)_0^{E_s^{max}} \qquad E_i = \left(\frac{\sqrt{a_i \alpha_i \beta_i \zeta_i \gamma^{th}} - \alpha_0 \zeta_i}{a_i \beta_i \left(\alpha_0 + \alpha_i\right)}\right)_0^{E_i^{max}}. \tag{41}$$

2) Limited feedback: In this case, the selection procedure and the optimization problem are the same as in Section III-B1, however the constraint on the end-to-end SNR changes to $\bar{\gamma}_i \geq \gamma^{th}$, where

$$\bar{\gamma}_i = E_s \left(k_{\alpha_0}\bar{\gamma}_{\alpha_0} + k_{\alpha_i}\bar{\gamma}_{\alpha_i} - \frac{k_{\alpha_i}\bar{\gamma}_{\alpha_i}\zeta_i^{k_{\beta_i}}}{a_i^{k_{\beta_i}}\bar{\gamma}_{\beta_i}^{k_{\beta_i}}} \frac{1}{E_i^{k_{\beta_i}}} e^{\frac{\zeta_i}{a_i E_i \bar{\gamma}_{\beta_i}}} \Gamma\left(1-k_{\beta_i}, \frac{\zeta_i}{a_i E_i \bar{\gamma}_{\beta_i}}\right)\right). \tag{42}$$

Again exploiting the equality of the constraint on $\bar{\gamma}_i$, we obtain

$$E_s = \frac{\gamma^{th} E_i^{k_{\beta_i}}}{\left(k_{\alpha_0}\bar{\gamma}_{\alpha_0} + k_{\alpha_i}\bar{\gamma}_{\alpha_i}\right) E_i^{k_{\beta_i}} - \frac{k_{\alpha_i}\bar{\gamma}_{\alpha_i}\zeta_i^{k_{\beta_i}}}{a_i^{k_{\beta_i}}\bar{\gamma}_{\beta_i}^{k_{\beta_i}}} e^{\frac{\zeta_i}{a_i E_i \bar{\gamma}_{\beta_i}}} \Gamma\left(1-k_{\beta_i}, \frac{\zeta_i}{a_i E_i \bar{\gamma}_{\beta_i}}\right)}. \tag{43}$$

Therefore, the optimization problem becomes

$$\min_{E_i} \quad \frac{\gamma^{th} E_i^{k_{\beta_i}}}{\left(k_{\alpha_0}\bar{\gamma}_{\alpha_0} + k_{\alpha_i}\bar{\gamma}_{\alpha_i}\right) E_i^{k_{\beta_i}} - \frac{k_{\alpha_i}\bar{\gamma}_{\alpha_i}\zeta_i^{k_{\beta_i}}}{a_i^{k_{\beta_i}}\bar{\gamma}_{\beta_i}^{k_{\beta_i}}} e^{\frac{\zeta_i}{a_i E_i \bar{\gamma}_{\beta_i}}} \Gamma\left(1-k_{\beta_i}, \frac{\zeta_i}{a_i E_i \bar{\gamma}_{\beta_i}}\right)} + E_i. \tag{44}$$

Taking the derivative and equating to 0 gives

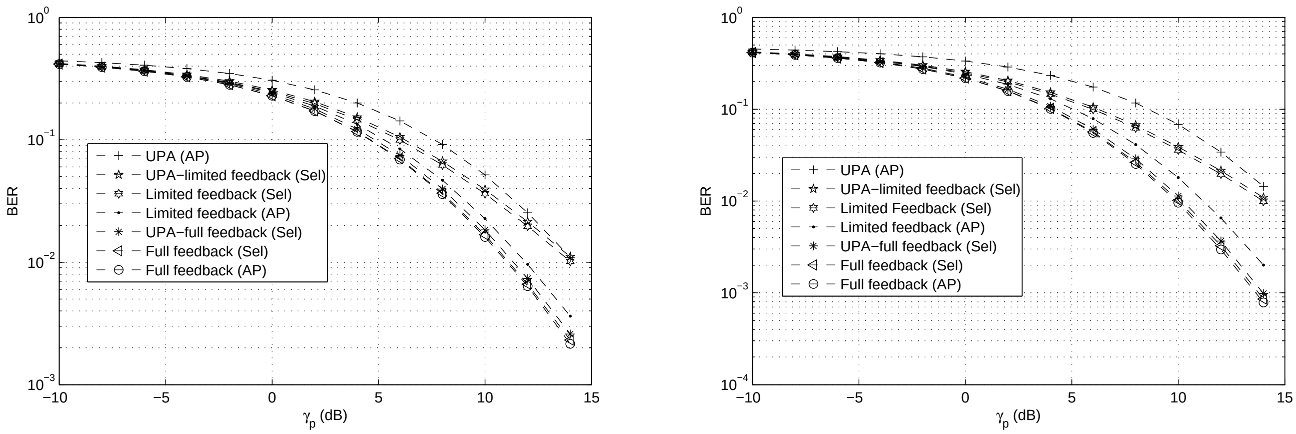
$$\begin{aligned}&\left(\left(k_{\alpha_0}\bar{\gamma}_{\alpha_0} + k_{\alpha_i}\bar{\gamma}_{\alpha_i}\right) E_i^{k_{\beta_i}} - \frac{k_{\alpha_i}\bar{\gamma}_{\alpha_i}\zeta_i^{k_{\beta_i}}}{a_i^{k_{\beta_i}}\bar{\gamma}_{\beta_i}^{k_{\beta_i}}} e^{\frac{\zeta_i}{a_i E_i \bar{\gamma}_{\beta_i}}} \Gamma\left(1-k_{\beta_i}, \frac{\zeta_i}{a_i E_i \bar{\gamma}_{\beta_i}}\right)\right)^2 - \frac{\gamma^{th} k_{\alpha_i} k_{\beta_i}\bar{\gamma}_{\alpha_i}\zeta_i^{k_{\beta_i}}}{a_i^{k_{\beta_i}}\bar{\gamma}_{\beta_i}^{k_{\beta_i}}} e^{\frac{\zeta_i}{a_i E_i \bar{\gamma}_{\beta_i}}} \times \\ &\Gamma\left(1-k_{\beta_i}, \frac{\zeta_i}{a_i E_i \bar{\gamma}_{\beta_i}}\right) E_i^{k_{\beta_i}-1} + \frac{\gamma^{th} k_{\alpha_i}\bar{\gamma}_{\alpha_i}\zeta_i}{a_i E_i^2 \bar{\gamma}_{\beta_i}} - \frac{\gamma^{th} k_{\alpha_i}\bar{\gamma}_{\alpha_i}\zeta_i^{k_{\beta_i}+1}}{a_i^{k_{\beta_i}+1}\bar{\gamma}_{\beta_i}^{k_{\beta_i}+1}} \Gamma\left(1-k_{\beta_i}, \frac{\zeta_i}{a_i E_i \bar{\gamma}_{\beta_i}}\right) e^{\frac{\zeta_i}{a_i E_i \bar{\gamma}_{\beta_i}}} E_i^{k_{\beta_i}-2} = 0.\end{aligned} \tag{45}$$

The above equation can be solved through bisection search to yield the value of E_i which can be substituted back into (43) to obtain E_s. The maximum of E_s and E_i is checked and if it exceeds its peak constraint, then it is set at it its peak constraint and the other power is obtained from the constraint. If no power exceeds its respective peak constraint, then the minimum power is checked and if it is below 0, it is set to zero and the other power is obtained from the constraint.

For Rayleigh fading, (46) simplifies to

$$\begin{aligned}\left(\left(\bar{\gamma}_{\alpha_0} + \bar{\gamma}_{\alpha_i}\right) E_i - \frac{\bar{\gamma}_{\alpha_i}\zeta_i}{a_i\bar{\gamma}_{\beta_i}} \, e^{\frac{\zeta_i}{a_i E_i \bar{\gamma}_{\beta_i}}} E_1\left(\frac{\zeta_i}{a_i E_i \bar{\gamma}_{\beta_i}}\right)\right)^2 - \frac{\gamma^{th}\bar{\gamma}_{\alpha_i}\zeta_i}{a_i\bar{\gamma}_{\beta_i}} e^{\frac{\zeta_i}{a_i E_i \bar{\gamma}_{\beta_i}}} E_1\left(\frac{\zeta_i}{a_i E_i \bar{\gamma}_{\beta_i}}\right) \\ + \frac{\gamma^{th}\bar{\gamma}_{\alpha_i}\zeta_i}{a_i E_i^2 \bar{\gamma}_{\beta_i}} - \frac{\gamma^{th}\bar{\gamma}_{\alpha_i}\zeta_i}{a_i E_i \bar{\gamma}_{\beta_i}} E_1\left(\frac{\zeta_i}{a_i E_i \bar{\gamma}_{\beta_i}}\right) e^{\frac{\zeta_i}{a_i E_i \bar{\gamma}_{\beta_i}}} = 0.\end{aligned} \tag{46}$$

The case of simple nodes can again be solved by using an upper bound on the average SNR and a simple closed-form solution can be obtained in a similar manner as in Section II-C2.



(a) (b)

Fig. 1: Comparison of BER (a) $m = 3$ (b) $m = 5$.

IV. Numerical Results and Discussions

We present numerical results for the schemes discussed in this section. In the numerical results, all the noise variances are taken to be equal, i.e. $\sigma_{sd}^2 = \sigma_{si}^2 = \sigma_{id}^2 = \sigma^2$. The average SNR of all the links are set at 0.5, i.e. $\bar{\gamma}_{\alpha_i} = \bar{\gamma}_{\beta_i} = 0.5$, for i. All the shape parameters are taken to be 1 except when indicated otherwise. The peak individual constraints are set as $E_s^{max} = 3$ and $E_i^{max} = 3$ for all i. For OPA, E_{tot} is taken to be 5.5 and for energy-efficiency, γ^{th} is taken to be 10 dB. Also, for the energy-efficiency problem, it might be the case that due to channel conditions the constraint on the end-to-end SNR cannot be met. In that case all transmitting relays and the source transmit at their maximum power. The relay gain is modeled as $a_i = \frac{1}{E_s^{max} k_{\alpha_0} \bar{\gamma}_{\alpha_0} + \sigma_{si}^2}$.

Figure 1 shows the comparison of the SER for the different schemes for BPSK as a function of $\gamma_p = \frac{E_{tot}}{\sigma^2}$, which is a measure of the SNR, for $m = 3$ and $m = 5$. Firstly, we compare among the AP and selection schemes for the different cases of CSI at the relays. It is evident from Figure 1 that the two OPA AP schemes comfortably provide better performance than uniform power allocation (UPA(AP)). Full feedback (AP) gives the greatest gain, as one would expect, of more than 4 dB and more than 3 dB for $m = 5$ and $m = 3$, respectively, over UPA (AP) at a BER of 10^{-2}, while limited feedback (AP) displays a gain of more than 2.5 dB and more than 2 dB for $m = 5$ and $m = 3$, respectively, at the same BER. Moreover, at a BER of 10^{-2}, the performance difference between full feedback (AP) and limited feedback (AP) is around 1.2 dB and 1 dB for $m = 5$ and $m = 3$, respectively. Also, with increase in the number of relays, the AP case gives better BER among all schemes as one would expect. However, the performance gain for full feedback is greater than the limited feedback case with increase in number of

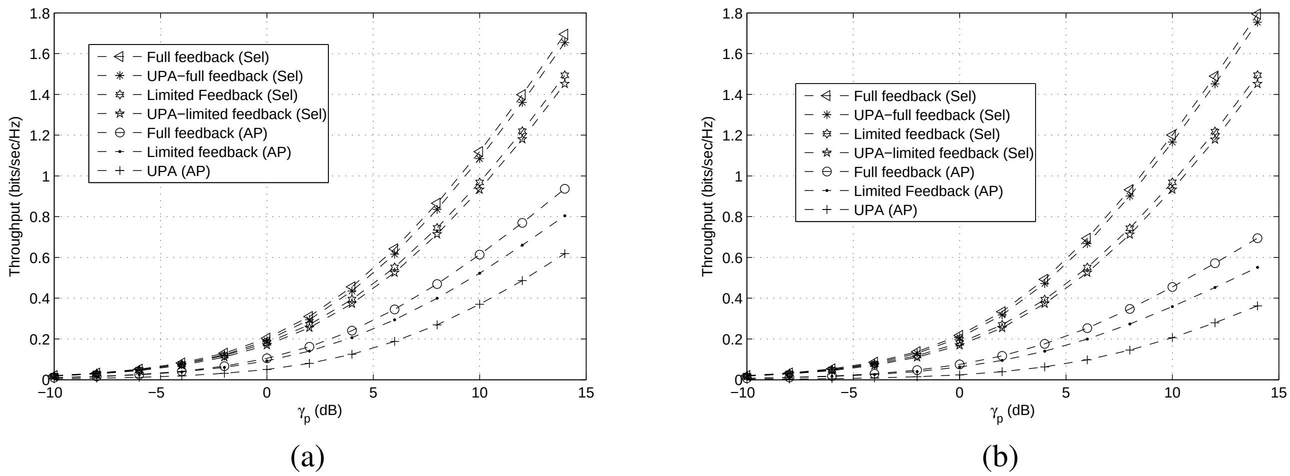
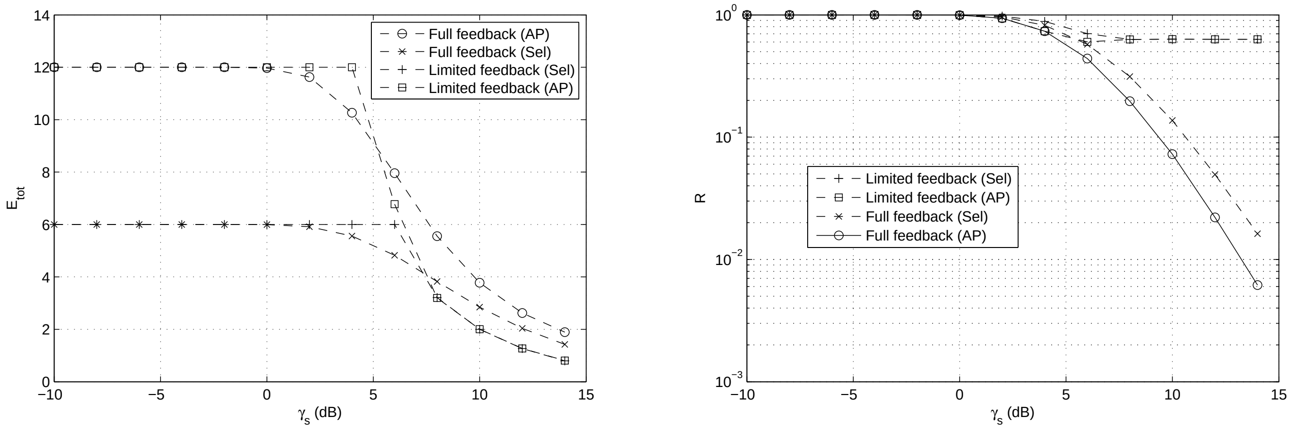


Fig. 2: Comparison of throughputs (a) $m = 3$ (b) $m = 5$.

relays which can be seen by comparing the increase in the performance gap between the two cases.

Similar observations can be made for selective relaying. The two OPA schemes perform better than their UPA (Sel) counterparts for both cases of $m = 3$ and $m = 5$. However, the gain of full feedback (Sel) and limited feedback (Sel) over their UPA counterparts is small as compared to the AP case. But the gain of full feedback (Sel) over limited feedback (Sel) is quite large as compared to the AP case. Full feedback (Sel) outperforms limited feedback (Sel) by around 4 dB and 2.9 dB for $m = 3$ and $m = 4$, respectively. Thus, the decrease in performance due to limited feedback is severe in the case of selective relaying. Furthermore, full feedback (Sel) gives only a small degradation over full feedback (AP), while a large degradation is seen for the limited feedback scenario when moving from AP to selective relaying. An interesting point to note is that UPA (Sel) outperforms UPA (AP). This is due to the fact that even though AP has more relays, the total power is the same for both AP and selective relaying. For UPA (AP), this power is equally distributed among the relays and the source, however, for selective relaying the power is shared between only two nodes and moreover, the relay which maximizes the end-to-end SNR is selected. Thus, more power allocated to the relay which has better channel conditions and UPA (Sel) performs better than UPA (AP). However, as γ_p increases, all the relays see good channel conditions, in general, and the gain of AP starts to manifest.

The throughputs of all the OPA schemes are shown in Figure 2 for $m = 3$ and $m = 5$. All the selective relaying schemes give better throughput than all the AP schemes. This is due to the orthogonal distribution of sources in AP. Hence, transmitting one packet of information requires $m+1$ time slots for AP while it requires only 2 slots for selection. Thus, there is a gain in throughput for selective relaying with increase



(a) (b)

Fig. 3: Energy-efficiency for $m = 3$ (a) E_{tot} (b) R.

in number of relays, while there is a decrease in throughput in the AP case. Furthermore, a similar pattern is observed for the throughputs as was seen for the SER case when comparing among the two relaying strategies, i.e full feedback achieves the best performance followed by limited feedback.

Jointly considering Figs. 1 and 2, it is clear that selective relaying is the preferred relaying strategy for the full feedback case. It provides the largest throughput and only leads to a small degradation in BER over full feedback (Sel). In addition, it requires less feedback then its AP counterpart, as in selective relaying only the selected relay needs to be informed of its power and no power feedback is required for the other relays. In the AP case, the power allocation is fed back to all the relays. In the case of limited feedback, the preference of relaying strategies depends upon the objective. If more robustness is required, then AP relaying should be performed. However, if higher data rates are required, then selective relaying seems to be the choice. Moreover, the from a feedback-performance trade-off point of view, there is a large gap in performance between the full feedback cases and limited feedback cases. However, the feedback is significantly reduced for limited feedback. Therefore, it depends mainly upon system specification which scheme to utilize. For example, if the quality of the feedback channels is quite poor on average, then feedback might not be possible in many scenarios. So, limited feedback can be utilized in this case.

Figs. 3 and 4 show the results for the energy-efficiency problem for $m = 3$ and $m = 5$, respectively, as a function of $\gamma_s = \frac{1}{\sigma^2}$, which is again a measure of the SNR. Except for some minor differences, the two figures show the same trends in performance. Hence, we will mainly focus on Fig. 3. Fig. 3(a) shows the total energy consumed while Fig. 3(b) shows R, where R is the ratio of channel realizations where the constraint on the end-to-end SNR is achieved. The same holds true for Fig. 4. For low values

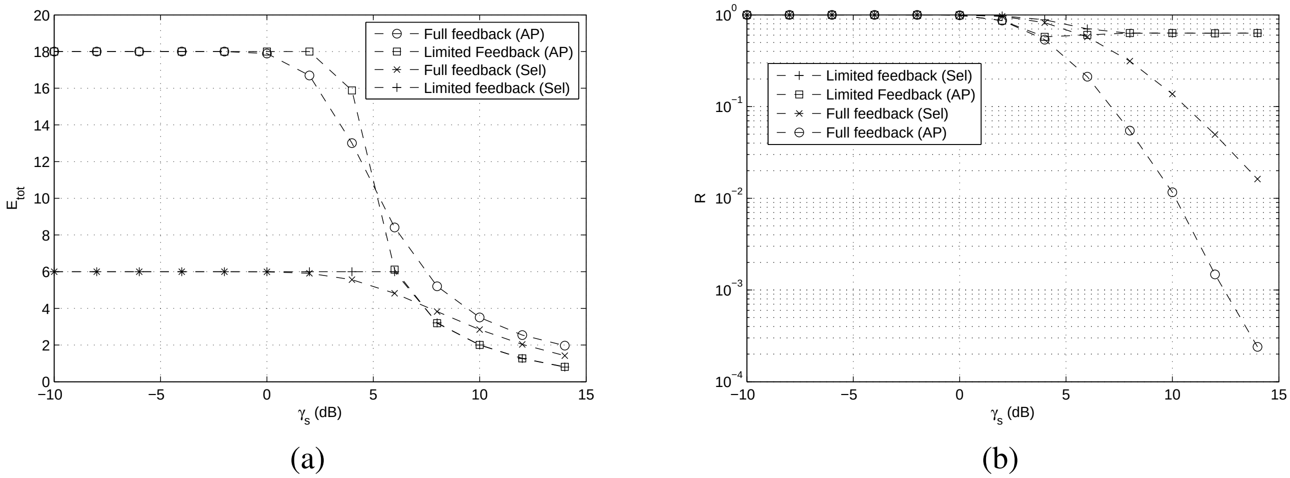


Fig. 4: Energy-efficiency for $m = 5$ (a) E_{tot} (b) R.

of the SNR, the system cannot achieve the minimum threshold on the end-to-end SNR for most channel realizations, hence, for AP relaying all the relays and the source transmit at their peak powers and for the selective relaying the source and the selected relay transmit at their peak powers. Thus in Fig. 3(a), the AP schemes transmit at 12 and the selection schemes transmit at 6 at low values of γ_s. Same in Fig. 4(a) where peak selective relaying value remains the same, but the peak AP value is changed to 18 due to the two additional relays. As the SNR increases all the cases start to fulfill the constraint on the end-to-end SNR on a more regular basis as can be seen from Figs. 3(b) and 4(b). However, at medium to high SNR, it would be expected that the two full feedback cases would consume less energy than their limited feedback counterparts. Moreover, among the two relaying schemes, one would expect AP relaying to consume less energy. However, this is not the case as evident from Figs. 3(a) and 4(a). Both the limited cases consume the lowest energy at medium-high SNR followed by full feedback (Sel) and lastly full feedback (AP). The reason for this behavior becomes apparent from Figs. 3(b) and 4(b). Both these figures show that, as expected, full feedback (AP) satisfies the constraint on the end-to-end SNR in most cases followed by limited feedback (Sel) and then the two limited feedback cases. Thus, the reason why full feedback (AP) consumes more energy on average than full feedback (Sel) is that, when full feedback (AP) cannot achieve the end-to-end SNR constraint, it transmits at 12 and 18 for $m = 3$ and $m = 5$, respectively, while full feedback (Sel) transmits at 6 in both instances. These cases make the average power consumed by full feedback (AP) more than full feedback (Sel). Moreover, these cases are also the reason why both full feedback schemes lag behind the two limited feedback schemes. The limited feedback schemes transmit at fixed powers, which is low at medium-high SNR, irrespective of the instantaneous channel conditions.

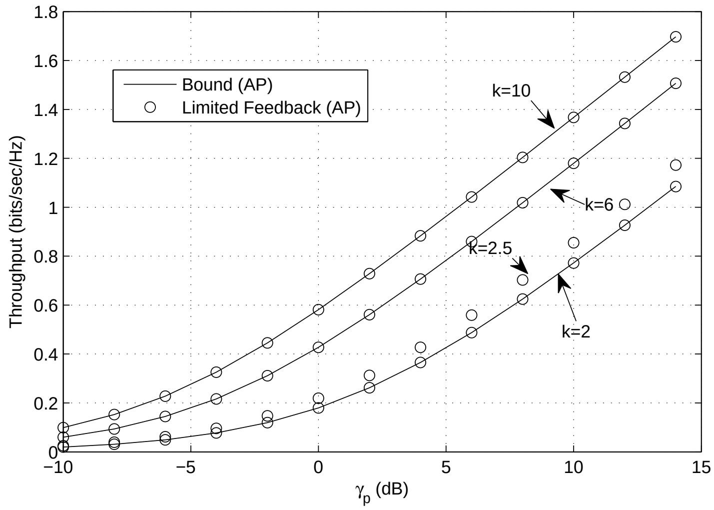


Fig. 5: Effect of bounding the SNR for $m = 3$ and $k_{\alpha_i} = k_{\beta_i} = k$.

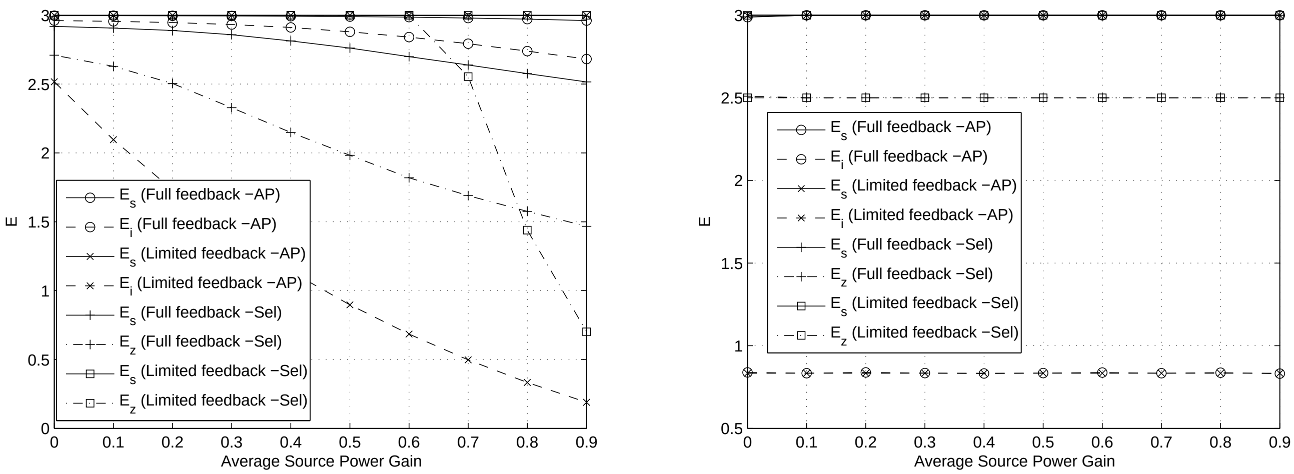
They meet the end-to-end SNR constraint on average, however, don't fulfill it instantaneously. Therefore, they don't transmit at peak power even when the instantaneous SNR is below the constraint while the full feedback schemes transmit at peak powers and hence, consume more energy on average.

From Figs. 3 and 4, it can be concluded that, at low SNR, limited feedback (Sel) is the best scheme in terms of energy-efficiency and complexity. It achieves the same performance on average as the other schemes and requires less feedback and complexity. From medium SNR and onwards, full feedback (AP) is the best scheme. Even though it consumes more power, it achieves the constraint on the end-to-end SNR more frequently. This is particularly true for a system with large number of relays.

Fig. 5 shows the effect of bounding the the average end-to-end SNR[10]. It can be seen from Fig. 5 that for integer values of the Nakagami parameter, bounding the SNR has almost no effect. The upper bound on the average end-to-end SNR and the actual average end-to-end SNR give the same performance for all three values of the Nakagami parameter. Thus, for integer values, instead of using convex numerical optimization techniques, the closed-form expressions for the upper bound can be utilized without loss of performance. For the case of non-integer Nakagami parameter, the non-integer is first rounded down to the nearest integer less than the actual value. In this case, the bound gives the same performance as the actual one at low SNR. However, the gap in performance starts to grow with increase in SNR as can be seen from Fig. 5 for $k = 2$ and $k = 2.5$. As can be seen from both Figs.

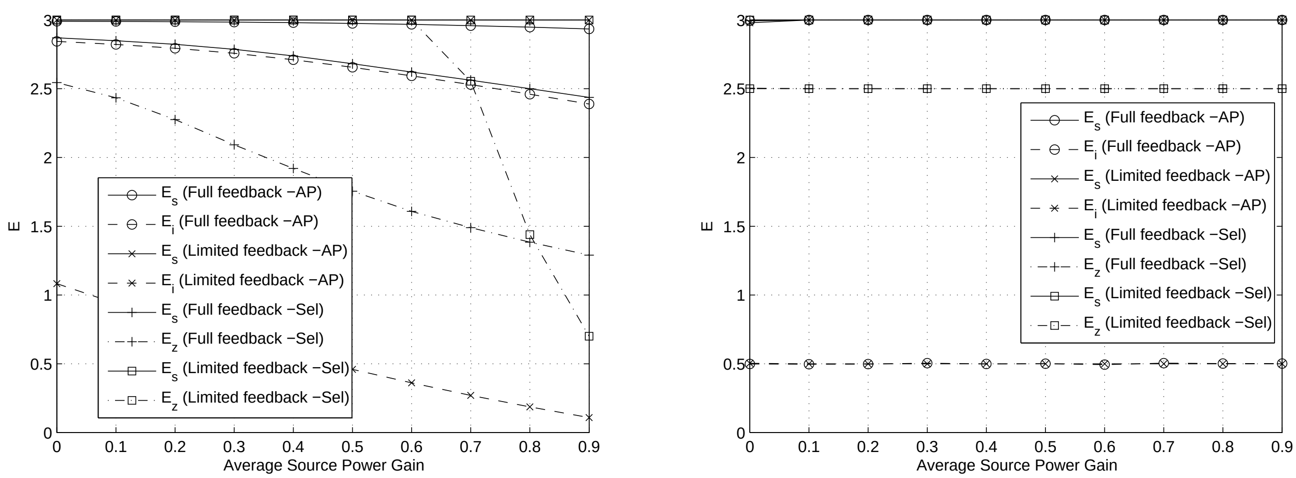
Power allocation for $m = 3$ and $m = 5$ is shown for both the problems as functions of the average

[10]As selective relaying displays the same performance for the bound, it is not shown here.



(a) (b)

Fig. 6: Power allocation for $m = 3$ (a) energy-efficiency (b) SNR maximization.



(a) (b)

Fig. 7: Power allocation for $m = 5$ (a) energy-efficiency (b) SNR maximization.

SNR of the direct link in Figs. 6 and 7, respectively. For the OPA problem, γ_p is set at 5 dB. Similarly, γ_s is also set as 5 dB. The average SNR gains of all the links except the direct link are set at 1. It can be seen from the both the figures that, as expected, the source power is the most important in all the cases as the signals the relays receive are dependent on it. Thus, even if the direct link is not good, the source power is high. Moreover, the source power decreases with increase in the strength of the direct for the energy-efficiency problem as less power is required to satisfy the constraint on the end-to-end SNR. The opposite is true for the SNR maximization where the source power increases with increase in the quality of the direct link. This is due to the opposite nature of the problems. In the SNR maximization problem, more power is allocated to the link which has good channel conditions.

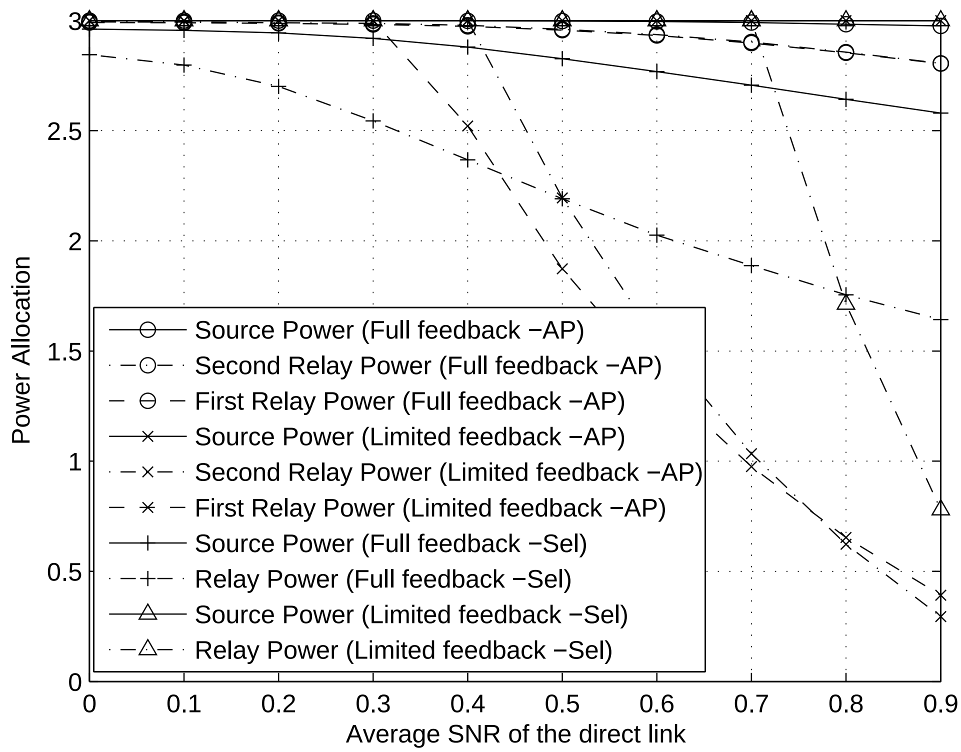


Fig. 8: Power Allocation for optimal power allocation as function of the average SNR of the direct link with $m = 2$ and the average SNRs of the first relay are set as 0.8 and 1 for the two hops, respectively and the average SNRs of the second relay are set as 1 and 0.8.

Also, it is interesting to note that the source power allocation is quite similar for both $m = 3$ and $m = 5$ for all the considered cases. However, the relay power allocation changes. For the AP cae, when there are a less number of relays, each relay is allocated more power and the relay power allocation decreases with increase in number of relays. For selective relaying, when there are greater number of relays, for the energy-efficiency problem, the power allocated is low on average as there is more choice from which to select the best relay.

Fig. 8 shows the power allocation for the energy-efficiency problem with $\gamma_s = 5$ dB. Fig. 8 clearly shows that, as expected, the source power is the most important and is the highest for all the cases for all the values of the average SNR of the direct link. Furthermore, it can also be observed that the allocated source power decreases with better direct link. One interesting observation from Fig. 8 is that, for the AP system, more power is allocated to the relay which has the better first hop when the direct link is of low quality and more power is allocated to the link which has the better second hop when the direct link has

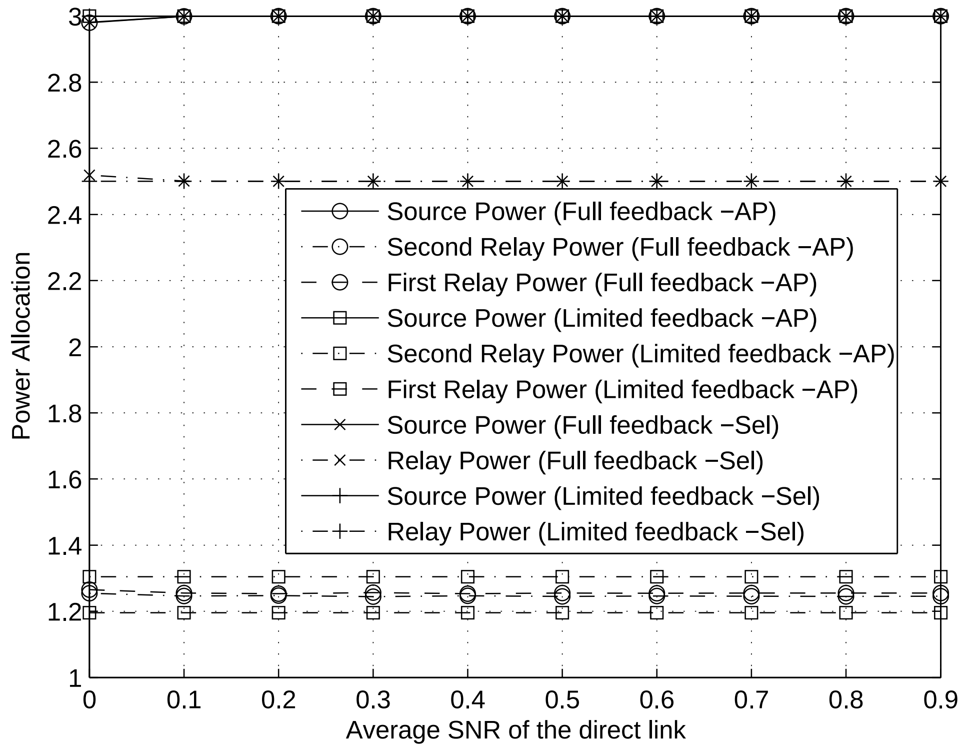


Fig. 9: Power Allocation for Energy-efficiency allocation as function of the average SNR of the direct link with $m = 2$ and the average SNRs of the first relay are set as 0.8 and 1 for the two hops, respectively and the average SNRs of the second relay are set as 1 and 0.8.

better quality[11]. The reason for this can be observed from equations (27) and (28) that the first term in (27) is a decreasing function of the direct link and an increasing function of the first hop. Hence, when the direct link is low, this term dominated. However, when the direct link becomes better, the second term which is an increasing function of the second hop starts to have more influence. A similar behaviour is shown in Fig. 9 for the problem of optimal power allocation to maximize the end-to-end SNR with $\gamma_p = 5$ dB. However, in Fig. 9, the source power increases with increase in the strength of the direct link due to more allocated to the link which has better channel conditions to maximize the end-to-end SNR.

V. Conclusions

We have studied power allocation to maximize the end-to-end SNR under a total power constraint and to minimize the the total power consumed while maintaining the end-to-end SNR above a required threshold for a fixed-gain AF relay network. We have studied both problems for the relay network operating in

[11]This is more noticeable for the limited feedback case. For the full feedback case, as the values are very close, it is not exactly clear from the plot.

AP mode where all the relays participate in signal forwarding and operating in selection mode in which only the selected relay forwards the signal to the destination. Furthermore, we have also considered the cases of full feedback and limited feedback for both modes of operation and for both optimization problems. We demonstrated the convexity/concavity of all the problems. Moreover, for the full feedback case, closed-form expressions have been obtained for all the problems. For the limited feedback case, the optimization problems have been solved using convex programming. To alleviate the complexity of convex programming, we have utilized an upper bound on the average end-to-end SNR to obtain closed-form expressions for the limited feedback case too which give the same performance as the convex programming at integer Nakagami parameter. Furthermore, we demonstrate the gain achieved by allocating power optimally over UPA. We also give insight into the performance of the system for both problems, for both AP relaying and selective relaying and for the two cases of feedback. Additionally, we also develop inequalities in Appendix B which may prove to be useful in future works. Thus, we believe that our work is a valuable contribution to the already available literature on power allocation strategies for fixed-gain AF relays.

Appendix A

Writing down the objective function

$$\gamma = E_s\left(\sum_{i=0}^{m}\alpha_i - \sum_{i=1}^{m}\frac{\alpha_i\zeta_i}{a_iE_i\beta_i+\zeta_i}\right). \tag{47}$$

The objective function, in general, not convex and concave. However, as we show below, it is concave (its negative is convex) for the domain we are interested in.

Define vector $\mathbb{E}$ as

$$\mathbb{E} = [E_s\ E_1\ E_2\ \cdots\ E_m]^T \qquad \mathbb{E} \succ \mathbf{0}. \tag{48}$$

Now let us define

$$f(\mathbb{E}) = [1\ 0\ 0\ldots0]\mathbb{E} = E_s \quad g(\mathbb{E}) = \sum_{i=0}^{m}\alpha_i - \sum_{i=1}^{m}\frac{\alpha_i\zeta_i}{a_iE_i\beta_i+\zeta_i}. \tag{49}$$

Both f and g are positive and increasing on their domain. For f to be concave

$$f(\theta x + (1-\theta)y) \geq \theta f(x) + (1-\theta)f(y), \tag{50}$$

where $0 \leq \theta \leq 1$. The left hand side (LHS) in the above is $\theta x_1 + (1-\theta)y_1$ and the right hand side (RHS) is equal to $\theta x_1 + (1-\theta)y_1$. As the LHS is equal to the RHS, f is concave. To show that g is concave, forming the Hessian

$$\mathbf{H}_g = \begin{bmatrix} 0 & 0 & 0 & \cdots & \cdots & 0 \\ 0 & -\frac{E_s\alpha_1\zeta_1 a_1^2\beta_1^2}{(a_1E_1\beta_1+\zeta_1)^3} & 0 & \cdots & \cdots & 0 \\ 0 & 0 & -\frac{E_s\alpha_2\zeta_2 a_2^2\beta_2^2}{(a_2E_2\beta_2+\zeta_2)^3} & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & \cdots & \cdots & -\frac{E_s\alpha_m\zeta_m a_m^2\beta_m^2}{(a_mE_m\beta_m+\zeta_m)^3} \end{bmatrix}. \tag{51}$$

As the eigenvalues of $\mathbf{H}_g$ are non-negative, $\mathbf{H}_g$ is negative semi-definite, and hence g is concave. Now let us define

$$h(\mathbb{E}) = f(\mathbb{E})g(\mathbb{E}) = E_s\left(\sum_{i=0}^{m}\alpha_i - \sum_{i=1}^{m}\frac{\alpha_i\zeta_i}{a_iE_i\beta_i+\zeta_i}\right). \tag{52}$$

For h to be concave ($-h$ to be convex)

$$h(\theta x + (1-\theta)y) \geq \theta h(x) + (1-\theta)h(y). \tag{53}$$

Therefore for concavity we have to show

$$\Delta \leq 0, \tag{54}$$

where

$$\Delta = \theta(fg)(x) + (1-\theta)(fg)(y) - (fg)(\theta x + (1-\theta)y) \tag{55}$$

As f and g are both positive and concave

$$(fg)(\theta x + (1-\theta)y) \geq (\theta f(x) + (1-\theta)f(y))(\theta g(x) + (1-\theta)g(y)) \tag{56}$$

Substituting (56) in the expression of Δ one has

$$\begin{aligned}\Delta \leq\ & \theta f(x)g(x) + (1-\theta)f(y)g(y) - \theta^2 f(x)g(x) - (1-\theta)^2 f(y)g(y) \\ & - \theta(1-\theta)f(x)g(y) - \theta(1-\theta)f(y)g(x).\end{aligned} \tag{57}$$

After some manipulation

$$\Delta \leq \theta(1-\theta)D(x,y), \tag{58}$$

where $D(x,y) = (f(x) - f(y))(g(x) - g(y))$. If $D(x,y) \leq 0$, then the proof of concavity is complete. For optimal power allocation, if $f(x) > f(y)$ then $\sum_{i \in y} E_i > \sum_{i \in x} E_i$ from the total power constraint. Therefore, with power allocation, $g(y) > g(x)$, as it $g(.)$ is a concave functions of the relay powers, implying $D(x,y) < 0$ and hence, concavity. Similarly, if $g(x) > g(y)$, then power allocation means that $\sum_{i \in x} E_i > \sum_{i \in y} E_i$, which in turns means $f(y) > f(x)$. Thus, $D(x,y) < 0$ and the objective function is concave.

Appendix B

Recall

$$\bar{\gamma} = E_s \sum_{i=0}^{m} k_{\alpha_i} \bar{\gamma}_{\alpha_i} - E_s \sum_{i=1}^{m} \frac{\bar{k_{\alpha_i}} \gamma_{\alpha_i} \zeta_i}{\Gamma(k_{\beta_i}) \bar{\gamma}_{\beta_i}^{k_{\beta_i}}} \int_0^\infty \frac{1}{a_i E_i y_i + \zeta_i} y_i^{k_{\beta_i}-1} e^{-\frac{y_i}{\bar{\gamma}_{\beta_i}}} dy_i \tag{59}$$

It is obvious that (59) is a concave function of E_s. To check concavity with respect to E_j, let

$$\nu(E_j) = \int_0^\infty \frac{1}{a_j E_j y_j + \zeta_j} y_j^{k_{\beta_j}-1} e^{-\frac{y_j}{\bar{\gamma}_{\beta_j}}} dy_j = \int_0^\infty \mathcal{P}_{y_j}(E_j) \mathcal{Q}(y_j) dy_j, \tag{60}$$

where

$$\mathcal{P}_{y_j}(E_j) = \frac{1}{a_j E_j y_j + \zeta_j} \qquad \mathcal{Q}(y_j) = y_j^{k_{\beta_j}-1} e^{-\frac{y_j}{\bar{\gamma}_{\beta_j}}}. \tag{61}$$

Therefore, to show that $\bar{\gamma}$ is concave with respect to E_j, one has to show that $\nu(E_j)$ is convex with respect to E_j. It is straightforward to see that $\mathcal{P}_{y_j}(E_j)$ is a convex and monotonically decreasing function of E_j and $\mathcal{Q}(y_j)$ is a non-negative function. Therefore, for two distinct values, E_j^1 and E_j^2, and $0 \leq \theta \leq 1$, it can be written

$$\nu(\theta E_j^1 + (1-\theta) E_j^2) = \int_0^\infty \mathcal{P}_{y_j}(\theta E_j^1 + (1-\theta) E_j^2) \mathcal{Q}(y_j) dy_j \leq \int_0^\infty \left(\theta \mathcal{P}_{y_j}(E_j^1) + (1-\theta) \mathcal{P}_{y_j}(E_j^2) \right) \mathcal{Q}(y_j) dy_j. \tag{62}$$

Hence

$$\nu(\theta E_j^1 + (1-\theta) E_j^2) \leq \theta \nu(E_j^1) + (1-\theta) \nu(E_j^2) \tag{63}$$

and the proof of convexity is complete. Therefore, $\bar{\gamma}$ is a concave function of E_j. The concavity of $\bar{\gamma}$ also implies from the second derivative condition

$$S(v, k_{\beta_j}) \geq 0 \qquad\qquad v > 0,\ k_{\beta_j} > 0, \tag{64}$$

where $v = \frac{\zeta_j}{a_j E_j \bar{\gamma}_{\beta_j}}$ and

$$S(v, k_{\beta_j}) = -(2 + k_{\beta_j}) v^{-k_{\beta_j}+1} - v^{-k_{\beta_j}+2} + \Gamma\left(1 - k_{\beta_j}, v\right) e^{v} \left(v^2 + 2k_{\beta_j} v + 2v + k_{\beta_j}^2 + k_{\beta_j}\right). \tag{65}$$

Now using a similar argument as in Appendix A, the joint concavity of $\bar{\gamma}$ can be established.

For the special case of Rayleigh fading, the proof of concavity establishes the relationship

$$E_1\left(v\right) e^{v} \left(v^2 + 2 + 4v\right) - v - 3 > 0 \qquad\qquad v > 0. \tag{66}$$

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