On the Outage Performance of Full-Duplex Selective Decode-and-Forward Relaying

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Abstract

We evaluate the outage performance in a three-terminal full-duplex relay channel that adopts a *selective decode-and-forward* protocol, taking relay self-interference into account. Previous work focused on coverage extension scenarios where direct source-destination transmissions are neglected or considered as interference. In this work, we account for the relay self-interference, and exploit the *cooperative diversity* offered by the independently fading source/relay message replicas that arrive at the destination. We present an approximate, yet accurate, closed-form expression for the end-to-end outage probability that captures their joint effect. With the derived expression in hand, we propose a relay transmit power optimization scheme that only requires the relay knowledge of channel statistics. Finally, we corroborate our analysis with simulations.

Index Terms

Full-duplex relay, self interference, selective decode-and-forward, cooperative diversity, outage probability.

I. INTRODUCTION

Multi-hop communication has been considered in recent cellular standards as a cost-effective solution for coverage extension and throughput enhancement [1]. Despite its offered performance gains, half-duplex relaying (HDR) is known to suffer from an inherent spectral efficiency loss when compared to direct transmission. This is owing to its nature of allocating orthogonal listening/forwarding phases at the relay. Full-duplex relaying (FDR), on the other hand, overcomes this inefficiency by allowing the relay to simultaneously transmit and receive on the same channel [2].

Early FDR performance evaluation attempts, however, assumed *perfect isolation* of the relay's receive antenna from its own overwhelming transmissions, which overestimates its actual performance merits [3]. In practice, even with recent advances in prototyping full-duplex nodes, all known analog, digital and spatial isolation/cancellation techniques cannot guarantee perfect isolation [4], [5]. As a result, a level of residual self-interference (RSI) persists. Moreover, the adverse effect of RSI proportionally grows with the relay transmit power. Hence, a clear tradeoff exists in FDR between its gained temporal efficiency and the corresponding degradation due to RSI, which is directly controlled by the relay power.

Recent FDR studies captured the effect of RSI in their models [6]–[11], and concluded the mentioned tradeoff. In [6]–[8], outage performance was studied for the Rayleigh-fading FDR channel with RSI, where closed-form expressions were presented for decode-and-forward (DF) and amplify-and-forward (AF) protocols. Further comprehensive capacity results were reported in [9], and addressed relay power optimization to locate the optimal tradeoff. As the relay forwards a delayed version of the source message while the destination employs symbol-by-symbol decoding, [6]–[10] focused on coverage extension scenarios with weak direct link gain, where the interfering source signal is treated as noise at the destination.

Beyond coverage extension scenarios, the existence of two independently-fading paths from source to destination motivates further work to exploit the available *cooperative diversity* via *combining*. In [9], a hybrid scheme was suggested to adaptively switch to direct transmission mode based on the relay knowledge of the end-to-end performance via both paths. This can be equivalently viewed at the destination as selective combining in which one of two signals is selected; direct source signal or relay signal with source interference. In a different approach to efficiently exploit the available diversity, the authors in [11] considered the communication over a block of symbols in AF relaying. Taking the relay's processing delay into account, the received vector at the destination over one block is a superposition of shifted versions of the source vector arriving through different paths. Despite addressing a different relaying strategy than the one we target, closed-form outage performance bounds could only be presented in [11] for two extreme cases; either RSI or direct link is considered at a time. Moreover, no relay power optimization was addressed.

In this work, we focus on regenerative FDR, and address scenarios where the source-destination link can be of non-negligible gain. Specifically, we study the performance of an FDR channel with RSI while adopting a *selective* relaying technique. Selective DF (SDF) was first proposed in [12] in the context of HDR, where it was shown to outperform its *fixed* counterpart in terms of outage probability. In SDF, the
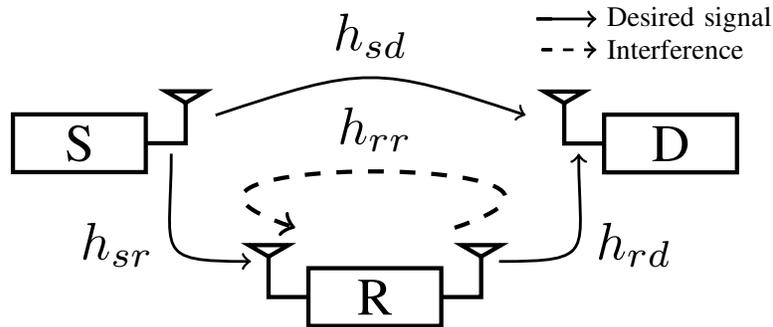
Fig. 1. System model.

relay node assists only when it is able to decode the source message. In FDR with direct link, SDF still offers considerable performance merits due to its ability to dynamically switch from *cooperation* mode to *direct transmission* mode whenever RSI prohibits proper decoding at the relay. This selection approach differs from that in [9], where SDF actually encourages the simultaneity of source/relay transmissions as long as the source-relay link is not in outage. Moreover, SDF only requires local channel state information (CSI) at the relay. With SDF relaying adopted, we present an approximate, yet accurate, closed-form expression for the end-to-end outage probability that captures the joint effect of RSI and direct link. Then, we propose a relay power optimization scheme that only requires relay knowledge of channel statistics. Finally, through simulations, we show the gains offered by the adopted SDF scheme over available schemes in the literature.

II. SYSTEM MODEL

We consider the FDR channel depicted in Fig. 1. As shown, a source (S) communicates with a destination (D) through a direct link with non-negligible gain, with possible assistance of a FD relay (R). We denote the channel between node $i \in \{s, r\}$ and node $j \in \{r, d\}$ by h_{ij}, and assume it to experience block fading. Thus, h_{ij} remains constant over one block, and varies independently from one block to another following a circularly symmetric complex Gaussian distribution with zero mean and variance π_{ij}. Accordingly, the channel envelope is Rayleigh distributed, and thus, the channel gain, $|h_{ij}|^2$, is an exponential random variable with mean parameter π_{ij}. As mentioned earlier, the relay loopback channel introduces self-interference that cannot be perfectly cancelled in practice. For notational convenience, we assume that h_{rr} denotes the RSI channel after undergoing all known practical isolation and cancellation techniques, see [4]–[7], [9] and the references therein. Also, without loss of generality, all additive white Gaussian noise components are assumed of unit variance.

At time t, node S transmits $x[t]$ with rate R bits/s/Hz, and $\mathbb{E}\{|x[t]|^2\} = 1$. Simultaneously, due to processing delay D, node R forwards $x[t - D]$, which imposes self-interference. Thus, the received signal at R is given by:

$$y_r[t] \quad = \quad h_{sr}x[t] + \sqrt{P}h_{rr}x[t - D] + n_r[t], \tag{1}$$

where P and $n_r[t]$ denote the relay power and noise, respectively, while the source power is absorbed into both the S-R and S-D channel coefficients without loss of generality. At time t, assuming complex Gaussian inputs and unit bandwidth, the mutual information between the input and output of the S-R channel, denoted by $\mathcal{I}_{sr}$, is given by:

$$\mathcal{I}_{sr} \quad = \quad \log_2 \left\{ 1 + \frac{|h_{sr}|^2}{P|h_{rr}|^2 + 1} \right\}. \tag{2}$$

Node R is assumed to adopt SDF relaying, in which it keeps silent whenever the S-R link is found in outage, and assists only when $\mathcal{I}_{sr} > R$. Now, we discuss the signal model in each of the two S-R link states.

A. No Outage in S-R Link $(\mathcal{I}_{sr} > R)$

It is assumed that the outage event dominates the error event, and thus, R successfully decodes the source message when $\mathcal{I}_{sr} > R$. The received signal at the destination is given by:

$$y_d[t] \quad = \quad h_{sd}x[t] + \sqrt{P}h_{rd}x[t - D] + n_d[t], \tag{3}$$

where $n_d[t]$ denotes the destination noise at time slot t.

All channel gains remain constant over a block duration of $L + D$ time slots corresponding to L successive codewords transmitted from the source, in addition to the D time slots delay due to relay processing. Hence, we rewrite (3) in vector form to jointly account for the $L + D$ received signals as:

$$\boldsymbol{y}_d = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{n}_d, \tag{4}$$

where $\boldsymbol{y}_d = (y_d[1], \ldots, y_d[L + D])^T$, $\boldsymbol{x} = (x[1], \ldots, x[L])^T$, $\boldsymbol{n}_d = (n_d[1], \ldots, n_d[L + D])^T$, and

$$\boldsymbol{H} = \quad h_{sd} \begin{bmatrix} \boldsymbol{I}_L \\ \boldsymbol{0}_{D \times L} \end{bmatrix} + \sqrt{P}h_{rd} \begin{bmatrix} \boldsymbol{0}_{D \times L} \\ \boldsymbol{I}_L \end{bmatrix}. \tag{5}$$

B. *S-R Link in Outage* $(\mathcal{I}_{sr} < R)$

In this case, the relay does not assist. Thus, the received signal at the destination at time t is given by:

$$y_d[t] \;\; = \;\; h_{sd}x[t] + n_d[t]. \tag{6}$$

For large $\frac{L}{D}$, in order to keep the same block structure adopted by the no outage case, it is equivalent to use the vector form in (4) with the value of P set to zero.

III. OUTAGE PERFORMANCE OF SDF-FDR

A. *Outage Analysis*

The overall SDF-FDR channel outage probability can be expressed in terms of individual link outage probabilities as:

$$\mathcal{P} = \mathcal{P}_{sr}\mathcal{P}_{sd} + (1 - \mathcal{P}_{sr})\,\mathcal{P}_{(s,r)\to d}, \tag{7}$$

where $\mathcal{P}_{sr}$ and $\mathcal{P}_{sd}$ are the outage probabilities of the S-R and S-D links, respectively, whose expressions are easily derived starting from (1) and (6), see [10], to be:

$$\mathcal{P}_{sr} = \mathbb{P}\left\{\mathcal{I}_{sr} < R\right\} = 1 - \frac{\pi_{sr}e^{-\frac{2^R-1}{\pi_{sr}}}}{(2^R - 1)P\pi_{rr} + \pi_{sr}}, \tag{8}$$

$$\mathcal{P}_{sd} = \mathbb{P}\left\{\mathcal{I}_{sd} < R\right\} = 1 - e^{-\frac{2^R-1}{\pi_{sd}}}. \tag{9}$$

In (7), $\mathcal{P}_{(s,r)\to d}$ is the probability of outage in the cooperative multiple access channel formed by S and R as the transmitter side and by D as the receiver side. Thus, it is defined as:

$$\mathcal{P}_{(s,r)\to d} = \mathbb{P}\left\{\frac{1}{L+D}\,\mathcal{I}_{(s,r)\to d} < \frac{L}{L+D}R\right\}, \tag{10}$$

where $\mathcal{I}_{(s,r)\to d} = \mathcal{I}(\boldsymbol{x}; \boldsymbol{y}_d)$ is the mutual information per block between $\boldsymbol{x}$ and $\boldsymbol{y}_d$ in (4). Assuming complex Gaussian inputs and unit bandwidth, it can be readily given by:

$$\mathcal{I}_{(s,r)\to d} = \log_2 \det\left\{\boldsymbol{I}_L + \boldsymbol{H}^H\boldsymbol{H}\right\} = \log_2 \prod_{i=1}^{L}(1 + \lambda_i), \tag{11}$$

where

$$\boldsymbol{H}^H\boldsymbol{H} \;\; = \;\; \alpha\boldsymbol{I}_L + \beta\boldsymbol{B}_L{}^D + \beta^*\boldsymbol{F}_L{}^D, \tag{12}$$

with $\alpha = |h_{sd}|^2 + P|h_{rd}|^2$ and $\beta = \sqrt{P}h_{sd}^* h_{rd}$, while $\boldsymbol{B}_L$ $(\boldsymbol{F}_L)$ denotes a square backward (forward) shift matrix of size L, with ones only on the first subdiagonal (superdiagonal) and zeros elsewhere. Also, $\{\lambda_i\}_{i=1}^L$ denote the eigenvalues of $\boldsymbol{H}^H \boldsymbol{H}$. For $D = 1$, $\boldsymbol{H}^H \boldsymbol{H}$ is a tridiagonal Toeplitz matrix, whose eigenvalues are given in closed-form, [13, pp. 80], by:

$$\lambda_i = \alpha + 2|\beta| \cos \frac{i\pi}{L+1}, \ i \in \{1, 2, \cdots, L\}. \tag{13}$$

It can be shown that the expression in (13) can be further generalized for $L = mD$, $m \in \mathbb{Z}^+$, to be on the form:

$$\lambda_{D(i-1)+1} = \alpha + 2|\beta| \cos \frac{iD\pi}{L+D}, \ i \in \{1, 2, \cdots, m\}, \tag{14}$$

with each eigenvalue of multiplicity D. This is owing to the special structure of $\boldsymbol{H}^H \boldsymbol{H}$ that can allow us to split it into D orthogonal sets of m equations each. Each set can be separately solved in a similar way to that of the $D = 1$ case, yielding identical sets of eigenvalues for all D sets. The proof is deferred to the appendix. Hence, (11) becomes:

$$\begin{aligned}
\mathcal{I}_{(s,r)\to d} &= \log_2 \prod_{i=1}^m \left(1 + \alpha + 2|\beta| \cos \frac{iD\pi}{L+D}\right)^D \\
&= D \sum_{i=1}^m \log_2 \left\{(1+\alpha)\left(1 + \frac{2|\beta| \cos \frac{i\pi}{m+1}}{1+\alpha}\right)\right\} \\
&= L\log_2(1+\alpha) + D\sum_{i=1}^m \log_2\left(1 + \frac{2|\beta|\cos\frac{i\pi}{m+1}}{1+\alpha}\right).
\end{aligned} \tag{15}$$

$|\beta| = \sqrt{P}|h_{sd}^* h_{rd}|$ is the envelope of a product of two independent complex Gaussian Random variables with zero means and variances of π_{sd} and $P\pi_{rd}$, whose distribution is given in [14], in terms of $K_0(.)$; the modified Bessel function of the second kind, as:

$$f_{|\beta|}(z) = \frac{4z}{P\pi_{sd}\pi_{rd}} K_0\left(\frac{2z}{\sqrt{P\pi_{sd}\pi_{rd}}}\right). \tag{16}$$

To analytically obtain the desired probability of outage, we need to derive the CDF of an m-fold product of functions comprising dependent random variables with non-elementary probability distributions, or alternatively, an m-fold product of even more complicated functions of the original simpler independent exponential random variables. Fortunately, with the aid of Taylor series expansion, we can reach an approximate compact form of this function product that additionally gets around any tedious analysis

involving Bessel functions. Yet, it is verified through simulations to be in close matching with the exact behavior. Thanks to the arithmetic-geometric mean inequality, we know that $\alpha \geq 2|\beta|$, and hence, $2|\beta| \cos \frac{i\pi}{m+1} < 2|\beta| \leq \alpha < \alpha + 1$. Thus, in the second term of (15), $|\frac{2|\beta| \cos \frac{i\pi}{m+1}}{1+\alpha}| < 1$. For mathematical tractability, we only use the first order Taylor expansion that $\ln(1 + x) \approx x$, or alternatively, $\log_2(1 + x) \approx \frac{x}{\ln(2)}$. Noting that $\sum_{i=1}^{m} \cos \frac{i\pi}{m+1} = 0$, the second term vanishes, and the desired outage probability can be approximated as:

$$\mathcal{P}_{(s,r)\to d} \approx \mathbb{P}\left\{\log_2\left(1 + \alpha\right) < R\right\} = F_\alpha\left(2^R - 1\right) \tag{17}$$

where $F_\alpha\left(z\right)$ is the cumulative distribution function (CDF) of α. Since $|h_{sd}|^2$ and $|\sqrt{P}h_{rd}|^2$ are independent exponential random variables with mean parameters π_{sd} and $P\pi_{rd}$, respectively, α is a hypoexponential random variable with two rate parameters, $\frac{1}{\pi_{sd}}$ and $\frac{1}{P\pi_{rd}}$. Thus, according to [15, Eq. (5.9)], its CDF is given by:

$$F_\alpha\left(z\right) = 1 - \frac{P\pi_{rd}e^{-\frac{z}{P\pi_{rd}}} - \pi_{sd}e^{-\frac{z}{\pi_{sd}}}}{P\pi_{rd} - \pi_{sd}}. \tag{18}$$

Now, substituting (8), (9), (17) and (18) into (7), and performing some manipulations, the overall outage probability is obtained as:

$$\mathcal{P} \approx \mathcal{P}_{sd} - g(P), \tag{19}$$

where

$$g(P) = \frac{\pi_{sr}e^{-\frac{2^R-1}{\pi_{sr}}} \times P\pi_{rd} \overbrace{\left(e^{-\frac{2^R-1}{P\pi_{rd}}} - e^{-\frac{2^R-1}{\pi_{sd}}}\right)}^{\mathcal{A}}}{\left((2^R - 1)P\pi_{rr} + \pi_{sr}\right) \times \underbrace{\left(P\pi_{rd} - \pi_{sd}\right)}_{\mathcal{B}}}. \tag{20}$$

In agreement with intuition, it can be easily seen from (19) that the end-to-end outage probability is less than that in the S-D link by a non-negative function[1] of the relay power, $g(P)$, $\forall P \geq 0$. Moreover, its value is exactly equal to zero when $P = 0$, i.e., when no cooperation takes place, leading to $\mathcal{P} = \mathcal{P}_{sd}$.

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[1]It is to be noted that the terms multiplied by $\mathcal{A}$ and $\mathcal{B}$ are clearly non-negative in P, $\forall P > 0$, while the signs of $\mathcal{A}$ and $\mathcal{B}$ are simultaneously flipped from negative to positive when $P\pi_{rd}$ crosses π_{sd}. Thus, the non-negativity of the ratio is preserved. Also, it can be verified by applying l'Hôpital's rule that $\frac{\mathcal{A}}{\mathcal{B}} > 0$ at $P\pi_{rd} = \pi_{sd}$. As a result, $g(P)$ is non-negative in P, $\forall P > 0$.

B. Relay Transmit Power Optimization

To minimize the end-to-end outage probability, it is equivalent to maximize $g(P)$ over all possible values of P, i.e., we need to solve the following optimization problem:

$$P_{opt} = \arg \max_{P} \quad g(P) \quad \text{s.t. } 0 \leq P \leq P_{max}, \tag{21}$$

where P_{max} is the maximum relay power. The optimization problem in (21) has a nonlinear objective function and a single linear, hence convex, constraint. Thus, the concavity of the problem under consideration solely depends on the concavity properties of $g(P)$. Although it is not straightforward to show due to its non-elementary form with the reciprocal of P in the exponent, we conjecture that $g(P)$ is quasi-concave in $P > 0$. This is primarily motivated by the observed quasi-concave shape of the function when evaluated numerically for numerous combinations of system parameters, i.e., rate and channel variances. Conjecturing the quasi-concavity of $g(P)$ allows us to use known numerical methods designed for the maximization of quasi-concave functions, e.g., the bisection method. Even for functions that are not quasi-concave, the bisection method always succeeds to locate a local optimum. To further investigate its quasi-concavity, we generated 10^5 random combinations of the system parameters drawn from the ranges: $\pi_{i,j} \in [0, 30]$ dB for $i \in \{s, r\}$ and $j \in \{r, d\}$, and $R \in]0, 5]$ bits/s/Hz. Over each of these combinations, we numerically compared the optimal points obtained using the bisection method with those distilled via grid search, in which the function was exhaustively evaluated over 5×10^6 equally spaced points covering the relay transmit power range. Interestingly, the bisection method always located the global optimum, which comes in favor of the original quasi-concavity conjecture.

IV. NUMERICAL RESULTS

In this section, we present simulation results for the proposed SDF-FDR scheme, and compare them to the theoretic results obtained in the previous sections. We consider two variants of the SDF-FDR scheme based on the adopted relay power allocation: 1) maximum power with P_{max} set to unity, and 2) optimal power, P_{opt}, via the bisection method. We compare the SDF-FDR scheme to existing schemes in the literature, namely, 1) non-cooperative direct transmission (DT) via the S-D link, 2) cooperative multi-hop DF (MHDF) in which the direct source transmissions are treated as interference at the destination and 3) a hybrid scheme (MHDF/DT) that was proposed in [9], in which adaptive switching between MHDF and DT occurs based on the available CSI at the relay. Two CSI scenarios are considered here for the
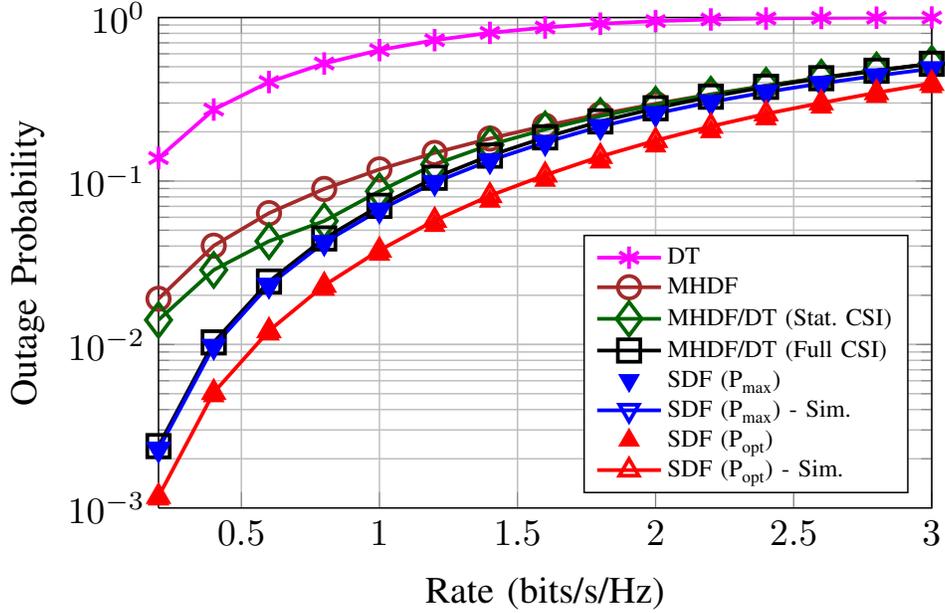
Fig. 2. Outage prob. vs. rate for π_{sd}=0dB, $\pi_{sr} = \pi_{rd} = 20$dB, and π_{rr}=10dB.

hybrid scheme, namely, a) full global CSI, in which the relay knows all instantaneous channel gains, and b) statistical global CSI, which is more practical, in which the relay only knows the average S-D and R-D channel gains/capacities, and the instantaneous local (receive) CSI. It is to be noted that the first variant of the proposed SDF scheme utilizes only local CSI, while the second additionally utilizes only statistical global CSI for power optimization. This makes the statistical CSI scenario more fair in comparison. We run the simulation over 10^6 channel realizations for $L = 20$ symbols per block and $D = 2$ symbols, i.e., $m = 10$.

First, we plot the outage probability over a range of source information rates. In Fig. 2, we can see that DT fails at $\pi_{sd} = 0$, i.e., at noise level, while all cooperative schemes result in close performance. This is due to the limited cooperative diversity offered in this scenario, where all cooperative schemes tend to rely solely on the multi-hop path. In Fig. 3, as π_{sd} increases, the performance gap grows between MHDF and SDF. Although all cooperative schemes yield almost the same performance for small values of π_{sd}, the performance gap grows between MHDF and SDF as π_{sd} increases, as shown in Fig. 3. This is due to the fact that the direct link strength adversely affects MHDF due to the higher level of interference introduced at the destination, while it actually improves the performance of SDF by boosting the cooperative diversity in the channel. With statistical global CSI, the MHDF/DT scheme performs at least as the better of MHDF and DT, yet remains with lower performance than that of SDF. If we further provide MHDF/DT with
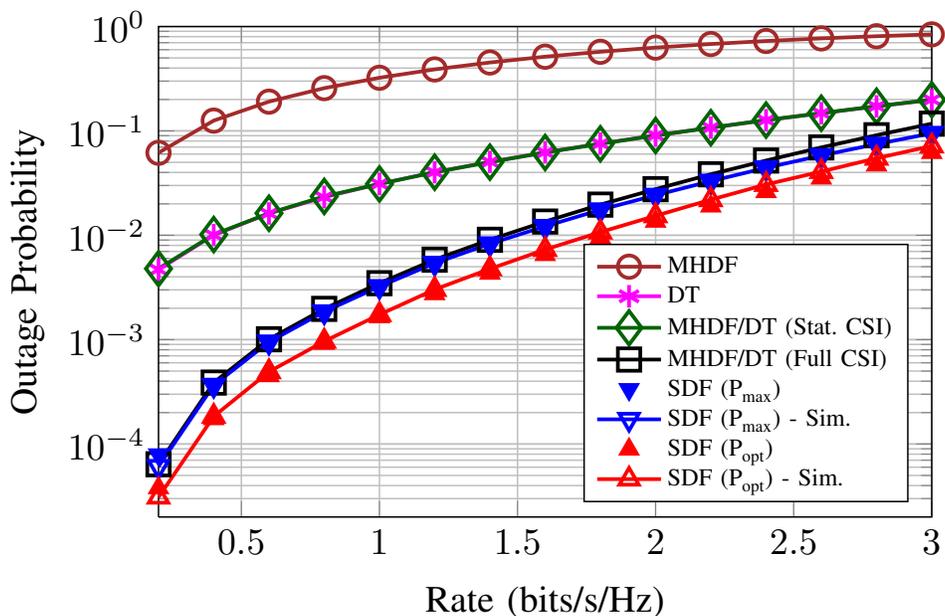
Fig. 3. Outage prob. vs. rate for π_{sd}=15dB, π_{sr}=π_{rd}=20dB, and π_{rr}=10dB.
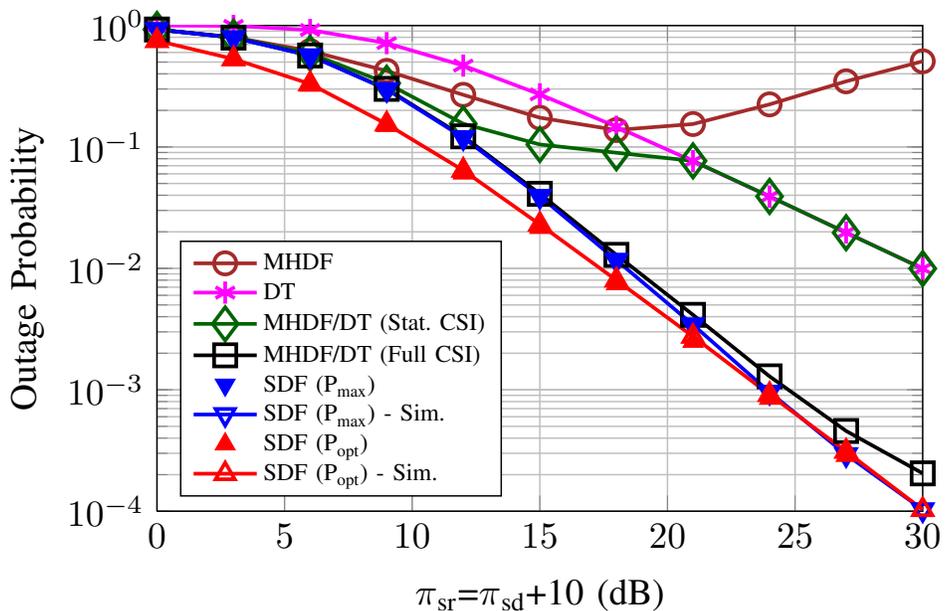

Fig. 4. Outage prob. vs. $\pi_{sr} = \pi_{sd} + 10$ dB, for $\pi_{rd} = 20$ dB, $\pi_{rr} = 6$ dB and $R = 1$ bit/s/Hz.

full global CSI at the relay, its performance improves and almost meets that of SDF under these channel conditions. Yet, under different channel conditions, SDF-FDR yields lower outage probability as will be shortly highlighted.

Next, we investigate the outage performance for different link strengths. In Fig. 4, the outage probability is plotted with varying source power. Equivalently, we sweep the S-R and S-D link variances together, and fix their relation to $\pi_{sr} = \pi_{sd} + 10$ dB. It can be clearly seen that the outage probability of SDF drops
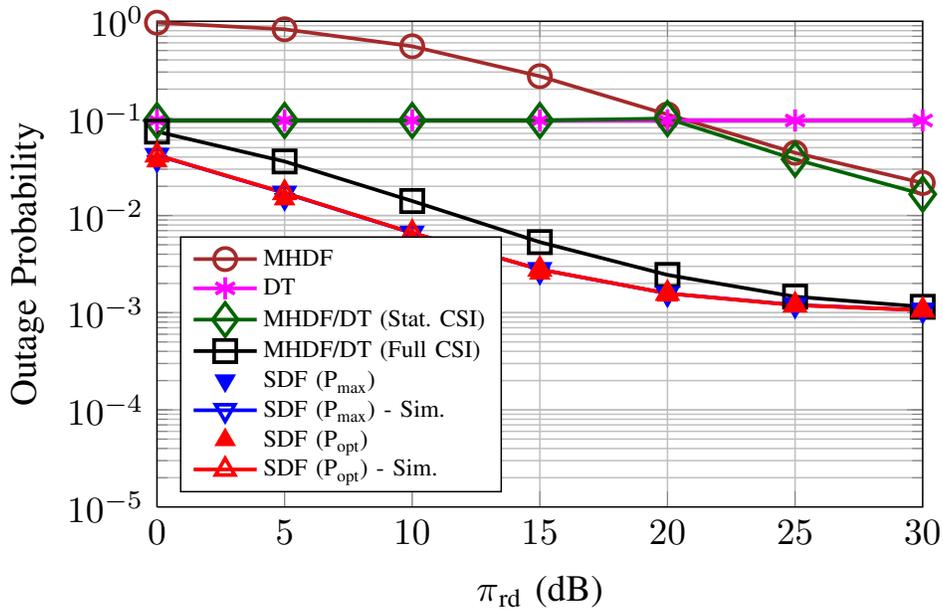
Fig. 5. Outage prob. vs. π_{rd}, for $\pi_{sd} = 10$ dB, $\pi_{sr} = 30$ dB, $\pi_{rr} = 10$ dB and $R = 1$ bit/s/Hz.

with the source power faster than MHDF due to the additionally offered cooperative diversity. Although the performance over the first hop is enhanced when the source power increases, the communication via MHDF starts to fail due to the growing level of interference at the destination. We can also notice that a performance gap starts to arise between MHDF/DT with full global CSI and SDF-FDR when the source transmit power is high. Specifically, this gap occurs when the R-D link becomes the bottleneck in the MHDF scheme. In this case, the MHDF/DT scheme switches to non-cooperative mode whenever the R-D link gain falls below that of the direct link. On the other hand, as long as the source messages are successfully delivered to the relay, performance of SDF-FDR will be enhanced due to combining. This effect can be further noticed in Fig. 5, where we plot the outage performance over a range of R-D link variances. We can notice that the performance gap exists over a wide range of π_{rd} when the S-R link happens to be in very good condition. Fig. 6 shows the effect of π_{rr} on the outage performance. As expected, the performance of all cooperative schemes degrades with the increase in π_{rr}. This is due to the increase in self-interference at the relay that leads the S-R link, and accordingly the whole multi-hop path, to outage states. However, the performance of selective schemes does not go worse than that of direct transmission, since selective relaying switches the relay node to non-cooperative mode in the case of high π_{rr}.

Discussion: When only local CSI is available at the relay, the SDF-FDR scheme is shown to outperform all existing schemes in terms of outage probability. When full global CSI is further fedback to the relay
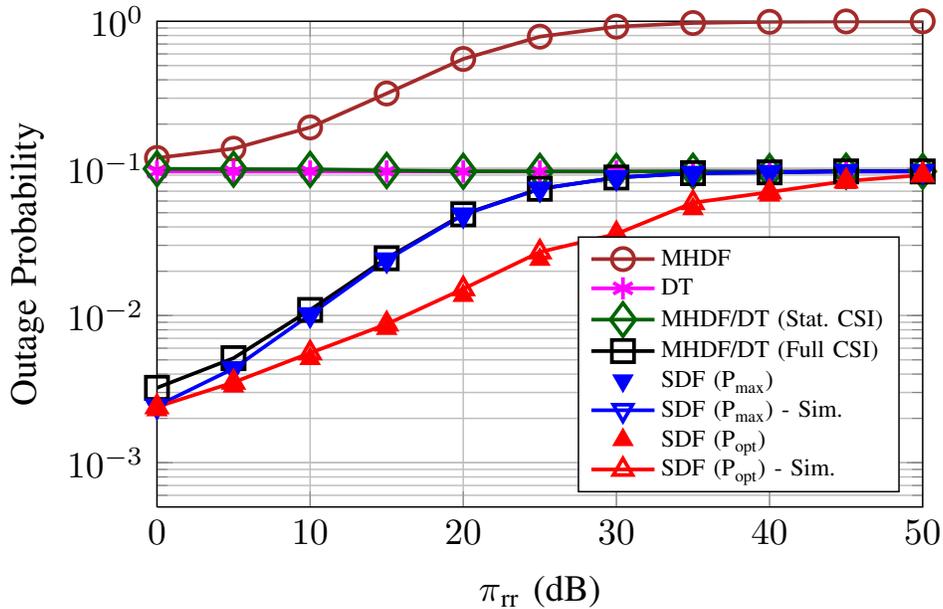
Fig. 6. Outage prob. vs. π_{rr}, for $\pi_{sd} = 10$ dB, $\pi_{sr} = 20$ dB, $\pi_{rd} = 20$ dB and $R = 1$ bit/s/Hz.

in MHDF/DT, its performance meets that of SDF-FDR under some channel conditions, yet yields higher outage probability in others. This owes to the suboptimality of the selection criterion in MHDF/DT, where the relay stops cooperation whenever the direct link capacity becomes higher than that of the multi-hop path, while the relay always cooperates in SDF-FDR as long as the S-R link is not in outage. It is to be noted that, since the idea of the proposed SDF-FDR scheme primarily depends on block-decoding, it would still hold its known limitations when compared to existing schemes that depend on symbol-by-symbol processing, namely, delayed and higher-complexity decoding. However, due to the special structure of the channel matrix, the source codewords per block can be grouped into disjoint sets that can be separately decoded without loss of performance, thus reducing significantly the decoding complexity. Further reduction in complexity can be achieved if we employ conditional detection techniques. Finally, for the SDF schemes, the results obtained via simulations are shown to be well-matching with the theoretical results for $m = 10$, and they further coincide for larger values of m.

V. CONCLUSION

In this work, we studied the outage performance of a selective decode-and-forward full-duplex relay channel with loopback interference at the relay. We presented an approximate end-to-end outage probability expression that captures the joint effect of relay self-interference, and the cooperative diversity offered by additionally exploiting the direct link with non-negligible gain. In order to minimize the outage probability,

we proposed a relay transmit power optimization scheme that only requires the relay knowledge of channel statistics. Through numerical results, selective relaying was shown to outperform its fixed relaying counterpart since it switches to non-cooperative mode whenever the relay's self-interference drives the source-relay channel to an outage state. In conclusion, selective relaying is shown to represent a promising approach that can boost the performance of practical full-duplex relay channels.

APPENDIX

Let $L = mD$, $m \in \mathbb{Z}^+$, and consider a family of square matrices $\boldsymbol{K}(\alpha; \beta; L; D) = (\boldsymbol{k}_1, \cdots, \boldsymbol{k}_L)$ on the form:

$$\boldsymbol{K}(\alpha; \beta; L; D) = \alpha \boldsymbol{I}_L + \beta \boldsymbol{B}_L{}^D + \beta^* \boldsymbol{F}_L{}^D, \tag{22}$$

where $\alpha \in \mathbb{R}^+$, $\beta \in \mathbb{C}$, while $\boldsymbol{B}_L$ ($\boldsymbol{F}_L$) denotes a square backward (forward) shift matrix of size L, with ones only on the first subdiagonal (superdiagonal) and zeros elsewhere. Consider the eigenvalue problem:

$$\boldsymbol{K}\boldsymbol{u} = \lambda \boldsymbol{u}. \tag{23}$$

It can be noticed that the nonzero elements of $\boldsymbol{K}$ are limited to positions on the form $(iD + j, j)$, $\forall\, j \in \{1, \cdots, L\}$, $i \in \{-1, 0, 1\}$, $1 \leq iD + j \leq L$. This makes a column linearly dependent only on its two neighboring D-spaced columns and orthogonal on all others. Motivated by this special structure, we can split $\boldsymbol{K}$ as the sum of D matrices with orthogonal column spaces:

$$\boldsymbol{K} = \sum_{j=1}^{D} \boldsymbol{K}_j, \tag{24}$$

where the matrix $\boldsymbol{K}_j$ holds only m nonzero columns corresponding to the m D-spaced columns of $\boldsymbol{K}$ with shift j, i.e., $\{\boldsymbol{k}_{(i-1)D+j}\}_{i=1}^{m}$, at their respective positions, while the remaining columns are all zeros. Similarly, let us project the $L \times 1$ eigenvector $\boldsymbol{u} = (u[1], \cdots, u[L])^T$ onto D orthogonal subspaces, such that:

$$\boldsymbol{u} = \sum_{j=1}^{D} \boldsymbol{u}_j, \tag{25}$$

where the vector $\boldsymbol{u}_j$ holds only m nonzero elements corresponding to the m D-spaced elements of $\boldsymbol{u}$ with shift j, i.e., $\{u[(i-1)D+j]\}_{i=1}^{m}$, at their respective positions, while the remaining elements are all

zeros. Thus, the eigenvalue problem can be rewritten as

$$\sum_{j=1}^{D} \boldsymbol{K}_j \sum_{j=1}^{D} \boldsymbol{u}_j = \lambda \sum_{j=1}^{D} \boldsymbol{u}_j, \tag{26}$$

It is clear that $\boldsymbol{u}_i$ lies in the nullspace of $\boldsymbol{K}_j \ \forall \ i \neq j$. Hence, due to orthogonality, the eigenvalue problem can be split into D eigenvalue problems on the form:

$$\boldsymbol{K}_j \boldsymbol{u}_j = \lambda \boldsymbol{u}_j, \ \forall j \in \{1, \cdots, D\} \tag{27}$$

Since $\boldsymbol{K}_j$ has zero rows and columns corresponding to the zero elements of $\boldsymbol{u}_j$, we can eliminate them and alternatively solve the reduced m-dimensional eigenvalue problem:

$$\tilde{\boldsymbol{K}}_j \tilde{\boldsymbol{u}}_j = \lambda \tilde{\boldsymbol{u}}_j, \ \forall j \in \{1, \cdots, D\} \tag{28}$$

We can notice that $\tilde{\boldsymbol{K}}_j = \boldsymbol{K}(\alpha; \beta; \frac{L}{D}, 1), \forall j \in \{1, \cdots, D\}$, which is a a tridiagonal Toeplitz matrix with known closed-form expression for its m eigenvalues. Thus, $\boldsymbol{K}(\alpha; \beta; L, D)$ has the m eigenvalues of $\boldsymbol{K}(\alpha; \beta; \frac{L}{D}, 1)$, each with multiplicity D. Accordingly, its determinant is that of $\boldsymbol{K}(\alpha; \beta; \frac{L}{D}, 1)$ raised to the Dth power.

REFERENCES

[1] Y. Yang, H. Hu, J. Xu, and G. Mao, "Relay technologies for WiMax and LTE-advanced mobile systems," *IEEE Commun. Mag.*, vol. 47, no. 10, pp. 100 –105, Oct. 2009.

[2] H. Ju, E. Oh, and D. Hong, "Catching resource-devouring worms in next-generation wireless relay systems: Two-way relay and full-duplex relay," *IEEE Commun. Mag.*, vol. 47, no. 9, pp. 58 –65, Sept. 2009.

[3] Y. Zhu, Y. Xin, and P.-Y. Kam, "Outage probability of Rician fading relay channels," *IEEE Trans. Veh. Technol.*, vol. 57, no. 4, pp. 2648 –2652, July 2008.

[4] M. Jain, J. Choi, T. Kim, D. Bharadia, S. Seth, K. Srinivasan, P. Levis, S. Katti, and P. Sinha, "Practical, real-time, full duplex wireless," in *Proc. ACM MobiCom*, Las Vegas, NV, Sept. 2011.

[5] M. Duarte and A. Sabharwal, "Full-duplex wireless communications using off-the-shelf radios: Feasibility and first results," in *Proc. ASILOMAR*, Pacific Grove, CA, Nov. 2010.

[6] T. Riihonen, S. Werner, R. Wichman, and J. Hamalainen, "Outage probabilities in infrastructure-based single-frequency relay links," in *Proc. IEEE WCNC*, Budapest, Hungary, Apr. 2009.

[7] T. Riihonen, S. Werner, and R. Wichman, "Optimized gain control for single-frequency relaying with loop interference," *IEEE Trans. Wireless Commun.*, vol. 8, no. 6, pp. 2801 –2806, June 2009.

[8] T. Kwon, S. Lim, S. Choi, and D. Hong, "Optimal duplex mode for DF relay in terms of the outage probability," *IEEE Trans. Veh. Technol.*, vol. 59, no. 7, pp. 3628 –3634, Sept. 2010.

[9] T. Riihonen, S. Werner, and R. Wichman, "Hybrid full-duplex/half-duplex relaying with transmit power adaptation," *IEEE Trans. Wireless Commun.*, vol. 10, no. 9, pp. 3074 –3085, Sept. 2011.

[10] H. Alves, G. Fraidenraich, R. D. Souza, M. Bennis, and M. Latva-aho, "Performance analysis of full duplex and selective and incremental half duplex relaying schemes," in *Proc. IEEE WCNC*, Paris, France, Apr. 2012.

[11] T. M. Kim and A. Paulraj, "Outage probability of amplify-and-forward cooperation with full duplex relay," in *Proc. IEEE WCNC*, Paris, France, Apr. 2012.

[12] J.N. Laneman, D.N.C. Tse, and G.W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inf. Theory*, vol. 50, no. 12, pp. 3062 – 3080, Dec. 2004.

[13] W.-C. Yueh and S. S. Cheng, "Explicit eigenvalues and inverses of several toeplitz matrices," *The ANZIAM Journal*, vol. 48, no. 01, pp. 73–97, July 2006.

[14] N. O'Donoughue and J.M.F. Moura, "On the product of independent complex Gaussians," *IEEE Trans. Signal Process.*, vol. 60, no. 3, pp. 1050 –1063, Mar. 2012.

[15] S. M. Ross, *Introduction to Probability Models*, Academic Press, New York, 10th edition, 2010.