Modeling and Management of InterCell Interference in Future Generation Wireless Networks

Thesis by
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ABSTRACT

There has been a rapid growth in the data rate carried by cellular services, and this increase along with the emergence of new multimedia applications have motivated the 3rd Generation Partnership (3GPP) Project to launch Long-Term Evolution (LTE) [1]. LTE is the latest standard in the mobile network technology and is designed to meet the ubiquitous demands of next-generation mobile networks. LTE assures significant spectral and energy efficiency gains in both the uplink and downlink with low latency. Multiple access schemes such as Orthogonal Frequency Division Multiple Access (OFDMA) and Single Carrier Frequency Division Multiple Access (SC-FDMA) which is a modified version of OFDMA have been recently adopted in 3GPP LTE downlink and uplink, respectively [1].

A typical feature of OFDMA is the decomposition of available bandwidth into multiple narrow orthogonal subcarriers. The orthogonality among subcarriers causes minimal intra-cell interference, however, the inter-cell interference (ICI) incurred on a given subcarrier is relatively impulsive and poses a fundamental challenge for the network designers. Moreover, as the number of interferers on a given subcarrier can be relatively limited it may not be accurate to model ICI as a Gaussian random variable by invoking the central limit theorem. The nature of ICI relies on a variety of indeterministic parameters which include frequency reuse factor, channel conditions, scheduling decisions, transmit power, and location of the interferers.

This thesis presents a combination of algorithmic and theoretical studies for efficient modeling and management of ICI via radio resource management. In the
preliminary phase, we focus on developing and analyzing the performance of several
centralized and distributed interference mitigation and rate maximization algorithms. 
These algorithms relies on optimizing the spectrum allocation and user’s transmission 
powers to maximize the system capacity. Even though, the developed algorithms 
possesses low complexity, the simulation run-time may become challenging in the 
practical scenarios with very large number of users and subcarriers.

Motivated by this fact, we then develop several statistical models that can ac-
curately capture the dynamics of interference with distinct applications in the per-
formance analysis of single carrier and multicarrier future wireless networks. The 
developed models can be customized for (i) various state-of-the-art coordinated and 
uncoordinated scheduling algorithms; (ii) slow and fast power control mechanisms;
(iii) partial and fractional frequency reuse systems; and (iv) various composite fading 
distributions. The developed framework is useful in evaluating important system 
performance metrics such as outage probability, ergodic capacity, and average fairness 
numerically without the need of time consuming Monte-Carlo simulations. The 
thoretical framework is expected to enhance the planning tools for OFDMA based 
wireless networks by providing fast estimates of the typical performance metrics.

Finally, we investigate and quantify the spectral and energy efficiency of two 
tier heterogeneous networks (HetNets) by employing power-control based interference 
mitigation technique. In particular, we analyze the performance of two tier HetNets 
deployment by deriving the theoretical bounds on the area spectral efficiency and 
exact analytical expressions for the energy efficiency by considering slow and fast 
power control mechanisms. The derived expressions are expected to be useful in 
providing insights for the design of efficient HetNet deployments.
ACKNOWLEDGEMENTS

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<th>Meaning</th>
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<tbody>
<tr>
<td>1G</td>
<td>First Generation</td>
</tr>
<tr>
<td>2G</td>
<td>Second Generation</td>
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<tr>
<td>3G</td>
<td>Third Generation</td>
</tr>
<tr>
<td>4G</td>
<td>Fourth Generation</td>
</tr>
<tr>
<td>OFDM</td>
<td>Orthogonal Frequency Division Multiplexing</td>
</tr>
<tr>
<td>3GPP</td>
<td>3rd Generation Partnership Project</td>
</tr>
<tr>
<td>LTE</td>
<td>Long Term Evolution</td>
</tr>
<tr>
<td>WIMAX</td>
<td>Worldwide Interoperability for Microwave Access</td>
</tr>
<tr>
<td>OFDMA</td>
<td>Orthogonal Frequency Division Multiple Access</td>
</tr>
<tr>
<td>SC-FDMA</td>
<td>Single Carrier Frequency Division Multiple Access</td>
</tr>
<tr>
<td>ICI</td>
<td>Intercell Interference</td>
</tr>
<tr>
<td>FDMA</td>
<td>Frequency Division Multiple Access</td>
</tr>
<tr>
<td>TDMA</td>
<td>Time Division Multiple Access</td>
</tr>
<tr>
<td>CDMA</td>
<td>Code Division Multiple Access</td>
</tr>
<tr>
<td>ISI</td>
<td>Inter-symbol interference</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal to Noise Ratio</td>
</tr>
<tr>
<td>PAPR</td>
<td>Peak-to-Average-Power Ratio</td>
</tr>
<tr>
<td>BS</td>
<td>Base Station</td>
</tr>
<tr>
<td>RF</td>
<td>Radio Frequency</td>
</tr>
<tr>
<td>PC</td>
<td>Power Control</td>
</tr>
<tr>
<td>SIR</td>
<td>Signal-to-Interference ratio</td>
</tr>
<tr>
<td>FFR</td>
<td>Fractional Frequency Reuse</td>
</tr>
<tr>
<td>UFR</td>
<td>Universal Frequency Reuse</td>
</tr>
<tr>
<td>HetNets</td>
<td>Heterogeneous Networks</td>
</tr>
<tr>
<td>LB</td>
<td>Lower Bound</td>
</tr>
<tr>
<td>UB</td>
<td>Upper Bound</td>
</tr>
<tr>
<td>ASE</td>
<td>Area Spectral Efficiency</td>
</tr>
<tr>
<td>AGE</td>
<td>Area Green Efficiency</td>
</tr>
<tr>
<td>GP</td>
<td>Geometric Programming</td>
</tr>
<tr>
<td>Symbol</td>
<td>Meaning</td>
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<td>--------</td>
<td>-------------------------------</td>
</tr>
<tr>
<td>RV</td>
<td>Random Variable</td>
</tr>
<tr>
<td>PDF</td>
<td>Probability Distribution Function</td>
</tr>
<tr>
<td>CDF</td>
<td>Cumulative Distribution Function</td>
</tr>
<tr>
<td>MGF</td>
<td>Moment Generating Function</td>
</tr>
<tr>
<td>PMF</td>
<td>Probability Mass Function</td>
</tr>
<tr>
<td>i.i.d.</td>
<td>independent and identically distributed</td>
</tr>
<tr>
<td>DoD</td>
<td>Degree of Difficulty</td>
</tr>
<tr>
<td>GPS</td>
<td>Global Positioning System</td>
</tr>
<tr>
<td>CEFA</td>
<td>Cell Edge Frequency Allocation</td>
</tr>
<tr>
<td>CCFA</td>
<td>Cell Centered Frequency Allocation</td>
</tr>
<tr>
<td>UFA</td>
<td>Uniform Frequency Allocation</td>
</tr>
<tr>
<td>FS</td>
<td>Forward Synchronization</td>
</tr>
<tr>
<td>NS</td>
<td>No Synchronization</td>
</tr>
<tr>
<td>RS</td>
<td>Reverse Synchronization</td>
</tr>
<tr>
<td>EPA</td>
<td>Equal Power Allocation</td>
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# LIST OF SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
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<tbody>
<tr>
<td>$\mathbb{R}^N$</td>
<td>Real vectors of dimension $N$</td>
</tr>
<tr>
<td>$\mathbb{C}^N$</td>
<td>Complex vectors of dimension $N$</td>
</tr>
<tr>
<td>$\mathcal{N}(0, \sigma^2)$</td>
<td>zero mean Gaussian random variable with variance $\sigma^2$</td>
</tr>
<tr>
<td>Exp($\lambda$)</td>
<td>Exponential distribution with parameter $\lambda$</td>
</tr>
<tr>
<td>Gamma($\kappa(\cdot), \Theta(\cdot)$)</td>
<td>Gamma distribution with shape $\kappa$ and scale $\Theta$ parameter</td>
</tr>
<tr>
<td>$\mathcal{K}<em>G(m</em>{c(\cdot)}, m_{s(\cdot)}, \Omega(\cdot))$</td>
<td>Generalized-$\mathcal{K}$ distribution</td>
</tr>
<tr>
<td>$m_c$</td>
<td>Fading parameter</td>
</tr>
<tr>
<td>$m_s$</td>
<td>Shadowing parameter</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>Average power</td>
</tr>
<tr>
<td>$\Gamma(a)$</td>
<td>Gamma function</td>
</tr>
<tr>
<td>$\Gamma_u(a; b)$</td>
<td>Upper incomplete Gamma function</td>
</tr>
<tr>
<td>$\Gamma_l(a; b)$</td>
<td>Lower incomplete Gamma function</td>
</tr>
<tr>
<td>$P(A)$</td>
<td>Probability of event $A$</td>
</tr>
<tr>
<td>$f(.)$</td>
<td>Probability distribution function (PDF)</td>
</tr>
<tr>
<td>$F(.)$</td>
<td>Cumulative distribution function (CDF)</td>
</tr>
<tr>
<td>$M(.)$</td>
<td>Moment generating function (MGF)</td>
</tr>
<tr>
<td>$\mathbb{E}[]$</td>
<td>Expectation operator</td>
</tr>
<tr>
<td>$[a, b]$</td>
<td>Discrete set of elements which ranges from $a$ to $b$</td>
</tr>
<tr>
<td>$\mathbb{U}(\cdot)$</td>
<td>Unit step function</td>
</tr>
<tr>
<td>$\delta(\cdot)$</td>
<td>Dirac delta function</td>
</tr>
<tr>
<td>$\mathcal{B}(a; b)$</td>
<td>Beta function</td>
</tr>
<tr>
<td>$r_{sel}$</td>
<td>Distance of the allocated user from its closest serving BS</td>
</tr>
<tr>
<td>$\tilde{r}_{sel}$</td>
<td>Distance of the allocated interfering user from the BS of interest.</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Composite shadowing and fading of the desired signal.</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Composite shadowing and fading of the interfering signal.</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Shadowing component of the desired signal.</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Fading component of the desired signal.</td>
</tr>
<tr>
<td>$r_k$</td>
<td>Distance of any user located in ring $k$ from its serving BS.</td>
</tr>
<tr>
<td>Symbol</td>
<td>Meaning</td>
</tr>
<tr>
<td>--------</td>
<td>---------</td>
</tr>
<tr>
<td>( \tilde{r}_{i,k} )</td>
<td>Distance of interferer located in ring ( k ) and ( i^{\text{th}} ) angular interval from the BS of interest.</td>
</tr>
<tr>
<td>( X_0 )</td>
<td>Desired signal received at the BS of interest.</td>
</tr>
<tr>
<td>( X_l )</td>
<td>Interfering signal received at the BS of interest from an interfering cell ( l ).</td>
</tr>
<tr>
<td>( Y )</td>
<td>Cumulative interference received at the BS of interest.</td>
</tr>
<tr>
<td>( R )</td>
<td>Radius of a given cell.</td>
</tr>
<tr>
<td>( D )</td>
<td>Diameter of a given cell.</td>
</tr>
<tr>
<td>( U )</td>
<td>Number of users in a given cell.</td>
</tr>
<tr>
<td>( K )</td>
<td>Number of rings in a given cell.</td>
</tr>
<tr>
<td>( N )</td>
<td>Number of subcarriers allocated to given cell.</td>
</tr>
<tr>
<td>( L )</td>
<td>Number of cells in a cellular system.</td>
</tr>
<tr>
<td>( F )</td>
<td>Average fairness measure.</td>
</tr>
<tr>
<td>( C )</td>
<td>Ergodic capacity.</td>
</tr>
<tr>
<td>( q )</td>
<td>Outage threshold.</td>
</tr>
<tr>
<td>( \mathcal{P} )</td>
<td>Average power preservation.</td>
</tr>
<tr>
<td>( u_k )</td>
<td>Number of users in each ring ( k ).</td>
</tr>
<tr>
<td>( P_0 )</td>
<td>Desired received signal power level</td>
</tr>
<tr>
<td>( P_{\text{max}} )</td>
<td>Maximum transmission power of a user</td>
</tr>
<tr>
<td>( r_t )</td>
<td>Threshold distance</td>
</tr>
<tr>
<td>( \mathcal{I} )</td>
<td>Number of uniform angular intervals</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

Evolution of wireless technology is now approaching its fourth generation (4G). In past, wireless systems have chased various evolutionary tracks targeting the performance and efficiency of mobile users. First generation (1G) successfully fulfilled the implementation of voice applications, while the second generation (2G) has introduced capacity and coverage. Third generation (3G) which has pursuit for high data rate open the gates for mobile broadband experience, which will later be realized by the 4G systems such as Long Term Evolution (LTE) and Worldwide Interoperability for Microwave Access (WiMaX). LTE is one of the recently developed standards in the mobile technology which is designed to meet the demands of next-generation mobile networks [1]. The flexibility of the recently proposed LTE standard allows the service providers to efficiently plan a system even with a limited amount of resources. Moreover, it allows service providers to select a suitable system design that fulfill their requirements by balancing the network spectral efficiency gains with the number of supported users and coverage.

1.1 Brief Technical Overview of LTE

In this section, we discuss some important technical aspects of LTE, i.e., multiple access schemes, resource allocation, and intercell interference (ICI) management issues.
Moreover, we will also present literature review to highlight the technical challenges involved in various aspects of LTE.

1.1.1 Multiple Access Techniques

Multiple access is a technique in which users can share a communication channel via some predefined protocol. Some of the well-known techniques include Frequency Division Multiple Access (FDMA), Time Division Multiple Access (TDMA), Code Division Multiple Access (CDMA), and hybrid combination of these methods. Due to the merits of high spectral efficiency, resiliency to radio frequency (RF) interference, lower multi-path distortion, Orthogonal Frequency Division Multiple Access (OFDMA) techniques are envisioned for both LTE uplink and downlink transmissions. Even though, a specific multiple access technique is typically employed for both the uplink and the downlink transmissions, LTE have selected different transmission schemes namely OFDMA and Single Carrier Frequency Division Multiple Access (SC-FDMA) for downlink and uplink, respectively.

Downlink-OFDMA

In comparison to the conventional access techniques such as TDMA and CDMA, OFDMA is proved to be a superior access technology for broadband wireless data networks. The main advantages of OFDMA arises from its scalability, robustness against multi-path delay spread, and its orthogonality [2]. Large bandwidths are typically sensitive to frequency-selective channel variations, however, in OFDMA this problem is alleviated by dividing the system bandwidth into several orthogonal small bands referred to as subcarriers. Division of data stream over several subcarriers reduces the symbol duration and in turn reduces the relative multipath delay spread, relative to symbol time. Transmission on orthogonal subcarriers is an efficient option in the ideal situations when there is no multi-path delay spread, however, in reality
a time guard in between the OFDM symbols is necessary to assure a inter-symbol interference (ISI) free transmission which can be done by making this time guard sufficiently larger than the maximum expected delay spread.

**Uplink SC-FDMA**

SC-FDMA has been emerged as a promising multiple access technique for the LTE uplink data transmissions. SC-FDMA is a modified version of OFDMA with similar throughput performance and overall complexity, however, in comparison to OFDMA the transmissions are performed sequentially rather than in parallel which reduces the envelope fluctuations in the transmitted waveform significantly [3]. As a result SC-FDMA signals have inherently lower peak-to-average-power ratio (PAPR) and reduced frequency diversity than OFDMA. The problem of ISI in this setup which results due to severe multi-path propagation in cellular systems, can be alleviated by employing adaptive frequency domain equalization at the base station (BS) which reduces the burden of linear amplification in portable terminals.

**1.1.2 Radio Resource Management**

Radio resource management has been evolved as an efficient tool to coordinate, mitigate and manage ICI while enhancing the network performance in OFDMA based cellular networks. More specifically, efficient spectrum/subcarrier allocation and power management solutions are needed to leverage the potential of OFDMA networks.

**Downlink OFDMA Networks**

Sum rate maximization problem via resource allocation is extensively studied for the downlink OFDMA networks. The optimal downlink strategy is to separately optimize subcarrier and power allocation, i.e., allocate a subcarrier to the user with best channel quality and then perform water-filling over the allocated subcarriers
In [5], the authors investigated scaling laws for upper and lower bounds of the downlink capacity in the asymptotic regime. However, the problem becomes more challenging in the uplink due to the individual power constraint at each user. Simply allocating a subcarrier to the user with best channel quality may degrade the network performance, as some active users may have better channel gains but low transmission powers on a specific subcarrier.

**Single Cell Uplink OFDMA Networks**

In order to achieve near optimal throughput in the uplink, a subcarrier shall be assigned to the user that possesses the maximum power-channel gain product[6, 7, 8]. In [6], a near optimal greedy scheme is proposed which is based on maximizing the power-channel gain product with an iterative water-filling on the subcarriers. The authors in [7] generalized the strategy presented in [6] by considering utility maximization and developed a polynomial time algorithm to compute an upper bound for the optimal solution. They also verified the near optimality of the solution in [6]. In [8], sum rate maximization is studied with and without proportional fairness and a transmission power-SNR product based ranking scheme is developed. The scheme is based on the statistical knowledge of the number of subcarriers assigned to each user. In [9, 10], the authors developed a sub-gradient based scheduling framework to compute the optimal solution of the relaxed problem. In the above mentioned literature, ICI is ignored completely which results in notable performance degradation in practical multi-cell scenarios.

**Multi-Cell Uplink OFDMA Networks**

Most of the recent work in the context of multi-cell OFDMA networks [11, 12], aims at minimizing the overall transmitted power, i.e., linear objective with predefined rate constraints. Furthermore, in some recent literature, low complexity distributed game
theoretic solutions are also studied. However, the schemes are iterative and optimality is not guaranteed [13]. An auction based approach is proposed in [14], where joint auction and dual decomposition based technique is considered. The technique is asymptotically optimal as the number of subcarriers in every cell goes to infinity. In summary, all these approaches are sub-optimal and no criteria are used to calibrate their performance gap with respect to the optimal solution.

1.1.3 Interference Modeling

Enhancing the coverage by allowing as many users to communicate reliably irrespective of their location and mobility appears to be a primary concern of network service providers. This task is typically fulfilled by doing aggressive spectrum reuse which on one side enhances the spectral efficiency, whereas on the other side it causes severe ICI among the users of same spectrum. The incurred ICI in OFDMA networks with universal frequency reuse (UFR) is severe and highly random due to its dependence on the channel statistics and on the dynamics of the multiuser scheduling decisions. Therefore, it is of utmost importance for the system designers to accurately characterize the behavior of the ICI in order to quantify various network performance metrics and to develop efficient resource allocation and interference mitigation schemes.

Modeling of Downlink ICI

Several recent studies considered the modeling of ICI in the downlink of cellular networks where the locations of the interferers is usually assumed to be deterministic. The authors in [15] present a semi-analytical approach to estimate the downlink ICI by modifying the Burr distribution considering path loss, Rayleigh fading and log-normal shadowing. In [16], a semi-analytical distribution for the signal-to-interference-noise ratio (SINR) is derived under path loss and log normal shadowing for femtocell networks. In [17], the network capacity of a single user and multiuser interference-limited
cellular scenario is derived in closed form with a Rayleigh fading channel model. In [18], the applicability of the Gaussian and binomial distributions for modeling the downlink ICI is investigated. It is shown that significant interference arises from two interfering cells irrespective of the user’s locations. Another interesting work appears in [19] where the authors derived the distribution of the downlink ICI for OFDMA networks under log-normal shadowing and Rayleigh fading. The distribution of ICI is shown to highly deviate from the Gaussian distribution, especially at moderate network loads. However, none of the above mentioned include the impact of scheduling schemes and power allocations in the modeling of ICI.

Modeling of Uplink ICI

In [20], the authors developed an analytical model for subcarrier collisions as a function of the cell load and frequency reuse pattern. They derived an expression for the SINR in the uplink and downlink, ignoring the effect of shadowing and fading. In [21], the authors developed an analytical expression for the subcarrier collision probability considering non-coordinated schedulers. In [22], the authors modeled uplink ICI in an OFDMA network as a function of the reuse partitioning radius and traffic load assuming arbitrary scheduling. In [23], the authors presented a semi-analytical method to approximate the distribution of the uplink ICI through numerical simulations without considering the impact of scheduling schemes. However, none of the above mentioned work consider the impact of channel based scheduling and power allocations in the modeling of ICI.

Fundamental Differences in the Modeling of Uplink and Downlink ICI

In contrast to the downlink, the modeling of ICI in the uplink is more challenging due to the following fundamental differences:

- The number of significantly contributing interferer’s typically remains the same
irrespective of the position of the desired user in the downlink. It is shown in [18] that the strongest interference is generated by two closest interfering BSs irrespective of the location of victim. On the contrary, the number of significantly contributing interferer’s to the BS cannot be determined in the uplink case which makes the uplink ICI more unstable.

- The uplink ICI is induced at a single location (i.e., at the BS) and is dependent on the location of the interferer’s which is highly varying based on the deployed scheduling scheme in the neighbor cells. Whereas, conditioned on the location of the desired user in its own cell, the location of interferer’s becomes deterministic in the downlink (See Fig. 1.1).

- Uplink cell-edge and cell center users are subject to same amount of interference which is received at the BS. Whereas the same is not true for the downlink case in which cell edge users continue to experience poor service due to the high interference coming from the nearby BSs [20].

- The nicer part of the uplink is that the receiver is fixed and all users will suffer from same amount of interference on a given subcarrier [20], therefore ignoring the interference while scheduling a given subcarrier makes no difference in the scheduling decisions. However, in multi-carrier case it might be possible to achieve some throughput gains by performing scheduling of different subcarriers based on the measured interference. This scheduling technique comes under the framework of ICI coordination mechanisms.

1.1.4 Power Control: ICI Mitigation Technique

Power control (PC) mechanisms in the uplink of cellular networks have been recognized as a class of interference mitigation techniques that can lead to notable reduction
in power consumption and increase in spectral efficiency while maintaining a desired quality of service, by controlling the users’ transmit powers [24, 25, 26]. It has been shown in [26] that PC is more important in the uplink due to the limited battery power of the mobile users and the nature of interference which is heavily based on the locations of the mobile users. In this context, several variations of uplink PC mechanisms have been developed and integrated in cellular standards such as closed-loop and open-loop PC, slow and fast PC, and fractional PC.

Typically, conventional slow PC compensates for the long term channel variations (shadowing and distance-based path loss) of a mobile user from its serving BS, whereas fast PC allows to compensate for fast fading effects as well. Uplink slow PC can be implemented by sending PC commands from BSs (closed-loop manner) or by estimating own transmit powers at mobile users based on channel measurements from downlink pilots (open-loop manner). On the other hand, uplink fast PC require frequent measurements and exchange of control data between the serving BS and its mobile users. Fractional PC allows fractional path loss compensation and can be implemented with either slow or fast mode. To reduce ICI, fractional PC normally mandate the cell-edge users to transmit with less power compared to the cell-center users with a trade-off cost in terms of cell-edge spectral efficiency.

Numerous simulation-based studies are available in the literature where the per-
formance of uplink PC has been analyzed for various network scenarios and simulation parameters [26, 27, 24, 28]. However, in parallel to these simulation-based studies, there is a need to develop tractable analytical models to gain more in depth intuition and extract new insights based on various performance trends. In addition, analytical models are particularly beneficial for rapid network performance assessment and help in clearly analyzing the inter-dependencies between system and design parameters. An effort in this regard has been carried out in [29] where the authors presented a theoretical approach to gain insights into the selection of fractional PC parameters. However, the impact of shadowing and fading was ignored and each user was considered to be capable of compensating fully its distance-based path loss ignoring the battery power constraints.

1.1.5 BS Cooperation: ICI Coordination Mechanisms

The motivation underlying the BS coordination is to concentrate the bulk of ICI in a specific portion of the total bandwidth, thereby preventing the performance degradation on other parts of the bandwidth. However, this specific portion of the bandwidth should be as small as possible. A series of research works on the modeling of ICI in multi-carrier networks dealt with the development of subcarrier collision models to calibrate several performance metrics [20, 21, 30]. In [19], the distribution of ICI is derived for downlink cellular networks considering Rayleigh fading channels. The impact of shadowing, user locations, and transmit powers is not considered. In [31], the authors presented a numerical method to analyze the distribution of the signal-to-interference ratio (SIR) and the capacity in uplink wireless networks in which a single user is assumed to be active per time instant. However, there is no previous work that has captured the impact of channel based scheduling, coordinated and uncoordinated scheduling, on the statistics of ICI in a multi-carrier cellular network scenario.
Fractional Frequency Reuse

Fractional frequency reuse (FFR) emerges recently as an efficient interference mitigation technique which promises significant reduction in ICI by increasing the frequency reuse factor at the cell-edges [32]. Many recent studies are focused on the simulation/optimization based strategies where typical network parameters such as network throughput, frequency reuse factor, and bandwidth partition are optimized [33, 34, 32]. However, in parallel to the intensive simulation campaigns and sophisticated optimization algorithms, the development of tractable analytical models is equally important in gaining theoretical insights behind various performance trends. In this regard, some worth mentioning research works are [35], [36] where the theoretical calculations are provided for important network performance metrics such as capacity and outage in downlink FFR-based OFDMA networks. In addition, an optimal distance threshold has been derived in [37] considering the impact of channel based scheduling in the downlink networks.

Unlike downlink, the arbitrary locations and transmit power of the mobile interferers deviate and complicate the nature of the uplink ICI significantly. Therefore the conclusions regarding the efficiency of FFR in the downlink may not be directly applicable to the uplink scenarios. In this context, an important observation has been provided in [20] where it is shown that the performance of downlink FFR schemes deviates from the uplink counterpart and may not necessarily dominates the performance of UFR under all scenarios. This fact emphasizes the importance of critically investigating the range of scenarios where FFR outperforms UFR especially in the uplink.

1.1.6 ICI Mitigation in Heterogeneous Networks

A core wireless network which is a combination of existing macro-cell infrastructure and small cells with varying transmit powers and coverage areas is referred to
as Heterogeneous network (HetNet). HetNets emerges as a cost-effective means of extending the cellular coverage, enhancing the overall network spectral and energy efficiency and providing improved indoor broadband wireless services [38, 39]. However, despite these benefits, the HetNet technology raise multiple new challenges in terms of network architecture, access policies, and interference management.

In general, there are three kind of access policies for small cell resources which allow or restrict its usage, namely: (i) **public access**: allow all users to access resources; (ii) **private access**: allow only subscribed users to access resources; (iii) **hybrid access**: allow all non-subscribers to use resources with in some special circumstances[40]. In addition, there are three fundamental spectrum allocation techniques for HetNet deployments, namely: (i) **Dedicated channel access**: in which channels allocated to small cells are orthogonal to the channels allocated to the macrocell users; (ii) **Co-channel access**: in which whole spectrum is shared by both macro-cell and small cell users; (iii) **Partial access**: in which the whole spectrum is divided into two parts, one dedicated to the macrocells and the other shared by both macrocells and small cells [41, 42].

Numerous research studies in the literature are focused on illustrating and optimizing the spectral efficiency of HetNets for specific scenarios through classical optimization techniques and algorithms. Most of these studies do not emphasize on the implications of realistic access policies, spectrum allocation and interference management techniques [43, 44, 45]. In this regard, some attention has been paid recently on the simulation [46, 47, 48] as well as theoretical modeling [49, 41, 42] of important network performance metrics in the downlink of heterogeneous cellular networks with a special focus on user association and access control policies.

Several HetNet deployment strategies are currently under consideration and their performance is commonly calibrated with respect to the achievable profitability, spectral efficiency, and outage probability [50]. However, energy efficiency has been re-
cently marked as one of the alarming bottleneck in the telecommunication growth paradigm mainly due to two major reasons i) dramatically varying global climate [51], and ii) slowly progressing battery technology [52, 53]. Therefore, it is of immense importance for the network designers to develop and utilize efficient interference mitigation technique that can (i) reduce ICI significantly; (ii) maintain spectral efficiency; and (iii) increase energy efficiency of a particular HetNet deployment.

1.2 Thesis Outline

The thesis is organized as follows:

1. Chapter 2 is focused on developing efficient rate-maximization algorithms in the uplink of a multi-cell OFDMA network. The problem has a non-convex combinatorial structure and is NP hard. Due to the inherent complexity of implementing the optimal solution, firstly, we compute an upper bound (UB) and lower bound (LB) to the optimal average network throughput. Also, we study the effect of ICI on the performance of the near-optimal single cell resource allocation scheme proposed in [6] which leads to another simple lower bound. We then propose less complex centralized and distributed schemes that are well-suited for practical scenarios. The computational complexity of all schemes is analyzed and performance is compared through simulations.

2. Chapter 3 presents a novel framework to derive analytical expressions for the ICI distribution on a given subcarrier as a function of both the channel statistics and multiuser scheduling decisions. The framework is generic in the sense that the derivations holds for generalized fading channels and various scheduling algorithms. Furthermore, the derivations are also extendible to downlink scenarios. Firstly, we derive the distribution of the locations of the scheduled user in a given cell considering a wide range of scheduling schemes. Based on
this, we derive the distribution and MGF of the ICI considering a single interfering cell. Consequently, we determine the MGF of the cumulative ICI observed from all interfering cells. Finally, we utilize the obtained expressions to evaluate important network performance metrics such as the outage probability, ergodic capacity and average fairness.

3. Chapter 4 presents a novel statistical model for uplink ICI as a function of various slow and fast PC mechanisms assuming round robin scheduling. The derived expressions are generic and can be utilized to quantify numerically key network performance metrics that include average resource fairness, average power savings, and ergodic capacity. Closed form expressions are derived for Generalized-$\mathcal{K}$ composite fading environments. Results are generated for multiple network scenarios, and insights are extracted to assess various PC mechanisms as a function of system parameters and design options.

4. Chapter 5 extends the statistical model for the uplink ICI derived in chapter 3 to a multiuser multi-carrier cellular network considering the impact of various uncoordinated scheduling schemes on the locations and transmit powers of the interferers. Furthermore, the derived model is extended to incorporate coordinated scheduling schemes and fractional frequency reuse schemes. A study is then presented to quantify the potential performance gains of coordinated over uncoordinated scheduling schemes under various BS coordination scenarios. Numerical results demonstrate that different frequency allocation patterns significantly impact the network performance depending on the coordination among neighboring BSs.

5. Chapter 6 analyzes the uplink spectral and energy efficiency of the HetNets by considering that each mobile user is capable of adapting its transmit power according to slow and fast PC mechanism while maintaining a certain signal
power received at the BS [26]. We consider dedicated channel access and public access policy for our deployment configuration. Taking into account the worst and best case interference scenarios, we derive analytical expressions to compute the bounds on the area spectral efficiency (ASE) of energy aware HetNet configuration. Moreover, exact analytical expressions for energy efficiency metrics are also obtained for considered HetNets configuration.

6. Chapter 7 presents a brief summary and outlines some ideas for possible future directions.
Chapter 2

Rate Maximization via Resource Allocation in OFDMA Networks

2.1 Introduction

Dynamic resource allocation plays a central role in the air interface design of state-of-the-art OFDMA-based cellular technologies. In this chapter, we focus our attention on maximizing the overall network throughput by optimizing the allocation of resources (i.e., subcarriers and powers) jointly in a multi-cell uplink OFDMA network. The goal is to develop efficient resource allocation schemes that takes into account the ICI while considering universal frequency reuse.

The solution of such problem is difficult to achieve optimally due to its NP hard combinatorial nature and high dimensionality, however, in this chapter firstly, we compute an upper bound (UB) and lower bound (LB) to the optimal average network throughput. Also, we study the effect of ICI on the performance of the near-optimal single cell resource allocation scheme proposed in [6] which leads to another simple LB. Simulation results show that this LB is slightly loose but can be computed easily. Since the computation of the optimal solution is exhaustive, we then propose a centralized sub-optimal resource allocation scheme which uses a geometric programming (GP) based power allocation phase in conjunction with a heuristic subcarrier
allocation phase. The proposed scheme possesses an iterative and computationally intensive subcarrier allocation phase. However, it can serve as an effective benchmark for the less complex schemes even without the power allocation phase. The power allocation phase is discussed for both high and general SINR regimes. Finally, we propose and evaluate less complex centralized and distributed schemes that are suitable for practical implementations.

The rest of the chapter is organized as follows: In Section 2.2, the system model is defined and problem is formulated. In Section 2.3 and 2.4, the optimal solution of the problem and bounds are presented, respectively. In Section 2.5 and Section 2.6, the proposed centralized and distributed schemes are explained. Section 2.7 presents numerical results followed by concluding remarks in Section 2.8.

**Notation:** Matrices are represented using boldface upper case letters while bold face lower case letters are used for vectors.

![Figure 2.1: Uplink cellular network.](image)
2.2 System Model and Problem Formulation

A network of $L$ cells with a set of $U$ users in each cell $l$ is considered. Full reuse of the spectrum is assumed in all the cells. Each BS is assumed to have $N$ orthogonal subcarriers, and each subcarrier can be allocated to a single user per cell. The average network throughput $C$ is a function of both subcarrier and power allocation variables. The sum rate maximization problem is formulated as follows using the standard Shannon capacity formula, $C_{n,u,l} = \log_2(1 + \gamma_{n,u,l})$, where $C_{n,u,l}$ and $\gamma_{n,u,l}$ represent the throughput and SINR of the $u^{th}$ user at $n^{th}$ subcarrier in cell $l$, respectively:

$$\text{maximize} \quad \sum_{l=1}^{L} \sum_{u=1}^{U} \sum_{n=1}^{N} \alpha_{n,u,l} \log_2 \left( 1 + \frac{p_{n,u,l} h_{n,u,l}}{\sigma^2 + I_{n,l}} \right)$$ (2.1)

subject to

$$\sum_{n=1}^{N} p_{n,u,l} \leq P_{u,\text{max}}, \quad \forall u, \forall l$$ (2.2)

$$\sum_{u=1}^{U} \alpha_{n,u,l} = 1, \quad \forall n, \forall l$$ (2.3)

$$\alpha_{n,u,l} \in \{0, 1\}, \quad \forall n, \forall l, \forall u.$$ (2.4)

In (2.1), $I_{n,l} = \sum_{j=1, j \neq l}^{L} \sum_{u=1}^{U} \alpha_{n,u,j} p_{n,u,j} g_{n,u,j}$ represents the cumulative interference at $n^{th}$ subcarrier in cell $l$ from the users in all other cells, $p_{n,u,l}$ denotes the power transmitted by $u^{th}$ user at the $n^{th}$ subcarrier in cell $l$, $\alpha_{n,u,l}$ represents the allocation of $u^{th}$ user at the $n^{th}$ subcarrier in cell $l$ and $h_{n,u,l}$ is the channel gain of $u^{th}$ user at the $n^{th}$ subcarrier in cell $l$. Constraint (2.2) implies that the power spent by $u^{th}$ user on its allocated subcarriers cannot exceed the maximum available power, $P_{u,\text{max}}$. For each cell, we collect the power allocation variables $p_{n,u,l}$ in a vector $p_{n,l} = [p_{n,1,l}, p_{n,2,l}, \ldots, p_{n,u,l}]$ and then stack all the vectors in a power matrix $P_l$ of cell $l$ where $P_l \in \mathbb{R}^{N \times U}$. Constraint (2.3) restricts the allocation of each subcarrier to only one user. The channel gains $h_{n,u,l}$ and binary allocation variables $\alpha_{n,u,l}$ are
stacked up similarly in the matrices $H_l$ and $A_l$, respectively, where $A_l, H_l \in \mathbb{R}^{N \times U}$. Moreover, we define $g_{n,u,lj}$ as the interfering gain from the $u^{th}$ user in cell $l$ to cell $j$, $\forall j \neq l$ at $n^{th}$ subcarrier. We collect these interfering gains into a vector $g_{n,lj} = [g_{n,1,lj}, g_{n,2,lj}, \ldots, g_{n,u,lj}]$ and then stack all the vectors in a matrix $G_{lj} \in \mathbb{R}^{N \times U}$.

### 2.3 Optimal Solution in High SINR Regime

Assuming perfect knowledge of channel gains at a centralized controller, the optimal solution for (2.1) can be computed in the high SINR regime by an exhaustive search over all possible combinations of the allocations. For each possible allocation, optimum powers can be computed by transforming (2.1) into a GP. Note that the power allocation problem is in itself a known non-convex problem for the general SINR regime [54]. However, in the high SINR regime the problem becomes a convex GP problem. For a given set of allocation variables and considering a high SINR regime, the objective function in (2.1) can be rewritten as follows:

$$\max_{p_{n,u,l}} \sum_{l=1}^{L} \sum_{u=1}^{U} \sum_{n=1}^{N} \alpha_{n,u,l} \log_2 \left( \frac{p_{n,u,l} h_{n,u,l}}{\sigma^2 + I_{n,l}} \right).$$ \quad (2.5)

Maximizing the SINRs is equivalent to minimizing the interference to signal ratio:

$$\min_{p_{n,u,l}} \sum_{l=1}^{L} \sum_{u=1}^{U} \sum_{n=1}^{N} \alpha_{n,u,l} \log_2 \left( \frac{\sigma^2 + I_{n,l}}{p_{n,u,l} h_{n,u,l}} \right).$$ \quad (2.6)

Equivalently, (2.1) can be reformulated for high SINR regime and given allocation variables as follows:

$$\min_{p_{n,u,l}} \log_2 \prod_{l=1}^{L} \prod_{u=1}^{U} \prod_{n=1}^{N} \left( \frac{\sigma^2 + I_{n,l}}{p_{n,u,l} h_{n,u,l}} \right)^{\alpha_{n,u,l}}$$ \quad (2.7)

subject to $\sum_{n=1}^{N} \alpha_{n,u,l} p_{n,u,l} \leq P_{u,max}, \ \forall u, \forall l$. 
Note that the numerator in (2.7) is a posynomial and the denominator is a monomial, hence (2.7) is a GP problem in standard form that can be solved optimally through efficient interior point methods [55] after performing the logarithmic transformation of variables [54]. However, even for small dimensions, it is not recommendable to compute the optimal solution, due to the huge computational complexity $O(U^{LN})$ associated with the exhaustive search based subcarrier allocation phase. In addition, the GP based power allocation method discussed above has two restrictions: high-SINR assumption and centralized time-consuming computations. Due to the mentioned facts, there is a need to develop bounds and sub-optimal resource allocation schemes.

2.4 Bounds on the Network Throughput

2.4.1 Lower Bound on the Optimal Network Throughput

A LB for the optimum multi-cell network throughput can be computed by considering worst case ICI. Observing the dependency of ICI on the subcarrier allocation and power allocation variables, we assume that each user in each cell is transmitting on each subcarrier with its maximum power. A simple LB for the average network throughput $C$ taking the worst case ICI into account can be written as follows:

$$C(A_l, P_l) \geq \frac{1}{L} \sum_{l=1}^{L} \sum_{u=1}^{U} \sum_{n=1}^{N} \alpha_{n,u,l} \log_2 \left( 1 + \frac{p_{n,u,l} h_{n,u,l}}{\sigma^2 + \xi_{n,l}} \right),$$

(2.8)

where $\xi_{n,l} = \sum_{j=1, j \neq l}^{L} \sum_{u=1}^{U} P_{u,\text{max}} g_{n,u,jl}$.

A tighter LB can be derived by using Algorithm 1 where each subcarrier is allocated to the user that maximizes $Q_{n,u,l}$ where:

$$Q_{n,u,l} = \frac{p_{n,u,l} h_{n,u,l}}{\xi_{n,l} + \sigma^2}.$$  

(2.9)
Thus, $Q_{n,u,l}$ is an SINR term for each user $u$ at each subcarrier $n$ in each cell $l$ assuming worst case interference. We collect these SINR terms into a vector $q_{n,l} = [q_{n,1,l}, q_{n,2,l}, \ldots, q_{n,u,l}]$ and then stack all the vectors in a matrix $Q_l \in \mathbb{R}^{N \times U}$. The resulting allocations based on this criteria are then used to compute the LB network throughput using (2.1).

Note that if $\xi_{n,l} = 0$, than $Q_{n,u,l}$ becomes the marginal rate which is shown to be a near-optimal criterion in single cell network scenarios without ICI [6]. Moreover, equal power allocation has insignificant performance loss in high SINR regime compared to the optimal water-filling solution [6, 8], thus power equalization is implemented in Algorithm 1. For the low SINR regime, we can incorporate water-filling rather than equalization in a straightforward manner.

### 2.4.2 Upper Bound on the Optimal Network Throughput

Establishing an UB is significantly important in order to calibrate the performance of sub-optimal resource allocation schemes with respect to the optimal solution. The UB can be derived by ignoring the effect of ICI in all the cells. This can be achieved by substituting $\xi_{n,l} = 0$ in Algorithm 1, i.e., $Q_{n,u,l} = \frac{p_{n,u,l} h_{n,u,l}}{\sigma^2}$:

$$C(A_l, P_l) \leq \frac{1}{L} \sum_{l=1}^{L} \sum_{u=1}^{U} \sum_{n=1}^{N} \alpha_{n,u,l} \log_2 \left( 1 + \frac{p_{n,u,l} h_{n,u,l}}{\sigma^2} \right).$$

(2.10)

The allocations computed by Algorithm 1 are near optimal since they are based on a criterion which is shown to be near optimal in the context of single cell scenarios [6, 7, 8]. The average network throughput revealed by these allocations could be highly optimistic for multi-cell scenarios. Thus, we can investigate the impact of ICI by simply computing the throughput using (2.1) instead of (2.10) with these allocations. Computing throughput this way quantify the performance degradation when the single cell near-optimal allocations are used in multi-cell network scenarios.
Complexity Analysis

The \((n, u)\) pair at which the term \(Q_{n,u,l}\) becomes maximum is allocated (Step 4), which has a complexity of a two dimensional search, i.e., \(O(UN)\). However, as soon as a subcarrier is assigned, each user updates its power as defined in Algorithm 1. This process iterates until all the subcarriers in all the cells are allocated and, thus, the time complexity of Algorithm 1 is \(O(UN^2)\).

**Algorithm 1** Computing LB and UB Allocations in Cell \(l\)

1. **Input:** \([H_l], [A_l], [P_l], [G_{jl}]\) where \(\alpha_{n,u,l} = 0, p_{n,u,l} = P_{u,max}/N \ \forall u, \forall n\)

2. For each user \(u\) in cell \(l\), power is divided equally over all of its allocated subcarriers and the remaining unallocated subcarriers of the system.

3. Using \([P_l]\) from step 2, \([H_l]\) and \([G_{jl}]\), compute the matrix \(Q_l\) for each cell \(l\).

4. Find the \((n, u)\) pair that has the maximum value of \(Q_{n,u,l}\). Allocate subcarrier \(n\) to user \(u\).

5. Delete the \(n^{th}\) subcarrier from the set of unallocated subcarriers.
   - **If** there are still unallocated subcarriers in the system go to step 2.
   - **else** terminate after distributing the maximum power equally at each user over all of its assigned subcarriers.

2.4.3 A Motivating Example

Consider an example with two cells, two users and two subcarriers. Each user can transmit with a maximum power of 1 W. Assume \(H_1 = [1 \ 0.9; 0.8 \ 0.7]\) and \(H_2 = [1 \ 0.9; 0.8 \ 0.7]\). Single cell allocation strategies that aim to maximize the local throughput of each cell suggest \(A_1, P_1\) and \(A_2, P_2 = [1 \ 0; 0 \ 1]\). Computing the UB using (2.10) results in 1.7655 bps/Hz/cell where \(\sigma^2 = 1\). Now, assume the knowledge of interfering link gains at each BS, i.e., \(G_{12} = [0.9 \ 0.2; 0.2 \ 0.9]\) and \(G_{21} = [0.7 \ 0.1; 0.1 \ 0.7]\). Computing the throughput again while keeping the single cell allocations and taking into account the interfering gains leads to an average network throughput of 1.1137 bps/Hz/cell. However, better allocations are possible if we consider \(A_1, P_1\)
and \( \mathbf{A}_2, \mathbf{P}_2 = [0 \ 1; \ 1 \ 0] \) as per the criterion which enhances the resulting average network throughput to 1.5977 bps/Hz/cell.

### 2.5 Centralized Resource Allocation Schemes

Considering the high intricacy of implementing the optimal solution, we develop a two-stage centralized scheme. The subcarrier allocation phase of the developed scheme is iterative and computationally intensive, however, the performance is better even without the power allocation phase. Therefore, it can provide an effective benchmark for low complexity schemes.

#### 2.5.1 Centralized Iterative Scheme

In the proposed scheme, we split the resource allocation procedure into two phases: subcarrier allocation phase and power allocation phase. It is important to note that the subcarrier allocation phase involves a power equalization step, thus, it is not totally independent of power allocation.

**Phase I: Subcarrier Allocation**

- **Initial Allocation**: Firstly, we define the term for the allocation of resources to the users as follows:
  \[
  \chi_{n,u,l} = \frac{p_{n,u,l} h_{n,u,l}}{\sum_{j=1,j \neq l}^{L} P_{u,\text{max}} g_{n,u,lj}}. \tag{2.11}
  \]
  This criterion guarantees the selection of the users who possess not only better power-gain product but also they offer less interference to the neighbor cells. The denominator \( \sum_{j=1,j \neq l}^{L} P_{u,\text{max}} g_{n,u,lj} \) accounts for the maximum aggregate interference that the \( u^{\text{th}} \) user in cell \( l \), may cause to all cells. Even though this criterion is heuristic, it improves the performance compared to the traditional C/I scheme (which gives nearly similar results as our LB). Once the initial
allocations are computed, we can calculate the initial throughput of the network \( C_o \) using (2.1).

- **Maximize Throughput Iteratively until Convergence:** In this step, we select any cell \( l \) and subcarrier \( n \) arbitrarily and re-perform the allocation at this subcarrier considering the other cell allocations fixed, i.e., \( I_{n,l} \) remains fixed. More explicitly, we compute \( C_{n,u,l} = \log_2(1 + \frac{p_{n,u,l}h_{n,u,l}}{\sigma^2 + I_{n,l}}) \) for all users in cell \( l \) one by one and select the user which gives the maximum incremental throughput at subcarrier \( n \), i.e., \( C_{n,u,l} - C_o \). Note that, in order to compute \( C_{n,u,l} \), we need to compute \( p_{n,u,l} \) which can be obtained simply by dividing \( P_{u,\text{max}} \) equally among all the fixed allocated subcarriers of user \( u \) and the new one which is currently under observation.

Once the reallocation is done at subcarrier \( n \), we move to the next subcarrier in cell \( l \) and so on. As the new allocations are computed for cell \( l \), we calculate the new increased network throughput \( C_{\text{new}} \) and move to another cell \( j \). The whole process is repeated again with \( C_o = C_{\text{new}} \) until convergence.

**Phase II: Power Allocation**

Once the subcarrier allocation is done, the optimal powers can then be calculated for the high SINR regime or for the general SINR regime through solving a series of GPs using successive convex approximation which is a provably convergent heuristic [54]. This approach is known to compute globally optimal power allocations in many cases. Thus, for given allocations, (2.1) can be formulated for the general SINR regime as:

\[
\begin{align*}
\text{minimize} & \quad \log_2 \prod_{l=1}^{L} \prod_{u=1}^{U} \prod_{n=1}^{N} \left( \frac{\alpha^2 + I_{n,l}}{p_{n,u,l}h_{n,u,l} + \sigma^2 + I_{n,l}} \right)^{\alpha_{n,u,l}} \\
\text{subject to} & \quad \sum_{n=1}^{N} \alpha_{n,u,l}p_{n,u,l} \leq P_{u,\text{max}}, \quad \forall u, \forall l.
\end{align*}
\]
Note that the numerator and denominator in (2.12) are posynomials and minimizing a ratio between two posynomials is referred to be a truly non-convex NP hard intractable problem known as complimentary GP. However, this problem can be transformed into GP by letting the denominator $f(p) = \sum_{l=1}^{L} \sum_{u=1}^{U} \varrho_{n,u,l}(p) + \sigma^2 + I_{n,l}$ and approximating the denominator $f(p)$ with a monomial using the arithmetic/geometric mean inequality as follows:

$$\sum_{l=1}^{L} \sum_{u=1}^{U} \varrho_{n,u,l}(p) \geq \prod_{l=1}^{L} \prod_{u=1}^{U} \left( \frac{\varrho_{n,u,l}(p)}{s_{n,u,l}} \right)^{s_{n,u,l}},$$

(2.13)

where $s_{n,u,l} = \frac{\varrho_{n,u,l}(p_0)}{f(p_0)}$. Thus, the problem can be solved by extending the single condensation method presented in [54] for multi-cell scenario. The details of centralized iterative scheme are presented in Algorithm 2.

**Complexity Analysis**

The initial allocation phase has a complexity of $O(U N^2)$ which is the same as Algorithm 1. Next, we perform a one dimensional search for the user in cell $l$ with maximum incremental throughput at subcarrier $n$. The process is repeated for each subcarrier and cell. Thus, the computational complexity of this step is $O(U N L)$. Since, the process continues until convergence, (i.e., $M$ iterations), the complexity of this step can be written as $O(U N L M)$. Finally, the total complexity of subcarrier allocation phase is $O(U N^2 + N U L M)$. The complexity of Phase II is difficult to determine, however, it can be measured in terms of the degree of difficulty (DoD) that in turn relies on the number of constraints and variables associated with the GP [56]. Since we are dealing with $L U$ power constraints and $L U N$ power variables, the total computational complexity of centralized iterative scheme is $O(U N^2 + N U L M) + DoD(L U N)$. Apparently it seems that implementing centralized GP/successive GP based schemes may not be a good choice for practical implementations. However, in order to re-
duce the complexity and DoD of the power allocation phase, we have developed the following less complex centralized scheme.

2.5.2 Centralized Non-Iterative Scheme

In this scheme, firstly the subcarriers are allocated in each cell $l$ using the heuristic criterion defined in (2.11). The allocation of each subcarrier is followed by the power allocation phase (based on equalization) as mentioned in the initial allocation phase of Algorithm 2 (i.e., Steps 1 to 5). Once the subcarrier allocations are finalized, we then compute GP based powers for the allocated users at any arbitrarily selected subcarrier $n$ in all cells. Setting the equalization based powers $p_{n,u,l,\text{eq}}$ as the UB on $p_{n,u,l}$ and considering a high SINR regime, we now define the following less complex GP problem with the objective to maximize the throughput as follows:

$$
\min_{p_{n,u,l}} \log_2 \prod_{l=1}^{L} \left( \frac{\sigma^2 + I_{n,l}}{p_{n,u,l} h_{n,u,l}} \right)^{\alpha_{n,u,l}}
$$

subject to $p_{n,u,l} \leq p_{n,u,l,\text{eq}}, \forall l.$

(2.14)

Clearly, the resulting GP based power of each competing user at subcarrier $n$ in the different cells may not succeed in achieving the UB, due to the ICI effect. We call this power as left-over power. The left-over power can then be distributed equally among the remaining allocated subcarriers of the user as detailed in Algorithm 3.

Since at the end of the initial allocation phase, the subcarrier allocations become fixed and the total power is distributed equally among the allocated subcarriers of a user, we cannot set an UB which depicts higher power than the previously allocated power. If we do so, this may cause power reduction or even no power at some other allocated subcarrier of that user in order to maintain the total power constraint. Thus, this may results in an invalid subcarrier allocation.

Next follows an example which demonstrates the significance of GP as well as
Algorithm 2 Centralized Iterative Scheme

1. **Input:** \([H_l], [A_l], [P_l], [G_{lj}]\) where \(\alpha_{n,u,l} = 0, p_{n,u,l} = P_{u,\text{max}}/N \ \forall u, \forall n\)

Subcarrier Allocation (Phase I)

**Initial Allocation:**

2. For each user \(u\) in cell \(l\), power is divided equally over all of its allocated subcarriers and the remaining unallocated subcarriers of the system.

3. Using \([P_l]\) from step 2, \([H_l]\) and \([G_{lj}]\), compute \(\chi_{n,u,l}\) for every \(u^{th}\) user at \(n^{th}\) subcarrier in cell \(l\).

4. Find the \((n,u)\) pair that has the maximum value of \(\chi_{n,u,l}\). Allocate subcarrier \(n\) to user \(u\).

5. Delete the \(n^{th}\) subcarrier from the set of unallocated subcarriers.
   
   **If** there are still unallocated subcarriers in the system go to step 2,
   **else** terminate after distributing the maximum power at each user over all of its assigned subcarriers

6. Compute \(C_o\)
   
   **Maximize Throughput Iteratively until Convergence**
   
   \[\text{do while}(C_{\text{new}} - C_o \geq \epsilon)\]
   
   \[\begin{align*}
   & l = 1, \ \text{do while } l \leq L, l = l + 1 \\
   & n = 1, \ \text{do while } n \leq N, n = n + 1 \\
   & u = 1, \ \text{do while } u \leq U, u = u + 1 \\
   \end{align*}\]

7. Allocate the subcarrier \(n\) to user \(u\).

8. Compute \(p_{n,u,l}\) by dividing \(P_{u,\text{max}}\) equally among the allocated subcarriers.

9. Compute \(C_{n,u,l} - C_o\)

10. Allocate subcarrier \(n\) to the user who maximizes \(C_{n,u,l} - C_o\)

11. Compute \(C_{\text{new}}\) using (2.1).

12. \(C_o = C_{\text{new}}\)

   **Power Allocation (Phase II)**

13. Compute the optimal powers \(P_l\) in the high SINR regime (2.7) given the allocations from Phase I.

14. For general SINR regime, take \(P_l\) from step 13 as an initial starting point.

15. Using \(P_l\), evaluate \(p_{n,u,l} h_{n,u,l} + \sigma^2 + I_{n,l}\) for each allocated user \(u\) in cell \(l\) at subcarrier \(n\).

16. Compute the weights \(s_{n,u,l}\) as follows:

\[s_{n,u,l} = \frac{\theta_{n,u,l}}{f(p)}\]

17. Approximate the posynomial using (2.13).
centralized non-iterative scheme over equal power allocation. Consider $H_1, H_2 = [0.30 \ 0.25; 0.04 \ 0.15] \times 10^{-9}$, $G_{12} = [0.06 \ 0.05; 0.16 \ 0.06] \times 10^{-11}$ and $G_{21} = [0.14 \ 0.69; 0.76 \ 0.1935] \times 10^{-11}$. The equal power allocations dictate $P_1 = [0 \ 0.5; 0 \ 0.5]$ and $P_2 = [0.5 \ 0; 0.5 \ 0]$ which leads to an average network throughput of 11.8392 bps/Hz/cell. However, computing the GP based powers results in $P_1 = [0 \ 0.53; 0 \ 0.47]$ and $P_2 = [0.38 \ 0; 0.62 \ 0]$ which lead to a maximum average network throughput of 17.2734 bps/Hz/cell.

Complexity Analysis

The initial allocation phase has a complexity of $O(UN^2)$ which is the same as Algorithm 1. Since (2.14) has $L$ constraints and variables, the complexity of the power allocation phase is significantly reduced. Although this procedure restricts the degree of freedom offered by GP, numerical results show that the network throughput remains comparable with reduced complexity.

Algorithm 3 Centralized Non-Iterative Scheme

1. Repeat Steps 1 to 5 of Algorithm 2, i.e., initial allocation phase. $n = 1$, do while $n \leq N$, $n = n + 1$

2. Compute the GP based powers $p_{n,u,l}$ of the allocated users at any subcarrier $n$ considering a high SINR regime using (2.14).

3. For each user allocated in a cell $l$ at any subcarrier $n$, divide the left-over power equally among the remaining allocated subcarriers of the user.

4. Remove the subcarrier $n$ from the set of unallocated subcarriers.

2.6 Distributed Resource Allocation Scheme

In the centralized strategy, we assume that $\chi_{n,u,l}$ is known, i.e., every BS knows the interfering gains offered by its users to the neighboring BSs. The interfering gains
are based on path loss, shadowing and fading. Assuming the knowledge of local user positions at each BS, the path loss of local users toward the first tier of interfering cells can be determined, however, the knowledge of shadowing and fading gains is difficult to assume in practical scenarios. Thus, in the distributed approach, we compute our results without using the knowledge of shadowing and fading interfering gains.

Each BS performs the subcarrier allocations without taking ICI into account. In other words we compute single cell near optimal allocations using Algorithm 1. The allocation decisions are locally made at each BS and do not need collaboration. Once the allocations are decided, each cell shares them with all other interfering cells. The GP based optimal powers in (2.7) can then be evaluated in a distributed way using dual decomposition methods by first performing the log transformation of the variables, i.e., \( \ln p_{n,u,l} = \tilde{p}_{n,u,l} \) and \( \ln p_{n,u,j} = \tilde{p}_{n,u,j} \), then adding auxiliary variable \( \ln z_{n,lj} = \tilde{z}_{n,lj} \) where \( z_{n,lj} = p_{n,u,j} \) in order to transfer the coupling in the objective to coupling in the constraints [54]. For given allocations, the problem in (2.7) can thus be written in a distributed way as follows:

\[
\begin{align*}
\text{minimize} & \quad \sum_{n=1}^{N} \sum_{l=1}^{L} \sum_{j=1}^{L} \ln p_{n,u,l} \ln p_{n,u,j} \\
\text{subject to} & \quad \sum_{n=1}^{N} e^{\tilde{p}_{n,u,l}} \leq P_{u,\text{max}}, \forall u, \forall l \\
& \quad z_{n,lj} = p_{n,u,j}, \forall n, \forall l.
\end{align*}
\]

(2.15)

Since the computational complexity of (2.15) is high as it has \( LU \) power constraints and \( LUN \) variables, we present the dual decomposition of (2.14) which is more suitable for practical scenarios and has a lower computational complexity. Moreover, the objective function in (2.15) not only depends on the powers of local users \( p_{n,u,l} \) but also on the power of users sharing the same subcarrier in neighboring cells \( p_{n,u,j} \). Thus, in order to minimize the objective in (2.15), each BS requires the knowledge of interfering gains and interfering transmit powers, that may lead to significant
overhead to exchange control information. Thus, in order to obtain a practical distributed solution, we keep a local copy of each of the effective received powers i.e., 

\[ z_{n,ij} = g_{n,ij}p_{n,ij} \]  

[54]. (2.14) can then be formulated in a distributed way as:

\[
\begin{align*}
\minimize & \quad \sum_{l=1}^{L} \log_{2} \left( \frac{\sigma^2 + \sum_{j=1,j\neq l}^{L} e^{\tilde{z}_{n,ij}}}{e^{\hat{p}_{n,u,l}h_{n,u,l}}} \right) \\
\text{subject to} & \quad e^{\hat{p}_{n,u,l}} \leq P_{n,u,l,eq}, \quad \forall l \\
\end{align*}
\]

(2.16)

The partial lagrange \( L(\tilde{p}_{n,u,l}, \tilde{z}_{n,ij}, \lambda_{l}, \eta_{n,ij}) \) for (2.16) can then be written as:

\[
\sum_{l=1}^{L} \log_{2} \left( \frac{\sigma^2 + \sum_{j=1,j\neq l}^{L} e^{\tilde{z}_{n,ij}}}{e^{\hat{p}_{n,u,l}h_{n,u,l}}} \right) + \sum_{l=1}^{L} \sum_{j=1,j\neq l}^{L} \eta_{n,ij} (\tilde{z}_{n,ij} - g_{n,ij} + \hat{p}_{n,ij}) + \sum_{l=1}^{L} \lambda_{l} (e^{\hat{p}_{n,u,l}} - P_{n,u,l,eq}) .
\]

(2.17)

Eq. (2.17) can be decomposed into \( L \) sub-problems with local variables \( \tilde{p}_{n,u,l}, \tilde{z}_{n,ij}, \lambda_{l} \) and coupling variable \( \eta_{n,ij} \). The simple lagrangian \( L_{l} \) for each cell \( l \) can then be written as follows:

\[
L_{l} = \log_{2} \left( \frac{\sigma^2 + \sum_{j=1,j\neq l}^{L} e^{\tilde{z}_{n,ij}}}{e^{\hat{p}_{n,u,l}h_{n,u,l}}} \right) + \sum_{j=1,j\neq l}^{L} \eta_{n,ij} \tilde{z}_{n,ij} - \left( \sum_{j=1,j\neq l}^{L} \eta_{n,ij} \right) \hat{p}_{n,u,l} + \lambda_{l} (e^{\hat{p}_{n,u,l}} - P_{n,u,l,eq}) ,
\]

(2.18)

where \( \lambda_{l} \) is the lagrange multiplier for the inequality constraints and \( \eta_{n,ij} \) are the consistency prices. Thus, the minimization of (2.18) with respect to the local variables can be done in a distributed way at all BSs. At every iteration, each cell \( l \) receives the term \( \left( \sum_{j=1,j\neq l}^{L} \eta_{n,ij} \right) \) by message passing and minimizes the local Lagrangian (2.18) with respect to the local variables \( \tilde{p}_{n,u,l}, \tilde{z}_{n,ij}, \lambda_{l} \) subject to the local constraints. In order to obtain \( \eta_{n,ij} \) the following master lagrange dual problem has to be solved:

\[
\maximize_{\eta_{n,ij}} \sum_{l=1}^{L} \minimize_{\hat{p}_{n,u,l},\tilde{z}_{n,ij},\lambda_{l}} L_{l} .
\]

(2.19)
A simple way to solve (2.19) is to use the following subgradient update for the consistency prices:

\[ \eta_{n,lj}(t+1) = \eta_{n,lj}(t) + (\delta/t) (\tilde{z}_{n,lj} - \log_2 p_{n,u,j} g_{n,u,jl}) \]  

(2.20)

In summary, each BS minimizes (2.18) in parallel with respect to the local variables after receiving the term \( \sum_{j=1,j\neq l}^{L} \eta_{n,jl} \). Each BS then estimates the received interference \( z_{n,lj} \) from each cell and update the local consistency prices using (2.20). Finally, each BS broadcast them by message passing to all BSs. Note that \( \delta \) in (2.20) represents the step size and is non-negative.

### 2.7 Performance Evaluation

A cellular OFDMA network is considered where the radius of each cell is assumed to be 1 km. The maximum user transmit power is considered to be 1 W. The channel gain is defined as:

\[ h_{n,u,l} = \left( -122 - 10 \gamma \log_{10} d_{u,l} \right) - \mathcal{N}(0, \sigma^2) + 10 \log_{10} F_{n,u,l} \]

where \( d_{u,l} \) is the distance between the \( u \)th user in the interfering cell \( j \) and the \( l \)th BS. The first term denotes the path loss where \( \gamma \) is the path loss exponent and is set equal to 3. The second term represents log-normal shadowing with a mean of 0 dB and a standard deviation of 8 dB. The last factor \( F_{n,u,l} \) corresponds to Rayleigh fading. The bandwidth of the system is assumed to be 20 MHz with a noise power spectral density of \( 8.6455 \times 10^{-15} \) W/Hz at each receiver. The channel conditions are assumed to be fixed during a frame.

The interfering gains from the \( j \)th interfering cell to the cell of interest \( l \) are computed as:

\[ g_{n,u,jl} = \left( -122 - 10 \gamma \log_{10} d_{u,l} \right) - \mathcal{N}(0, \sigma^2) + 10 \log_{10} F_{n,u,jl} \]

where \( d_{u,l} \) is the distance between the \( u \)th user in the interfering cell \( j \) and the \( l \)th BS. We consider two typical simulation scenarios, (i) users are equidistant from the BS and placed at
equally spaced angles, (ii) users are assumed to be uniformly distributed across the whole cellular area.

In Table 2.1, we compare the performance and complexity of the centralized and distributed schemes with the derived bounds and the optimal solution in high SINR regime. The optimal solution is computed by the exhaustive search based subcarrier allocation phase detailed in Section 2.3. All users are placed at equal distance $d$ from the BS and at equally spaced angles (i.e., scenario A). The results are taken after averaging over 100 channel realizations. The simulation results show that the performance gap between the benchmark centralized iterative scheme (with power allocation) and the optimal solution is negligible compared to the low complexity centralized and distributed schemes. However, this observation may not remain valid for bigger network scenarios. Moreover, as $d$ increases the degradation of the average network throughput is evident.

Table 2.1: Average network throughput (in bps/Hz/cell) of the derived bounds, centralized and distributed schemes for $L=2$ cells and $N = 6$ subcarriers/cell.

<table>
<thead>
<tr>
<th></th>
<th>$U=2$ Users</th>
<th>$U=4$ Users</th>
<th>$U=6$ Users</th>
<th>Computation Complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$d=0.5km$</td>
<td>$d=0.9km$</td>
<td>$d=0.5km$</td>
<td>$d=0.9km$</td>
</tr>
<tr>
<td>UB</td>
<td>44.2642</td>
<td>33.1294</td>
<td>55.7414</td>
<td>42.8390</td>
</tr>
<tr>
<td>Optimal</td>
<td>37.1168</td>
<td>29.8642</td>
<td>47.9975</td>
<td>35.6520</td>
</tr>
<tr>
<td>Centralized A</td>
<td>36.8061</td>
<td>28.6973</td>
<td>46.4765</td>
<td>34.0713</td>
</tr>
<tr>
<td>Centralized B</td>
<td>36.4755</td>
<td>27.0352</td>
<td>45.6239</td>
<td>33.1280</td>
</tr>
<tr>
<td>Distributed</td>
<td>35.3623</td>
<td>25.9976</td>
<td>43.5918</td>
<td>31.9231</td>
</tr>
<tr>
<td>LB</td>
<td>35.0966</td>
<td>25.8635</td>
<td>42.5509</td>
<td>31.0261</td>
</tr>
</tbody>
</table>
Figure 2.2: Comparison of all proposed schemes for $L=2$ cells, (a) Scenario A (b) Scenario B: Users are placed at 0.9 km from BS.

Figure 2.3: Comparison of all proposed schemes for $L=4$ cells, (a) Scenario A (b) Scenario B: Users are placed at 0.9 km from BS.
In Fig. 2.2, Fig. 2.3 and Fig. 2.4, we present the performance of the centralized iterative scheme, centralized non-iterative scheme and the distributed scheme for two cells, four cells and seven cells, respectively. The results have been taken after averaging over 10,000 channel realizations and are shown for both simulation scenarios. The performance of all schemes is calibrated using the established upper and LBs. Since the centralized iterative scheme has computationally exhaustive power allocation phase, the results are presented for the subcarrier allocation phase of Algorithm 2 only. However, it can be observed that the scheme still has the capability to serve as a suitable benchmark for the developed low complexity schemes. In order to highlight the significance of the less complex GP problem defined in (2.14), we also present the performance of the centralized non-iterative scheme without power allocation.

For the two cell scenario, the performance gap between the centralized schemes is negligible and they give nearly similar results. However, as the number of cells increases the performance gain of the centralized iterative scheme is evident over all schemes even without power allocation. Moreover, it is also important to note...
the significant degradation in the performance of centralized non-iterative scheme without power allocation phase. This degradation is found to be increasing with the increase in number of cells. It is also worth to mention here that the proposed less complex GP problem (2.14) can be implemented in a distributed way using the techniques explained in [54] and can be used with any set of subcarrier allocations. Thus, in the distributed approach we use the near optimal single cell allocations in conjunction with the less complex GP problem (2.14). The significance of the power allocation phase can be observed easily from the results which becomes more evident for high number of cells. Moreover, the presented results depict the reduction in the average network throughput as the number of interfering cells increases. The performance gap of the proposed schemes increases with respect to the evaluated UB. Even though the UB is not tight and reflects an over optimistic average network throughput, it provides an idea on the performance gap between the proposed schemes and the optimal solution.

2.8 Conclusion

In this chapter, we proposed less complex centralized and distributed schemes that are well-suited for practical scenarios. The computational complexity of all schemes was analyzed and performance were compared through simulations. Simulation results demonstrated that the proposed low complexity schemes can achieve comparable performance to the centralized sub-optimal scheme in various scenarios. Moreover, comparisons with the upper and LBs provided insight on the performance gap between the proposed schemes and the optimal solution. Now, in Chapter 3 we will look more deeply into the theoretical modeling and management of ICI.
Chapter 3

An Interference Model with Channel-based Scheduling

3.1 Introduction

Explosive growth in the demand of high quality wireless data services compel the network designers to utilize spectrum more aggressively which on one side enhances the spectrum efficiency, whereas on the other side it enhances the ICI which is an alarming bottleneck in the telecommunication growth paradigm. The allocation of the same frequency bands across neighboring cells produces ICI which is highly dependent on the statistics of the channel characteristics and on the dynamics of the multiuser scheduling decisions. In this context, it is of immense importance for the system designers to accurately characterize the behavior of the ICI which helps in gaining more theoretical insights, quantifying various network performance metrics and developing efficient resource allocation and interference mitigation schemes.

The chapter presents a novel framework for modeling the uplink and downlink ICI in a multiuser cellular network. The proposed framework assists in quantifying the impact of various fading channel models and multiuser scheduling schemes on the uplink and downlink ICI. Firstly, we derive a semi-analytical expression for the distribution of the location of the scheduled user in a given cell considering a wide
range of scheduling schemes. Based on this, we derive the distribution and MGF of the ICI considering a single interfering cell. Consequently, we determine the MGF of the cumulative ICI observed from all interfering cells and derive explicit MGF expressions for three typical fading models. Finally, we utilize the obtained expressions to evaluate important network performance metrics such as the outage probability, ergodic capacity and average fairness numerically. Monte-Carlo simulation results are provided to demonstrate the efficacy of the derived analytical expressions.

3.1.1 Proposed vs Stochastic Cellular Network Modeling

The increased complexity of modern cellular networks require a theoretical approach to calculate the coverage, rate and reliability accurately. The tractable expressions for the coverage probability or outage probability are typically unavailable even for a randomly located mobile user, thus making it more difficult to derive general results which help in efficiently identifying the key design parameters.

Utilizing stochastic geometry to model cellular networks, has attracted much attention recently in the research community [57, 49, 58, 59]. Compared with the traditional Wyner or grid model, the approach models the BS locations by Poisson point process and then utilizes the mathematical results in stochastic geometry to study the average behavior over several spatial realizations. It has been shown that such network models are as accurate as the traditional grid models for both single and multiple tier scenarios, and provide tractable analytical tools to derive the pessimistic lower bound on coverage. Additionally, the geometry characteristics of unplanned heterogeneous networks are also well captured by the stochastic geometry model[60]. However, the stochastic modeling approach is (i) characterized for an entire system not for a given cell. As cellular networks are already built out, network providers will often want to know the performance they achieve in some given cells by adding additional infrastructure (and thus interference) in the rest of the network; (ii)
It is also challenging to incorporate more complex kinds of heterogeneous infrastructure like fixed relays or distributed antennas into the signal and interference models; (iii) focused mainly for downlink networks where a typical user is considered to be located at origin. Whereas perfect power control is assumed for uplink networks; (iv) mandates a specific user association policy; (v) difficult to analyze the interference statistics for various resource allocation schemes and frequency reuse patterns.

The remainder of this chapter is organized as follows. Section 3.2 presents the system model and the main steps of the proposed framework. In Section 3.3, the distribution of the scheduled user location is derived for different scheduling algorithms. In Section 3.4, the distribution of the uplink ICI from one neighboring cell is derived. The MGF of the cumulative ICI from all interfering cells is determined in Section 3.5 and utilized in Section 3.6 to evaluate three network performance metrics. Finally, numerical and simulation results are presented and analyzed in Section 3.7, and conclusions are drawn in Section 3.8.

### 3.2 System Model and Proposed Framework

#### 3.2.1 Description of the System Model

We consider a given cell surrounded by $L$ neighboring cells. For analytical convenience, the cells are assumed to be circular with radius $R^1$. Each cell $l$ is assumed to have $U$ uniformly distributed users. The frequency reuse factor is assumed to be unity with each subcarrier reused in all cells. The bandwidth of a subcarrier is assumed to

---

1The circular shape interference models can be extended to hexagonal models using the analytical approach presented in [61]. The chapter discussed, evaluated and compared two common modeling assumptions for the shape of the cells in a wireless cellular network, i.e., the hexagonal and the circular cell shapes. In practice, use of the hexagonal cells instead of the circular cell approximation provide more suitable results for the planning of wireless networks. However, it was concluded that the inradius circular approximation gives results closer to the hexagonal approach compared to the circumradius one. The chapter provides a review of analytical co-channel interference models and the derived formulation allows the determination of the impact of cell shape on system performance.
be less than the channel coherence bandwidth, thus, each subcarrier experiences flat fading. Time is divided into time slots of length smaller than the channel coherence time and, thus, the channel variation within a given time slot is negligible.

Generally, the scheduling strategies can be broadly categorized into two classes; (i) rate maximization (i.e., rate adaptation) while transmitting with constant/maximum power; (ii) power minimization (i.e., power adaptation) while achieving a fixed data rate. In this work, we focus on rate adaptive schemes where users transmit with their maximum power in order to maximize their rate depending on the existing channel and interference conditions. Therefore, for the scope of this chapter, we assume that all users transmit with their maximum power $P_{\text{max}}$ on a given subcarrier with rate adaptation depending on their channel qualities. At this point, it is also important to emphasize that this is not a limitation and the approach can be extended for various uplink power control mechanisms. The instantaneous SNR $\gamma$ of each user can then be written as follows:

$$\gamma = P_{\text{max}} C\frac{r^{-\beta}\psi\eta}{\sigma^2} = \bar{K} r^{-\beta} \zeta,$$

(3.1)

where $\bar{K} = \frac{P_{\text{max}} C}{\sigma^2}$, $C$ is the path loss constant, $r$ is the user distance from its serving BS, $\psi$ and $\eta$ denotes the shadowing and small scale fading coefficient between user and BS on a given subcarrier, respectively, $\beta$ is the path loss exponent, $\sigma$ denotes the thermal noise at the receiver and $\zeta$ is the composite fading. Note that all users are assumed to be associated with their closest BS [20, 62], therefore $r \leq R$. For simplicity we consider a system with single antenna at transmitter and receiver, however, the derived expressions are expected to be extendible for multiple antenna systems \(^2\)

Each cell is divided into $K$ concentric circular regions. Since path loss decays exponentially from cell center to cell edge, therefore, we consider discretization of

\(^2\)Multiple antenna systems increase the ability to obtain diversity gains which helps in increasing the data rate without increasing transmit power or bandwidth. For SIMO systems, i.e., several omnidirectional antennas are used at the BS and each antenna has the same frequency band allotted than the capacity gains are expected to be calculated by extending the results presented in the thesis.
cellular region in such a way that the path loss decay within each circular region remains constant or uniform. The main motivation for dividing the cell into a discrete set of circular regions relies on the fact that the channel statistics of the users located in a given circular region become relatively similar especially for large values of $K$. More explicitly, the characterization of the circular regions can be demonstrated as:

$$\kappa = 10^{\beta \log_{10} r_k} - 10^{\beta \log_{10} r_{k-1}}, \quad r \leq R,$$

(3.2)

where $\kappa$ is the path loss decay within each circular region [dB]. Due to the exponential nature of the path loss, it varies rapidly near the cell center than at the cell edge, therefore, (i) each of the $k^{th}$ circular region bounded by two adjacent rings, i.e., $r_k$ and $r_{k-1}$ possess non-uniform width $\Delta_k = r_k - r_{k-1}$; (ii) the number of circular regions are high in the cell center than at the cell edge; (iii) the average number of users located within $k^{th}$ circular region bounded by ring $r_k$ and $r_{k-1}$ are considered to be located at $r_k$. Note that, this is an approximation which is required for deriving the analytically tractable model of ICI and in any case it is not required for the Monte-Carlo simulations. The average number of users in each ring $k$ are given by:

$$u_k = \frac{U(r_k^2 - r_{k-1}^2)}{R^2}, \quad k = 1, 2, \cdots, K.$$

(3.3)

### 3.2.2 Main Steps of the Proposed Framework

In order to characterize the statistics of the uplink and downlink ICI for generalized fading channels and various scheduling schemes, the proposed framework mandates the following steps:

1. Derive the distribution $f_{r_{\text{sel}}}(r)$ of the distance of the allocated user $r_{\text{sel}}$ in a given
Figure 3.1: Geometrical illustration of dividing the cellular network into multiple rings of non-uniform width $\Delta_k$.

cell from its serving BS based on the deployed scheduling scheme. Without loss of generality, we assume the same scheduling scheme is implemented in all cells; therefore, $f_{r_{\text{sel}}}(r)$ remains the same for all cells.

2. Derive the distribution $f_{\tilde{r}_{\text{sel}}} (\tilde{r})$ of the distance between the allocated user in a neighboring interfering cell and the BS of the cell of interest $\tilde{r}_{\text{sel}}$.

3. Derive the distribution $f_{X_{l}}(x)$ of the interference from the neighboring cell $l$.

Finally, derive the MGF of the cumulative ICI, i.e., $Y = \sum_{l=1}^{L} X_{l}$.

3.3 Distribution of the Scheduled User Location

Considering the high dependence of the uplink ICI on the location of the scheduled users in the neighbor interfering cells which in turn depends on the deployed scheduling schemes, we derive in this section the distribution of the distance between the scheduled user and its serving BS in a given cell (i.e., the probability mass function (PMF) of $r_{\text{sel}}$) considering the following five scheduling algorithms: greedy schedul-
ing, proportional fair scheduling, round robin scheduling, location based round robin scheduling, and greedy round robin scheduling.

3.3.1 Greedy Scheduling Scheme

Greedy scheduling is an opportunistic scheme that aims at maximizing the network throughput by taking full advantage of multiuser diversity. However, it suffers from low fairness among users which makes it less attractive for network operators. The procedure for determining the PMF of \( r_{sel} \) considering greedy scheduling is divided into two steps:

**Step 1 (Selecting the user with the highest SNR in ring \( k \)):** Since the path loss decay within each circular region is considered to be uniform, we approximate the distance of all users located within \( k^{th} \) circular region by ring \( r_k \) for analytical tractability as we already mentioned before. In this step, we select a user with maximum SNR in each ring \( k \) which posses \( u_k \) users. Thus, selecting a user in a ring \( k \) is equivalent to selecting the user with maximum channel gain among all the users in ring \( k \), i.e.,

\[
\zeta_k = \max\{\zeta_1, \zeta_2, \ldots, \zeta_i, \ldots, \zeta_{u_k}\},
\]

where \( \zeta_i \) is the composite fading channel gain between user \( i \) and its BS on a given subcarrier. The CDF and PDF of the maximum channel gain \( \zeta_k \) can be written as:

\[
F_{\zeta_k}(\zeta_k) = \prod_{i=1}^{u_k} F_{\zeta_i}(\zeta_k)^{i.i.d} = (F_{\zeta}(\zeta_k))^{u_k},
\]

\[
f_{\zeta_k}(\zeta_k) = \sum_{j=1}^{u_k} f_{\zeta_j}(\zeta_k) \prod_{i=1, i\neq j}^{u_k} F_{\zeta_i}(\zeta_k)^{i.i.d} = u_k f_{\zeta}(\zeta_k) (F_{\zeta}(\zeta_k))^{u_k-1}.
\]

To consider path loss, we now perform a transformation of RVs using (5.17), \( \gamma_k = K r_k^{-\beta} \zeta_k \), where, \( \gamma_k \) is the selected user SNR in each ring \( k \). The CDF and PDF of \( \gamma_k \)
can then be written as follows:

\[ F_{\gamma_k}(\gamma_k) = \prod_{i=1}^{u_k} F_{\zeta_i}(\bar{K}^{-1} \gamma_k r_k^\beta) = \left( F_{\zeta}(\gamma_k r_k^\beta \bar{K}^{-1}) \right)^{u_k}, \]  

(3.7)

\[ f_{\gamma_k}(\gamma_k) = \frac{1}{r_k^{-\beta}} \sum_{j=1}^{u_k} f_{\zeta_j}(\gamma_k r_k^\beta \bar{K}^{-1}) \prod_{i=1, i \neq j}^{u_k} F_{\zeta_i}(\gamma_k r_k^\beta \bar{K}^{-1}) = \frac{u_k}{r_k^{-\beta}} f_{\zeta}(\gamma_k r_k^\beta \bar{K}^{-1}) \left( F_{\zeta}(\gamma_k r_k^\beta \bar{K}^{-1}) \right)^{u_k-1}. \]  

(3.8)

**Step 2 (Selecting the user with maximum SNR among K rings):** In this step, we compute the probability of selecting the \( k \)th ring among all other rings. It is important to note that this is equivalent to selecting the ring \( k \) which possesses the user with the highest SNR among all rings. Conditioning on \( \gamma_k \), the PDF of \( r_{\text{sel}} \) can be written explicitly as follows:

\[ P(r_{\text{sel}} = r_k | \gamma_k) = \prod_{i=1, i \neq k}^{K} p(\gamma_i \leq \gamma_k) = \prod_{i=1, i \neq k}^{K} F_{\gamma_i}(\gamma_k). \]  

(3.9)

By averaging over the distribution of \( \gamma_k \), the final expression for the PMF of \( r_{\text{sel}} \) is

\[ P(r_{\text{sel}} = r_k) = \int_0^{\infty} \left( \prod_{i=1, i \neq k}^{K} F_{\gamma_i}(\gamma_k) \right) f_{\gamma_k}(\gamma_k) d\gamma_k. \]  

(3.10)

Using (3.7), (3.10) can be written for i.i.d. case as follows:

\[ P(r_{\text{sel}} = r_k) = \int_0^{\infty} \prod_{i=1, i \neq k}^{K} \left( F_{\zeta}(\gamma_k r_k^\beta) \right)^{u_i} \frac{u_k}{r_k^{-\beta}} f_{\zeta}(\gamma_k r_k^\beta) \left( F_{\zeta}(\gamma_k r_k^\beta) \right)^{u_k-1}, \]  

(3.11)

where \( r_{\text{sel}} \in [0, R] \). The results in (3.11) are generalized for any shadowing and fading statistics. Even though (3.11) is not a closed form expression, the integration can be solved accurately using standard mathematical software packages such as MAPLE and MATHEMATICA.
3.3.2 Proportional Fair Scheduling Scheme

The proportional fair scheduling scheme allocates the subcarrier to the user with the largest normalized SNR \((\gamma / \bar{\gamma})\) [63], where \(\gamma\) and \(\bar{\gamma}\) denote the instantaneous SNR and the short term average SNR of a given user, respectively. In other words, the selection criterion is based on selecting a user who has maximum instantaneous SNR relative to its own average SNR. The distribution of \(r_{sel}\) can be derived as follows:

**Step 1** *(Selecting the user with maximum normalized SNR in ring \(k\)):* In this step, the performance of proportional fair scheduling scheme is independent of the path loss factor if users are moving relatively slowly, i.e., their path loss remains nearly the same on a short term basis. In this case, the problem of selecting the maximum normalized SNR in a ring \(k\) can be written as:

\[
\zeta_k = \max \left\{ \frac{\zeta_1}{\bar{\zeta}_1}, \frac{\zeta_2}{\bar{\zeta}_2}, \ldots, \frac{\zeta_i}{\bar{\zeta}_i}, \ldots, \frac{\zeta_{u_k}}{\bar{\zeta}_{u_k}} \right\},
\]

where \(\bar{\zeta}_i = \int_0^\infty \zeta_i f_{\zeta_i}(\zeta_i)d\zeta_i\) is the average of the composite fading channel and \(\zeta_k\) is the maximum normalized composite fading channel gain in ring \(k\). For i.i.d. composite fading gains of the users located in ring \(k\), i.e., \(\bar{\zeta} = \bar{\zeta}_1 = \bar{\zeta}_2 = \cdots \bar{\zeta}_{u_k}\), the problem of selecting the user with maximum normalized channel gain reduces to selecting the user with the maximum channel gain, i.e., \(\zeta_k = \max \{\zeta_1, \zeta_2, \ldots, \zeta_i, \ldots, \zeta_{u_k}\}\). Thus, for any ring \(k\), the CDF and PDF of the selected SNR \(\gamma_k = K r_k^{\beta} \zeta_k\) is given as:

\[
F_{\gamma_k}(\gamma_k) = \prod_{i=1}^{u_k} F_{\zeta_i}(\bar{\zeta}_i \gamma_k r_k^{\beta} K^{-1})^{i.i.d} (F_{\zeta_i}(\bar{\zeta}_i \gamma_k r_k^{\beta} K^{-1}))^{u_k},
\]

\[
f_{\gamma_k}(\gamma_k) = \frac{1}{r_k^{-\beta}} \sum_{j=1}^{u_k} f_{\zeta_j}(\bar{\zeta}_j \gamma_k r_k^{\beta} K^{-1}) \prod_{i=1, i \neq j}^{u_k} F_{\zeta_i}(\bar{\zeta}_i \gamma_k r_k^{\beta} K^{-1})^{i.i.d} \left( \frac{u_k}{r_k^{-\beta}} (F_{\zeta_i}(\bar{\zeta}_i \gamma_k r_k^{\beta} K^{-1}))^{u_k-1} f_{\zeta_i}(\bar{\zeta}_i \gamma_k r_k^{\beta} K^{-1}) \right).
\]
The short term average SNR of the selected user in ring $k$, i.e., $\bar{\gamma}_k$ can then be computed as $\bar{\gamma}_k = \int_0^\infty \gamma_k f_{\gamma_k}(\gamma_k)d\gamma_k$. Finally, the normalized selected SNR in each ring $k$ can be defined as $\xi_k = \frac{\gamma_k}{\bar{\gamma}_k}$ and performing a transformation of RVs, the CDF and PDF of $\xi_k$ can be given as $F_{\xi_k}(\xi_k) = F_{\gamma_k}(\bar{\gamma}_k \xi_k)$; and $f_{\xi_k}(\xi_k) = \frac{1}{\bar{\gamma}_k} f_{\gamma_k}(\bar{\gamma}_k \xi_k)$, respectively.

**Step 2** *(Selecting the ring $k$ with maximum normalized SNR from the $K$ rings)*: Once we characterize the PDF and CDF of $\xi_k$, the probability of selecting any ring $k$ can be written using (3.9) as follows:

$$P(r_{\text{sel}} = r_k) = \int_0^\infty \left( \prod_{i=1, i\neq k}^{K} F_{\xi_i}(\xi_k) \right) f_{\xi_k}(\xi_k)d\xi_k.$$  \hfill (3.15)

### 3.3.3 Round Robin Scheduling Scheme

Round robin scheduling is a non-opportunistic scheme where a user is selected randomly within a time slot. As each user has equal probability of allocation, the scheme can be referred to as a strictly fair scheduling scheme. The round robin scheme provides maximum fairness among users and may serve as a lower bound in terms of network throughput which is useful in calibrating the performance of other scheduling schemes, however, the resulting network throughput is significantly low which makes it less attractive for practical implementations. The PMF of the scheduled user location can then be given as:

$$P(r_{\text{sel}} = r_k) = \frac{u_k}{U}.$$  \hfill (3.16)

### 3.3.4 Location Based Round Robin Scheduling Scheme

Location based round robin is another non-opportunistic scheduling scheme which does not require any channel state information, however, it requires the location information of the users. Even though location based scheduling is not common in practice,
the location of each mobile user can be determined at the BS using global positioning system (GPS) or estimate based on a power measurement of pilot signals from the surrounding beacons, e.g., using triangulation based techniques. In this regard, there are variety of techniques available in the literature which demonstrate how the location of users can be evaluated at the BS (see[64, 65] and the references therein). Moreover, the users located in different circular regions can also be classified based on the long term average SNRs, i.e., by computing SNR thresholds for different distances (rings) [37] which is a common technique in fractional frequency reuse (FFR) systems to distinguish between cell-edge and cell center users.

In this context, we consider $W$ time slots during which the distance of the users from their serving BS will remain approximately the same. For simplicity, the number of time slots $W$ is set equal to $K$. At a given time slot $T_w$, we select any arbitrary user from a specific ring (analysis) and circular region (simulations) starting from the cell center. We continue to allocate the users by accessing the circular rings sequentially from cell center to cell edge. At this point, it is important to emphasize that all cells are considered to be time synchronous in allocating the users from particular rings, i.e., at a given time slot $T_w$ all cells are selecting the $w^{th}$ ring. Thus, the PMF of $r_{sel}$ for a given time slot $w$ denoted by $P(r_{sel} = r_k^{T_w})$ can be given as:

$$P(r_{sel} = r_k^{T_w}) = \begin{cases} 
1, & \text{if } k = w \\
0, & \text{else.} 
\end{cases}$$

(3.17)

The scheme can produce relatively high capacity gains on average compared to the traditional round robin scheme. Moreover, the average fairness measure is very close to the traditional round robin.
3.3.5 Greedy Round Robin Scheduling Scheme

Greedy round robin is an opportunistic scheduling scheme which captures the multiuser diversity while maintaining some fairness among users. In this scheme, we consider $W = K$ time slots during which the distance of the users from their serving BSs remain nearly the same, however, the small scale fading gain on the considered subcarrier may vary from one time slot to the other. We select the user with maximum SNR in each time slot $T_w$, however once a user is selected from a ring, all users located in that ring will not be scheduled for transmission for the next $K - 1$ time slots. Note that all BSs are considered to be time synchronized in terms of scheduling.

Clearly, the probability of allocating a ring $k$ at $T_1$ can simply be given by (3.11). However, the probability of selecting a ring $k$ at $T_2$ is a dependent event and can be derived using Bayes theorem as follows:

$$P(r_{sel} = r_{k}^{T_2}) = \sum_{j=1}^{K} \left( P\left( \frac{r_{sel} = r_{k}^{T_2}}{r_{sel} = r_{j}^{T_1}} \right) P(r_{sel} = r_{j}^{T_1}) \right), \quad (3.18)$$

where,

$$P\left( \frac{r_{sel} = r_{k}^{T_2}}{r_{sel} = r_{j}^{T_1}} \right) = \int_0^{\infty} \prod_{i \neq k, i \neq j} P_{\gamma_k r_i^\beta} \frac{u_k f_{\gamma_k r_k^\beta}}{r_k^\beta} \left( F_{\gamma_k r_i^\beta} \right)^{u_k - 1} \gamma_k. \quad (3.19)$$

Since the probability of allocating any ring $k$ within time slot $T_w$ depends on all previous states, therefore, the principle of Markov chain transition probabilities is not directly applicable. For more clarity, the probability of selecting a ring $k$ at $T_3$ is given as follows:

$$P(r_{sel} = r_{k}^{T_3}) = \sum_{m \neq k}^{K} \sum_{j \neq m}^{K} P\left( \frac{r_{sel} = r_{k}^{T_3}}{r_{sel} = r_{j}^{T_2} \cap r_{sel} = r_{m}^{T_1}} \right) P\left( \frac{r_{sel} = r_{j}^{T_2}}{r_{sel} = r_{m}^{T_1}} \right) P(r_{sel} = r_{m}^{T_1}), \quad (3.20)$$
where,

\[
P \left( \frac{\mathbf{r}_{\text{sel}} = r_k T_3}{\mathbf{r}_{\text{sel}} = r_j T_2 \cap \mathbf{r}_{\text{sel}} = r_m T_1} \right) = \int_0^\infty \prod_{\substack{i \neq k \\
i \neq j, i \neq m}}^K \left( F_{\zeta}^\prime(\gamma_k r_i^\beta) \right)^{u_i} \frac{u_k f_{\zeta}^\prime(\gamma_k r_k^\beta)}{r_k^{-\beta}} \left( F_{\zeta}^\prime(\gamma_k r_k^\beta) \right)^{u_k-1} d\gamma_k.
\]

(3.21)

In general, the probability of selecting any ring \( k \) at a time slot \( T_w \) can be given as:

\[
P(r_{\text{sel}} = r_k^{T_N}) = \sum_{n \neq k}^{K} \sum_{s \neq n}^{K} \cdots \sum_{j \neq s, n \ldots}^{K} P \left( \frac{\mathbf{r}_{\text{sel}} = r_k^{T_w}}{\mathbf{r}_{\text{sel}} = r_j^{T_{w-1}} \cap \mathbf{r}_{\text{sel}} = r_s^{T_2} \cap \mathbf{r}_{\text{sel}} = r_n^{T_1}} \right) \times \cdots \times P \left( \frac{\mathbf{r}_{\text{sel}} = r_s^{T_2}}{\mathbf{r}_{\text{sel}} = r_n^{T_1}} \right) \times P(r_{\text{sel}} = r_m^{T_1}).
\]

(3.22)

**Computational Efficiency:** The time complexity of the greedy round robin scheme is heavily based on the computational time of the \texttt{NIntegrate} operation in \textsc{Mathematica}. One \texttt{NIntegrate} operation requires around 0.95 sec which is equivalent to the computational complexity of (i) greedy scheme and (ii) the first time slot of the greedy round robin scheme. Monte-Carlo simulation time required for 100,000 trials in greedy scheme requires around 150.67 sec which is more than 150 times the computational complexity of \texttt{NIntegrate} operation. This fact demonstrates the computational efficiency of greedy scheme in comparison to Monte-Carlo simulations. However, in the second time slot, greedy round robin scheme requires \( K - 1 \) integrations whereas, for the third time slot \( K - 2 \) integrations are required. Therefore, in general the time complexity of greedy round robin scheme at any time slot \( w \) is given as:

\[
\text{Computational time[sec]} = 0.95 + \sum_{i=2}^{w} 0.95(K - i + 1); \quad w \leq K,
\]

(3.23)

where \( K \) denotes the number of rings. Therefore considering \( w = 15 \) and \( K = w \) for greedy round robin, the analytical time complexity is around 113 sec which is still lower than the Monte-Carlo simulation time required for the greedy scheme.
Therefore, even though the greedy round robin scheme is computationally complex for large $W$, the evaluation time remains comparable to the Monte-Carlo simulations.

### 3.3.6 Evaluating the Joint PMF of $r_{\text{sel}}$ and $\theta$

Note that, $P(r_{\text{sel}} = r_k)$ derived for all of the above scheduling schemes is the marginal PMF of $P(r_{\text{sel}} = r_k, \theta = \theta_i)$ where $\theta$ denotes the angle of the allocated user with respect to the serving BS and it is uniformly distributed from 0 to $2\pi$ (see Fig. 1). Although the PDF of $\theta$ is continuous we can discretize it for analytical consistency and complexity reduction. Consider discretizing the range of RV $\theta$ in $I^3$ uniform angular intervals of desired accuracy. Thus $P(\theta = \theta_i) = \frac{1}{I}$ where $\theta_i$ denotes any discrete value that the RV $\theta$ can take. Since $r_{\text{sel}}$ and $\theta$ are independent\(^4\), their joint PMF is:

\[
P(r_{\text{sel}} = r_k, \theta = \theta_i) = P(r_{\text{sel}} = r_k)P(\theta = \theta_i) = \frac{P(r_{\text{sel}} = r_k)}{I}.
\] (3.24)

### 3.4 Distribution of ICI from One Cell

The derivation for the distribution of the ICI from an interfering cell $l$, i.e., $f_{X_l}(x)$, depends on the distribution of the distance between the allocated user in the interfering cell $l$ and the BS of interest, i.e., $f_{\tilde{r}_{\text{sel}}}(\tilde{r})$. As mentioned earlier, each interfering cell is assumed to have identical conditions in a given time slot. Therefore, $f_{\tilde{r}_{\text{sel}}}(\tilde{r})$ applies to all interfering cells and, thus, we will drop the subscript $l$ in the sequel to simplify notation. Using the cosine law (see Fig. 3.1), we can write:

\[
\tilde{r}_{\text{sel}}^2 = r_{\text{sel}}^2 + D^2 - 2r_{\text{sel}}D \cos \theta,
\] (3.25)

\(^3\)The number of angular intervals can vary within the range of 72 to 720 with negligible loss of accuracy

\(^4\)As the users are uniformly distributed in a two dimension circle around the origin, therefore, the location information of any user in terms of radius and polar angle is completely independent
where $\tilde{r}_{\text{sel}}$ is the distance of the allocated user in the interfering cell $l$ from the BS of interest, $r_{\text{sel}}$ is the distance of the allocated user from its serving BS, i.e., (BS $l$), $\theta \in [0, 2\pi]$ and $D = 2R$ since we consider universal frequency reuse with one tier of interfering cells. The approach can be extended to any number of tiers in a straightforward manner. In order to determine the PMF of $\tilde{r}_{\text{sel}}$ where $\tilde{r}_{\text{sel}} \in [D - R, D + R]$, first of all we compute $\tilde{r}_{i,k}$ for given $\theta_i$ and $r_k$ using (3.25) as follows:

$$
\tilde{r}_{i,k}^2 = r_k^2 + D^2 - 2r_kD \cos \theta_i \quad \forall r_k, \forall \theta_i,
$$

(3.26)

where $\tilde{r}_{i,k}$ denotes the interfering distance from a specified polar coordinate $(r_k, \theta_i)$ in the interfering cell to the BS of interest located at a distance $D$ (see Fig. 3.1). In addition, it is worth to mention that $\tilde{r}_{i,k}$ are the points at which $P(\tilde{r}_{\text{sel}} = \tilde{r}_{i,k})$ can be defined using (3.24) as follows:

$$
P(\tilde{r}_{\text{sel}} = \tilde{r}_{i,k}) = \frac{P(r_{\text{sel}} = r_k)}{I}.
$$

(3.27)

The two dimensional data set of $\tilde{r}_{\text{sel}}$, at which $P(\tilde{r}_{\text{sel}} = \tilde{r}_{i,k})$ is defined, can then be grouped into $M$ segments of any arbitrary width $\Delta$. This can be done by dividing the distance between $D - R$ and $D + R$ into $M$ equal segments of width $\Delta$ and mapping $\tilde{r}_{i,k}$ accordingly. Clearly, by adding all the probabilities for which $\tilde{r}_{\text{sel}}$ lies in the $m^{th}$ segment we get the probability of $\tilde{r}_{\text{sel}} = \tilde{r}_m$:

$$
P(\tilde{r}_{\text{sel}} = \tilde{r}_m) = \sum_{\tilde{r}_{i,k} \in [\tilde{r}_m - \frac{\Delta}{2}, \tilde{r}_m + \frac{\Delta}{2}]} P(\tilde{r}_{\text{sel}} = \tilde{r}_{i,k}),
$$

(3.28)

where $\tilde{r}_m$ denotes any discrete value that the RV $\tilde{r}_{\text{sel}}$ can take. Recall $X = \tilde{K} \tilde{r}^{-\beta} \chi$, therefore the PDF of $X$ conditioned on $\tilde{r}_{\text{sel}}$ can be determined by RV transformation
as follows:

\[ f_{X|\tilde{r}_{sel}} = \frac{f_X(x\tilde{r}_{sel}^\beta K^{-1})}{K\tilde{r}_{sel}^{-\beta}}. \]  \hspace{1cm} (3.29)

Averaging over the PMF of \( \tilde{r}_{sel} \), the distribution of the ICI, \( f_X(x) \), from any cell \( l \) can be given as:

\[ f_X(x) = \tilde{r}_M \sum_{\tilde{r}_m = \tilde{r}_1}^{\tilde{r}_M} \frac{f_X(x\tilde{r}_m^\beta K^{-1})}{K\tilde{r}_m^{-\beta}} \mathbb{P}(\tilde{r}_{sel} = \tilde{r}_m). \]  \hspace{1cm} (3.30)

It is important to emphasize that the derivation of the distribution of ICI is based on the scheduling decisions of interfering cells at a given time slot. Therefore, the parameter \( \tilde{r}_m \) of the ICI distribution varies from one time slot to the other for the location based round robin and greedy round robin schemes.

### 3.5 MGF of the Cumulative ICI

Computing the distribution of the cumulative ICI \( Y \) requires the convolution of the PDF of \( L \) RVs \( X_l, \forall l = 1, 2, \cdots L \), which is a tedious task for many practical scenarios. To avoid the convolutions, we utilize an MGF based approach and derive the expression for the MGF of the cumulative ICI \( Y \).

#### 3.5.1 Derivation for the Uplink ICI

Since each cell is considered to have same scheduling scheme deployed, therefore, the MGF of the cumulative interference considering i.i.d. interferers can be calculated as:

\[ \mathcal{M}_Y(t) = \prod_{l=1}^{L} \mathcal{M}_{X_l}(t) = (\mathcal{M}_{X}(t))^L = \left(\mathbb{E}[e^{tx}]\right)^L. \]  \hspace{1cm} (3.31)

Looking at the structure of (3.30), we can write \( \mathcal{M}_{X}(t) \) as:

\[ \mathcal{M}_{X}(t) = \int_0^{\infty} e^{tx} f_X(x) dx = \sum_{\tilde{r}_m = \tilde{r}_1}^{\tilde{r}_M} \frac{P(\tilde{r}_{sel} = \tilde{r}_m)}{K\tilde{r}_m^{-\beta}} \int_0^{\infty} e^{tx} f_X(x\tilde{r}_m^\beta K^{-1}) dx. \]  \hspace{1cm} (3.32)
The derived expression is generic and applies to any composite fading distribution. Next, we will present explicit MGF expressions for the uplink ICI considering three typically used practical fading models.

**Special Case 1: Rayleigh fading** $-\zeta, \chi \sim \text{Exp}(\lambda)$: In this case, the small scale fading coefficient on a given subcarrier is considered to be Rayleigh distributed whereas the effect of shadowing is not considered. The distribution of interference considering a single interfering cell can then be derived as:

$$f_X(x) = \sum_{\tilde{r}_m = \tilde{r}_1}^{\tilde{r}_M} \tilde{K}^{-1} \lambda \tilde{r}_m^\beta e^{-\lambda \tilde{r}_m^\beta} K^{-1} x P(\tilde{r}_{\text{sel}} = \tilde{r}_m), \quad (3.33)$$

Note that (3.33) is a Hyper-Exponential distribution with parameter $\tilde{K}^{-1} \lambda \tilde{r}_m^\beta$. Thus, using the MGF of the Hyper-Exponential distribution, $M_Y(t)$ can be derived as follows:

$$M_Y(t) = \left( \sum_{\tilde{r}_m = \tilde{r}_1}^{\tilde{r}_M} \frac{\tilde{K}^{-1} \lambda \tilde{r}_m^\beta}{\tilde{K}^{-1} \lambda \tilde{r}_m^\beta - t} P(\tilde{r}_{\text{sel}} = \tilde{r}_m) \right)^L. \quad (3.34)$$

**Special Case 2: Generalized-$\mathcal{K}$ composite fading** $-\zeta, \chi \sim \mathcal{K}_G(m_s, m_c, \Omega)$: In wireless channels, shadowing and fading across the channel between a user and BS can be jointly modeled by a composite fading distribution. A closed form composite fading model, namely Generalized-$\mathcal{K}$ also referred to as Gamma-Gamma distribution, has been recently introduced in [66] which is general enough to model well-known shadowing and fading distributions such as log-normal, Nakagami-$m$, Rayleigh etc. Using (5.30), $f_X(x)$ in this case can be derived as follows:

$$f_X(x) = \sum_{\tilde{r}_m = \tilde{r}_1}^{\tilde{r}_M} \frac{2(x \tilde{r}_m^\beta \tilde{K}^{-1})^{m_c+m_s-2}}{K^{-1} \tilde{r}_m^\beta \Gamma(m_c) \Gamma(m_s)} K_{m_s-m_c} b \sqrt{x \tilde{r}_m^\beta \tilde{K}^{-1}} \left( b \right) \frac{m_c+m_s}{2} P(\tilde{r}_{\text{sel}} = \tilde{r}_m), \quad (3.35)$$

where, $K_v(\cdot)$ denotes the modified Bessel function of second kind with order $v$, $b = 2 \sqrt{\frac{m_c+m_s}{\tilde{K}}}$. Performing some algebraic manipulations and using [67, Eq. 6.643/3], the
expression for $\mathcal{M}_X(t)$ can be derived as:

$$
\mathcal{M}_X(t) = \sum_{\tilde{r}_m = \tilde{r}_1}^{\tilde{r}_M} P(\tilde{r}_{sel} = \tilde{r}_m^{\beta}) \mathbb{W}_{\frac{1-m_s}{2}, \frac{m_s}{2}} \left( \frac{-\tilde{r}_m^{\beta} b^2}{4Kt} \right) e^{r_s \tilde{r}_m^{\beta}} \left( \frac{-b^2 \tilde{r}_m^{\beta}}{4Kt} \right)^{\frac{m_s+m_c-1}{2}},
$$

(3.36)

where $\mathbb{W}(\cdot)$ denotes the Whittaker function. Finally, $\mathcal{M}_Y(t)$ can be written as follows:

$$
\mathcal{M}_Y(t) = \sum_{\tilde{r}_m = \tilde{r}_1}^{\tilde{r}_M} P(\tilde{r}_{sel} = \tilde{r}_m^{\beta}) \mathbb{W}_{\frac{1-m_s}{2}, \frac{m_s}{2}} \left( \frac{-\tilde{r}_m^{\beta} b^2}{4Kt} \right) e^{r_s \tilde{r}_m^{\beta}} \left( \frac{-b^2 \tilde{r}_m^{\beta}}{4Kt} \right)^{\frac{m_s+m_c-1}{2}},
$$

(3.37)

Integrating the CDF and MGF of Generalized-$\mathcal{K}$ RV which involves Meijer-G and Whittaker functions, respectively, in MATHEMATICA and MAPLE can be time consuming.

**Special Case 3: Gamma Composite Fading** - $\zeta, \chi \sim \text{Gamma}(m_s, m_c)$: In [66], the authors proposed an accurate approximation of the Generalized-$\mathcal{K}$ RV by the more tractable gamma distribution using moment matching method. The approximation provides a simplifying model for the composite fading in wireless communication systems. Using (5.30), $f_X(x)$ can be written in this case as:

$$
f_X(x) = \sum_{\tilde{r}_m = \tilde{r}_1}^{\tilde{r}_M} \frac{e^{\frac{x \tilde{r}_m^{\beta} K^{-1}}{m_c} (x \tilde{r}_m^{\beta} K^{-1})^{m_s-1}}}{K \tilde{r}_m^{\beta} \Gamma(m_s) m_c^{m_s}} P(\tilde{r}_{sel} = \tilde{r}_m).
$$

(3.38)

Performing some algebraic manipulations and letting $y = x(\frac{\tilde{r}_m^{\beta}}{m_c} - t)$, $\mathcal{M}_X(t)$ can be derived as follows:

$$
\mathcal{M}_X(t) = \sum_{\tilde{r}_m = \tilde{r}_1}^{\tilde{r}_M} \frac{P(\tilde{r}_{sel} = \tilde{r}_m) (K^{-1} \tilde{r}_m^{\beta})^{m_s-1}}{K \tilde{r}_m^{\beta} \Gamma(m_s) (\frac{\tilde{r}_m^{\beta} K^{-1}}{m_c} - t)^{m_s}} \int_0^\infty e^{-y} y^{m_s-1} dy
$$

$$
= \sum_{\tilde{r}_m = \tilde{r}_1}^{\tilde{r}_M} P(\tilde{r}_{sel} = \tilde{r}_m) \left( \frac{K^{-1} \tilde{r}_m^{\beta}}{K^{-1} \tilde{r}_m^{\beta} - m_c t} \right)^{m_s}.
$$

(3.39)
Finally, $\mathcal{M}_Y(t)$ can be written as follows:

$$\mathcal{M}_Y(t) = \left( \sum_{\tilde{r}_m = \tilde{r}_1}^{\tilde{r}_M} P(\tilde{r}_{sel} = \tilde{r}_m) \left( \frac{\tilde{K}^{-1} m_t^{\beta}}{\tilde{K}^{-1} m_t^{\beta} - m_c t} \right)^m_s \right)^L. \quad (3.40)$$

### 3.5.2 Derivation for the Downlink ICI

The downlink interference $X_l$ considering a single interfering cell $l$ can be written as:

$$X_l = \tilde{r}_l^{-\beta} \chi_l, \quad (3.41)$$

where $\chi_l$ is the interfering statistics from the $l^{th}$ neighboring BS, and $\tilde{r}_l$ is the distance of $l^{th}$ interfering BS from the scheduled mobile receiver. At this point, it is important to highlight that for a given $r_k$ and $\theta_i$ of a scheduled mobile user in the cell of interest, the distance of all interfering BSs can be calculated using cosine law which is not the same as in the uplink, where all $\tilde{r}_l$ are independent. Therefore, conditioned on the location of the mobile receiver, the distribution of the downlink cumulative ICI, i.e., $Y|r_k, \theta_i$, is simply a weighted sum of the distribution of the interfering channel statistics $\chi$. More precisely and by using the symmetry of the grid model the PDF of the cumulative ICI ($Y = \sum_{l=1}^{L} X_l$) can be given as:

$$f_Y(y) = \sum_{r_k=x_1}^{r_K} \sum_{\theta_i=0}^{2\pi} P(r_{sel} = r_{i,k}) f_{Y|r_k,\theta_i}(y|r_k, \theta_i), \quad (3.42)$$

where

$$Y|r_k, \theta_i = \sum_{l=1}^{L} \tilde{r}_l^{-\beta} \chi_l, \quad (3.43)$$

and the distance of all interferers can be determined using cosine law as follows:

$$\tilde{r}_l = \sqrt{D^2 + r_k^2 - 2r_k D \cos((l-1)\pi/3 + \pi/6 - \theta_i)} \quad \forall l = 1, \ldots, L. \quad (3.44)$$

---

5The location of interferers from a given cell is random and completely independent from the location of an interferer in any other cell.
Given the distance of the interferers, the conditional MGF of the cumulative interference can be calculated as follows:

$$M_{Y|r_k,\theta_i}(t) = \prod_{l=1}^{L} M_{\chi}(\bar{r}_l^{-\beta} t). \quad (3.45)$$

Finally the MGF of the cumulative ICI can be calculated as follows:

$$M_Y(t) = \sum_{r_k=r_1 \theta_i=0}^{r_K} 2\pi \sum_{r_k} P(r_{set} = r_{i,k}) \prod_{l=1}^{L} M_{\chi}(\bar{r}_l^{-\beta} t). \quad (3.46)$$

The expression for the MGF of the cumulative ICI in (3.46) is general for any kind of composite channel fading models. The explicit expressions for three above discussed practical fading models can also be obtained in a straightforward manner.

### 3.6 Evaluation of Network Performance Metrics

In this section, we demonstrate the significance of the derived MGF expressions in quantifying important network performance metrics such as the outage probability $P_{out}$, ergodic capacity $C$ and average fairness $F$ among users numerically.

#### 3.6.1 Evaluation of Outage Probability

The outage probability is typically defined as the probability of the instantaneous interference-to-signal-ratio to exceed a certain threshold. In order to evaluate $P_{out}$ numerically, we use the MGF based technique introduced in [68] for interference limited systems. Firstly, we define a new RV, $Z = q \sum_{l=1}^{L} X_l - X_0 = qY - X_0$, where $q$ is the outage threshold and $X_0$ is the corresponding signal power of the scheduled user in the central cell. An outage event occurs when $p(Z \geq 0)$, i.e., when the interference exceeds the corresponding signal power. This decision problem is solved in [68] by combining the characteristic function of $Z$ and residue theorem. The
characteristic function of $Z$ is defined as $\phi_Z(j\omega) = \mathbb{E}[e^{j\omega Z}]$. Considering interference $Y$ and signal power $X_0$ to be independent\(^6\), the expression for $\phi_Z(j\omega)$ can be given as, $\phi_Z(j\omega) = \phi_Y(jq\omega)\phi_{X_0}(-j\omega)$; where $\phi_Y(jq\omega)$ can be given by (3.34), (3.37), and (3.40) for different fading models. In general, the characteristic function of $X_0$ can be calculated as follows [69]:

$$\phi_{X_0}(\omega) = \mathbb{E}(e^{j\omega x_0}) = \int_{0}^{\infty} e^{j\omega x_0} f_{X_0}(x_0) dx_0 = j\omega \int_{0}^{\infty} e^{j\omega x_0} F_{X_0}(x_0) dx_0,$$

(3.47)

where $F_{X_0}(x_0) = \prod_{i=1}^{K} F_{r_i}(x_0)$ for opportunistic schemes and compact closed form expressions of $\phi_{X_0}(j\omega)$ are available in the literature [70, Eq. 19]. For non-opportunistic scheduling schemes $\phi_{X_0}(\omega) = \sum_{r_k=r_1}^{r_K} \phi_{\zeta|r_k}(j\omega)P(r_{sel} = r_k)$, where $\phi_{\zeta|r_k}(j\omega)$ is the characteristic function of $\zeta$ in ring $k$. The outage probability can then be computed by using the classical lemma introduced in [68] as follows:

$$P_{out} = \frac{1}{2} + \frac{1}{\pi} \int_{0}^{\infty} \Im\left(\frac{\phi_Z(\omega)}{\omega}\right) d\omega,$$

(3.48)

where $\Im(\phi_Z(\omega))$ denotes the imaginary part of $\phi_Z(\omega)$. Using (3.48), the outage probability can be evaluated using any standard mathematical software packages such as MATHEMATICA.

### 3.6.2 Evaluation of Ergodic Network Capacity

Another important performance evaluation parameter is the network ergodic capacity $C$, i.e.,

$$C = \mathbb{E} \left[ \log_2 \left( 1 + \frac{X_0}{\sum_{l=1}^{L} X_l + \sigma^2} \right) \right].$$

(3.49)

Usually, the computation of (3.49) requires $(L + 1)$-fold numerical integrations. To avoid this, we utilize the efficient lemma derived in [71] with a slight modification to

\(^6\)It is a well-known fact that the desired and interfering signals are completely independent in macro-cellular systems due to the typical far distances and diverse channel fading conditions.
take thermal noise into account and compute $C$ as follows:

$$
\mathbb{E} \left[ \ln \left( 1 + \frac{X_0}{\sum_{l=1}^{L} X_l + \sigma^2} \right) \right] = \int_0^\infty \frac{M_Y(t) - M_{X_0,Y}(t)}{t} e^{-\sigma^2 t} dt,
$$

(3.50)

where $M_Y(t) = \mathbb{E}[e^{-t \sum_{l=1}^{L} X_l}]$ and $M_{X_0,Y}(t) = \mathbb{E}[e^{-t (X_0 + \sum_{l=1}^{L} X_l)}] = \mathbb{E}[e^{-t (X_0 + Y)}]$. Note that this is the definition of MGF as defined in [71] which is not the same as our definition. Thus, we can use $M_Y(t)$ from (3.34), (3.37) and (3.40) directly with a sign change of $jw$. Moreover, (3.49) can also be solved efficiently by expressing it in terms of the weights and abscissas of a Laguerre orthogonal polynomial [71]:

$$
\mathbb{E} \left[ \ln \left( 1 + \frac{X_0}{\sum_{l=1}^{L} X_l + 1} \right) \right] = \sum_{\epsilon=1}^{E} \alpha_{\epsilon} \frac{M_Y(\xi_{\epsilon}) - M_{X_0,Y}(\xi_{\epsilon})}{\xi_{\epsilon}} + R_E,
$$

(3.51)

where $\xi_{\epsilon}$ and $\alpha_{\epsilon}$ are the sample points and the weight factors of the Laguerre polynomial, tabulated in [67], and $R_E$ is the remainder. Note that the MGF of $X_0$ can be calculated as explained in (5.36).

### 3.6.3 Evaluation of Average Fairness

In order to quantify the degree of fairness among different scheduling schemes, we use the notion developed in [72]. The average fairness of a scheduling scheme with $U$ users can be given as,

$$
F = -\sum_{i=1}^{U} p_i \log_{10} p_i,
$$

where $p_i$ is the proportion of resources allocated to a user $i$ or the access probability of user $i$. A system is strictly fair if each user has equal probability to access the channel and in such case the average fairness becomes one. The other extreme occurs when the channel access is dominated by a single user; in such case, the average fairness reduces to zero. The average fairness can be easily computed using our derived results as follows:

$$
F = -\sum_{k=1}^{K} P(r_{\text{sel}} = r_k) \frac{\log_{10} P(r_{\text{sel}} = r_k) - \log_{10} u_k}{\log_{10} U},
$$

(3.52)
where \( u_k \) denotes the number of users in a ring \( k \).

### 3.7 Results and Discussions

In this section, we first define the system parameters and describe the Monte-Carlo simulation setup which is required to demonstrate the accuracy of the derived expressions. We then address some important insights and study the performance trends of different scheduling schemes.

#### Parameter Settings and Simulation Setup

The radius \( R \) of the cell is set to 500m and the cell is decomposed into non-uniform circular regions of width \( \Delta_k \). The path loss variation within each circular region is set to \( \kappa = 2 \text{dB} \).

For each Monte-Carlo simulation trial, we generate \( U \) uniformly distributed users in a circular cell of radius \( R \). Each user has instantaneous SNR given by (5.17) and short term average SNR \( (\bar{\gamma}) \). We allocate a user with maximum instantaneous SNR in the greedy scheme whereas in the proportional fair scheme we allocate a user based on the maximum normalized SNR. For the round robin scheme, we select any user arbitrarily. For location based round robin we select a user randomly from the \( w^{th} \) ring in a time slot \( w \) whereas we select a user with maximum SNR without considering the users of the previously allocated \( w - 1 \) rings for the greedy round robin scheme.

Next, we calculate the distances of the selected users, i.e., \( r_{\text{sel}} \), from their serving BS and compute the distances to the BS of interest, i.e., \( \tilde{r}_{\text{sel}} \) for all scheduling schemes. The process repeats for large number of Monte-Carlo simulation trials. The distance data is then analyzed by creating a histogram whose bins are given by \([1, \cdots, r_{k-1}, r_k, r_{k+1}, \cdots, R]\).
Results and Discussions

Fig. 3.2 depicts the PMF of the location of the scheduled user in a given cell based on the proportional fair, greedy and round robin scheduling schemes. Since the proportional fair scheme exhibits some fairness among users in a cell, the PMF of the allocated user locations is expected to be more flat compared to the greedy scheme. Since the cell edge has more users due to the large area and each user has equal probability to be allocated on a given subcarrier, therefore the round robin scheme exhibits high probability at the cell-edge.

It is important to note that the numerical results for the derived PMF in Fig. 3.2 nearly coincide with the exhaustive Monte-Carlo simulation results with a small number of rings $K = 10$ and $\kappa = 2$dB. Moreover, it can also be noticed that the width of the circular regions tend to increase from cell center to cell edge which is due to the exponentially decaying path loss. The number of required rings is expected to decrease by reducing $\beta$ and increasing the amount of power decay within each circular region and vice versa.

Another important point to explain with reference to Fig. 3.2 is that with the increase in the number of competing users on a given subcarrier, the PMF of opportunistic scheduling schemes tends to get skewed which is due to the fact that the higher the number of users in the cell center, the higher is the probability of allocating a subcarrier in the cell center. In order to get an integer number of users within a ring, we perform rounding in the analysis, i.e., we consider zero active users in the rings where $u_k \leq 0.5$. In Monte-Carlo simulations, we consider the probability of allocating a user in these rings to be zero which can also be verified from Fig. 3.2.

In Fig. 3.3, the PMF of the distance between the allocated user in interfering cell $l$ and the BS of interest, i.e., $P(\tilde{r}_{sel} = \tilde{r}_m)$, is presented. Numerical results are found to be in close agreement with the Monte-Carlo simulation results. For the opportunistic scheduling schemes, it is likely that a user close to its serving BS can get a subcarrier,
thus, the PMF of the distance of allocated interfering users is expected to have high density in the middle. However, the slight descend in the central region in Fig. 3.3 is due to ignoring users that lie within the rings where the average number of users is less than half.

Moreover, we can observe that the round robin scheduler is highly vulnerable to interference compared to the other schemes as high interference is expected to come from the cell edge users in the interfering cells. On the other hand, the greedy scheduler is expected to have allocations near the cell center and, thus, leads to less interference from neighboring cells. The proportional fair scheme lies in between the two extremes. Fig. 3.4 illustrates the CDF of the ICI considering different number of interfering cells and path loss exponents $\beta$ for the greedy scheduling scheme. With the increase in the number of interferers, the interference level increases. Moreover, as $\beta$ increases, the signal degrades rapidly and thus interference level is reduced considerably. At this point, it is important to mention that in this chapter we derive
Figure 3.3: PMF of the distance at which the users in the interfering cells are allocated (i.e., PMF of $\tilde{r}_{\text{sel}}$) for proportional fair, greedy and round robin scheduling schemes with path loss exponent $\beta = 2.6$, $U = 50$, $I = 720$, $\chi \sim \text{Gamma}(3/2, 2/3)$, Number of Monte-Carlo simulations $=100,000$, $C=60$ dB, $P_{\text{max}}=1$W, $\sigma^2=-174$ dBm/Hz, $\Delta=50$ m.

and utilize the MGF of the cumulative ICI rather than the CDF of the cumulative ICI in order to evaluate important network performance metrics. Therefore, the analytical part of the provided figure of the CDF of the cumulative ICI is plotted using a technique mentioned in [73] to convert MGF into CDF numerically.

Fig. 3.5 investigates the effect of increasing the number of competing users on a given sub-carrier considering all scheduling schemes. It can be observed that the increase in the number of users enhances the performance of the opportunistic scheduling schemes due to additional multiuser diversity gains. The greedy scheme achieves the best performance whereas the round robin scheme achieves the worst performance. As expected, the proportional fair scheme lies in between the two extremes. The average capacity of location based round robin over $W = K$ time slots has been shown to be better than the conventional round robin scheme. The average capacity results of the greedy round robin scheme is presented for $W = 3$ and $W = 6$. Clearly, for $W = 1$, the scheme is equivalent to the greedy scheme; however, with the increase
of time slots, performance degradation takes place due to the reduction of multiuser diversity caused by ignoring the users from previously allocated rings.

Fig. 3.6 quantifies the average resource fairness of all presented scheduling schemes. As expected, round robin is a strictly fair scheme. The proportional fair scheme possesses the ability to enhance the network throughput compared to round robin scheduling while providing a high degree of fairness. The greedy scheme is observed to be the most unfair scheme. Considering $K$ time slots, the average fairness of the location based round robin scheme is investigated and found to be very close to the round robin scheme, however, with degradation in performance as can be observed in Fig. 3.5. For the greedy round robin scheme, we plotted the fairness metric considering $W = 3$ and $W = 6$; it is shown that as the number of time slots increases, the fairness improves with a trade-off price in terms of ergodic capacity.

In Fig. 3.7, we evaluate the network outage probability as a function of the outage threshold; $q = (Z + X_0)/Y$ for (i) $U = 50$ users; (ii) $U = 100$ users. The numerical
and simulation results are nearly identical for most cases. The higher the outage threshold for a given signal and interference power, the greater outage is expected. Moreover, for larger number of users the outage probability is observed to reduce for all opportunistic scheduling schemes except the round robin scheme. Since increasing the number of users on a given subcarrier in non-opportunistic schemes does not directly affect the access probability of a ring $k$, therefore its impact on the ICI is almost negligible. This fact can also be verified from Fig. 3.5.

Finally, in Fig. 3.8, we evaluate the network ergodic capacity as a function of the fading severity parameter and average power of Gamma fading interference channels for different scheduling schemes. Firstly, it can be observed that increasing the average power of the interference channel which is given by $\Omega = m_c m_s$ for a given fading severity parameter $m_s$, the capacity degrades significantly for all schemes. Moreover, it is also shown that increasing the fading severity $m_s$ while keeping the average power $\Omega = 3$ fixed has minimal impact on the system capacity. Therefore, the lower average power of interference channel $\Omega$, the better is the overall system performance.\hspace{1cm} \text{7}
Figure 3.6: Average system fairness for different number of users considering different scheduling schemes with $\beta = 2.6, C = 60$ dB, $P_{\text{max}} = 1W, \sigma^2 = -174$ dBm/Hz.

### 3.8 Conclusion

In this chapter, we proposed a novel approach to model the uplink ICI considering various scheduling schemes and composite fading channel models. The proposed approach is not dependent on a particular shadowing and fading statistics, hence, extensions to different models is possible. The provided numerical results and help in gaining insights into the behavior of ICI considering different scheduling schemes and composite fading models. Moreover, they provide quantitative assessment of the relative performance of various scheduling schemes which is important for network design and assessment. Now, we will look in the next chapter into modeling and management of ICI via different PC mechanisms along with the scheduling schemes.

---

7 The small gap between the analytical and simulation results is mainly due to assuming in the analytical derivations that users located within a ring are at the boundary of the ring. This gap can be further reduced by increasing the number of rings in the evaluated expressions.
Figure 3.7: Impact of different scheduling schemes on the network outage probability for different number of users and various outage thresholds, $\beta=2.6$, $\chi \sim \text{Gamma}(3/2, 2/3)$, $C=60$ dB, $P_{\text{max}}=1$W, $\sigma^2=-174$ dBm/Hz.

Figure 3.8: Ergodic network capacity of different scheduling schemes as a function of different parameters of the interference statistics $\chi \sim \text{Gamma}(m_s, m_c)$ with $U = 50$ users, $\beta=2.6$, $\sigma^2=-174$ dBm/Hz.
Chapter 4

Statistics of the Interference with Slow and Fast Power Control

4.1 Introduction

Uplink PC is an interference mitigation technique that aims at limiting the ICI in cellular networks while maintaining target received signal levels at base stations. PC mechanisms directly impact the interference dynamics and, thus, affect the overall achievable capacity and consumed power in cellular networks. Due to the stochastic nature of wireless channels and mobile user’s locations, it is important to derive analytical statistical models for ICI that can capture the impact of design alternatives related to PC mechanisms.

In this chapter, firstly we focus on location based slow PC with greedy scheduling considering that each user is capable of adapting its transmit power autonomously either by measuring its location through a global positioning system (GPS) or estimating its distance based on the power measurement of pilot signals from the surrounding BSs [65]. We then present a general mathematical framework to derive the statistics of the uplink ICI on a given subcarrier considering more practical PC mechanisms where users can possibly compensate for their shadowing and fading effects along with their distance-based path loss from the serving BS while maintaining a target received
power level. In this context, we consider three different PC mechanisms which include conventional fast PC in addition to modified versions of conventional slow and fast PC. Closed form expressions for the statistics of ICI are derived for all cases considering Generalized-$K$ composite fading channels and round robin scheduling. Finally, we demonstrate the importance of the obtained ICI expressions by utilizing them to derive analytical expressions for several key network performance metrics.

The chapter is organized as follows: In Section 4.2 we derive the statistics of ICI considering location based PC mechanism and greedy scheduling. In Section 4.3, we formulate and define three channel-based PC mechanisms. In Section 4.4, we derive the distribution of the ICI considering slow and fast modified PC mechanisms and the statistics of the received signal level at a given BS. In Section 4.5, we derive expressions for conventional fast PC mechanisms. Network performance metrics are derived for channel-based PC mechanisms in Section 4.6. Section 4.7 presents selected results followed by concluding remarks in Section 4.8.

4.2 Location-based PC with Greedy Scheduling

We consider a given cell surrounded by $L$ interfering cells. For analytical convenience, the cells are assumed to be circular with radius $R$. Each cell contains $U$ uniformly distributed users where each user is assumed to have perfect knowledge of its distance to the serving BS. The rate adaptation and allocation of users on a given subcarrier, therefore, depend on the channel qualities as well as the transmit powers of the users. The instantaneous SNR of any user can then be written as:

$$\gamma = \frac{\min(P_{\text{max}}, P_0 r^\beta) r^{-\beta} \zeta}{\sigma^2},$$

(4.1)

where $P_{\text{max}}[W]$ is the maximum transmit power of a user, $P_0[W]$ is the desired power level at the receiver, $r[\text{m}]$ is the user distance from its serving BS, $\beta$ is the path loss
exponent, $\sigma^2$ denotes the thermal noise at the receiver which is considered to be unity without loss of generality and $\zeta$ represents the combined shadowing and fading RV. More explicitly, (4.1) can be re-written as:

$$
\gamma = \begin{cases} 
P_0\zeta, & P_0r^\beta < P_{\text{max}} \\
\max r^{-\beta}\zeta, & P_0r^\beta \geq P_{\text{max}}. 
\end{cases}
$$

(4.2)

The distance at which users need their maximum power to compensate path loss completely is referred to as threshold distance ($r_t$) and can be computed as follows:

$$
r_t = \left( \frac{P_{\text{max}}}{P_0} \right)^{1/\beta}.
$$

(4.3)

Users located within $r_t$ can compensate path loss completely while saving some proportion of their power, whereas the users located beyond $r_t$ transmit with their maximum power to achieve a certain rate that is less than their desired target.

Each cell is decomposed into $K$ concentric circular rings. The average number of users in a given ring $k$ can be computed as explained in Chapter 3. The proposed approach to model ICI is detailed in the following steps:

- Derive the distribution of the allocated desired user location $f_{r_{\text{sel}}}(r)$ and allocated interfering user location $f_{\tilde{r}_{\text{sel}}}$. 

- Derive the distribution of the ICI $f_{X_l}(x)$ from the allocated user in neighboring cell $l$ to the BS of interest. Since the allocated interfering user can transmit with different power levels depending on the distance from its own serving BS, the incurred interference can be modeled as

$$
X_l = \begin{cases} 
P_0r^\beta \tilde{r}^{-\beta} \chi & \tilde{r} \in [D-r_t, D+r_t], r \in [0, r_t] \\
P_{\text{max}} \tilde{r}^{-\beta} \chi & \text{otherwise},
\end{cases}
$$

(4.4)
where \( D = 2R \) and \( \chi \) denotes the combined shadowing and fading component of the interference statistics.

- Derive the MGF of the cumulative interference \( Y = \sum_{l=1}^{L} X_l \) caused by the allocated interfering users in all neighboring cells.

### 4.2.1 Statistics of the ICI

**Distribution of Allocated User Locations**

Since each circular region has uniform path loss variation, the users within a ring \( k \) are assumed to be subject to approximately the same path loss. Thus, selecting a user in a ring \( k \) is equivalent to selecting the user with maximum channel gain among all the users in ring \( k \). The CDF and PDF of the maximum channel gain \( \zeta_k \) can be written as follows, respectively,

\[
F_{\zeta_k}(\zeta_k) = \prod_{i=1}^{u_k} F_{\zeta_i}(\zeta_k) = (F_{\zeta}(\zeta_k))^{u_k};
\]

\[
f_{\zeta_k}(\zeta_k) = \sum_{j=1}^{u_k} f_{\zeta_j}(\zeta_k) \prod_{i=1, i \neq j}^{u_k} F_{\zeta_i}(\zeta_k) = u_k f_{\zeta}(\zeta_k) (F_{\zeta}(\zeta_k))^{u_k-1}.
\]

Considering the model in (4.2), we split the analysis into two regions, namely the region within the threshold distance and the region beyond the threshold distance. After performing the RV transformation, we can write the CDF of the selected user SNR in each ring \( k \) as follows:

\[
F_{\gamma_k}(\gamma_k) = \begin{cases} \left( F_{\zeta}(\frac{\gamma_k}{P_0}) \right)^{u_k}, & r_k < r_t \\ \left( F_{\zeta}(\gamma_k r_k^\beta) \right)^{u_k}, & r_k \geq r_t. \end{cases}
\]  

(4.5)

Now the conditional probability of selecting \( k \)'th ring among all other rings can be derived as \( P(r_{sel} = r_k | \gamma_k) = \prod_{i=1}^{K} F_{\gamma_k}(\gamma_k). \) By averaging over \( \gamma_k \), the final expression for the PMF of \( r_{sel} \) can be written as follows:

\[
P(r_{sel} = r_k) = \int_0^{\infty} P(r_{sel} = r_k | \gamma_k) f_{\gamma_k}(\gamma_k) d\gamma_k.
\]  

(4.6)
The result in (4.6) can be evaluated accurately using standard mathematical software packages such as MAPLE and MATHEMATICA and is valid for any composite fading statistics. Note that $P(r_{sel} = r_k)$ in (4.6) is the marginal PMF of $P(r_{sel} = r_k, \theta = \theta_i)$ which is derived as in Chapter 3.

**Distribution of the ICI**

Since, each cell is assumed to have identical conditions, $f_{\tilde{r}_{sel}}(\tilde{r})$ remains the same for all interfering cells. Defining $\tilde{r}_{i,k}$ for given $\theta_i$ and $r_k$ using cosine law as $\tilde{r}_{i,k}(r, \theta) = \sqrt{r_k^2 + D^2 - 2r_kD \cos \theta_i}$, $P(\tilde{r}_{sel} = \tilde{r}_{i,k})$ can be given as $P(\tilde{r}_{sel} = \tilde{r}_{i,k}) = P(r_{sel} = r_k)/I$.

As the interfering users can transmit with different power levels depending on their distance from their serving BS, the interference $X$ can be categorized into two regions mentioned as follows:

$$X = \begin{cases} P_0 r_k^{\beta} \chi \tilde{r}_{i,k} \in [D-r_t \ D+r_t], r_k \in [0 \ r_t] \\ P_{\text{max}} \tilde{r}_{i,k}^{\beta} \chi \text{ otherwise}, \end{cases} \quad (4.7)$$

where $\chi$ denotes the interference channel statistics. The PDF of $X$ conditioned on $\tilde{r}_{i,k}(r, \theta)$ can be determined by RV transformation as follows:

$$f_{X|\tilde{r}_{i,k}}(x) = \begin{cases} \frac{\tilde{r}_{i,k}^{\beta} f_{\chi}(\frac{x}{P_0 r_k^{\beta}})}{P_0 r_k^{\beta}} \tilde{r}_{n,k} \in [D-r_t \ D+r_t], r_k \in [0 \ r_t] \\ \frac{\tilde{r}_{i,k}^{\beta} f_{\chi}(\frac{x}{P_{\text{max}} \tilde{r}_{i,k}^{\beta}})}{P_{\text{max}} \tilde{r}_{i,k}^{\beta}} \text{ otherwise}. \end{cases} \quad (4.8)$$

Simply averaging over $\tilde{r}_{i,k}$ and letting $A = \frac{1}{P_0} \tilde{r}_{i,k}^{\beta} (r, \theta) r_k^{-\beta}$ and $B = \frac{1}{P_{\text{max}}} \tilde{r}_{i,k}^{\beta} (r, \theta)$ we can write the distribution of interference, i.e., $f_X(x)$ as shown below:

$$f_X(x) = \begin{cases} \sum_{r_k} \sum_{\theta_i} A P(\tilde{r}_{sel} = \tilde{r}_{i,k}) f_{\chi}(Ax) \tilde{r}_{i,k} \in [D-r_t \ D+r_t], r_k \in [0 \ r_t] \\ \sum_{r_k} \sum_{\theta_i} B P(\tilde{r}_{sel} = \tilde{r}_{i,k}) f_{\chi}(Bx) \text{ otherwise}. \end{cases} \quad (4.9)$$
Finally, $f_X(x)$ can be written explicitly as follows:

$$f_X(x) = \sum_{r_k \in [0, r_t]} \sum_{\theta_i} A P(\tilde{r}_{sel} = \tilde{r}_{i,k}) f_X(Ax) + \sum_{r_k \notin [0, r_t]} \sum_{\theta_i} B P(\tilde{r}_{sel} = \tilde{r}_{i,k}) f_X(Bx).$$

(4.10)

Since the scheduling scheme is considered to be identical in all cells, the interferers are i.i.d. and therefore the MGF of the cumulative interference $Y$ can be given as $M_Y(t) = \prod_{l=1}^{L} M_{X_l}(t) = (M_X(t))^L = \left(\mathbb{E}[e^{tx}]\right)^L$. Looking at the structure of (4.10), we can derive MGF of any composite fading model as $M_X(t) = \int_0^\infty e^{tx} f_X(x) dx$. The expression applies to any kind of composite fading models. As the Generalized-K distribution can be approximated by a simple Gamma distribution \[66\], therefore in this case $M_X(t)$ can be derived as follows:

$$M_X(t) = \sum_{\tilde{r}_k \in [0, r_t]} \sum_{\theta_i} \frac{A^{m_s} P(\tilde{r}_{sel} = \tilde{r}_{n,k})}{(A - m_c t)^{m_s}} + \sum_{\tilde{r}_k \notin [0, r_t]} \sum_{\theta_i} \frac{B^{m_s} P(\tilde{r}_{sel} = \tilde{r}_{n,k})}{(B - m_c t)^{m_s}}. \tag{4.11}$$

### 4.2.2 Network Performance Metrics Derivation

In this section, we will utilize the derived expressions to evaluate the network ergodic capacity, average fairness, and average power savings per user.

#### Evaluation of Network Ergodic Capacity

Using the lemma derived in [71], the capacity can be calculated as $\mathbb{E} \left[ \ln \left( 1 + \frac{X_0}{\sum_{l=1}^{K} X_l} \right) \right] = \int_0^\infty \frac{M_Y(t) - M_{X_0,Y}(t)}{t} dt$, where $M_Y(t) = \mathbb{E}[e^{-t\sum_{l=1}^{L} X_l}]$ is the MGF of the cumulative interference and $M_{X_0,Y}(t) = \mathbb{E}[e^{-t(X_0 + \sum_{l=1}^{L} X_l)}] = \mathbb{E}[e^{-t(X_0 + Y)}]$ is the joint MGF of the corresponding signal power of the scheduled user $X_0$ and cumulative interference $Y$. Since $X_0$ and $Y$ are independent\(^1\), $M_{X_0,Y}(t) = M_{X_0}(t) M_Y(t)$. The expression for $M_{X_0}(t)$ can be given as $M_{X_0}(t) = \int_0^\infty e^{tx_0} f_{X_0}(x_0) dx_0 = \int_0^\infty e^{tx_0} F_{X_0}(x_0) dx_0$ where $F_{X_0}(x_0) = \prod_{i=1}^{K} F_{\gamma_i}(x_0)$, $f_{X_0}(x_0) = \frac{\partial}{\partial x_0} F_{X_0}(x_0)$. Closed form expressions are also

\(^1\)Typically, the desired and interfering signal are received on significantly diverse fading channels, therefore, it is practically reasonable to assume the independence between them.
available for $\mathcal{M}_{X_0}(t)$ in the literature [70].

Evaluation of Average Fairness and Power Savings

The average fairness can be easily computed using our derived results as explained in Chapter 3. Moreover, the average power savings per subcarrier can be given as:

$$
\bar{P} = \sum_{r_k = r_1}^{r_t} P(r_{\text{sel}} = r_k) \left( P_{\text{max}} - P_0 r_k^\beta \right) .
$$

(4.12)

4.3 Problem Formulation for Channel based PC Mechanisms

We consider a given cell surrounded by $L$ interfering cells. For analytical tractability, each cell is considered to be circular with radius $R$ and $U$ uniformly distributed mobile users. In general, conventional PC mechanisms adjust the transmit power of a mobile user on a given subcarrier as follows:

$$
\min(P_{\text{max}}, P_0 + PL) \ [\text{dBm}],
$$

(4.13)

where PL denotes the overall path loss estimate (including distance-based path loss, shadowing, and fading), $P_0$ is a user specific or cell specific parameter which sets the desired received signal power level at the BS of interest, and $P_{\text{max}}$ is the maximum transmit power capability per user on a given subcarrier. In this section, we consider three PC mechanisms, namely: (i) modified fast PC; (ii) modified slow PC; (iii) conventional fast PC. The modified PC mechanisms are different from their conventional counterpart in that the users that can transmit are only those capable of fully compensating their overall path loss while maintaining the desired target signal power level at the BS. The remaining users that cannot fully compensate their overall path
loss will remain silent and save their power instead of transmitting with $P_{\text{max}}$ which is the case in conventional PC.

Even though the modified PC mechanisms do not favor the cell-edge users at high values of $P_0$, they do ensure notable throughput gains by reducing ICI for a wide range of scenarios as will be discussed later. Moreover, the modified PC mechanisms provide a special form of fractional PC in which cell-edge users reduce their transmit powers as well as their achieved rates when they cannot achieve their target quality. We assume round robin scheduling, i.e., each user in conventional PC and each user who is able to compensate for its overall path loss in modified PC has equal probability of allocation on a given subcarrier. The following is a mathematical description for each of the considered PC mechanisms:

1. **Modified Fast PC**: schedule the transmission of only those users who can compensate their complete path loss which is based on their distance from the serving BS, shadowing and fading conditions. The received signal power at the BS from any user can be modeled as:

$$\gamma = \begin{cases} 
P_0, & P_{\text{max}} > \frac{P_0 r^\beta}{\xi} \\
0, & \text{otherwise} 
\end{cases}$$

where $\beta$ is the path loss exponent, $r$ is the distance of a user from its serving BS and $\xi$ is the combined shadowing and fading statistics.

2. **Modified Slow PC**: schedule the transmission of a set of users who can compensate their long term channel variations which include their distance from the serving BS and shadowing. The received signal power at the BS from any user can be modeled as:

$$\gamma = \begin{cases} 
P_0 \eta, & P_{\text{max}} > \frac{P_0 r^\beta}{\xi} \\
0, & \text{otherwise} 
\end{cases}$$
where $\eta$ is the fading component and $\xi$ denotes the shadowing component of the composite shadowing and fading statistics $\zeta$, i.e., $\zeta = \xi \eta$.

3. **Conventional Fast PC:** allow all users to compensate their distance based path loss, shadowing and fading; however, in case of failure in achieving the target power level, these users will transmit with their maximum power $P_{\text{max}}$. The signal power received at the BS from any user can then be modeled as:

$$\gamma = \min \left( P_{\text{max}}, \frac{P_0 r^\beta}{\zeta} \right) r^{-\beta} \zeta = \begin{cases} P_0, & P_{\text{max}} > \frac{P_0 r^\beta}{\zeta} \\ P_{\text{max}} r^{-\beta} \zeta, & \text{otherwise} \end{cases}. \quad (4.16)$$

**Proposed ICI Modeling Approach**

To characterize the statistics of the uplink ICI with different PC mechanisms, each cell is decomposed into $K$ concentric circular regions such that the area of each circular region remains the same (See Fig. 4.1). Each cell has $U$ uniformly distributed users with equal number of users on average in each circular region. For analytical tractability, the radius of each circular region is selected such that it possesses a single user on average, i.e., $U = K$. The radius of each ring $k$ with a single user on average can then be calculated as follows:

$$r_k = \sqrt{\frac{R^2 - r_0^2}{K} + r_{k-1}^2}, \quad k = 1, 2, \ldots, K, \quad 0 \leq r_k \leq R, \quad (4.17)$$

where $r_0$ is the reference distance, $r_k$ denotes the radius of ring $k$ and the distance of $k^{\text{th}}$ user from its serving BS. The user located within $k^{\text{th}}$ circular region bounded by ring $r_k$ and $r_{k-1}$ is considered to be located at $r_k$. The main steps of the solution are concisely detailed as follows:

1. Derive the distribution $f_{r_{\text{sel}}}(r)$ of the distance of the allocated user denoted by $r_{\text{sel}}$ in a given cell from its serving BS. Without loss of generality, we assume
Figure 4.1: Graphical illustration of dividing the cellular network into multiple circular regions of equal area.

same PC mechanism deployed in each cell therefore, \( f_{r_{sel}}(r) \) remains the same for all cells. Also, derive the distribution of the signal received at the BS of interest from a scheduled user defined as follows:

Modified Fast PC:  
\[
X_0 = \begin{cases} 
  P_0, & \zeta > \frac{P_0 r_{sel}^\beta}{P_{\text{max}}}, \\
  0, & r_{sel} \in \emptyset 
\end{cases}, 
\]  
(4.18)

Modified Slow PC:  
\[
X_0 = \begin{cases} 
  P_0 \eta, & \xi > \frac{P_0 r_{sel}^\beta}{P_{\text{max}}}, \\
  0, & r_{sel} \in \emptyset 
\end{cases}, 
\]  
(4.19)

Conventional Fast PC:  
\[
X_0 = \begin{cases} 
  P_0, & \zeta > \frac{P_0 r_{sel}^\beta}{P_{\text{max}}}, \\
  P_{\text{max}} \tilde{r}_{sel}^{-\beta} \zeta, & \text{otherwise} 
\end{cases}, 
\]  
(4.20)

The condition \( r_{sel} \in \emptyset \) denotes the event when no mobile user is scheduled for transmission. This event takes places only with the modified PC mechanisms.

2. Derive the distribution \( f_{\tilde{r}_{sel}}(\tilde{r}) \) of the distance between the allocated user in a neighbor interfering cell and the BS of the cell of interest denoted by \( \tilde{r}_{sel} \).

3. Derive the distribution \( f_{X_l}(x) \) of the interference from a selected user in interfering cell \( l \) to the BS of interest. The received interference power is given
as:

Modified Fast PC: \( X = \begin{cases} 
P_0 \alpha \tilde{r} - \beta \chi, & \zeta > \frac{P_0 \beta}{P_{max}}, \\
0 & \text{otherwise}
\end{cases} \) \hspace{1cm} (4.21)

Modified Slow PC: \( X = \begin{cases} 
P_0 \alpha \tilde{r} - \beta \chi, & \xi > \frac{P_0 \beta}{P_{max}}, \\
0 & \text{otherwise}
\end{cases} \) \hspace{1cm} (4.22)

Conventional Fast PC: \( X = \begin{cases} 
P_0 \alpha \tilde{r} - \beta \chi, & \zeta > \frac{P_0 \beta}{P_{max}}, \\
P_{max} \tilde{r} - \beta \chi, & \text{otherwise}
\end{cases} \) \hspace{1cm} (4.23)

where \( \chi \) denotes the composite fading of the interfering channel.

4. Derive the MGF of the cumulative ICI, i.e., \( Y = \sum_{l=1}^{L} X_l \) from the scheduled users on a given subcarrier in all \( L \) interfering cells.

4.4 Conventional PC with Round Robin Scheduling

4.4.1 Distribution of the Received Signal Power at BS

The conventional fast PC scheme allows the scheduling of all users on a given subcarrier, either with their maximum power or the power required to achieve the target received signal level at the BS. Considering round robin scheduling, all users possess equal probability of allocation on a given subcarrier and, thus, the probability of selecting a ring or a user \( k \) can be expressed as follows:

\[ P( r_{sel} = r_k ) = \frac{1}{K} \] \hspace{1cm} (4.24)
From Chapter 3, we can write \( P(r_{sel} = r_k, \theta = \theta_i) = P(r_{sel} = r_k)/I \). Conditioned on \( r_{sel} \), the PDF and CDF of \( X_0 \) is derived using (4.20) as follows, respectively:

\[
 f_{X_0|r_{sel}}(x_0) = \frac{r^\beta_{sel}}{P_{max}} P \left( \frac{r^\beta_{sel} x_0}{P_{max}} \right) + \delta (x_0 - P_0) \left( 1 - F_\zeta \left( \frac{P_0 r^\beta_{sel}}{P_{max}} \right) \right).
\] (4.25)

\[
 F_{X_0|r_{sel}}(x_0) = F_\zeta \left( \frac{x_0 r^\beta_{sel}}{P_{max}} \right) + \delta (x_0 - P_0) \left( 1 - F_\zeta \left( \frac{x_0 r^\beta_{sel}}{P_{max}} \right) \right).
\] (4.26)

**Proof.** See Appendix A. □

Finally, averaging over \( r_{sel} \), the PDF and MGF of \( X_0 \) can be obtained as follows, respectively:

\[
 f_{X_0}(x_0) = \sum_{k=1}^{K'} \frac{r^\beta_k}{P_{max}} P \left( \frac{x_0}{P_{max}} \right) + \delta (x_0 - P_0) \left( 1 - F_\zeta \left( \frac{P_0 r^\beta_k}{P_{max}} \right) \right) P(r_{sel} = r_k) +
\] (4.27)

\[
 M_{X_0}(s) = \sum_{k=1}^{K'} e^{-sP_0} \left( 1 - F_\zeta \left( \frac{P_0 r^\beta_k}{P_{max}} \right) \right) P(r_{sel} = r_k) +
\] (4.28)

\[
 \frac{r^\beta_k}{P_{max}} \int_0^{P_0} e^{-s x_0} f_\zeta \left( \frac{r^\beta_k x_0}{P_{max}} \right) P(r_{sel} = r_k) dx_0.
\]

Note that the MGF is not in closed form for generalized fading channels, however, a closed form is derived later for Generalized-\( K \) fading channels.

### 4.4.2 Statistics of the ICI

The PDF, CDF and MGF of the Generalized-\( K \) distribution involve time consuming special functions such as Meijer-G and Whittaker functions. Therefore, to avoid the computational difficulties associated with the Generalized-\( K \) distribution, an accurate approximation has been proposed recently in [66] via a more tractable Gamma distri-
bution using the moment matching method, i.e., \( K_G(m_c, m_s, \Omega) \approx \text{Gamma}(\kappa, \Theta) \). By matching the first and second moments of the two distributions, the corresponding values of \( \kappa \) and \( \Theta \) can be determined as follows [66]:

\[
\kappa = \frac{m_m m_s}{m_m + m_s + 1 - m_m m_s \epsilon}, \quad \Theta = \frac{\Omega}{\kappa},
\]

(4.29)

where \( \epsilon \) is the adjustment factor. In this chapter, we approximate the Generalized-
\( K \) composite fading of the desired channel and the interference channel by the more
tractable Gamma distribution and proceed with deriving the statistics of the incurred
interference for all PC mechanisms.

It is clear from (4.21), (4.22) and (4.23) that the distribution of interference mainly
relies on the distribution of the ratio of two RVs given by \( \chi \) and \( \zeta \) conditioned on
\( \zeta > \frac{P_{yr, \beta}}{P_{max}} \) for fast PC schemes, whereas \( \chi \) and \( \xi \) conditioned on \( \xi > \frac{P_{yr, \beta}}{P_{max}} \) for slow
PC schemes, where \( \frac{P_{yr, \beta}}{P_{max}} \) is a constant when we condition on \( r_{sel} \). As we approximate
the composite fading statistics \( \chi \) and \( \zeta \) by a Gamma distribution, we prove first
Theorem 1 and Lemma 1 which are required to derive the statistics of the ICI in the
following subsections.

**Theorem 1 (PDF of the Ratio of Gamma and Truncated Gamma RVs).** Let \( W = P/Q \) be a RV with \( P \sim \text{Gamma}(\kappa_p, \Theta_p) \) and \( Q \sim \text{Gamma}(\kappa_q, \Theta_q) \), conditioned on
\( Q > c \) where \( c \in \mathbb{R} \) is a constant. Denoting the event \( \mathbb{B} \rightarrow Q > c \), the conditional
PDF of \( W \) is given by

\[
f_{W|\mathbb{B}}(w) = \frac{w^{\kappa_p - 1} \Theta_q^{\kappa_q} (\Theta_p + w \Theta_q)^{-\kappa_p - \kappa_q}}{P(Q > c) \Gamma(\kappa_p) \Gamma(\kappa_q)} \Gamma_u \left( \kappa_p + \kappa_q; \frac{w \Theta_q + \Theta_p}{\Theta_p \Theta_q} \right),
\]

(4.30)

Proof. See Appendix B.

**Lemma 1 (MGF of the Ratio of Gamma and Truncated Gamma RVs).** The condi-
tional MGF of $W$ for integer values of $\kappa_p + \kappa_q$ is given by

$$
\mathcal{M}_{W|B}(s) = \frac{\Gamma(\kappa_q + \kappa_p) e^{-c/\Theta_q}}{(\Gamma(\kappa_q) - \Gamma(\kappa_q; c/\Theta_q))} \sum_{k=0}^{\kappa_p + \kappa_q-1} \frac{(c/\Theta_q)^k}{k!} U\left(\kappa_p; 1 + k - \kappa_q; \frac{c + s\Theta_p}{\Theta_q}\right),
$$

(4.31)

where $U$ is a confluent Hypergeometric function which is also referred to as a Tricomi Hypergeometric function given by

$$
U(a; b; z) = \frac{1}{\Gamma(a)} \int_0^\infty e^{-zt} t^{a-1}(1 + t)^{b-a-1} dt.
$$

(4.32)

Proof. See Appendix B. \hfill \square

**Lemma 2** (MGF of the ratio of two Gamma RVs). As $c \to 0$, $f_{W|B}(w) \to f_W(w)$, i.e., $P(Q > c) \to 1$ and, thus, $f_W(w)$ reduces to the $F$-distribution given by

$$
f_W(w) = \frac{w^{\kappa_p-1}\Theta_q^{\kappa_p}\Theta_p^{\kappa_q} (\Theta_p + w\Theta_q)^{-\kappa_p-\kappa_q}}{\mathcal{B}(\kappa_p, \kappa_q)}.
$$

(4.33)

Using $\sum_{k=0}^{n} \frac{x^k}{k!} = \frac{e^x \Gamma(n+1, x)}{n!}$ and simplifying (4.31), the expression of the MGF can be derived as follows:

$$
\mathcal{M}_W(s) = \lim_{c \to 0} \frac{\Gamma_a(\kappa_p + \kappa_q; c/\Theta_q)}{(\Gamma(\kappa_q) - \Gamma(\kappa_q; c/\Theta_q))} \frac{\Gamma(\kappa_p) \Gamma(1 + k - \kappa_q; s\Theta_p)}{B(\kappa_p, \kappa_q) \Gamma(1 - \kappa_q; s\Theta_p)}.
$$

(4.34)

By defining a RV $W = \chi/\zeta$ with $\mathcal{B} \to \zeta > \frac{P_{\text{sel}}}{P_{\text{max}}}$, the distribution of $X$ conditioned on $\tilde{r}_{\text{sel}}$ using (4.23) is given by

$$
f_X(x|\tilde{r}_{\text{sel}}) = Af_{W|B}(Ax)p_k + (1 - p_k)Bf_X(Bx)
$$

(4.35)

where $B = \tilde{r}_{i,k}^\beta / P_{\text{max}}$. Note that the second term arises from the event when the
mobile user is unable to compensate its path loss and transmit with its maximum transmission power. Finally, averaging over the distribution of $\tilde{r}_{\text{sel}}$, the PDF and MGF of interference can be derived as:

$$f_X(x) = \sum_{\tilde{r}_{\text{sel}}=\tilde{r}_{i,k}} Af_W|\mathbb{B}(Ax)p_k P(\tilde{r}_{\text{sel}} = r_{i,k}) + (1 - p_k)B f_X(Bx) P(\tilde{r}_{\text{sel}} = r_{i,k}). \quad (4.36)$$

$$M_X(s) = \sum_{\tilde{r}_{\text{sel}}=\tilde{r}_{i,k}} M_{W|\mathbb{B}}(s/A)p_k P(\tilde{r}_{\text{sel}} = r_{i,k}) + (1 - p_k)M_X(s/B) P(\tilde{r}_{\text{sel}} = r_{i,k}). \quad (4.37)$$

### 4.5 Modified Slow and Fast PC with Round Robin Scheduling

In this section, we derive the discrete distribution of the distance of the allocated user in a given cell, i.e., the PMF of $r_{\text{sel}}$ for two types of PC. We also derive the explicit expressions for the PDF and MGF of the received signal power at the BS $X_0$ from any scheduled user which is useful in quantifying several performance metrics.

#### 4.5.1 Distribution of the Received Signal Power at BS

**Modified Fast PC**

The modified fast PC mechanism does not allocate resources to users who are unable to compensate their overall path loss on a given subcarrier. Consequently, it may lead to some cases with no transmission which on one hand may degrade the system capacity, whereas, on the other hand it may reduce ICI depending on the value of $P_0$ and other system parameters. The probability of the $k^{th}$ user to compensate its
The overall path loss on a given subcarrier can be calculated using (4.14) as:

\[ p_k = P \left( \zeta_k > \frac{P_0 r_k^\beta}{P_{\text{max}}} \right) = 1 - F_{\zeta_k} \left( \frac{P_0 r_k^\beta}{P_{\text{max}}} \right). \]  

(4.38)

Using (4.14), it is clear that the received signal at the BS remains \( P_0 \) irrespective of the channel conditions of the allocated user. As users cannot be distinguished based on their received powers at the BS, therefore each user who is able to compensate its path loss, shadowing and fading at a given time instant, has equal probability of allocation on a given subcarrier irrespective of the scheduling scheme.

The probability of selecting a user in a ring \( k \) is critically dependent on two important factors; (i) \( k^{\text{th}} \) user able to compensate its overall path loss with probability \( p_k \) greater than zero; (ii) the remaining number of users who can compensate their overall path loss with some probability \( p_k \) greater than zero. Note that it is not necessary that all users will have \( p_k > 0 \), i.e., if some users have \( p_k = 0 \) then these users will not be considered and have zero probability of allocation. The number of competing users will then reduce to \( K' \) in a cell.

To derive the probability of selecting a user \( k \) or ring \( r_k \), it is important to consider all possible combinations of the remaining \( K' - 1 \) users who possess non-zero probability of overall path loss compensation. Consider a scenario where all \( K' \) users can compensate their overall path loss, therefore the probability of selecting a user \( k \) can be given as \( \frac{1}{K'} \prod_{i=1}^{K'} p_i, \forall p_i > 0 \). Another extreme situation is when no user is able to compensate its overall path loss except the \( k^{\text{th}} \) user, therefore, the probability of selecting \( k^{\text{th}} \) user in this case will be \( p_k \prod_{i=1,i\neq k}^{K'} (1 - p_i), \forall p_i > 0 \). Clearly, the total number of combinations required to be considered is \( 2^{K'-1} \); the ability of a given user to overcome its overall path loss is denoted by the binary variable \( n(i) \) where \( n(i) \in \{0, 1\}, 1 \leq i \leq K' - 1 \). Note that '1' represents successful path loss compensation and '0' represents path loss compensation failure. The probability of
selecting $k^{th}$ ring or user $k$ can then be derived as follows:

$$P(r_{sel} = r_k) = \sum_{n \in \mathcal{N}} \frac{p_k}{1 + \sum_{i=1, i \neq k}^{K'} n(i)} \prod_{i=1, i \neq k}^{K'} p_i^{n(i)} (1 - p_i)^{1-n(i)} \quad (4.39)$$

where $\mathcal{N}$ denotes the set of $2^{K'-1}$ combinations of which each combination has $K' - 1$ bits, e.g., for $K' = 3$, $\mathcal{N}$ possesses four elements, i.e., $n \in \{00',01',10',11'\}$, where $n$ denotes a binary vector of size two. The factor of $\frac{1 + \sum_{i=1, i \neq k}^{K'} n(i))^{-1}}$ in (4.39) is the probability of allocating the $k^{th}$ user for each possible combination. For a given combination ‘01’, $n(1) = 0$ and $n(2) = 1$, therefore $(1 + \sum_{i=1, i \neq k}^{K'} n(i))^{-1} = 0.5$, as only two users are transmitting. Note that $\sum_{k=1}^{K'} P(r_{sel} = r_k)$ may not necessarily be equal to one as there exists a certain probability of not allocating a subcarrier to any user (no transmission case), i.e., $\sum_{k=1}^{K'} P(r_{sel} = r_k) + P(r_{sel} = \emptyset) = 1$, where

$$P(r_{sel} = \emptyset) = \prod_{k=1}^{K'} P(r_{sel} \neq r_k) = \prod_{k=1}^{K'} F_{\zeta_k} \left( \frac{P_0 r_k^\beta}{P_{max}} \right). \quad (4.40)$$

Note that as $P_0 \to \infty$ in (4.40), then $P(r_{sel} = \emptyset) \to 1$, and in turn $\sum_{k=1}^{K'} P(r_{sel} = r_k) \to 0$. This is the case when the probability of silent time instants approaches unity and no user can be scheduled on a given subcarrier in a given cell.

Conditioning on the location of the allocated user in a given cell mandates the condition $\zeta > \frac{P_0 r_{sel}^\beta}{P_{max}}$ to be fulfilled. Therefore, using (4.1) the PDF of the received signal power at the BS conditioned on $r_{sel}$ can be derived as follows:

$$f_{X_0|r_{sel}}(x_0) = \delta(x_0 - P_0). \quad (4.41)$$

Averaging over $r_{sel}$ in (4.41), the PDF and MGF of $X_0$ can be derived as follows,
respectively:

\[ f_{X_0}(x_0) = \delta(x_0 - P_0) \sum_{k=1}^{K'} P(r_{\text{sel}} = r_k) + \delta(x_0)P(r_{\text{sel}} = \emptyset), \quad (4.42) \]

\[ \mathcal{M}_{X_0}(s) = \mathbb{E}[e^{sx_0}] = e^{-sp_0} \sum_{k=1}^{K'} P(r_{\text{sel}} = r_k) + P(r_{\text{sel}} = \emptyset). \quad (4.43) \]

Note that the second terms in (4.42) and (5.33) accommodate the probability of no transmission.

**Modified Slow PC**

Due to the rapidly varying fading channels especially at high mobile user speeds, the performance of fast PC may degrade due to the lack of measurements or outdated feedback control information. Therefore, slow PC is normally utilized in practice at high mobile speeds in order to compensate only for the long term channel variation (distance based pathloss and shadowing). The probability of path loss compensation by the \( k^{\text{th}} \) user located in the \( k^{\text{th}} \) ring can then be expressed as:

\[ \tilde{p}_k = P \left( \xi_k > \frac{P_0r_k^\beta}{P_{\text{max}}} \right) = 1 - F_{\xi_k} \left( \frac{P_0r_k^\beta}{P_{\text{max}}} \right). \quad (4.44) \]

The remaining analysis and expressions to compute \( P(r_{\text{sel}} = r_k) \) will remain the same as in (4.39). Since conditioning on the location of the allocated mobile user in a given cell mandates the condition \( \xi > \frac{P_0r_{\text{sel}}^\beta}{P_{\text{max}}} \) to be fulfilled and the RVs \( \eta \) and \( \xi \) are independent, therefore, using (4.19) the PDF of \( X_0 \) conditioned on \( r_{\text{sel}} \) can be derived as follows:

\[ f_{X_0|r_{\text{sel}}}(x_0) = \frac{1}{P_0} f_{\eta} \left( \frac{x_0}{P_0} \right). \quad (4.45) \]
Averaging over $r_{\text{sel}}$, the PDF and MGF of $X_0$ can be derived as follows, respectively:

$$f_{X_0}(x_0) = \sum_{k=1}^{K'} \frac{1}{P_0} f_{\eta_k} \left( \frac{x_0}{P_0} \right) P(r_{\text{sel}} = r_k) + \delta(x_0) P(r_{\text{sel}} = \emptyset). \quad (4.46)$$

$$\mathcal{M}_{X_0}(s) = \sum_{k=1}^{K'} \mathcal{M}_{\eta_k} (sP_0) P(r_{\text{sel}} = r_k) + P(r_{\text{sel}} = \emptyset). \quad (4.47)$$

From Chapter 3, we can write $P(r_{\text{sel}} = r_k, \theta = \theta_i) = \frac{P(r_{\text{sel}} = r_k)}{I}$. 

### 4.5.2 Statistics of ICI

In this section, we derive the distribution of the ICI for the different PC mechanisms.

**Modified Fast PC Mechanism**

Conditioned on the location of allocated mobile user $\tilde{r}_{\text{sel}}$ in an interfering cell mandates the condition $\zeta > \frac{P_0 r_{\beta_{\text{sel}}}^\beta}{P_{\text{max}}}$ to be fulfilled. Therefore, by defining a RV as $W = \chi/\zeta$ conditioned on $I$, where $I \rightarrow \zeta > \frac{P_0 r_{\beta_{\text{sel}}}^\beta}{P_{\text{max}}}$, the interference $X$ conditioned on $\tilde{r}_{\text{sel}}$ can be calculated using (4.21) as $X = P_0 r_{\beta_{\text{sel}}}^\beta \tilde{r}_{\beta_{\text{sel}}} W$. The distribution of $X$ can then be derived by RV transformation of the result derived in (4.30) as follow:

$$f_X(x|\tilde{r}_{\text{sel}}) = A f_{W|I}(A x), \quad (4.48)$$

where $A = r_{\beta_{\text{sel}}}^\beta / P_0$. Averaging at $\tilde{r}_{\text{sel}}$ and using (4.21), the PDF of $X$ is given as:

$$f_X(x) = \sum_{\tilde{r}_{\text{sel}} = \tilde{r}_{i,k}}^{} A f_{W|I}(A x) P(\tilde{r}_{\text{sel}} = \tilde{r}_{i,k}) + \prod_{k=1}^{K'} (1 - p_k) \delta(x). \quad (4.49)$$

where $p_k$ is given by (4.38). Note that there is a non-zero probability of $X = 0$ which is the case when none of the users is able to compensate its path loss, therefore the second term in (4.49) denotes the probability of silent time instants with no
transmission. Consequently, the MGF of the interference from an interfering cell can be obtained as follows:

\[
M_X(s) = \sum_{\tilde{r}_{sel}=\tilde{r}_{i,k}} M_{W|B}(s/A)P(\tilde{r}_{sel}=\tilde{r}_{i,k}) + \prod_{k=1}^{K'} (1 - \tilde{p}_k). \tag{4.50}
\]

### Modified Slow PC Mechanism

Consider a RV \( W = \chi/\xi \) conditioned on \( B \), where \( B \rightarrow \xi > \frac{P_o r_{sel}}{P_{\text{max}}} \), the interference \( X \) conditioned on \( \tilde{r}_{sel} \) can be given as \( X = P_0 r_{sel}^{-\beta} W \). Similar to the previous case, the PDF and MGF of \( X \) can be derived using (4.22) as follows, respectively:

\[
f_X(x) = \sum_{\tilde{r}_{sel}=\tilde{r}_{i,k}} A f_{W|B}(Ax)P(\tilde{r}_{sel}=\tilde{r}_{i,k}) + \prod_{k=1}^{K'} (1 - \tilde{p}_k) \delta(x). \tag{4.51}
\]

\[
M_X(s) = \sum_{\tilde{r}_{sel}=\tilde{r}_{i,k}} M_{W|B}(s/A)P(\tilde{r}_{sel}=\tilde{r}_{i,k}) + \prod_{k=1}^{K'} (1 - \tilde{p}_k). \tag{4.52}
\]

### MGF of the Cumulative ICI

Since each cell is considered to have the same PC mechanism deployed, the MGF of the cumulative interference can be calculated as \( M_Y(s) = \prod_{l=1}^{L} M_{X_l}(s) = (M_X(s))^L \).

### 4.6 Network Performance Metrics for Channel based PC Mechanisms

In this section, we will utilize the obtained MGF expressions of the cumulative ICI \( Y \) and the received signal \( X_0 \) to derive numerical expressions for network ergodic capacity \( C \), the average resource fairness among users \( F \), and the average network power savings \( P \), as a function of the three considered PC mechanisms.
Evaluation of Network Ergodic Capacity

Again, we follow the efficient lemma proposed in [71] to compute $C$ as $\mathbb{E} \left[ \ln \left( 1 + \frac{X_0}{Y + \sigma^2} \right) \right] = \int_0^\infty \frac{M_Y(s)-M_{X_0,Y}(s)}{s} e^{-s} ds$, where $M_Y(s) = \mathbb{E}[e^{-s \sum_{i=1}^L X_i}]$ is the MGF of the cumulative interference and $M_{X_0,Y}(s) = \mathbb{E}[e^{-s(X_0+\sum_{i=1}^L X_i)}] = \mathbb{E}[e^{-s(X_0+Y)}]$ is the joint MGF of the signal power received at the BS $X_0$ and cumulative interference $Y$. Since $X_0$ and $Y$ are independent, $M_{X_0,Y}(s) = M_{X_0}(s)M_Y(s)$. The closed-form expression for $M_{X_0}(s)$ for the modified fast PC mechanism is given in (5.33). The closed form expression for the modified slow PC mechanism, as given in (5.34) can be derived for approximated Generalized-$K$ fading as follows:

$$M_{X_0}(s) = \sum_{k=1}^{K'} (1 + sP_0\Theta_{\zeta_k})^{-\kappa_{\zeta_k}} P(r_{sel} = r_k) + P(r_{sel} = \emptyset).$$ (4.53)

The closed form expressions for the conventional fast PC mechanism, as given in (4.28), can be derived for approximated Generalized-$K$ fading as:

$$M_{X_0}(s) = \sum_{k=1}^{K'} \left( \frac{r_k^\beta}{r_k^\beta + s\Theta_{\zeta_k} P_{\max}} \right)^{\kappa_{\zeta_k}} \left( 1 - \frac{\Gamma_u(\kappa_{\zeta_k}, P_0(r_k^\beta P_{\max} + s))}{\Gamma(\kappa_{\zeta_k})} \right) P(r_{sel} = r_k) + e^{-sP_0p_k} P(r_{sel} = r_k)$$ (4.54)

Evaluation of Average Fairness among Users

To measure the degree of resource fairness among users, we follow the notion as mentioned in Chapter 3 for the different PC mechanisms.

Evaluation of Average Network Power Savings

The conventional fast PC mechanism leads to power savings when the allocated user on a given subcarrier is able to compensate its overall path loss, i.e., $\mathbb{B} \rightarrow \zeta > \frac{P_{\text{link}}}{P_{\max}}$;
otherwise, each user will transmit with $P_{\max}$. Therefore, a new RV $T$ which captures the instantaneous power savings can be defined as follows:

$$T = \begin{cases} 
  P_{\max} - \frac{P_{0r_{sel}}}{\zeta} & \zeta > \frac{P_{0r_{sel}}}{P_{\max}} \\
  0 & \zeta \leq \frac{P_{0r_{sel}}}{P_{\max}} 
\end{cases}$$

(4.55)

The average power savings per subcarrier conditioned on $r_{sel}$, i.e., $\mathbb{E}[T|r_{sel}]$ can be derived by using the property of expectations $\mathbb{E}[g(x)] = \int_0^\infty g(x)f(x)dx$ as follows:

$$\mathbb{E}[T|r_{sel}] = \int_{\frac{P_{max}}{P_{0r_{sel}}}}^{\infty} \left( P_{\max} - \frac{P_{0r_{sel}}}{\zeta} \right) f_\zeta(\zeta)d\zeta ,$$

(4.56)

Finally, averaging over $r_{sel}$, the closed form expression for the average network power savings $\mathcal{P}$ can be derived as follows:

$$\mathcal{P} = \sum_{k=1}^{K'} P(r_{sel} = r_k) \frac{P_{\max} \Gamma_u \left( \kappa_\zeta, \frac{P_{0r_k}}{P_{\max} \Theta_\zeta} \right)}{\Gamma(\kappa_\zeta)} - P(r_{sel} = r_k) \frac{P_{0r_k}}{\Theta_\zeta \Gamma(\kappa_\zeta)} \Gamma_u \left( \kappa_\zeta - 1, \frac{P_{0r_k}}{P_{\max} \Theta_\zeta} \right).$$

(4.57)

The average power savings for the modified fast PC can then be calculated as:

$$\mathcal{P} = \sum_{k=1}^{K'} \frac{P(r_{sel} = r_k) P_{\max} \Gamma_u \left( \kappa_\zeta, \frac{P_{0r_k}}{P_{\max} \Theta_\zeta} \right)}{P(\bar{r}) \Gamma(\kappa_\zeta)} - \frac{P(r_{sel} = r_k) P_{0r_k}}{P(\bar{r}) \Theta_\zeta \Gamma(\kappa_\zeta)} \Gamma_u \left( \kappa_\zeta - 1, \frac{P_{0r_k}}{P_{\max} \Theta_\zeta} \right) + P_{\max} P(r_{sel} = \emptyset).$$

(4.58)

Note that there is an additional power saving which results from the event of no transmission. The closed form expression of the average power savings in the modified slow PC mechanism can be derived in a straightforward manner by simply replacing the RV $\zeta$ in (4.56) with the RV $\xi$. 
4.7 Results and Discussions

Location-based PC

In this section, we aim to validate the accuracy of the derived expressions through Monte-Carlo simulations. The results are presented for Gamma composite fading, i.e., \( \zeta \sim \text{Gamma}(1, 1) \), \( \chi \sim \text{Gamma}(3/2, 2/3) \).

In Fig. 4.2 the impact of the maximum transmit power limit is shown on the PMF of allocated user locations. The obtained PMF results fit well with exhaustive Monte-Carlo simulations. Since the users located within the threshold region \( r_t \) can compensate their distance based path loss, each user has on average equal probability of allocation within \( r_t \). The increasing trend of PMF within \( r_t \) is therefore simply due to an increase in the number of users in each ring from cell center to the cell edge. It is important to note that the users located beyond \( r_t \) transmit with their maximum power as they cannot compensate path loss. These users are therefore scheduled based on their relative channel gains which prioritizes close users over the far users and hence causes rapid decay of allocation probability beyond \( r_t \).

In greedy scheduling, the cell center users have higher priority to be allocated over the cell edge users. On the other side, round robin scheduling provides equal probability of allocation to each user, hence high probability of allocation near the cell edge due to the large area and large number of users at the cell-edge. By observing the result, it can be concluded easily that greedy scheduling with PC follows the trend of round robin within \( r_t \) whereas the trend of greedy scheduling beyond \( r_t \). The performance of greedy scheduling with PC is therefore expected to lie in between the two extremes. Two different transmit power limits are also studied in Fig. 4.2 which yields two threshold distances, i.e., \( r_t = 400 \)m and \( r_t = 260 \)m, respectively. It can be observed that the greater the maximum transmit power, the greater is the threshold distance and more users located farther from the BS become capable to compensate
path loss which increases fairness and in turn the incurred ICI. The slight mismatch in the simulations and analysis demonstrates the impact of discretization which is dominant for channel based scheduling beyond $r_t$. However, this error can be reduced by increasing the number of rings.

The CDF of the ICI for different transmit power budgets and different path loss exponents for greedy scheduling with and without PC is plotted in Fig. 4.3. High values of path loss exponents cause rapid signal degradation, hence, reduces ICI. Moreover, it can be observed clearly that with low user transmit powers, there is a significant reduction in ICI compared to the high transmission powers. *It is further interesting to note that the performance of greedy scheduling with PC always remain better than the greedy scheme in terms of incurred ICI, average fairness (see Fig. 4.4part(a)) and average power consumption of the users.* The top figure in Fig. 4.4 quantifies the average fairness of the greedy with and without PC and round robin schedulers. With the increase of transmit powers, far users can also adapt their power which increases the average fairness among users. For high user transmit powers, the greedy scheduling
Figure 4.3: (a) CDF of the ICI for different transmit power levels and path loss exponents considering greedy scheduling with and without PC, $P_0 = -23$ dBm, $R = 500$ m.

with PC achieves the fairness of round robin scheme as is also evident from Fig. 4.2. However, the capacity (see Fig. 4.4part(b)) and power preservation remains better than the round robin scheme in which power savings are zero.

The bottom figure in Fig. 4.4 demonstrate the network capacity of interference limited systems (i.e., thermal noise is neglected). Without PC, the performance of greedy and round robin scheduling remains independent of the transmit power as the factor of $P_{\text{max}}$ cancels out in the capacity calculation. However, since the greedy scheduling with PC have less ICI then the greedy scheduler, the network capacity is expected to increase which is not the case as the corresponding user transmit powers are also lowered along with the interfering powers. The main reason of the capacity degradation with the increase in transmit power budget is that the greater transmission power more users can compensate path loss which reduces the number of users transmitting with their maximum powers. This phenomena on one hand increase average power savings whereas on the other hand degrades system capacity.
Figure 4.4: (a) Average fairness among users considering greedy scheduler with and without PC and round robin scheduler without PC (b) Ergodic capacity considering greedy scheduler with and without PC and round robin scheduler without PC, $U = 50$, $P_0 = -23$ dBm, $R = 500$ m, $\beta = 2.2$.

Channel-based PC Mechanisms

The radius $R$ of the cell is set equal to 500 m and the number of competing users on a given subcarrier is $U = 20$ with path loss exponent $\beta = 3$. Generalized-$K$ composite fading distribution is approximated by the tractable Gamma distribution using (4.29), i.e., the interference channel composite fading $f_\chi(\chi)$ is approximated as $\text{Gamma}(3/2, 2/3)$ unless stated otherwise, whereas the desired channel composite fading $f_\zeta(\zeta)$ is approximated as $K_G(4, 3/4, 2) \approx \text{Gamma}(0.5, 3.8)$, i.e., $K_G(4, 3/4, 2)$ is a product of shadowing $\xi \sim \text{Gamma}(4, 0.5)$ and fading $\eta \sim \text{Gamma}(3/4, 4/3)$.

In the Monte-Carlo simulations, we start by generating $U$ uniformly distributed users per cell. The instantaneous power received at the BS from each user is generated as given in (4.14), (4.15) or (4.16). We then select any user arbitrarily from the set of users who are able to compensate their overall path loss for the modified PC mechanisms, whereas, we select any user randomly irrespective of its channel conditions for the conventional PC mechanism. The distances of the selected users, i.e., $r_{sel}$ from
Figure 4.5: PMF of the distance of allocated users (i.e., PMF of \( r_{\text{sel}} \)) for different values of \( P_0 \) for the modified fast PC mechanism.

The derived analytical PMF results fit nearly perfectly with the corresponding Monte-Carlo simulation results which verifies the accuracy of the obtained expressions. The PMF results are not shown for the conventional PC mechanism as each ring has equal probability to get a subcarrier allocated. Firstly, it can be observed that as the desired signal power received at the BS, i.e., \( P_0 \) increases, the probability of allocating a subcarrier or the probability of transmission reduces which is due to the reduction in the number of users who can successfully compensate their overall path loss. More precisely, the increase in \( P_0 \) encourages more cell-edge users to remain silent which ultimately results in reducing the proportion of served cell-edge users.

Moreover, the probability of allocating subcarriers to cell-edge users is higher for
Figure 4.6: PMF of the distance of allocated users (i.e., PMF of $r_{\text{sel}}$) for different values of $P_0$ for the modified slow PC mechanism.

the modified fast PC mechanism since fast fading makes the channels more arbitrary and, thus, reduces the bias between cell-center and cell-edge users. As $P_0$ becomes relatively low (e.g., case of -53 dBm), the performance of both modified PC mechanisms converges towards fair round robin allocation across the cell area. Having low $P_0$ corresponds indirectly to low loaded network scenarios with limited interference and, thus, all users in the cell can achieve their target quality when allocated resources.

Fig. 4.7 investigates the impact of $P_0$ on the CDF of the ICI from a single interfering cell for the modified and conventional fast PC mechanisms. Increasing $P_0$ with modified PC enhances the probability of no transmission, whereas with conventional PC it enhances the probability of selecting a user who is transmitting with $P_{\text{max}}$. Therefore, the observed trend of incurred ICI is almost reverse for both PC mechanisms as an increase in $P_0$ will reduce ICI for modified PC whereas increase ICI for conventional PC. For the ICI CDF plots, it can be also seen that the Monte-Carlo simulation results fit perfectly with the derived numerical results.

Fig. 4.8 compares the CDF of the cumulative ICI of all considered PC mechanisms for different values of $P_0$. Moreover, a comparison is also provided for round robin
Figure 4.7: CDF of the ICI for different values of $P_0$ for modified fast PC and conventional fast PC with a single neighboring cell.

Figure 4.8: CDF of the cumulative ICI for different values of $P_0$ considering modified fast PC, modified slow PC, conventional fast PC, and round robin scheduling without PC.
scheduling without PC where all allocated users transmit with their maximum power irrespective of their channel conditions. As in this chapter we derive and utilize the MGF of the cumulative ICI instead of the CDF, Fig. 4.8 is plotted using simulations. It can be seen that the ICI is notably higher in the case of no PC and, thus, Fig. 4.8 can be used to quantify the ICI reduction gains of the various PC mechanisms with respect to no PC. Moreover, conventional PC reduces the ICI only at low values of $P_0$ and converges ultimately to the case of no PC at higher values of $P_0$ (e.g., conventional PC and no PC nearly overlap when $P_0 = -33$ dBm).

Fig. 4.9(a) investigates the percentage of users served by the BS with slow and fast modified PC as a function of $P_0$. The average number of users served by the BS in modified fast PC remains slightly lower compared to its counterpart slow PC. This is due to the fact that fast PC mandates to serve only the users who can fully compensate their overall path loss whereas slow PC is more flexible by allowing the users who can only compensate their shadowing and distance-based pathloss components. In Fig. 4.9(b), the average network fairness is studied as a function of $P_0$. Conventional PC with round robin scheduling is strictly fair; however, the fairness of modified PC reduces with the increase of $P_0$. Moreover, the fairness of modified fast PC is slightly better than modified slow PC in coherence with the results presented in Fig. 4.5 and Fig. 4.6. Note that the average fairness of modified PC is higher than greedy scheduling without PC for a specific range and then becomes worse due to the low number of allocated users at high values of $P_0$.

Fig. 4.9(c) quantifies the average power savings on a given subcarrier considering conventional and modified PC. In most cases, the average power efficiency of slow and fast PC is found to be nearly the same; however, significant gains are observed for modified PC over conventional PC. With modified PC, the number of users transmitting with maximum power is less and the percentage of users who are allowed to transmit is also less, especially for large values of $P_0$. 
Figure 4.9: (a) Percentage of users served by the BS; (b) Average resource fairness ($F$); (c) Average power savings ($P$) as a function of $P_0$ considering different PC mechanisms.

More precisely, the increase in $P_0$ reduces the average power savings for modified PC as users are transmitting with more power to achieve their target. However, afterwards a gradual rise in the average power savings is observed due to higher percentage of users not transmitting since they cannot meet their target. On the other hand, as $P_0$ increases the probability of users transmitting with $P_{\text{max}}$ increases in conventional PC which reduces the power savings.

Fig. 4.10 presents the network ergodic capacity of the considered PC mechanisms in addition to greedy and round robin scheduling without PC, where selected users transmit with maximum power irrespective of their channels. Significant gains are observed for modified PC compared to conventional PC especially for a specific range in which the probability of allocating users is high. These gains are achieved with a trade-off cost in terms of the average resource fairness. Moreover, it is interesting to observe that as the proportion of silent time instants with no transmissions increases beyond a certain limit, the gains tend to reduce and a fall back in the ergodic capacity is observed. Conventional PC converges to the performance of round robin scheduling without PC for high values of $P_0$. The performance of fast PC is observed to be better
Figure 4.10: Ergodic capacity (C) as a function of $P_0$ considering modified fast PC, modified slow PC, conventional fast PC, greedy scheduling without PC and round robin scheduling without PC.

than slow PC for low values of $P_0$ which is in agreement with the observed ICI CDF in Fig. 4.8.

Note that the accuracy of the analytical derivations in case of modified PC tends to reduce slightly as the probability of allocating users decreases for high values of $P_0$. In this case, it is highly probable that the selected user in the Monte-Carlo simulations is located within the first ring whereas it is assumed to be at the ring boundary (at distance $r_1$) in the analytical derivations. Fig. 4.11 captures the impact of different wireless channel conditions on the network ergodic capacity for different PC mechanisms. The ergodic capacity is plotted as a function of the average shadowing power $\Omega_\xi = \kappa_\xi \Theta_\xi$. A larger performance gap between fast and slow PC is observed for high fading severity parameters, whereas, the performance gap tends to diminish for mild fading severity conditions. Moreover, with the increase in the power of the shadowing channel statistics, the network ergodic capacity increases as expected.
Mean of Received Shadowing Power ($\Omega_\xi = \kappa_\xi \Theta_\xi$)

Network Ergodic Capacity [bps/Hz]

Greedy w/o PC
Modified Fast PC; $\kappa_\eta = 3/4$, $\kappa_\xi = 4$
Modified Fast PC; $\kappa_\eta = 4$, $\kappa_\xi = 3/4$
Modified Slow PC; $\kappa_\eta = 4$, $\kappa_\xi = 3/4$
Modified Slow PC; $\kappa_\eta = 3/4$, $\kappa_\xi = 4$
Round Robin w/o PC
Conventional Fast PC

Figure 4.11: Ergodic capacity ($C$) as a function of the power $\Omega_\xi$ of the shadowing statistics considering modified fast PC, modified slow PC, conventional fast PC, greedy scheduling without PC and round robin scheduling without PC, $\Omega_\eta = 1$, $P_0 = -33$dBm.

4.8 Conclusion

This chapter presented a mathematical framework to derive numerical expressions for the distribution of uplink ICI as a function of different PC mechanisms assuming round robin scheduling. The accuracy of the derived expressions is validated via Monte-Carlo simulations. The importance of the derived expressions is demonstrated by utilizing them to evaluate key network performance metrics that include average network fairness, average power savings, and network ergodic capacity. Regarding the performance trends of the various PC mechanisms discussed in this chapter, the following observations can be extracted from the presented results: (i) The performance trends remain almost similar for the modified slow and fast PC mechanisms in most cases. Fast PC typically outperforms slow PC in terms of ergodic capacity and ICI levels especially for low and medium values of the target power level $P_0$. The difference between the two depends notably on the parameters of the shadowing and fading statistics. (ii) Taking into account multiple performance metrics, modified PC significantly outperforms conventional PC for a medium range of $P_0$ values with a
trade-off cost in terms of average fairness. Nonetheless, at extremely high values of $P_0$, this observation reverses as conventional PC and no PC outperform modified PC.

(iii) Compared to no PC, conventional PC is observed to be a better choice in terms of power savings with a trade-off cost in terms of ergodic capacity. (iv) As a summary, the obtained results motivate the design of hybrid fractional PC mechanisms that combine conventional PC, modified PC, and no PC based on the channel distribution characteristics and the target received signal level $P_0$ which can be indirectly mapped to the load in the network and the quality of service requirements of the users. Now we will look in the next chapter into the modeling and management of ICI in multicarrier networks with coordinated, uncoordinated scheduling and FFR based resource allocation schemes.

APPENDIX A - PROOF of (4.25) and (4.26)

In order to simplify notations, we consider $c = P_0 r_{sel}^\beta / P_{\text{max}}$ in (4.20) and rewrite (4.20) as given below:

$$X_0 = P_0 U (\zeta - c) + \frac{P_0}{c} \zeta U (c - \zeta) = \left( \frac{P_0}{c} \zeta - P_0 \right) U (c - \zeta) + P_0, \quad (4.59)$$

As Dirac Delta functions can be used to evaluate the PDF of transformed RVs [74], the PDF of $X_0$ can be stated as follows:

$$f_{X_0}(X_0) = \int_0^\infty \delta (X_0 - g(\zeta)) f_\zeta(\zeta) d\zeta, \quad (4.60)$$

where $g(\zeta)$ is given by the right hand side of (4.59) and by substituting it in (4.60), we can write:

$$f_{X_0}(X_0) = \int_0^\infty \delta \left( X_0 - \left( \frac{P_0}{c} \zeta - P_0 \right) U (c - \zeta) - P_0 \right) f_\zeta(\zeta) d\zeta. \quad (4.61)$$
Note that the integral in (4.61) can be decomposed into two integrals as shown below:

\[ f_{X_0}(X_0) = \int_0^c \delta \left( X_0 - \frac{P_0}{c} \right) f_\zeta(\zeta)d\zeta + \int_c^\infty \delta \left( X_0 - P_0 \right) f_\zeta(\zeta)d\zeta. \] \hspace{1cm} (4.62)

Applying the transformation property of Dirac Delta distribution [74] on the first integral and solving the second integral, (4.62) can be simplified as follows:

\[ f_{X_0}(X_0) = \delta \left( X_0 - P_0 \right) \left( 1 - F_\zeta(c) \right) + \int_0^c \frac{c}{P_0} \delta \left( \zeta - \frac{c}{P_0} X_0 \right) f_\zeta(\zeta)d\zeta. \] \hspace{1cm} (4.63)

The definite integral of (4.63) can be rewritten as \( \int_0^\infty \frac{c}{P_0} \delta \left( \zeta - \frac{c}{P_0} X_0 \right) \mathbb{U}(c - \zeta) f_\zeta(\zeta)d\zeta \) and can be solved by using the property of Dirac delta functions as \( \frac{c}{P_0} \mathbb{U}(P_0 - X_0) f_\zeta \left( \frac{c}{P_0} X_0 \right) \).

Substituting this solution into (4.63), the PDF of \( X_0 \) can be finally given as in (4.25).

Note that the CDF of \( X_0 \) defined as \( F_{X_0}(X_0) = \int_0^{X_0} f_{X_0}(u)du \) can be written as:

\[ F_{X_0}(X_0) = \mathbb{U}(X_0 - P_0)(1 - F_\zeta(c)) + \frac{c}{P_0} \int_0^{X_0} \mathbb{U}(P_0 - u) f_\zeta \left( \frac{c}{P_0} u \right) du. \] \hspace{1cm} (4.64)

The definite integral in (4.64) can be simplified as \( \frac{c}{P_0} \int_0^\infty \mathbb{U}(P_0 - u) \mathbb{U}(X_0 - u) f_\zeta \left( \frac{c}{P_0} u \right) du = \frac{c}{P_0} \int_0^\infty \mathbb{U}(\min(P_0, X_0) - u) f_\zeta \left( \frac{c}{P_0} u \right) du \). Finally, the integral can be simplified and solved as:

\[ \frac{c}{P_0} \int_0^{\min(P_0, X_0)} f_\zeta \left( \frac{c}{P_0} u \right) = F_\zeta \left( \frac{\min(P_0, X_0)c}{P_0} \right). \] \hspace{1cm} (4.65)

Substituting (4.65) into (4.64), the CDF of \( X_0 \) is derived as

\[ F_{X_0}(X_0) = F_\zeta \left( \frac{\min(P_0, X_0)c}{P_0} \right) + \mathbb{U}(X_0 - P_0)(1 - F_\zeta(c)). \] \hspace{1cm} (4.66)

Moreover, the expression in (4.66) can be further simplified as shown in (4.26).
APPENDIX B - Proof of (4.30) and (4.31)

Consider a RV \( W = P/Q \) which is a ratio of two independent RVs such that \( P > 0 \) and \( Q > c \), where \( c \) is a real constant. Introducing an auxiliary RV \( V \), letting \( V = Q \) and applying bivariate transformation, the joint distribution of \( V \) and \( W \) can be given as \( f_{W,V}(w,v) = f_{P,Q}(wv,v) |v|, \ v > c \). As \( P \) and \( Q \) are independent \( f_{P,Q}(p,q) = f_P(p)f_Q(q) \), therefore \( f_{W,V}(w,v) = f_P(wv)f_Q(v) |v| \), and finally by averaging over \( V \) the PDF of \( W \) can be derived as:

\[
f_W(w) = \int_c^{\infty} f_P(wv)f_Q(v) |v| dv. \tag{4.67}
\]

Now consider \( P \) and \( Q \) are gamma distributed RVs, i.e., \( P \sim \text{Gamma}(\kappa_p, \Theta_p) \) and \( Q \sim \text{Gamma}(\kappa_q, \Theta_q) \), then using (4.67), the distribution of \( W \) can be written as:

\[
f_W(w) = \frac{(wv)^{\kappa_p-1}e^{-wv/\Theta_p}}{\Theta_p^{\kappa_p} \Gamma(\kappa_p) \Gamma(\kappa_q)} \int_c^{\infty} e^{-u \left( \frac{w}{\Theta_p} + \frac{1}{\Theta_q} \right) + u^{\kappa_q+\kappa_p}} |v| dv. \tag{4.68}
\]

Using \([67][3.381/3]\), the integral in (4.68) can be solved and simplified as

\[
\int_c^{\infty} e^{-u \left( \frac{w}{\Theta_p} + \frac{1}{\Theta_q} \right) + u^{\kappa_q+\kappa_p}} |v| dv = e^{-w \left( \frac{1}{\Theta_p} + \frac{1}{\Theta_q} \right) + \kappa_q - \kappa_p} \Gamma_u \left( \kappa_q + \kappa_p, \left( \frac{w}{\Theta_p} + \frac{1}{\Theta_q} \right) c \right). \tag{4.69}
\]

Finally, substituting (4.69) in (4.68) and doing some algebraic manipulations, the PDF of \( W \) can be written as follows:

\[
\frac{w^{\kappa_p-1} \Gamma_u \left( \kappa_q + \kappa_p, \left( \frac{w\Theta_q + \Theta_p}{\Theta_p \Theta_q} \right) c \right)}{\Theta_p^{-\kappa_q} \Theta_q^{-\kappa_p} \Gamma(\kappa_p) \Gamma(\kappa_q) (w\Theta_q + \Theta_p)^{\kappa_q+\kappa_p}}. \tag{4.70}
\]
To derive the MGF of $W$, i.e. $\mathcal{M}_W(s) = \int_0^\infty e^{-sw} f_W(w)dw$, we can write:

$$
\mathcal{M}_W(s) = \frac{\Theta_p^{\kappa_p} \Theta_q^{\kappa_q}}{\Gamma(\kappa_p) \Gamma(\kappa_q)} \int_0^\infty \frac{e^{-sw} w^{\kappa_p - 1}}{(\Theta_p + w\Theta_q)^{\kappa_p + \kappa_q}} \Gamma \left( \kappa_q + \kappa_p, \left( \frac{w}{\Theta_p} + \frac{1}{\Theta_q} \right) c \right) dw.
$$

(4.71)

Note that the integral in (4.71) contains an incomplete upper gamma function which can be expressed for integer values of $\kappa_p + \kappa_q$ as $\Gamma_a(n, x) = \Gamma(n) e^{-x} \sum_{k=0}^{n-1} \frac{x^k}{k!}$. Note that for non-integer case, Gaussian quadrature rule can be applied to decompose incomplete gamma function. Using this relation, $\mathcal{M}_W(s)$ in (4.71) can be given as:

$$
\mathcal{M}_W(s) = \frac{\Gamma(\kappa_q + \kappa_p) \Theta_p^{\kappa_p} \Theta_q^{\kappa_q} e^{-c/\theta_q}}{\Gamma(\kappa_p) \Gamma(\kappa_q)} \int_0^\infty \frac{e^{-w\left(\frac{c}{\Theta_p} + s\right)} w^{\kappa_p - 1}}{(\Theta_p + w\Theta_q)^{\kappa_p + \kappa_q}} \sum_{k=0}^{\kappa_p + \kappa_q - 1} \frac{\left( \frac{w}{\Theta_p} + \frac{1}{\Theta_q} \right)^k}{k!} dw.
$$

(4.72)

By interchanging the summation and integral functions (4.72) can be simplified as:

$$
\mathcal{M}_W(s) = \frac{\Gamma(\kappa_q + \kappa_p) \Theta_p^{\kappa_p} \Theta_q^{\kappa_q} e^{-c/\theta_q}}{\Gamma(\kappa_p) \Gamma(\kappa_q)} \sum_{k=0}^{\kappa_p + \kappa_q - 1} \frac{c^k}{k!(\Theta_p \Theta_q)^k} \int_0^\infty \frac{e^{-w\left(\frac{c}{\Theta_p} + s\right)} w^{\kappa_p - 1}}{(\Theta_p + w\Theta_q)^{\kappa_p + \kappa_q - k}} dw.
$$

(4.73)

Note that the solution of integral in (4.73) and its closed-form expression can be given in terms of HypergeometricU function defined in (4.32) as follows:

$$
\int_0^\infty \frac{e^{-w\left(\frac{c}{\Theta_p} + s\right)} w^{\kappa_p - 1}}{(\Theta_p + w\Theta_q)^{\kappa_p + \kappa_q - k}} dw = \Theta_p^{-\kappa_p} \Theta_q^{-\kappa_q} \Gamma(\kappa_p) U \left( \kappa_p, 1 + k - \kappa_q, \frac{c + s\Theta_p}{\Theta_q} \right).
$$

(4.74)

Finally substituting (4.74) in (4.73) and doing some algebraic manipulations, the final expression for the MGF of $W$ can be obtained as follows:

$$
\mathcal{M}_W(s) = \frac{\Gamma(\kappa_q + \kappa_p) e^{-c/\theta_q}}{\Gamma(\kappa_p) \Gamma(\kappa_q)} \sum_{k=0}^{\kappa_p + \kappa_q - 1} \frac{c^k}{k! \Theta_q^k} U \left( \kappa_p, 1 + k - \kappa_q, \frac{c + s\Theta_p}{\Theta_q} \right).
$$

(4.75)
Chapter 5

Interference Modeling for
Multicarrier Networks with
Resource Allocation

5.1 Introduction

Due to the high spectral efficiency requirements and the scarcity of wireless spectrum resources, the implementation of universal frequency reuse (UFR) in LTE cellular networks is gaining momentum in the wireless industry and research communities. However, the implementation of UFR leads to technical challenges that need to be addressed for successful operation. In this context, ICI has been recognized as a primary performance limitation which poses a significant impact on the overall performance of UFR networks. Moreover, fractional frequency reuse (FFR) networks emerge as an efficient solution to suppress ICI in future wireless networks. In comparison to conventional cellular networks, the dynamics of ICI in FFR networks is highly uncertain depending on the partition criteria of cellular region and the allocation of users and spectrum within each partition.

In this chapter, firstly we extend the single carrier framework presented in Chap-
ter 3 to characterize the statistics of ICI in multi-carrier networks\(^1\). The developed framework captures the impact of different uncoordinated scheduling schemes (such as greedy, location based round robin, and round robin) with equal power allocation (EPA) on the allocated subcarriers of a given user; EPA was shown to possess near-optimal performance, i.e., close to water-filling, in uplink OFDMA networks [6]. Moreover, we extend the derived ICI model to design and analyze the performance of different BS coordination schemes and frequency reuse patterns. Finally, we focus on the ICI modeling of FFR multicarrier networks considering greedy and round robin scheduling assuming EPA on the allocated subcarriers of a given user. The developed theoretical model of ICI is adaptive for a varying range of cell and spectrum partitions. The derived expressions are used to evaluate the network capacity in order to gain theoretical insight into the performance of coordinated and uncoordinated scheduling schemes in various network scenarios. Simulation results validate the accuracy of the derived mathematical formalism and highlight the network conditions where FFR schemes outperform the conventional UFR schemes.

The chapter is organized as follows: Section 5.2 and Section 5.3 derive the statistics of uplink ICI considering uncoordinated and coordinated scheduling schemes, respectively. In Section 5.4, we derive the statistics of ICI in FFR networks and obtain closed form expressions for Rayleigh fading channels. Finally, in Section 5.5, we utilize the derived expressions to evaluate network ergodic capacity. Section 5.6 presents numerical and simulation results and Section 5.7 concludes the chapter.

5.2 Uncoordinated Scheduling Schemes

We consider the uplink of a multi-carrier cellular network, where a given cell is surrounded by \(L\) neighboring cells. For analytical tractability, each cell is considered to

\(^1\)Considering a single subcarrier environment implicitly assume the same behavior and performance gains, on average, associated with all subcarriers which may lead to extreme conclusions, i.e., it leads to an upper bound on the actual results as demonstrated in the results section.
be circular in shape with radius $R$ and $U$ uniformly distributed users. We assume frequency reuse factor of unity and the number of available subcarriers in each cell is equal to $N$. The instantaneous channel gain $\gamma^j$ of any user at subcarrier $j$ is defined as $\gamma^j = \xi (r/r_0)^{-\beta} \zeta^j$, where, $r_0$ is the path loss model reference distance, $\xi$ is the path loss constant which is set to unity, i.e., $r_0 = 1$ m, $r$ is the distance of a user from its serving BS, $\beta$ is the path-loss exponent, and $\zeta^j$ is the combined shadowing and fading, i.e., composite fading RV. The maximum available transmission power of each user is $P_{\text{max}}$. The scheduling decision on each subcarrier is considered to be independent\(^2\) of the decisions on other subcarriers. We consider three uncoordinated scheduling schemes, namely: (i) greedy, (ii) round robin, and (iii) location based round robin (LBRR).

For the purpose of analytical modeling, each cell is divided into $K$ circular regions with average number of users in each ring $k$ as in Chapter 3. The instantaneous channel gain $\gamma^j_k$ of any user located in ring $k$ at subcarrier $j$ can be given as: $\gamma^j_k = r_k^{-\beta} \zeta^j_k$. Note that, without loss of generality we consider i.i.d. composite fading statistics of the users located in a ring $k$ on all subcarriers.

**Statistical Interference Modeling Approach**

The main steps of the proposed ICI statistical modeling approach are:

1. Derive the distribution $f_{r_{\text{sel}}}^j(r)$ of the distance $r_{\text{sel}}$ of a scheduled user from its serving BS on a given subcarrier $j$ for each scheduling scheme.

2. Derive the average number of subcarriers allocated to a ring given by $n_k$.

3. Derive the average transmit power on each subcarrier $j$ allocated to a user located in ring $k$, i.e., $\tilde{p}^j_k$.

\(^2\)This assumption is considered for simplicity and mathematical tractability, however, in most cases (such as greedy and random scheduling) it is practically logical that the scheduling decision on a subcarrier is independent of the other subcarrier.
4. Derive the distribution $f_{\tilde{r}_{\text{sel}}}^{j}(\tilde{r})$ of the distance $\tilde{r}_{\text{sel}}$ between the allocated user on subcarrier $j$ in a neighbor interfering cell and the BS of the cell of interest.

5. Derive the distribution $f_{X_{l}^{j}}(x)$ of the interference from the neighboring cell $l$ on subcarrier $j$.

6. Derive the MGF of the cumulative ICI on subcarrier $j$, i.e., $Y^{j} = \sum_{l=1}^{L} X_{l}^{j}$.

7. Derive the MGF of the received signal power at the BS of interest $Z^{j}$ from the scheduled user at subcarrier $j$ to derive the network ergodic capacity.

This section focuses on the statistical characterization of uplink ICI assuming uncoordinated schemes where each BS performs scheduling independent of other BSs. More precisely, each BS can allocate a given subcarrier to any part of its cell without considering the ICI impact on neighboring cells.

### 5.2.1 Distribution of the Allocated User Locations

- **Greedy Scheduler:** selects a user with best channel gain on each subcarrier in any given cell. To derive the probability of allocating a subcarrier $j$ to a user in ring $k$, firstly we select the user with best channel gain $\gamma_{s,k}$ in each ring and then
we select the user with best channel gain among the selected users. Therefore, the probability of selecting a user in ring $k$ can be derived as:

$$P^j(r_{sel} = r_k) = \int_{0}^{\infty} \prod_{i=1, i \neq k}^{K} F_{\gamma_{s,k}^j} (\gamma_{s,k}) f_{\gamma_{s,k}^j} (\gamma_{s,k}) d\gamma_{s,k}$$  \hspace{1cm} (5.1)

where using (5.17), $F_{\gamma_{s,k}^j} (\gamma_{s,k}) = (F_{\zeta_{r_k}^j}(\gamma_{s,k}^{\beta}))^{u_k}$. For details on the derivation of (5.1), the readers can refer to Chapter 3.

- **Round Robin Scheduler**: selects any user on a given subcarrier $j$ with equal probability, therefore the probability of allocating a subcarrier $j$ to a user in $k^{th}$ circular ring is given as follows:

$$P^j(r_{sel} = r_k) = \frac{u_k}{U}$$  \hspace{1cm} (5.2)

- **Location based Round Robin Scheduler**: selects a ring with equal probability on any subcarrier $j$ which in turn implies equal number of subcarriers allocated to all rings. The probability of allocating a subcarrier $j$ to $k^{th}$ circular ring can then be given as follows:

$$P^j(r_{sel} = r_k) = \frac{1}{K}$$  \hspace{1cm} (5.3)

As we consider the same scheduling scheme deployed in each cell and on all subcarriers. Therefore, $P^j(r_{sel} = r_k)$ remains the same on all subcarriers which is the marginal PMF of $P^j(r_{sel} = r_k, \theta = \theta_i)$ given as $P^j(r_{sel} = r_k, \theta = \theta_i) = \frac{P^j(r_{sel} = r_k)}{\mathcal{I}}$.

### 5.2.2 Average Number of Allocated Subcarriers to a User

As we consider independent scheduling decision on all subcarriers, i.e., irrespective of the scheduling decisions on other subcarriers, thus, $P^j(r_{sel} = r_k)$ remains the same for all subcarriers. The average number of subcarriers allocated to a ring $k$, i.e., $n_k$ can
then be calculated as \( n_k = NP^j (r_{sel} = r_k) \). Given \( n_k \) and considering i.i.d. shadowing and fading statistics within a ring \( k \), the probability of allocating a given subcarrier \( j \) to any user in ring \( k \) is given as \( 1/u_k \) for greedy scheduling. This is due to the fact that \( P(\gamma_g > \gamma_f) = P(\gamma_f > \gamma_g) \), \( \forall f, g \), where \( \gamma_f \) and \( \gamma_g \) is the channel gain of any pair of users within a ring \( k \). Based on this, the distribution of the number of subcarriers \( \tilde{n}_k \), allocated to a user in ring \( k \) can be characterized by Binomial distribution as:

\[
P(\tilde{n}_k = m) = \left( \frac{n_k}{m} \right) \left( \frac{1}{u_k} \right)^m \left( 1 - \frac{1}{u_k} \right)^{n_k - m} = \left( \frac{n_k}{m} \right) (u_k - 1)^{n_k - m} u_k^{-n_k}, \quad u_k > 1, \forall m = 0, 1, \cdots, n_k \quad (5.4)
\]

where \( P(\tilde{n}_k = n_k) = 1 \) for \( u_k = 1 \). Moreover, \( n_k \) can be a fraction and therefore, we perform rounding to remove the fractional part of the average number of subcarriers per ring, as applicable. Note that (5.24) also applies to round robin and location based round robin scheduling.

### 5.2.3 Equal Power Allocation (EPA) on the Subcarriers

Conditioned on the number of subcarriers allocated to a user in each ring \( k \), i.e., \( \tilde{n}_k \), the transmit power of that user can be given for the EPA scheme, which is an appropriate substitute of the water-filling in uplink OFDMA networks [6], as \( P_{\text{max}}/\tilde{n}_k, \forall j \in n_k \). Now by averaging over the distribution of \( \tilde{n}_k \) given in (5.24), the average transmit power of a user in ring \( k \) at subcarrier \( j \), i.e., \( \tilde{p}_j^k \) can be derived as:

\[
\tilde{p}_j^k = \begin{cases} 
\sum_{\tilde{n}_k=1}^{n_k} \frac{P_{\text{max}}}{m} P(\tilde{n}_k = m), & m \neq 0, u_k > 1 \\
P_{\text{max}}/n_k, & u_k = 1
\end{cases} \quad (5.5)
\]

where (5.25) remains valid for all non-zero values of \( m \). Moreover, when no subcarrier is allocated to a given user, i.e., \( m = 0 \), the user transmit power \( \tilde{p}_k^j = 0 \), therefore we
ignore this condition in (5.25).

5.2.4 MGF of the Cumulative ICI and Received Signal

As each cell is assumed to have identical conditions, \( f_{\tilde{r}_{\text{sel}}} (r) \) and in turn \( f_{\tilde{r}_{\text{sel}}} (\tilde{r}) \) remain the same for all interfering cells. With reference to Chapter 3, \( P(\tilde{r}_{\text{sel}} = \tilde{r}_{i,k}) \) can be given as \( P(j(\tilde{r}_{\text{sel}} = \tilde{r}_{i,k}) = \frac{P(j(\tilde{r}_{\text{sel}} = \tilde{r}_{i,k}))}{T} \). Moreover, interference on a given subcarrier \( j \), i.e., \( X^j_l \) depends on the location, transmit power, and interfering channel statistics of the selected user in the interfering cell \( l \) which is given as:

\[
X^j_l = \tilde{r}_{\text{sel}}^j \tilde{r}_{j}^j \chi^j_l
\]  

(5.6)

where \( \chi^j_l \) is the interfering channel composite fading RV and \( \tilde{p}_{\text{sel}}^j \) is the transmit power of the selected user in interfering cell at subcarrier \( j \). Conditioned on \( \tilde{r}_{\text{sel}} \), the PDF of \( X^j_l \) can be derived by RV transformation as follows:

\[
f_{X^j_l | \tilde{r}_{\text{sel}}}(x) = \frac{\tilde{r}_{\text{sel}}^j}{\tilde{p}_{\text{sel}}^j} f_{\chi} \left( \frac{x \tilde{r}_{\text{sel}}^j}{\tilde{p}_{\text{sel}}^j} \right)
\]  

(5.7)

Averaging over the PMF of \( \tilde{r}_{\text{sel}} \), the distribution of the ICI received from the scheduled user located in cell \( l \) is given as:

\[
f_{X^j_l}(x) = \sum_{\tilde{r}_{\text{sel}}=\tilde{r}_{i,k}} \tilde{r}_{i,k}^j \tilde{r}_{j}^j \chi^j_l
\]  

(5.8)

As the transmit power of the interferer implicitly depend on its location with respect to its serving BS, therefore \( \tilde{p}_{\text{sel}}^j \) can be calculated against each value of \( r_k \) using (5.25). Finally, \( \mathcal{M}_{X^j_l}(s) \) can be derived as follows:

\[
\mathcal{M}_{X^j_l}(s) = \sum_{\tilde{r}_{\text{sel}}=\tilde{r}_{i,k}} \frac{P(j(\tilde{r}_{\text{sel}} = \tilde{r}_{i,k}))}{\tilde{p}_{\text{sel}}^j \tilde{r}_{i,k}} \int_0^\infty e^{sx} f_{\chi} \left( \frac{x \tilde{r}_{i,k}^j}{\tilde{p}_{\text{sel}}^j} \right) dx
\]  

(5.9)
The derived expression in (5.9) is unified in terms of any composite fading channel model and various scheduling schemes. As an example, we will discuss the Gamma composite fading case for which $\mathcal{M}_{X_j}(s)$ can be derived in closed form as follows:

$$\mathcal{M}_{X_j}(s) = \sum_{\tilde{r}_{sel} = \tilde{r}_{i,k}} \frac{P_j(\tilde{r}_{sel} = \tilde{r}_{i,k})}{1 - \tilde{p}_j \Theta_j \Theta_j s}$$  \hspace{1cm} (5.10)

As each interfering cell is considered to have the same scheduling scheme deployed, therefore, the MGF of $Y_j$ can be calculated as $\mathcal{M}_{Y_j}(s) = \prod_{l=1}^L \mathcal{M}_{X_j}(s) = (\mathcal{M}_{X_j}(s))^L$.

The MGF of the received signal power from the scheduled user at $j^{th}$ subcarrier, i.e., $Z_j$ varies for different subcarriers depending on the transmit powers which implicitly depend on the location of the rings where the subcarriers are allocated. Considering that $\gamma_j = \text{max}\{\gamma_s, 1, \gamma_s, 2, \cdots \gamma_s, K\}$ for greedy scheduling and $p_k^j$ is the transmit power of the desired user in the cell of interest, the MGF of $Z_j = p_k^j \gamma_j$ can then be given by using the scaling property of MGF as follows:

$$\mathcal{M}_{Z_k}(s) = \mathcal{M}_{\bar{Z}_k}(p_k^j s) = \int_0^\infty e^{p_k^j \gamma_j} f_{\gamma_j} (\gamma_j) d\gamma_j = sp_k^j \int_0^\infty e^{sp_k^j \gamma_j} F_{\gamma_k} (\gamma_j) d\gamma_j$$ \hspace{1cm} (5.11)

where $F_{\gamma_j}(\gamma_j) = \prod_{k=1}^K F_{\gamma_{s,k}} (\gamma_j) = \prod_{k=1}^K (F_{\gamma_{s,k}} (\gamma_j))^\mu_k$. Closed form expression for (5.11) in Gamma fading environments are available in [70]. Similarly, for location based round robin and round robin scheduling the MGF of the received signal power at $j^{th}$ subcarrier allocated in ring $k$ is given for Gamma fading channels as follows:

$$\mathcal{M}_{Z_k}(s) = \mathcal{M}_{\bar{Z}_k}(p_k^j s) = \left(1 - p_k^j \tilde{r}_{k}^{-\beta} \Theta_k s\right)^{\kappa_k}$$ \hspace{1cm} (5.12)

where $\mathcal{M}_{\bar{Z}_k}(p_k^j s)$ is the MGF of $\bar{Z}_k$ at subcarrier $j$ in ring $k$. Note that $p_k^j = \tilde{p}_k^j$ for uncoordinated scheduling schemes and can be calculated using (5.25).
5.3 Coordinated Scheduling Schemes

This section focuses on location based coordination schemes in which BSs are assumed to cooperate by coordinating the allocation of subcarriers to different subareas within the cells. In this context, we consider three coordination schemes, namely:

(i) **Forward Synchronization (FS):** in which all BSs allocate the same set of subcarriers to each circular ring in a synchronized manner starting from the cell center (see Fig. 5.2). This scheme maintains a higher frequency reuse distance at cell center subcarriers whereas lower frequency reuse distance at cell edge subcarriers. Therefore, cell edge subcarriers are susceptible to a significant amount of ICI.

(ii) **Reverse Synchronization (RS):** in which all interfering BSs perform frequency assignment in the reverse order compared to the BS of interest (see Fig. 5.2). Compared to FS, the RS scheme increases the frequency reuse distance reasonably at cell edge subcarriers, whereas, it lowers the frequency reuse at cell center subcarriers.

(iii) **No Synchronization (NS):** allows no frequency coordination between the BSs.

Therefore to critically evaluate the impact of BS coordination and frequency allocation on ICI mitigation and capacity enhancement, we consider three frequency
Figure 5.3: Percentage of allocated subcarriers as a function of the distance from the serving BS for uncoordinated and coordinated scheduling schemes; $R = 500m$.

allocation schemes, namely:

(i) **Uniform Frequency Allocation (UFA):** in which each ring has equal number of subcarriers allocated. Note that UFA with NS is equivalent to uncoordinated location based round robin.

(ii) **Cell Edge Frequency Allocation (CEFA):** in which the majority of the subcarriers are allocated in the cell edge rings of the cell of interest. The number of subcarriers in each ring is calculated as follows:

\[
 n_{CEFA}^k = n_1 + (k - 1) \frac{2 - 2n_1K}{K(K - 1)}
\]  

(5.13)

where $n_1$ represent the number of subcarriers allocated to the first ring of all cells for FS whereas for RS $n_1$ denote the number of subcarriers allocated to the first ring of the cell of interest and the last ring of the interfering cells.

(iii) **Cell Centered Frequency Allocation (CCFA):** in which the majority of the subcarriers are allocated in the cell center rings of the cell of interest. The number of allocated subcarriers in each ring is simply the reverse of (5.13), i.e., $n_{CCFA}^k = n_{CEFA}^{K-k}$.

These schemes help in determining the best BS coordination strategy for different
subcarrier allocation patterns. The quantitative representation of subcarrier division in different rings for UFA, CCFA, and CEFA is illustrated in Fig. 5.3. Firstly, it can be observed that the CCFA pattern closely follows the pattern of greedy scheduling where a high percentage of subcarriers is allocated in the cell center rings. On contrary, CEFA follows round robin scheduling where a large number of subcarriers is allocated to the cell edge users. The small uniform steps in UFA ensure equal percentage of subcarriers in all rings; however, as we have more rings in the cell center, the percentage of allocated subcarriers in UFA slightly tends towards CCFA.

The following are the modifications required to the proposed approach in Section 5.2 in order to accommodate the different coordinated scheduling schemes. Knowing the number of subcarriers allocated in each ring \( k \) for UFA, CCFA, and CEFA (as illustrated in Fig. 5.3), the power on each subcarrier can be calculated using (5.25). However, it is important to mention that as \( u_k \) and \( n_k \) are the same in all cells (i.e., either cell of interest or interfering cell) for the NS and FS scenarios, therefore, \( p^j_k = \tilde{p}^j_k \). However, in the RS scenario, \( n_k \) in the cell of interest is opposite to the interfering cells, therefore, \( p^j_k \neq \tilde{p}^j_k \) and needs to be re-calculated using (5.25) for the interfering cells. In addition, coordinated schemes create different interference levels on different sets of subcarriers depending on their locations and transmit powers, i.e., \( X^j_k = \tilde{p}^j_k \tilde{\tau}^{-\beta} \chi^j \). Therefore, by averaging over \( \theta \), the MGF of the cumulative ICI on a subcarrier \( j \) located in ring \( k \) for gamma fading channels is derived as:

\[
M_{Y^j_k}(s) = \left( \sum_{i=1}^{T} \frac{1 - \tilde{p}^j_k \tilde{\tau}^{-\beta} \Theta \chi^j s^{-\kappa \chi}}{T} \right)^L
\]

The MGF of the desired signal at subcarrier \( j \) which is allocated in ring \( k \), i.e., \( Z^j_k \) can be given as in (5.12).
5.4 Fractional Frequency Reuse (FFR) Networks

We consider a cellular network with \( L \) circular cells, each of radius \( R \). We implement FFR technique which partitions the cellular region into an area centered near the base station (BS) with frequency reuse factor \( \mathcal{D} = 1 \) and cell-edge area with \( \mathcal{D} > 1 \) (in this paper, we take \( \mathcal{D} = 3 \)), to reduce interference from neighboring cells. The cellular network is considered to have \( N = N_{\text{cent}} + \mathcal{D} N_{\text{edge}} \) available subcarriers, where, \( N_{\text{cent}} \) and \( N_{\text{edge}} \) denote the allocated cell-center and cell-edge subcarriers, respectively. Each cell possesses \( U \) uniformly distributed users which are classified into cell-center and cell-edge groups based on their locations from the closest BS [36]. The average number of cell-center and cell-edge users is given by \( U_{\text{cent}} = U(R_{\text{th}}^2 - R_0^2)/R^2 \) and \( U_{\text{edge}} = U(R^2 - R_{\text{th}}^2)/R^2 \), respectively, where \( R_0 \) denotes the reference distance and \( R_{\text{th}} \) denotes the boundary between the cell-center and cell-edge partitions.

We consider decomposing each cell into \( K \) concentric circular zones such that the path loss decay \( \kappa \) [dB] within each circular region remains uniform. The set of the circular regions in cell-center and cell-edge groups denoted by \( \mathcal{K}_{\text{cent}} \) and \( \mathcal{K}_{\text{edge}} \) can be characterized as,

\[
10\beta \log_{10} r_k = \kappa + 10\beta \log_{10} r_{k-1}; \quad \mathcal{K}_{\text{cent}} : R_0 \leq r_k \leq R_{\text{th}}, \quad \mathcal{K}_{\text{edge}} : R_{\text{th}} \leq r_k \leq R \quad (5.15)
\]

where \( \beta \) is the path loss exponent and \( r_k \) denotes the radius of ring \( k \). The main motivation for dividing the cell into a discrete set of circular regions relies on the fact that the channel statistics of the users located within a given circular region becomes relatively similar especially for large values of \( K \) (For more details, check [75, 76, 77]). The average number of users located within \( k^{th} \) circular region is:

\[
u_k = \begin{cases} 
U_{\text{cent}}(r_k^2 - r_{k-1}^2)/(R_{\text{th}}^2 - R_0^2); & k \in \mathcal{K}_{\text{cent}} \\
U_{\text{edge}}(r_k^2 - r_{k-1}^2)/(R^2 - R_{\text{th}}^2); & k \in \mathcal{K}_{\text{edge}} 
\end{cases} \quad (5.16)
\]
The instantaneous channel gain $\gamma^j_k$ of any user located in ring $k$ at the subcarrier $j$ can be represented as:

$$\gamma^j_k = r^{-\beta} \zeta^j,$$  \hspace{1cm} (5.17)

where $\zeta^j$ is the combined shadowing and fading random variable (RV), i.e., composite fading of a given user at $j^{th}$ subcarrier. We consider independent and identically distributed (i.i.d.) composite fading statistics of all users within a given circular region$^3$. Moreover, the scheduling decision on each subcarrier is considered to be independent irrespective of the decisions on other subcarriers. Without loss of generality, we assume that the same scheduling scheme is employed in all cells and the set of subcarriers allocated to cell-center and cell-edge users is denoted by $N_{\text{cent}}$ and $N_{\text{edge}}$, respectively.

To characterize the statistics of the uplink CCI, the proposed framework includes the following steps:

- Derive the probability mass function (PMF) of the distance ($r^{j}_{\text{sel}}$) between the allocated user and serving BS on cell-center and cell-edge subcarriers.

- Determine the number of subcarriers allocated to a given ring $k$, i.e., $n_k$, and derive the transmit power on each subcarrier $j$ allocated to a user in ring $k$, i.e., $p^{j}_{k}$.

- Derive the PMF of the distance ($\tilde{r}^{j,l}_{\text{sel}}$) between the BS of interest and the allocated user on a given subcarrier $j$ in an interfering cell $l$, taking into account the corresponding co-channel cells for the cell-center and cell-edge subcarriers (See Fig. 1).

$^3$Due to the vicinity and uniform path loss decay within each circular region, it is reasonable to assume i.i.d. statistics within a given circular region.
• Derive the MGF of the cumulative interference $Y$ on a given subcarrier $j$, i.e.,

$$Y^j = \begin{cases} 
\sum_{l \in L_{\text{cent}}} X^j_l, & j \in N_{\text{cent}} \\
\sum_{l \in L_{\text{edge}}} X^j_l, & j \in N_{\text{edge}} 
\end{cases}$$

(5.18)

where $X^j_l$ denotes the interference received from a single cell on subcarrier $j$, $L_{\text{cent}}$ and $L_{\text{edge}}$ denotes the set of interfering BSs for cell-center and cell-edge subcarriers, respectively, as illustrated in Fig. 1. Note that the interferers in set $L_{\text{cent}}$ can be further distinguished into three groups $L_{\text{cent,1}}$, $L_{\text{cent,2}}$ and $L_{\text{cent,3}}$ based on their distance from the BS of interest as is demonstrated clearly in Fig. 1.

**PMF of the Allocated Cell Center and Cell Edge User Locations**

Firstly, we derive the statistics of ICI considering two state-of-the-art scheduling schemes, namely greedy and round robin. These two schemes provide a potential benchmark to the extreme network performance and therefore helps in analyzing the system performance.

**Greedy Scheduling Scheme**

Greedy scheduling is an opportunistic scheduling scheme which ensures the selection of a user with best channel gain on all cell-center and cell-edge subcarriers. To derive the probability of allocating a subcarrier $j$ to a user in ring $k$, we first select the user with the best channel gain $\gamma_{s,k}$ in each ring and then we select the user with the best channel gain among the selected users of all rings. The probability of selecting a user
in ring $k$ can then be derived as follows:

$$P(r_{sel}^j = r_k) = \begin{cases} 
\int_0^{\infty} \prod_{i \in \mathcal{K}_{cent}} F_{\gamma_{s,i}^j}(\gamma_{s,k}^j) f_{\gamma_{s,i}^j}(\gamma_{s,k}^j) d\gamma_{s,k}^j & j \in \mathcal{N}_{cent}, k \in \mathcal{K}_{cent} \\
\int_0^{\infty} \prod_{i \in \mathcal{K}_{edge}} F_{\gamma_{s,i}^j}(\gamma_{s,k}^j) f_{\gamma_{s,i}^j}(\gamma_{s,k}^j) d\gamma_{s,k}^j & j \in \mathcal{N}_{edge}, k \in \mathcal{K}_{edge} 
\end{cases}, \quad (5.19)$$

where $F_{\gamma_{s,i}^j}(\gamma_{s,k}^j) = (F_{\zeta_i^j}(\gamma_{s,k}^j))^u_k = (F_{\zeta_i^j}(\gamma_{s,k}^j))^u_k$. Note that, $P(A)$ denote the probability of event $A$, $f(\cdot)$ and $F(\cdot)$ denote the probability distribution function (PDF) and cumulative distribution function (CDF), respectively. The details on the derivation of (5.19) can be found in [77]. A closed form solution for (5.19) is derived and presented in the following special case.

**Special Case** (Rayleigh Fading and No Shadowing): For i.i.d. Rayleigh fading channels such that $\zeta$ is an exponential RV with parameter $\lambda$, i.e., $\zeta \sim \text{Exp}(\lambda)$, the closed form solution of (5.19) can be derived as

$$P(r_{sel}^j = r_k) = \sum_{m_1=0}^{u_1-1} \sum_{m_2=0}^{u_2-1} \cdots \sum_{m_{K_{cent}}=0}^{u_{K_{cent}}-1} \sum_{g=0}^{\infty} (-1)^{K_{cent} - 1} m_i \left( \begin{array}{c} u_1 \\ m_1 \end{array} \right) \left( \begin{array}{c} u_2 \\ m_2 \end{array} \right) \cdots \left( \begin{array}{c} u_{K_{cent}} \\ m_{K_{cent}} \end{array} \right) \left( (g + 1) \lambda_k + \sum_{i=1}^{K_{cent}} \lambda_i m_i \right) \quad (5.20)$$

where $K_{cent}$ is the number of rings in the cell-center, $\lambda_k = \lambda r_{s,i}^j$ and the infinite series in (5.20) can be truncated and generate results with the accuracy of up to four decimal places with $g = 0$ to $g = 1000$.

**Proof:** See Appendix.

Note that the first part of expression in (5.19) is derived in (5.20). However, using (5.20) the expression for second part of (5.19) can be written in a straightforward manner.

**Round Robin Scheduling Scheme**

Round robin is a non-opportunistic scheduling scheme which allocates equal number of subcarriers to all users, therefore the probability of allocating a subcarrier $j$ to a
ring $k$ is given by:

$$P(r_{sel}^j = r_k) = \begin{cases} 
  u_k/U_{cent} & j \in N_{cent}, k \in K_{cent} \\
  u_k/U_{edge} & j \in N_{edge}, k \in K_{edge} 
\end{cases} . \quad (5.21)$$

Note that, $P(r_{sel}^j = r_k)$ is the marginal PMF of $P(r_{sel}^j = r_k, \theta^j = \theta_i)$, where $\theta^j$ denotes the angle of the allocated user at subcarrier $j$ with respect to the serving BS and it is uniformly distributed from 0 to $2\pi$. Discretizing the range of RV $\theta^j$ in $I$ uniform angular intervals, we can write $P(\theta^j = \theta_i) = \frac{1}{I}$, where $\theta_i$ denotes any discrete value that the RV $\theta$ can take. Since $r_{sel}^j$ and $\theta^j$ are independent, their joint PMF is

$$P(r_{sel}^j = r_k, \theta^j = \theta_i) = P(r_{sel}^j = r_k)/I. \quad (5.22)$$

**Average Number of Allocated Subcarriers**

As we consider independent scheduling decision on all subcarriers, i.e., irrespective of the scheduling decisions on other subcarriers, thus, $P(r_{sel}^j = r_k)$ remains the same for all subcarriers. The average number of subcarriers allocated to a ring $k$, i.e., $n_k$ can then be calculated as follows:

$$n_k = \begin{cases} 
  N_{cent}P(r_{sel}^j = r_k) & j \in N_{cent}, k \in K_{cent} \\
  N_{edge}P(r_{sel}^j = r_k) & j \in N_{edge}, k \in K_{edge} 
\end{cases} . \quad (5.23)$$

Given $n_k$ and considering i.i.d. shadowing and fading statistics within a ring $k$, the probability of allocating a given subcarrier $j$ to any user in ring $k$ is given as $1/u_k$ for greedy scheduling. This is due to the fact that $P(\gamma_e > \gamma_f) = P(\gamma_f > \gamma_e), \forall f, e$, where $\gamma_f$ and $\gamma_e$ are the channel gains of any pair of users within a ring $k$. Based on this, the distribution of the number of subcarriers allocated to a user in ring $k$, i.e.,

---

4The reason is provided in Chapter 3.
\( \tilde{n}_k \) can be characterized by a Binomial distribution as follows:

\[
P(\tilde{n}_k = m) = \binom{n_k}{m} \left( \frac{1}{u_k} \right)^m \left( 1 - \frac{1}{u_k} \right)^{n_k-m} = \left( \frac{n_k}{m} \right) (u_k-1)^{n_k-m} u_k^{m-n_k}, \quad u_k > 1, \forall m = 0, 1, \cdots, n_k,
\]

(5.24)

where \( P(\tilde{n}_k = n_k) = 1 \) for \( u_k = 1 \). Moreover, \( n_k \) can be a fraction and therefore, we perform rounding to remove the fractional part of the average number of subcarriers per ring, as applicable. Note that (5.24) remain applicable to round robin scheduling scheme. Conditioned on the number of subcarriers allocated to a user in each ring \( k \), i.e., \( \tilde{n}_k \), the transmit power of that user can be given for the EPA scheme, which is an appropriate substitute of the water-filling in uplink OFDMA networks [6], as \( P_{\text{max}}/\tilde{n}_k, \forall j \in n_k \), where \( P_{\text{max}} \) denote the maximum transmission power of a user.

Now by averaging over the distribution of \( \tilde{n}_k \) given in (5.24), the transmit power of a user in ring \( k \) at subcarrier \( j \), i.e., \( p^j_k \) can be derived as follows:

\[
p^j_k = \begin{cases} 
\sum_{\tilde{n}_k=1}^{n_k} \frac{P_{\text{max}}}{m} P(\tilde{n}_k = m) & m \neq 0, \quad u_k > 1 \\
P_{\text{max}}/n_k & u_k = 1
\end{cases}, \quad (5.25)
\]

where (5.25) remains valid for all non-zero values of \( m \). Moreover, when no subcarrier is allocated to a given user, i.e., \( m = 0 \), the user transmit power \( p^j_k = 0 \), and we therefore ignore this condition in (5.25).

**PMF of the Allocated Interfering User Locations**

The distance between the allocated interfering user on the subcarrier \( j \) located in cell \( l \) and the BS of interest can be given, in general, using the cosine law as follows:

\[
\tilde{r}^{j,l}_{\text{sel}} = \sqrt{\left( r^j_{\text{sel}} \right)^2 + (D^{j,l})^2 - 2r^j_{\text{sel}}D^{j,l} \cos \theta^j}, \quad (5.26)
\]
where \( \tilde{r}^{j,l}_{sel} \) is the distance of the selected interfering user on subcarrier \( j \) in cell \( l \) from the BS of interest, \( r^j_{sel} \) is the distance of the selected user from its own BS, \( \theta^j \in \{0, 2\pi\} \), and \( D^{ji} \) is the distance between the serving BS \( l \) of the interferer and BS of interest, defined more explicitly as follows:

\[
D^{ji} = \begin{cases} 
2R & \forall l \in L_{\text{cent},1}, j \in N_{\text{cent}} \\
3.46R & \forall l \in L_{\text{cent},2}, j \in N_{\text{cent}} \\
4R & \forall l \in L_{\text{cent},3}, j \in N_{\text{cent}} \\
3.46R & \forall l \in L_{\text{edge}}, j \in N_{\text{edge}} 
\end{cases}
\]  

(5.27)

where \( L_{\text{cent},1} \) denote the first tier interfering BSs and \( L_{\text{cent},2} \) and \( L_{\text{cent},3} \) denote the second tier interfering BSs. The distance of the interferer which is located in the \( k \)th ring and \( i \)th angular interval of cell \( l \) at subcarrier \( j \), i.e., \( \tilde{r}^{j,l}_{i,k} \) can be computed using (5.26) as

\[
\tilde{r}^{j,l}_{i,k} = \sqrt{r^2_{k} + (D^{ji})^2 - 2r_{k}D^{ji}\cos\theta^j_i} \quad \forall i \quad \forall k.
\]

Knowing the PMF of the allocated user location on subcarrier \( j \) in each cell, the PMF of \( \tilde{r}^{j,l}_{sel} \) can be derived by using (5.22) as,

\[
P(\tilde{r}^{j,l}_{sel} = \tilde{r}^{j,l}_{i,k}) = P(r^j_{sel} = r_{k})/\mathcal{I}.
\]

**Statistics of the Cumulative ICI**

Interference on a given subcarrier \( j \), i.e., \( X^j_l \) depends on the location, transmit power, and interfering channel statistics of the selected user in the interfering cell \( l \) and can be defined as follows:

\[
X^j_l = P^j_{\text{sel}}(\tilde{r}^{j,l}_{sel})^{-\beta} \lambda^j_l,
\]  

(5.28)

where \( \lambda^j_l \) is the composite fading statistics of the interference channel at subcarrier \( j \) from interfering user in cell \( l \) and \( P^j_{\text{sel}} \) is the transmit power of the selected user in interfering cell \( l \) at subcarrier \( j \). The PDF of \( X^j_l \) conditioned on \( \tilde{r}^{j,l}_{sel} \) can then be
derived by RV transformation as follows:

\[ f_{X_j|\tilde{r}_{sel}}(x_j) = \frac{f_X(x_j)}{p_{\tilde{r}_{sel}}^{\beta}} \quad (5.29) \]

Averaging over the PMF of \( \tilde{r}_{sel} \), the distribution of the CCI from any cell \( l \) can be given as follows:

\[ f_{X_j}(x_j) = \sum_{\tilde{r}_{sel} = \tilde{r}_{i,k}} f_{X_j}\left(\frac{x_j^{\beta} p_k^{\beta}(\tilde{r}_{i,k})^{\beta}}{p_k^{\beta}(\tilde{r}_{i,k})^{\beta}}\right) P(\tilde{r}_{sel} = \tilde{r}_{i,k}). \quad (5.30) \]

As the transmit power of the interferer implicitly depend on its location with respect to its serving BS, therefore \( p_k \) can be calculated against each value of \( r_k \) using (5.25). Finally, using (5.28) and the scaling property of MGF, \( M_{X_j}(s) \) can be derived as follows:

\[ M_{X_j}(s) = \sum_{\tilde{r}_{sel} = \tilde{r}_{i,k}} M_{X_j}\left((\tilde{r}_{i,k})^{-\beta} p_k^s\right) P(\tilde{r}_{sel} = \tilde{r}_{i,k}). \quad (5.31) \]

As a special case, we now provide the MGF expressions considering Generalized-K composite fading channels in which shadowing and fading statistics are modeled by a Gamma and Nakagami distribution [78], respectively. Recently, it has been proposed that Generalized-K distribution can be accurately approximated by more tractable Gamma distribution [66]. In this context, we approximate \( \chi \) with a Gamma RV with shape parameter \( \kappa \) and scale parameter \( \Theta \), i.e., \( \chi \sim \text{Gamma}(\kappa, \Theta) \) and \( M_{X_j}(t) \) can therefore be given as:

\[ M_{X_j}(s) = \sum_{\tilde{r}_{sel} = \tilde{r}_{i,k}} \left(1 - p_k^{\beta}(\tilde{r}_{i,k})^{-\beta} \Theta \right)^{-m_\chi} P(\tilde{r}_{sel} = \tilde{r}_{i,k}). \quad (5.32) \]
Computing the distribution of the cumulative CCI $Y^j$ on a subcarrier $j$ requires the convolution of the PDF of $L$ RVs $X^j_l$, $\forall l = 1, 2, \cdots L$, which is a tedious task in practice. To avoid the convolutions, we utilize an MGF based approach and derive the expression for the MGF of the cumulative CCI. The cumulative MGF of the CCI on cell-center allocated subcarrier can then be given as:

$$
\mathcal{M}_{Y^j}(s) = \prod_{l \in \mathcal{L}_{cent,1}} \mathcal{M}_{X^j_l}(s) \prod_{l \in \mathcal{L}_{cent,2}} \mathcal{M}_{X^j_l}(s) \prod_{l \in \mathcal{L}_{cent,3}} \mathcal{M}_{X^j_l}(s), \quad \forall j \in N_{cent}. \quad (5.33)
$$

Considering i.i.d. composite fading statistics of interfering channel, i.e., $\chi$ and noting that within each group of the interfering cells, i.e, $\mathcal{L}_{cent,1}$ or $\mathcal{L}_{cent,2}$ or $\mathcal{L}_{cent,3}$, the distance $D^j_l$ between the BS of interest and the BS of interfering users is same, as shown in (5.27), therefore (5.33) can be simplified as follows:

$$
\mathcal{M}_{Y^j}(s) = \prod_{l \in \mathcal{L}_{cent,1}} \left( \mathcal{M}_{X^j_l}(s) \right)^{L/3}, \quad \forall j \in N_{cent}. \quad (5.34)
$$

However, note that the allocated interfering cell-edge subcarriers are originating from $\mathcal{L}_2$ only and therefore the BSs of interfering users are all at same distance from the BS of interest. In this case, the cumulative MGF of the CCI on allocated subcarrier $j$ at cell-edge can be given as follows:

$$
\mathcal{M}_{Y^j}(s) = \prod_{l \in \mathcal{L}_{edge}} \mathcal{M}_{X^j_l}(s) = \left( \mathcal{M}_{X^j_l}(s) \right)^{L/3}, \quad \forall j \in N_{edge}. \quad (5.35)
$$

### 5.5 Ergodic Capacity in Multi-Carrier Networks

Using the efficient lemma derived in [71], the capacity of each subcarrier $j$ located in ring $k$ can be calculated as $C^j_k = \mathbb{E} \left[ \ln \left( 1 + \frac{Z^j_k}{Y^j + \sigma^2} \right) \right] = \int_0^{\infty} \frac{\mathcal{M}_{Y^j}(s) - \mathcal{M}_{Z^j_k Y^j}(s)}{s} e^{-\sigma^2 s} ds$.

Since $Z^j_k$ and $Y^j$ are independent\(^5\) therefore $\mathcal{M}_{Z^j_k Y^j}(s) = \mathcal{M}_{Z^j_k}(s) \mathcal{M}_{Y^j}(s)$. The MGF

\(^5\)The reason is explained in Chapter 3.
of the received signal power from the scheduled user at \text{j}th subcarrier, i.e., \( Z^j_k \) varies for different subcarriers depending on their locations and transmit powers and can therefore be characterized for the greedy scheduling as given below:

\[
\mathcal{M}_{Z^j_k}(s) = \mathcal{M}_{\gamma_s^j}(p^j_k s) = e^{p^j_k \gamma_s^j} \int_0^{\infty} e^{s \gamma_s^j} F_{\gamma_s^j} d\gamma_s^j = s p^j_k \int_0^{\infty} e^{s \gamma_s^j} F_{\gamma_s^j} d\gamma_s^j, \quad (5.36)
\]

where \( \gamma_s^j = \max\{\gamma_{s,1}^j, \gamma_{s,2}^j, \ldots, \gamma_{s,K}^j\} \), i.e., \( F_{\gamma_s^j}(\gamma_s) = \prod_{i \in K_1} (F_{\gamma_i^j}(\gamma_i^j))^{w_i}, \forall j \in N_{\text{cent}} \) and similarly \( F_{\gamma_s^j}(\gamma_s) = \prod_{i \in K_2} (F_{\gamma_i^j}(\gamma_i^j))^{w_i}, \forall j \in N_{\text{edge}} \). For round robin scheduling, \( \mathcal{M}_{Z^j_k}(s) = \mathcal{M}_{\zeta_k^j}(p^j_k s) = \left(1 - p^j_k \gamma_s^j \Theta \zeta s\right)^{-\kappa \zeta} \).

Finally, knowing the average number of subcarriers \( n_k \) in each ring, the aggregate network capacity of FFR-based uplink OFDMA networks can be calculated as follows:

\[
C_{\text{FFR}} = \sum_{j \in N_{\text{center}}} n_k C^j_k + \sum_{j \in N_{\text{edge}}} n_k C^j_k, \quad (5.37)
\]

This section is mainly focused on demonstrating the importance of the derived MGF expressions in calculating the network ergodic capacity of uncoordinated as well as coordinated scheduling schemes. The ergodic capacity on any subcarrier \( j \) which is allocated in ring \( k \) can be calculated for uncoordinated scheduling schemes by using the lemma presented in [71] as follows:

\[
C^j_k = E \left[ \ln \left(1 + \frac{Z^j_k}{Y^j_s + \sigma^2}\right) \right] = \int_0^{\infty} \mathcal{M}_{Z^j_k/Y^j_s}(s) - \mathcal{M}_{Z^j_k}(s) \frac{1}{s} e^{-\sigma^2 s} ds
\]

where \( \sigma^2 \) is the thermal noise power. Since \( Z^j_k \) and \( Y^j_s \) are independent\(^6\) therefore \( \mathcal{M}_{Z^j_k/Y^j_s}(s) = \mathcal{M}_{Z^j_k}(s) \mathcal{M}_{Y^j_s}(s) \). Knowing the average number of subcarriers \( n_k \) in each ring, the aggregate network capacity on all subcarriers can be calculated as follows:

\[
C = \sum_{k=1}^{K} n_k C^j_k \quad (5.38)
\]

The cumulative ICI in coordinated scheduling varies on different subcarriers based on

\(^6\)As mentioned earlier in Chapter 3, it is reasonable to assume independence between desired and interfering channel fading conditions
their locations in the interfering cells. In FS, a subcarrier allocated in ring $k$ of the BS of interest is surely allocated in the same ring of interfering cells. Therefore, the capacity for FS can be calculated using the capacity lemma and (5.38) by replacing $Y^j$ with $Y^j_k$. On the other side, in RS a subcarrier allocated in ring $k$ of the cell of interest will be allocated in the ring $K - k$ of the interfering cells. Therefore, the capacity on any subcarrier $j$ in RS can be computed using lemma and (5.38) by replacing $Y^j$ with $Y^j_{K-k}$.

An Approximation for the Ergodic Capacity in Multi-Carrier Networks: By considering that each user transmits with its maximum power on each subcarrier, i.e., $\tilde{p}_k^j = P_{\text{max}}$ and assuming similar interference conditions on all subcarriers, the capacity of uncoordinated scheduling schemes in multi-carrier networks can be deduced directly from the capacity of single carrier networks, i.e., $C = NC^j$, where $C^j$ is the capacity of single carrier system which can be calculated by replacing $Z_k^j$ with $Z^j$ in (3.50). However, as this approximation does not capture the dynamics of power allocations, its results are expected to deviate from the actual performance in multi-carrier networks, as demonstrated in results section.

5.6 Results and Discussions

The radius $R$ of the cell is taken 500 m and the total number of users per cell is $U=50$ with path loss exponent $\beta = 3$. Generalized-K composite fading distribution is approximated by the tractable Gamma distribution, i.e., the interference channel composite fading $f_k(\chi)$ is approximated as Gamma($3/2, 2/3$), whereas the desired channel composite fading $f_\zeta(\zeta)$ is approximated as Gamma($2/3, 3/2$). The maximum transmission power per user is taken as $P_{\text{max}} = 1$ W. The number of available subcarriers are taken as $N = 400$, unless stated otherwise.
Coordinated and Uncoordinated Scheduling Schemes

Fig. 5.4 presents ergodic capacity results for the different uncoordinated scheduling schemes. It is shown that the numerical results calculated using (3.50) and (5.38) are very close to the Monte-Carlo simulation results which validates the accuracy of the derivations. Moreover, the increase in the number of subcarriers increases the overall network capacity for all scheduling schemes due to multi-carrier diversity. In addition, the increase in the number of users enhances the performance in case of greedy scheduling. However, this is not true for round robin and location based round robin scheduling due to their non-opportunistic nature; therefore, their results are skipped to maintain the clarity of the figures. Furthermore, the network capacity approximation is observed to be loose for greedy scheduling as more users are located in the cell center and, therefore, the assumption of best signal power dominates the worst interference power. However, for the non-opportunistic scheduling schemes, the worst interference power becomes dominant and makes the approximation closer to the actual results.

Fig. 5.5 investigate the performance of UFA and CCFA in three BS coordination scenarios, i.e, (i) NS; (ii) FS; (iii) RS. Note that UFA with NS is equivalent to uncoordinated location based round robin. Firstly, it can be observed that CCFA outperforms UFA in NS and FS scenarios. The main reason underlying this observation is that CCFA maximizes the percentage of subcarriers at the cell-center compared to UFA, therefore CCFA-NS is more beneficial compared to UFA-NS. Moreover, as FS favors a higher frequency reuse distance in the cell center, therefore CCFA-FS mandates to limit ICI on higher percentage of subcarriers and is therefore observed to outperform UFA-FS. Another important observation is that, UFA outperforms CCFA in the RS scenario. This is due to the fact that RS reverses the frequency allocation pattern and therefore, a higher percentage of the subcarriers gets allocated at the edge of the interfering cells in CCFA compared to UFA. Therefore, the fre-
quency reuse distance decreases on a majority of subcarriers which enhances ICI and degrades the network performance in CCFA-RS compared to UFA-RS.

Fig. 5.6 compares the performance of UFA with CEFA for different BS coordination scenarios. Firstly, it can be observed that UFA outperforms CEFA in the NS case as UFA allocates a higher percentage of subcarriers at cell center than CEFA. Moreover, UFA-FS also outperforms CEFA-FS as FS maintains a high frequency reuse distance at cell center subcarriers which are more in UFA compared to CEFA. Another important point to note is that CEFA-RS appears to be a better solution for ICI mitigation compared to UFA-RS as RS reverses the frequency allocation pattern in the interfering cells and, therefore, only a minority of the subcarriers are allocated at the cell edge of interfering cells. This fact emphasizes the significance of exposing only a small portion of the bandwidth to high ICI.

Finally, it can be concluded that the higher the number of resources allocated near the cell center of the cell of interest: (i) the larger gains are expected with FS, and (ii) performance degradation is expected with RS. Whereas, the higher the number of resources allocated near the cell edge of the cell of interest: (i) the larger gains are expected with RS, and (ii) performance degradation is expected with FS. Moreover, (i) CCFA-NS outperforms UFA-NS and CEFA-NS, due to the high number of subcarriers allocated at the cell center; (ii) CCFA-FS outperforms UFA-FS and CEFA-FS, due to higher frequency reuse distances for most of the cell center subcarriers; (iii) CEFA-RS outperforms UFA-RS and CCFA-RS due to the high number of subcarriers at the cell edge of the cell of interest and the relatively low number of subcarriers allocated at the cell edge of interfering cells.

Among all considered schemes, CCFA with FS is observed to be the best solution to mitigate ICI in UFR networks as CCFA confines large portion of the spectrum in the cell center and FS maintains a higher reuse distance on this part of the spectrum with a trade-off cost in terms of average fairness among users.
Figure 5.4: A throughput comparison of different uncoordinated scheduling schemes with EPA, namely: (i) greedy and its approximation; (ii) location based round robin (LBRR); (iii) round robin.

Figure 5.5: A throughput comparison of the coordinated scheduling schemes with different frequency allocation patterns and BS synchronizations, namely: (i) UFA with NS, FS, and RS; (ii) CCFA with NS, FS, and RS.
Fractional Frequency Reuse Schemes

This section is focused on analyzing the performance of uplink FFR networks while validating the accuracy of the derived analytical results through Monte Carlo simulations. The radius $R$ of the cell is taken $500 \text{ m}$ and the total number of users per cell is $U=50$ with path loss exponent $\beta = 3$. The maximum transmission power per user is taken as $P_{\text{max}} = 1 \text{ W}$. Generalized-K composite fading distribution is approximated by the tractable Gamma distribution, i.e., the interference channel composite fading $f_\chi(\chi)$ is approximated as Gamma($3/2, 2/3$), whereas the desired channel composite fading $f_\zeta(\zeta)$ is approximated as Gamma($2/3, 3/2$). The number of available subcarriers are taken as $N = 400$, unless stated otherwise.

Fig. 5.9 demonstrate the probability of allocating a subcarrier $j \in \mathcal{N}_{\text{cent}}$ to a cell-center user and another subcarrier $j \in \mathcal{N}_{\text{edge}}$ to a cell-edge user considering greedy and round robin scheduling. As greedy scheduling is more favorable for close users, the PMF of greedy allocation in both regions is highly skewed toward the cell-center. Moreover, as we consider uniformly distributed users, the edge areas in both partitions have higher user density and thus round robin scheduling has high possibility to
allocate a user which lies far from the BS of interest in both partitions. The PMF of the round robin scheduling is therefore observed to be skewed toward the cell-edge in both partitions.

\textit{C. Impact of Cell Partition and Bandwidth Partition on CCI}

Fig. 5.10 captures the impact of cell and bandwidth partition, i.e., $R_{th}$ and $N_{\text{cent}}/N$, on the 90th percentile of the CDF of (i) CCI on a given cell-center subcarrier; (ii) CCI on a given cell-edge subcarrier; and (iii) average CCI per subcarrier in FFR and UFR networks, with greedy scheduling. Firstly, it can be observed that the higher number of subcarriers allocated to the cell-center or cell-edge leads to higher CCI per cell-edge subcarrier or higher CCI per cell-center subcarrier, respectively. Clearly, the low number of allocated subcarriers in a particular region (either cell-center or cell-edge) eventually reduces the allocated subcarriers per user. Therefore the resulting transmit power on each subcarrier is relatively higher compared to the case when the number of allocated subcarriers per user is large.

Next important observation is that the average CCI per subcarrier is much more reduced when the number of allocated cell-center subcarriers is high. Note that due to the high spectrum weightage at the cell-center the average CCI per subcarrier is extremely skewed toward the CCI per cell-center subcarrier. Whereas, the same intensity of skewness is not observed when more spectrum is allocated to the cell-edge region due to the spectrum splitting factor $D$ (e.g., $DN_{\text{edge}}=340$ and $N_{\text{edge}}=120$). Finally, it can be concluded that the incurred average CCI per subcarrier in FFR remains comparable or slightly lower to UFR with extremely higher values of $N_{\text{cent}}/N$ in conjunction with lower values of $R_{th}$.

Fig. 5.11 investigate the performance of UFR and FFR networks with round robin scheduling. The interplay of the different network parameters and CCI remains the same as observed in Fig. 5.10. However, the degree of CCI reduction per subcarrier in round robin is significant compared to greedy scheduling with same cell and spectrum
partitions as shown in Fig. 5.10. This fact illustrates the usefulness of FFR in the scheduling schemes, which are highly vulnerable to interference.

D. Impact of Spectrum Partition on the Network Capacity

Fig. 5.12 quantifies the performance gains of UFR and FFR networks as a function of $N_{\text{cent}}/N$ for greedy and round robin scheduling schemes and $R_{\text{th}} = 230.55$ m. It is observed that, FFR scheme outperforms UFR when round robin scheduling is deployed. Whereas, UFR proves to be a better solution with greedy scheduling. The cell-center capacity increases with the increasing proportion of subcarriers in the cell-center (which also implies the reduction of CCI per subcarrier) with a trade-off cost in terms of cell-edge capacity. Finally, Fig. 5.13 demonstrate the impact of increasing number of subcarriers on the performance of greedy and round robin scheduling in FFR relative to UFR scheme. It has been concluded that the potential performance gains can be even further maximized by increasing the total number of subcarriers for round robin scheduling schemes.

5.7 Conclusion

This chapter provided a framework to analytically model the uplink ICI statistics in multiuser multi-carrier cellular networks while capturing the impact of both uncoordinated, coordinated scheduling and FFR schemes on the locations and transmit powers of the interferers. The impact of BS coordination on various frequency allocation patterns was investigated compared to the standard case without coordination. Ergodic capacity results demonstrated that forward BS synchronization is more beneficial for the frequency allocation schemes in which the majority of the subcarriers are likely to be allocated at the cell center (e.g., the CCFA scheme). However, reverse BS synchronization is an attractive solution for the frequency allocation schemes in which the majority of the subcarriers are likely to be allocated at the cell edge (e.g.,
the CEFA scheme). The derived ICI expressions can be also utilized to compare performance gains in different fading channel conditions and to derive other performance metrics such as network outage, fairness, and capacity-coverage tradeoffs. In addition, high proportion of allocated cell center subcarriers with low threshold distances are quite promising in maximizing the performance gains, especially in scheduling schemes where large number of users are likely to be allocated at the cell-edge regions. In next chapter, we will look into the performance of heterogeneous networks by applying slow and fast PC mechanisms.

Appendix

A Closed Form Expression of (5.19) in Rayleigh Fading Channels

By dropping the superscript \( j \) to avoid complexity of notations and considering that \( \zeta \) is an exponentially distributed RV, we can write the CDF of the selected user SNR in a ring \( k \), i.e., \( F_{\gamma_{s,k}}(\gamma_{s,k}) \) as:

\[
F_{\gamma_{s,k}} (\gamma_{s,k}) = (F_{\zeta}(\gamma_{s,k}r_{\beta}^k))^u_k = (1 - e^{-\lambda_{\gamma_{s,k}}u_{\beta}^k})^u_k. \tag{5.39}
\]

Taking \( \lambda_k = \lambda r_k^\beta \), we can rewrite (C.80) as \( F_{\gamma_{s,k}} (\gamma_{s,k}) = (1 - e^{-\lambda_k \gamma_{s,k}})^u_k \). Note that \( \int_{\gamma_{s,k}} (\gamma_{s,k}) = u_k(1 - e^{-\lambda_k \gamma_{s,k}})^u_k - 1 \). Therefore (5.19) can be rewritten for Rayleigh fading channels as

\[
P(r_{\text{sel}} = r_k) = \int_0^\infty \frac{u_k}{1 - e^{-\lambda_k \gamma_{s,k}}} d\gamma_{s,k}, \tag{5.40}
\]
Now by applying binomial expansion on part (I) of (C.82), we can write
\[ K \prod_{i=1}^{K} (1 - e^{-\lambda_i \gamma_{s,k}})^{u_i} = K \prod_{i=1}^{K} \left( \sum_{m_i=0}^{u_i-1} (-1)^{m_i} \binom{u_i}{m_i} e^{-\lambda_i m_i \gamma_{s,k}} \right) . \] (5.41)

The product of sum in (C.83) can be written as sum of products using algebraic manipulations as
\[ K \prod_{i=1}^{K} (1 - e^{-\lambda_i \gamma_{s,k}})^{u_i} = \sum_{m_1=0}^{u_1-1} \sum_{m_2=0}^{u_2-1} \cdots \sum_{m_K=0}^{u_K-1} (-1)^{\sum_{i=1}^{K} m_i} \binom{u_1}{m_1} \binom{u_2}{m_2} \cdots \binom{u_K}{m_K} e^{-\gamma_{s,k} \sum_{i=1}^{K} \lambda_i m_i} . \] (5.42)

Substituting (C.84) in (C.82), we can write
\[ P(\text{sel} = r_k) = \int_{0}^{\infty} \sum_{m_1=0}^{u_1-1} \sum_{m_2=0}^{u_2-1} \cdots \sum_{m_K=0}^{u_K-1} (-1)^{\sum_{i=1}^{K} m_i} \binom{u_1}{m_1} \binom{u_2}{m_2} \cdots \binom{u_K}{m_K} \times \left( u_k \lambda_k e^{-\gamma_{s,k} \left( \lambda_k + \sum_{i=1}^{K} \lambda_i m_i \right)} \right) \left( \frac{1}{1 - e^{-\lambda_k \gamma_{s,k}}} \right) d\gamma_{s,k} . \] (5.43)

Interchanging the integral and summation signs, we can write (C.85) as
\[ P(\text{sel} = r_k) = \sum_{m_1=0}^{u_1-1} \sum_{m_2=0}^{u_2-1} \cdots \sum_{m_K=0}^{u_K-1} (-1)^{\sum_{i=1}^{K} m_i} \binom{u_1}{m_1} \binom{u_2}{m_2} \cdots \binom{u_K}{m_K} \times \int_{0}^{\infty} u_k \lambda_k e^{-\gamma_{s,k} \left( \lambda_k + \sum_{i=1}^{K} \lambda_i m_i \right)} \left( \frac{1}{1 - e^{-\lambda_k \gamma_{s,k}}} \right) d\gamma_{s,k} . \] (5.44)

Using the identity [67][Eq. 3.311/4], the integral in (5.44) can be represented by an infinite series given as:
\[ u_k \lambda_k \int_{0}^{\infty} \frac{e^{-\gamma_{s,k} \left( \lambda_k + \sum_{i=1}^{K} \lambda_i m_i \right)}}{1 - e^{-\lambda_k \gamma_{s,k}}} d\gamma_{s,k} = \sum_{g=0}^{\infty} \left( (g + 1) \lambda_k + \sum_{i=1}^{K} \lambda_i m_i \right)^{-1} . \] (5.45)

The infinite series in (5.45) can be truncated and generate results with the accuracy of up to four decimal places with \( m = 1000 \). Substituting the value from (5.45) to
Figure 5.7: Graphical illustration of the (i) cell-center and cell-edge spectrum allocation in FFR networks with frequency reuse factor $D = 3$; (ii) distance of the serving BSs of the interferers at cell-center and cell-edge allocated subcarriers.

(5.44), the final result can be given as in (5.20).
Figure 5.8: Graphical illustration of (i) spectrum and cell partition within FFR networks; (ii) discretization of the cellular region into several concentric circular rings for analytical interference modeling in cellular networks.

Figure 5.9: Joint PMF of the allocated user locations on a given cell-center and cell-edge subcarrier for greedy and round robin scheduling schemes, with $R_{th} = 197.75$ m.
Figure 5.10: Three dimensional illustration of the 90th percentile of the CDF of cumulative CCI incurred on (i) cell-center subcarrier; (ii) cell-edge subcarrier; (iii) average CCI per subcarrier in FFR and UFR networks as a function of $N_{\text{cent}}/N$ and $R_{\text{th}}$ considering greedy scheduling.
Greedy Scheduling

90th Percentile of CCI

UFR: Average CCI per Subcarrier
FFR: Cell-Center Subcarrier
FFR: Cell-Edge Subcarrier
FFR: Average CCI per Subcarrier

Figure 5.11: Three dimensional illustration of the 90th percentile of the CDF of cumulative CCI incurred on (i) cell-center subcarrier; (ii) cell-edge subcarrier; (iii) average CCI per subcarrier in FFR and UFR networks as a function of $N_{cent}/N$ and $R_{th}$ considering round robin scheduling.
Figure 5.12: Network ergodic capacity attained by cell-edge and cell-center users considering greedy and round robin scheduling for different spectrum partitions considering $R_{th} = 230.55$ m.

Figure 5.13: Aggregate network ergodic capacity considering greedy and round robin scheduling schemes as a function of the number of subcarriers for FFR and UFR networks considering $R_{th} = 230.55$ m and $N_{cent}/N = 0.9$. 
Chapter 6

Interference Mitigation via Power Control in Heterogeneous Networks

6.1 Introduction

Heterogeneous networks (HetNets) are considered as a striking solution to the challenging demands such as high spectral and energy efficiency of mobile communications networks. HetNets are typically composed of multiple radio access technologies (RATs) where multiple low-power, low-cost user deployed BSs are complementing the existing macrocell network. A key recent trend in this regard is the use of small cells overlaid throughout the traditional macro cellular network which facilitates the operation and transmission of the macrocell BSs in various perspectives. Firstly, the small cell deployment shares the traffic load of macrocells whilst alleviating the capital and operational expenditures (CAPEX and OPEX) of macrocell BSs [79, 80]. Moreover, the mobile users located within a given small cell enjoys high quality transmission due to their short distance from the serving BS, hence ensuring enhanced network capacity. The small cell technology has also been praised as greener; i.e., more energy
efficient technology due to the short distance communication [81, 82, 50]. However, the interference conditions become worse due to the frequency reuse in the randomly deployed small cells.

This chapter develops a spectral and energy efficient deployment of small-cells referred to as cell-on-edge (COE), where the small-cell base stations (SBSs) are arranged around the edge of the macrocell. The COE configuration mandates to improve the network energy and spectral efficiency, and thereby reducing the interference, global carbon footprint, and capital and operational expenditures. This is achieved by considering slow and fast PC in the uplink where the mobile users are transmitting with adaptive power to compensate the path loss, shadowing and fading. A new performance metric, called area green efficiency (AGE), is introduced to characterize the aggregate energy savings in the uplink per unit macrocell area. We present a mathematical analysis to calibrate the area spectral efficiency (ASE) and AGE of the HetNets. Furthermore, we derive analytical bounds on the ASE and demonstrate that the derived bounds are useful in evaluating the ASE under the worst and the best case interference scenarios. Simulation results are presented to demonstrate the ASE and AGE improvements with respect to macro-only networks (MoNets) and other traditional small-cell deployment strategies.

The rest of the chapter is organized as follows. In Section 6.2, we present the system model of two tier HetNets. In Section 6.3, we analyze the system performance in terms of ASE and AGE by considering location-based PC only. In particular, we derive analytical bounds on ASE and exact AGE expressions. Later, in Section 6.4, we derive the ASE and AGE of the energy aware COE configuration by considering fast conventional PC mechanism. Section 6.5 presents numerical and simulation results followed by the concluding remarks in Section 6.6.
6.2 Network Layout

We consider two tier energy aware HetNet in the uplink as illustrated in Fig. 6.1 where the integration of macro and small cell networks has been shown. The first tier of the considered HetNet comprises of circular macrocells each of radius $R_m$ [m] with a BS $B_m$ deployed at the center and equipped with an omni-directional antenna. Each macrocell is assumed to have $H$ mobile users uniformly distributed over the region bounded by $R_0$ and $R_m$, where $R_0$ denotes the minimum distance between the macrocell mobile user and its serving BS.

The second tier of the HetNet comprises of $N$ circular small cells each of radius $R_n$ [m] with low-power low-cost user deployed small cell BSs $B_n$ located at the center. We consider that the small cells are distributed around the edge of the reference macrocell such that the resultant small cell deployment is referred to as COE configuration. For practical reasons, we calculate the number of small cells per macrocell as follows

$$N = \begin{cases} \mu \frac{(R_2^2 - R_1^2)}{R_n^2} = \mu \frac{4R_m}{R_n} & R_m > R_n \\ 0 & R_m \leq R_n \end{cases}$$

where $R_1 = R_m - R_n$, $R_2 = R_m + R_n$ and the factor $0 < \mu \leq 1$ is referred to as the small cell population factor (FPF) which control the number of small cells per macrocell, i.e.,

$$\mu = \begin{cases} 0 & \text{off-load small cells} \\ 1 & \text{maximum number of small cells per macrocell} \end{cases}$$  (6.1)

The number of mobile users in each small cell can be given as $F = (H - L)/N$ where $L = (H(R_1^2 - R_0^2))/R_m^2$. To be precise, in COE configuration, $L$ out of $H$ mobile users are uniformly distributed over the region bounded by $R_0$ and $R_1$ whereas the remaining mobile users, i.e., $H - L$ are reserved for $N$ small cells. The bandwidth
allocated to a macrocell is reused throughout the macrocell network at a distance $D' = R_u(R_m + R_n)$ [m], where $R_u$ represents the network traffic load and $R_u=2$ for a fully loaded cellular network. The total bandwidth allocated to the small cell tier is reused in each of the $N$ small cells within a macrocell.

We consider the spectrum partition based on the proportion of the number of mobile users in the macrocell and the small cells. Let $w_t$ [Hz] is the total bandwidth of the available spectrum per cell then the total bandwidth may be divided as $w_t = w_m + w_n$ where $w_m = w_t(L/H)$ [Hz] and $w_n = w_t(NF/H)$ [Hz] are the amount of the spectrum dedicated to the macrocell and the small cells respectively based on the proportion of the number of active mobile users. The allocated bandwidth of each mobile user in the macro and the small cell networks may be respectively calculated as $w_{t,m} = w_m/N_m$ and $w_{f,n} = w_n/N_n$; where $N_m$ and $N_n$ are the number of active serviced channels available per macrocell and small cell respectively. Moreover, for simplicity we consider that each channel can be allocated to one mobile user at a time and there will not be any mobile user which cannot be serviced by the respective
macro or small cell BS such that \( N_n = F \) and \( N_m = L \).

All mobile users in macrocell and small-cell networks are considered as mutually independent and uniformly distributed in their respective cells. The PDF of the location of a macrocell mobile user located at \((r, \theta)\) from the serving macrocell BS is expressed as

\[
f_r(r) = \frac{2r}{R_1^2 - R_0^2}, \quad f_\theta(\theta) = \frac{1}{2\pi},
\]

where \( R_0 \leq r \leq R_1 \) and \( 0 \leq \theta \leq 2\pi \). Similarly, the PDF of the location of a small-cell mobile user which is located at \((\tilde{r}, \tilde{\theta})\) from the serving SBS can be expressed by

\[
f_{\tilde{r}}(\tilde{r}) = \frac{2\tilde{r}}{R_n^2}, \quad f_{\tilde{\theta}}(\tilde{\theta}) = \frac{1}{2\pi},
\]

where \( 0 \leq \tilde{r} \leq R_n \) and \( 0 \leq \tilde{\theta} \leq 2\pi \) (see Fig. 6.1 for the geometrical representation of distances \( R_2 \) and \( R_1 \)).

### 6.3 Performance Analysis with Location-based PC

#### 6.3.1 Energy Aware Channel Propagation Model

Wireless channel is usually modeled by (i) distance dependent path-loss, (ii) shadowing, and (iii) multipath fading. In this section, we only consider path-loss effect since we assume a scenario where an efficient antenna diversity combining system is employed at the BS to eliminate the effects of multipath fading [83]. We consider two slope path-loss model to obtain the mean received power as a function of the distance between the mobile user and the respective serving BS. In general, the path-loss model can be written as follows

\[
P_{\text{rx}}(r) = K \frac{1}{r^\alpha(1 + r/g)^\beta} P_{\text{tx}}
\]

(6.4)
where $P_{\text{rx}}[W]$ denotes the average received signal power at the reference BS from the desired mobile user which is located at a distance $r$ from the same reference BS; $K$ is the constant which is depending on path-loss factor; $P_{\text{tx}} = P_{\text{max}}[W]$ is the maximum transmit power of each of the mobile user; $\alpha$ is the basic path-loss exponent; $\beta$ is the additional path-loss exponent; and $g = \frac{4h_{\text{tx}}h_{\text{rx}}}{\lambda_c} [\text{m}]$ is the breakpoint of a path-loss curve and it strictly depends on the BS antenna height $h_{\text{rx}} [\text{m}]$ and the antenna height of mobile user $h_{\text{tx}} [\text{m}]$ and $\lambda_c$ is the wave length of the carrier frequency. As each mobile user is considered to be capable of adapting its transmit power autonomously while maintaining a certain signal power received at the BS, the adaptive transmit power can be written as

$$P_{\text{tx}} = \min\left(P_{\text{max}}, P_0 \frac{r^\alpha (1 + r/g)^\beta}{K}\right)$$

(6.5)

where $P_0 [W]$ denotes the signal power received at the BS. Note that (4.1) is a simplified version of the conventional uplink PC (PC) which is recently approved by 3 GPP in LTE networks.

The distance at which mobile users require their maximum power $P_{\text{max}}$ to fully compensate path-loss while maintaining a certain signal power received at the BS $P_0$ is referred to as the \textit{threshold distance} ($R_t$). Since the mobile users located within $R_t$ can compensate their path-loss while saving some proportion of their power, therefore the region within $R_t$ is referred to as \textit{green area}. However, the mobile users located beyond $R_t$ may transmit with their maximum power to achieve some throughput gains. $R_t$ can be computed by solving the following equation numerically.

$$R_t^\alpha (g + R_t)^\beta = \frac{KP_{\text{max}}}{P_0} g^\beta$$

(6.6)
For special cases, in which $\beta = \alpha$, $R_t$ can be given as follows

$$R_t = \frac{1}{2} \left( -g \pm \sqrt{g \left( 4 \left( \frac{P_{\text{max}} K}{P_0} \right)^{1/\alpha} + g \right)} \right)$$

(6.7)

Moreover, in the scenarios where $\beta = m\alpha$ and $m$ is an integer, $R_t$ can be determined using standard mathematical software packages such as MATHEMATICA.

### 6.3.2 Area Spectral Efficiency of COE Configuration

The ASE is typically defined as the sum of the maximum achievable rates per unit bandwidth per unit macrocell area [83]. We express ASE with reference to the macrocell as the allocated frequencies are reused in the macrocells which are located at a distance $D'$ [m]. Therefore, we calculate it over the portion of the area covered by macrocell given as $\pi(R_m + R_n)^2$. The ASE can then be expressed as follows

$$\eta = \frac{C_h}{\pi w_h (D'/2)^2} = \frac{4C_h}{\pi w_h R_u^2 (R_m + R_n)^2}$$

(6.8)

where $R_u$ is the normalized reused factor and $R_u = 2$ represents a fully loaded cellular network. $C_h$ denotes the capacity of HetNet and can be written as

$$C_h = C_m + C_n = \sum_{l=1}^{L} C_{l_m} + \sum_{n=1}^{N} \sum_{f=1}^{F} C_{f_n}$$

(6.9)

where $C_m$ and $C_n$ [bits/sec] are the mean achievable capacity of $m^{th}$ macrocell and $N$ small cells, respectively and $C_{l_m} = w_{l_m} \log_2 (1 + \gamma_{l_m})$ is the Shannon capacity of $l^{th}$ mobile user in the $m^{th}$ macrocell whereas $C_{f_n} = w_{f_n} \log_2 (1 + \gamma_{f_n})$ is the Shannon capacity of $f^{th}$ mobile user in the $n^{th}$ small cell. Since we consider an interference limited system where the thermal noise power is negligible relative to the co-channel interference power [83], the received SIR of the $l^{th}$ macrocell mobile user in cell $m$,
i.e., $\gamma_{lm}$ can be given as

$$\gamma_{lm} = \frac{P_{rx_{lm},B_m}(r)}{\sum_{i=1,i\neq m}^{M} P_{rx_{li},B_m}(r)}$$  \hspace{1cm} (6.10)$$

where $P_{rx_{lm},B_m}(r)$ is the received power level at $B_m$ from the $l^{th}$ mobile user in cell $m$ and can be written explicitly by substituting (6.29) in (6.10) as

$$P_{rx_{lm},B_m}(r) = \begin{cases} 
P_{max}Kr_{lm,B_m}^{-\alpha} \left(1 + \frac{r_{lm,B_m}}{g}\right)^{-\beta} & \text{if } r_{lm,B_m} > R_t \\
\tilde{P}_0 & \text{if } r_{lm,B_m} < R_t
\end{cases}$$  \hspace{1cm} (6.11)$$

and $P_{rx_{li},B_m}(r)$ is the interfering power level received at $B_m$ from the $l^{th}$ mobile user in the $i^{th}$ interfering macrocell. Similarly, the uplink SIR of the $f^{th}$ mobile user in the $n^{th}$ small cell can be expressed as follows

$$\gamma_{fn} = \frac{P_{rx_{fn},B_n}(\tilde{r})}{\sum_{j=1,j\neq n}^{N} P_{rx_{fj},B_n}(\tilde{r})}$$  \hspace{1cm} (6.12)$$

where $P_{rx_{fn},B_n}(\tilde{r})$ is the received power at $B_n$ from the $f^{th}$ mobile user located in cell $n$ given as

$$P_{rx_{fn},B_n}(\tilde{r}) = \begin{cases} 
P_{max}K\tilde{r}_{fn,B_n}^{-\alpha} \left(1 + \frac{\tilde{r}_{fn,B_n}}{g}\right)^{-\beta} & \text{if } \tilde{r}_{fn,B_n} > R_t \\
\tilde{P}_0 & \text{if } \tilde{r}_{fn,B_n} < R_t
\end{cases}$$  \hspace{1cm} (6.13)$$

and $P_{rx_{fj},B_n}(\tilde{r})$ is the interfering power level received from the $f^{th}$ mobile user in the $j^{th}$ interfering small cell.

### 6.3.3 Bounds on the ASE of COE Configuration

We will now focus on deriving the analytical bounds on the ASE of two tier energy aware COE configuration. It is important to note that determining the statistics of cumulative interference for two slope path-loss model is computationally intensive
due to the arbitrary location of the interferers in the uplink. Therefore, we simplified our analysis by fixing the distance of the interferers and their transmit powers which helps in evaluating the bounds for the worst case and best case interference scenarios. The developed bounds provide useful insights in determining the worst case and best case SIR of HetNet mathematically.

1. **Upper Bound:** In order to evaluate the upper bound, we consider macrocell best interference configuration which mandates all cochannel interferers to be located on the far boundary of their respective cells, i.e., at a distance of $D' + R_1[m]$ from the desired mobiles BS while transmitting with a power of $P_m = \min(P_{\text{max}}, P_0KR_1^\alpha(1 + \frac{R_1}{g})^\beta)$. On the other hand, the small cell interferers are considered to be located at the edge of their respective cells as illustrated in Fig. 1, while transmitting with the power $P_n = \min(P_{\text{max}}, P_0KR_n^\alpha(1 + \frac{R_n}{g})^\beta)$. However, determining the exact best edge with respect to the BS of interest is not simple in small cell setup, therefore for sake of simplicity we categorized the small cell interferers roughly into two groups based on their distances from the BS of interest (See Fig. 1).

2. **Lower Bound:** In order to evaluate the lower bound, we consider macrocell worst interference configuration which mandates all cochannel interferers to be located on the near boundary of their respective cells, i.e., at a distance of $D' - R_1[m]$ from the desired mobiles BS while transmitting with $P_m$. Whereas the small cell interferers are considered to be located at the edge of their respective cells as illustrated in Fig. 1, while transmitting with $P_n$.

In order to determine the worst/best edge with respect to the small BS of interest we will now explain the details of the two interferer group. The first group (Group 1) corresponds to the group of interferers which are relatively located near the small BS of interest (See Fig. 1), therefore by applying the cosine law, the worst and best
case distance $d_{1,n}^\pm$ for Group 1 small cell interferers can be given as

$$d_{1,n}^\pm = R\sqrt{2 \pm 1} - n \sqrt{1 - \cos \left(\frac{2\pi n}{N} \pm \frac{\pi}{N}\right)} \quad (6.14)$$

where $n$ denotes the integer number by which small cell interferer BS is angularly spaced from the small BS of interest, $1 \leq n \leq \frac{N}{4} \forall N \in \text{Even}$ and $1 \leq n \leq \frac{N-1}{4} \forall N \in \text{Odd}$. The second group (Group 2) corresponds to the group of interferers which are relatively located far from the small BS of interest, therefore $d_{2,n}^\pm$ for Group 2 small cell interferers is given as

$$d_{2,n}^\pm = \sqrt{R_m^2 + (R_m \pm R_n)^2 - 2R_m(R_m \pm R_n)\cos \frac{n2\pi}{N}} \quad (6.15)$$

where $\frac{N}{4} + 1 \leq n \leq \frac{N}{2} - 1 \forall N \in \text{Even}$ and $\frac{N-1}{4} + 1 \leq n \leq \frac{N-1}{2} - 1 \forall N \in \text{Even}$. By using the symmetry of the COE configuration, the distances of the remaining half interferers can be computed using (6.14) and (6.15). Conditioned on the location of the desired mobile user in macrocell, the worst and best case SIR $\tilde{\gamma}_{l,m}^\pm$ is given as:

$$\tilde{\gamma}_{l,m}^\pm = \begin{cases} \frac{(D' \pm R_l)^\alpha (g + D' \pm R_l)^\beta P_0}{K(M-1)P_m^\gamma} & r_{l,m,B_m} \leq R_t \\ \frac{P_{\max}(D' \pm R_l)^\alpha (g + D' \pm R_l)^\beta}{r_{l,m,B_m}^\gamma (1 + r_{l,m,B_m})^\gamma (M-1)P_m} & r_{l,m,B_m} > R_t \end{cases} \quad (6.16)$$

Similarly, the worst case and best case SIR which is conditioned on the location of the desired mobile user in small cell is given as follows:

$$\tilde{\gamma}_{f,n}^\pm = \begin{cases} \frac{P_k}{g^\delta P_n KS^\pm} & r_{f,n,B_n} \leq R_t \\ \frac{P_{\max}r_{f,n,B_n}^\alpha (1 + r_{f,n,B_n})^{-\beta}}{g^\delta P_n KS^\pm} & r_{f,n,B_n} > R_t \end{cases} \quad (6.17)$$

where

$$S^\pm = \sum_{n=1}^{N/4} \frac{2(d_{1,n}^\pm)^{-\alpha}}{(g + d_{1,n}^\pm)^\beta} + \sum_{n=1+\frac{N}{4}}^{N-1} \frac{2(d_{2,n}^\pm)^{-\alpha}}{(g + d_{2,n}^\pm)^\beta} \quad (6.18)$$
The SIR of desired macro and small cell mobile user in (6.16) and (6.17) can be written in more compact form as

\[ \tilde{\gamma}_{l,m} = \left( \frac{A_m^\pm P_0}{P_{\text{max}} K} - A_m^\pm r_{l,m,B_m}^\alpha \left( 1 + \frac{r_{l,m,B_m}}{g} \right)^{-\beta} \right) \mathbb{U}(R_t - r_{l,m,B_m}) + A_m^\pm r_{l,m,B_m}^\alpha \left( 1 + \frac{r_{l,m,B_m}}{g} \right)^{-\beta} \]

\[ \tilde{\gamma}_{f,n} = \left( \frac{P_0}{g^\beta P_n K S^\pm} - \frac{P_{\text{max}} r_{f,n,B_n}^\alpha}{g^\beta P_n K S^\pm} \right) \mathbb{U}(R_t - r_{f,n,B_n}) + \frac{P_{\text{max}} r_{f,n,B_n}^\alpha}{g^\beta P_n K S^\pm} \left( 1 + \frac{r_{f,n,B_n}}{g} \right)^{-\beta} \]

where \( A_m^\pm = \frac{P_{\text{max}} (D' \pm R_t)^\alpha (g + D' \pm R_t)^{\beta}}{g^\beta P_m (M - 1)} \), and \( \mathbb{U}(.) \) is the unit step function. Finally, averaging over the distribution of the location of the desired mobile user in macrocell and small cell, the worst case and best case Shannon capacity can be given as

\[ C_{l,m}^\pm = \int_0^{R_m - R_n} \log_2(1 + \tilde{\gamma}_{l,m}) p_r(r) dr \]

\[ C_{f,n}^\pm = \int_0^{R_n} \log_2(1 + \tilde{\gamma}_{f,n}) p_r(\tilde{r}) d\tilde{r} \]

The integration can be solved using mathematical packages such as MATHEMATICA.

Finally, the bounds on \( \eta \) of the COE configuration can be given using (6.9) as

\[ \eta^\pm = \frac{4 \left( w_m C_{l,m}^\pm + N w_n C_{f,n}^\pm \right)}{w_t \pi R_u^2 (R_m + R_n)^2} \]

6.3.4 Exact Area Green Efficiency of COE Configuration

The AGE of a two tier HetNet is defined as the aggregate power savings per unit macrocell area while maintaining a certain signal power received at the BS receiver. Mathematically, the AGE of two tier heterogeneous network is

\[ \text{AGE} = \frac{P_m + P_n}{\pi (R_m + R_n)^2} \]
In order to calibrate the AGS of the energy aware COE configuration, we guarantee the received signal power of $P_0$ at both small cell and macrocell BSs. The total power savings of $L$ macrocell mobile users $\mathcal{P}_m$ can be determined as $\mathcal{P}_m = \sum_{l=1}^{L} P_{lm}$ where $P_{lm}$ is the power saving of the $l^{th}$ mobile user located in cell $m$ and is given by

$$P_{lm} = \begin{cases} 0 & r_{lm,B_m} > R_t \\ P_{\text{max}} - P_0 \frac{r_{lm,B_m}^{\alpha} (1 + r_{lm,B_m}/g)^{\beta}}{K} & r_{lm,B_m} < R_t \end{cases}$$  \hspace{1cm} (6.25)$$

Similarly the total power savings of $NF$ small cell mobile users $\mathcal{P}_n$ can be given as $\mathcal{P}_n = \sum_{n=1}^{N} \sum_{f=1}^{F} P_{fn}$ where $P_{fn}$ is the power saving of the $f^{th}$ mobile user located in cell $n$ and can be given as follows

$$P_{fn} = \begin{cases} 0 & \tilde{r}_{fn,B_n} > R_t \\ P_{\text{max}} - P_0 \frac{\tilde{r}_{fn,B_n}^{\alpha} (1 + \tilde{r}_{fn,B_n}/g)^{\beta}}{K} & \tilde{r}_{fn,B_n} < R_t \end{cases}$$  \hspace{1cm} (6.26)$$

In general, a closed form expression for $P_{lm}$ given in (6.25) can be derived as:

$$P_{lm} = P_{\text{max}} R_t - \frac{2(-1)^{-\alpha}}{R_{lm}^2} \left( \frac{P_0}{K} g^{2+\alpha} B(-R_t/g, 2 + \alpha, 1 + \beta) \right)$$  \hspace{1cm} (6.27)$$

where $B(\cdot, \cdot, \cdot)$ is the incomplete beta function. Similarly, closed form expression for (6.26) can be derived as:

$$P_{fn} = P_{\text{max}} R_t - \frac{2(-1)^{-\alpha}}{R_{fn}^2} \left( \frac{P_0}{K} g^{2+\alpha} B(-R_t/g, 2 + \alpha, 1 + \beta) \right)$$  \hspace{1cm} (6.28)$$
6.4 Performance Analysis with Fast PC

6.4.1 Energy Aware Channel Propagation Model

The received signal power at macrocell BS $B_m$ from the mobile user $l_m$ is

$$P_{l_m,B_m}^{rx}(r) = r^{-\alpha_m}(1 + r/g_m)^{-\beta_m}P_{l_m,B_m}^{tx}(r)\zeta_{l_m,B_m},$$

(6.29)

where, $P_{l_m,B_m}^{rx}(r)\text{[W]}$ denotes the average received signal power at the reference macrocell BS from the desired mobile user which is located at a distance $r$ from the same reference macrocell BS, $\zeta_{l_m,B_m}$ is the composite shadowing and fading component over the link between the mobile user and respective macrocell BS, $\alpha_m$ and $\beta_m$ are the basic and additional path loss exponents for macrocell, respectively, $g_m = \frac{4h_B h_l}{\lambda_c} \text{[m]}$ is the breakpoint of a path-loss curve for macrocell which depends on the macrocell BS antenna height $h_B$, the antenna height of the mobile user $h_l$ in the same macrocell and wavelength of the carrier frequency $\lambda_c$. $P_{l_m,B_m}^{tx}(r)\text{[W]}$ defines the mobile user transmit power for physical uplink shared channel (PUSCH) such that each of mobile users in the macrocell network adapts its transmit power according to a fast PC mechanism given by [84, 26, 85, 86]:

$$P_{l_m,B_m}^{tx}(r) = \min (P_{\text{max}}, P_0 PL(r)).$$

(6.30)

$P_{\text{max}}\text{[W]}$ is the maximum transmit power of each of the mobile users in HetSNet, $PL(r)$ is the combined total path-loss\(^1\) for the macrocell link, which is expressed by using the two-slope path-loss model as

$$PL(r) = \frac{r^{\alpha_m}(1 + r/g_m)^{\beta_m}}{\zeta_{l_m,B_m}}.$$

(6.31)

\(^1\)Here, total path-loss includes path-loss, shadowing and fading, where the shadowing and path-loss can be estimated by the mobile users and the fading statistics can be transmitted by the base station using feedback to the mobile users.
\( \alpha \) is the path loss compensation factor. \( P_0 \) is the cell specific parameter which is used to control the target SINR.

Using (6.30) and (6.31), (6.29) can be expressed as

\[
P_{rx_{lm,B_m}}(r) = \begin{cases} 
\frac{P_{\text{max}}}{PL(r)} & P_{\text{max}} < P_0PL(r) \\
\frac{P_0}{PL(r)} & \text{otherwise.}
\end{cases} 
\]

(6.32)

The received power at small-cell BS \( B_n \) from the mobile user in any of the small-cell \( f_n \) may be written as [84, 26, 85, 86]

\[
P_{rx_{f_n,B_n}}(\tilde{r}) = \tilde{r}^{\beta_n}P_{tx_{f_n,B_n}}(\tilde{r})\zeta_{f_n,B_n},
\]

(6.33)

where \( P_{rx_{f_n,B_n}}(\tilde{r}) \) denotes the average received signal power at the reference SBS from the desired mobile user which is located at a distance \( \tilde{r} \) from the reference SBS, \( \zeta_{f_n,B_n} \) is the composite shadowing and fading component experienced by the link between the mobile user and SBS in small-cell, \( \alpha_n \) and \( \beta_n \) are the basic and additional path loss exponents in small-cell, \( g_n = \frac{4h_{\text{B}_n}h_{\text{f}_n}}{\lambda_c} \) [m] is the breakpoint of a path-loss curve for small-cell and it strictly depends on the SBS antenna height \( h_{\text{B}_n} \) [m] and the antenna height of mobile user \( h_{\text{f}_n} \) [m] in the same small-cell, \( P_{tx_{f_n,B_n}}(\tilde{r}) \) [W] denote the mobile user transmit power for PUSCH such that each mobile user in the small-cell network adapts its transmit power according to fast PC [84, 85],

\[
P_{tx_{f_n,B_n}}(\tilde{r}) = \min \left( P_{\text{max}}, P_0 PL(\tilde{r})^\alpha \right).
\]

(6.34)

\( PL(\tilde{r}) \) is the combined total path-loss for the small-cell link, which is expressed by using the two-slope path-loss model as

\[
PL(\tilde{r}) = \frac{\tilde{r}^{\alpha_n}(1 + \tilde{r}/g_n)^{\beta_n}}{\zeta_{f_n,B_n}}.
\]

(6.35)
Using (6.34) and (6.35), (6.33) can be re-expressed as

\[
P_{f_n,B_n}^{rx}(\tilde{r}) = \begin{cases} 
    \frac{P_{\text{max}}}{PL(\tilde{r})} & P_{\text{max}} < P_0 PL(\tilde{r}) \\
    P_0 & \text{otherwise}. 
\end{cases}  
\]  

(6.36)

6.4.2 Area Spectral Efficiency of HetNets

The area spectral efficiency \( \eta \) of the HetNets is typically defined as follows [83]

\[
\eta = \frac{C_h}{\pi w_h(D'/2)^2} = \frac{4C_h}{\pi w_h R_u^2(R_m + R_n)^2}, 
\]  

(6.37)

where \( D' = R_u(R_m + R_n) \), \( R_u \) is the frequency reuse factor and \( C_h \) stands for the total achievable Shannon capacity of two tier HetNets and it is given by

\[
C_h = C_m + C_n = \sum_{l=1}^{L} C_{l,m} + \sum_{n=1}^{N} \sum_{f=1}^{F} C_{f,n}, 
\]  

(6.38)

where \( C_m \) [bps/Hz] and \( C_n \) [bps/Hz] stand for the ergodic capacity of the \( m^{th} \) macrocell and \( N \) small-cells, respectively. Variable \( C_{l,m} \) is the Shannon capacity of \( l^{th} \) mobile user in the \( m^{th} \) macrocell whereas \( C_{f,n} \) is the capacity of \( f^{th} \) mobile user in the \( n^{th} \) small-cell. More explicitly, \( C_{l,m} \) is expressed as

\[
C_{l,m} = w_{l,m} E[\log_2(1 + \gamma_{l,m})],
\]  

(6.39)

\[
= w_{l,m} \int_{0}^{\infty} \log_2(1 + \gamma_{l,m}) f_\gamma(\gamma_{l,m}) d\gamma_{l,m},
\]  

(6.40)

where \( f_\gamma(\gamma_{l,m}) \) denotes the PDF of \( \gamma_{l,m} \); \( \gamma_{l,m} \) is the SINR of the \( l^{th} \) macrocell mobile user in the \( m^{th} \) macrocell and it is expressed as

\[
\gamma_{l,m} = \frac{P_{l,m,B_m}^{rx}(r)}{\sum_{i=1,i \neq m}^{M} P_{l_i,B_m}^{rx}(r) + \sigma^2},
\]  

(6.41)
where $\sigma^2$ is the thermal noise power, $P_{tx_{l,m}}(r)$ is the received power at the macrocell BS from the $l^{th}$ desired mobile user and $\sum_{i=1,i\neq m}^{M} P_{tx_{i,B_{m}}(r)}$ is the sum of the individual interfering power levels received at the reference macrocell BS $B_{m}$ from the interferers $\{l_{i}\}_{i=1}^{M}$ which are located in each of the $(i^{th})$ interfering macrocells. Substituting (6.29) into (6.41), the SINR of the macrocell mobile user is given by:

$$
\gamma_{l,m} = \frac{P_{tx_{l,m}}(r)\zeta_{l,m,B_{m}}(r) \Big(g_{m} + r_{l,m,B_{m}}\Big)^{-\beta_{m}}}{\sum_{i=1,i\neq m}^{M} P_{tx_{i,B_{m}}(r)}\zeta_{l,m,B_{m}}(r) \Big(g_{m} + r_{l,m,B_{m}}\Big)^{-\beta_{m}} + \sigma^2},
$$

(6.42)

where $\zeta_{l,m,B_{m}}$ is the composite fading of interference from the $l^{th}$ mobile user in the $i^{th}$ macrocell to the $m^{th}$ BS of interest.

Similarly, for the small-cell networks, $C_{f,n}$ in (6.38) is expressed as

$$
C_{f,n} = w_{f,n} E\left[\log_{2}(1 + \gamma_{f,n})\right],
$$

(6.43)

$$
= w_{f,n} \int_{0}^{\infty} \log_{2}(1 + \gamma_{f,n}) f_{\gamma}(\gamma_{f,n}) d\gamma_{f,n},
$$

(6.44)

where $\gamma_{f,n}$ is the SINR of the $f^{th}$ mobile user located in the $n^{th}$ small-cell given by

$$
\gamma_{f,n} = \frac{P_{tx_{f,n,B_{n}}(\tilde{r})}}{\sum_{j=1,j\neq n}^{N} P_{tx_{f,j,B_{n}}(\tilde{r})} + \sigma^2},
$$

(6.45)

where $P_{tx_{f,n,B_{n}}(\tilde{r})}$ is the received power level at the reference small-cell BS $B_{n}$ from the $f^{th}$ desired mobile user and $\sum_{j=1,j\neq n}^{N} P_{tx_{f,j,B_{n}}(\tilde{r})}$ is the sum of the individual interfering power levels received at the reference small-cell BS $B_{n}$ from the interferers located in the $j^{th}$ interfering small-cell. Substituting (6.33) into (6.45), $\gamma_{f,n}$ is given as

$$
\gamma_{f,n} = \frac{P_{tx_{f,n,B_{n}}(\tilde{r})}\zeta_{f,n,B_{n}}(\tilde{r}) \Big(g_{n} + \tilde{r}_{f,n,B_{n}}\Big)^{-\beta_{n}}}{\sum_{j=1,j\neq n}^{N} P_{tx_{f,j,B_{n}}(\tilde{r})}\zeta_{f,j,B_{n}}(\tilde{r}) \Big(g_{n} + \tilde{r}_{f,j,B_{n}}\Big)^{-\beta_{n}} + \sigma^2},
$$

(6.46)
6.4.3 Ergodic Capacity Based on MGF Approach

The closed-form solution for the capacity are less tractable due to the randomness associated with the location of the interferers and the composite fading statistics. Moreover, in many typical fading scenarios, the evaluation of the distribution of SINR requires multi-fold convolutions which is a tedious task, however, recently an MGF based generalized framework [71] enables the evaluation of the system capacity given the MGF of the desired and interfering signals. Therefore, in this chapter we will focus on utilizing the capacity lemma proposed in [71] to derive the exact analytical expressions for the bounds on the ASE for two tier HetNet. Using [71, Eq.5], the capacity for macrocell network can be expressed as follows

\[
C_{lm} = w_l \int_0^\infty \cdots \int_0^\infty \frac{M_{I_m}(t) - M_{S_m,I_m}(t)}{t} e^{-\sigma^2 t} \times f_{r_{lm},B_m}(r) \prod_{i=1,i\neq m}^M f_{r_{lm},B_m}(r) dr_{lm,B_m} dr_{lm,B_m} dt \quad (6.47)
\]

where \(M_{I_m}(t)\) and \(M_{S_m,I_m}(t)\) denote the MGF of the macrocell interference and joint MGF of the received signal and interference, respectively. \(M_{I_m}(t)\) and \(M_{S_m}(t)\) can be evaluated as follows

\[
M_{I_m}(t) = \mathbb{E}[e^{-t \sum_{i=1,i\neq m} P_{r_{lm},B_m}}], \quad (6.48a)
\]

\[
M_{S_m}(t) = \mathbb{E}[e^{-t P_{r_{lm},B_m}}]. \quad (6.48b)
\]
The joint MGF $M_{S_n,I_n}(t) = M_{S_n}(t)M_{I_n}(t)$. Similarly, for the small-cell network, (6.47) can be expressed as

$$C_{fn} = w_{fn} \int_0^\infty \left( \prod_{j=1}^N f_{\tilde{r}_{fn,B_n}}(\tilde{r}_f) \prod_{j=1,j\neq n}^N f_{\tilde{r}_{fj,B_n}}(\tilde{r}) \right) \frac{M_{I_n}(t) - M_{S_n,I_n}(t)}{t} e^{-\sigma^2 t} \times$$

$$f_{\tilde{r}_{fn,B_n}}(\tilde{r}) \prod_{j=1,j\neq n}^N f_{\tilde{r}_{fj,B_n}}(\tilde{r}) d\tilde{r}_{fn,B_n} d\tilde{r}_{I_n,B_n} dt \quad (6.49)$$

where $M_{I_n}(t)$ and $M_{S_n,I_n}(t)$ denote the MGF of the small-cell interference and joint MGF of the received signal and interference, respectively, and are evaluated as follows

$$M_{I_n}(t) = E[e^{-t \sum_{j=1,j\neq n}^N P_{fj,B_n}^{rx}}], \quad (6.50a)$$

$$M_{S_n}(t) = E[e^{-t P_{fn,B_n}^{rx}}]. \quad (6.50b)$$

The joint MGF $M_{S_n,I_n}(t) = M_{S_n}(t)M_{I_n}(t)$.

The CDF and the MGF of the Generalized-$K$ distribution involves Meijer-G and Whittaker functions, respectively, which reduce the analytical tractability and are computationally intensive. However, in order to avoid the associated computational difficulties, the authors in [66] proposed an accurate approximation of the Generalized-$K$ distribution by a more tractable Gamma distribution using the moment matching method, i.e., $K_G(m_e, m_s, \Omega) \approx \Gamma(m_e, \theta_e)$. Let $m_d$ and $\Omega_d$ denote the fading severity parameter and local mean power, respectively, for the desired mobile users which are located in both the macrocell and the small-cell networks. Similarly, $m_i$ and $\Omega_i$ represent the fading severity parameter and local mean power for the interfering mobile users, which are located in both the macrocell and the small-cell networks.
Assumptions to Derive Upper and Lower Bounds

The derivation of bounds on ASE is based on the assumptions that the interferers are (i) located at the worst and the best locations in each of the cell in both macrocell and small-cell networks and (ii) transmitting with fixed powers $P_m$ for the macrocell interferers and $P_n$ for the small-cell interferers.

**Upper Bound:** We consider a macrocell best interference configuration which corresponds to the case where all cochannel interferers are located on the far boundary of their respective cells, i.e., at a distance given by

$$d_m^+ = D' + (R_m - R_n),$$  \hspace{1cm} (6.51)

from the desired mobile BS and transmitting with power $P_m = \mathbb{E}[P^x_{lm,B_m}]$ which can be expressed as

$$P_m = \frac{P_{\max} \left( \theta_d \Gamma_l(m_d, \frac{C_m}{\theta_d}) + C_m \Gamma_u(m_d - 1, \frac{C_m}{\theta_d}) \right)}{\theta_d \Gamma(m_d)}$$  \hspace{1cm} (6.52)

where $C_m = P_0 \frac{r_m (1 + r/g_m)^{\delta m}}{P_{\max}}$. The desired macrocell mobile user is considered to transmit with *adaptive* power. Similarly, the small-cell interferers are considered to be located at a distance given by

$$d_n^+ = \sqrt{2R^2_m \left( 1 - \cos \left( \frac{2\pi n}{N} + \frac{\pi}{N} \right) \right)}, \hspace{1cm} N \in \text{odd} \hspace{1cm} (6.53)$$

whereas for even $N$, one interferer is considered to be at the opposite side of the BS of interest with $d_n^+ = 2R_m + R_n$. The interferers are considered to transmit with fixed average transmitting powers, i.e., $P_n = \mathbb{E}[P^x_{fn}]$ which is expressed as

$$P_n = \frac{P_{\max} \left( \theta_d \Gamma_l(m_d, \frac{C_n}{\theta_d}) + C_n \Gamma_u(m_d - 1, \frac{C_n}{\theta_d}) \right)}{\theta_d \Gamma(m_d)}.$$  \hspace{1cm} (6.54)
where $C_n = P_0^{\frac{1}{\alpha n}}(1+g_{an})^{\beta n}$. Note that the desired small-cell mobile user is considered to transmit with an *adaptive* transmission power.

**Proof of $P_m$ and $P_n$:** A general proof of (6.52) and (6.54) is presented in Appendix C.

- **Lower Bound:** We consider a macrocell worst interference configuration which corresponds to the case where all the cochannel interferers are located on the boundary of their respective cells, i.e., at a distance given by

$$d_m^{-} = D' - (R_m - R_n), \quad (6.55)$$

from the desired mobile BS, and transmit with power $P_m$. Similarly, small-cell interferers are considered to be located at a distance given by

$$d_n^{-} = \sqrt{2R_m^2 \left(1 - \cos \left(\frac{2\pi N}{N} - \frac{\pi}{N}\right)\right)}, \quad N \in \text{odd} \quad (6.56)$$

whereas for even $N$, one interferer is considered to be at the opposite side of the BS of interest with $d_n^{-} = 2R_m - R_n$. Each small-cell interferer is considered to be transmitting with $P_n$.

### 6.4.4 Bounds on ASE of Energy Aware COE configuration

In order to derive the bounds on the mean achievable ASE of HetNet, we first derive a bound on the ergodic capacity of the macrocell and the small-cell networks, i.e., the analytical bound on the (6.47) and (6.49) are derived in the following sections.

#### Bounds on the Ergodic Capacity of Macrocell Network

By assuming the worst and the best interfering mobile users in a macrocell network, the SINR of the macrocell mobile user can be derived by substituting (6.51), (6.55)
and (6.52) into (6.42) as

$$\gamma_l^\pm = \frac{S_m}{I_m^\pm + \sigma^2} = \frac{P_{tx,m,B_m}(r)\zeta_{l,m,B_m}r_m^{-\alpha_m}(g_m + r_l,m,B_m)^{-\beta_m}}{\sum_{i=1,i\neq m}^{M} P_m\zeta_{l,i,B_m}(d_m^\pm)^{-\alpha_m}(g_m + d_m^\pm)^{-\beta_m} + \sigma^2},$$

where $S_m$ denote the desired signal power and $I_m$ denotes the bounded cumulative interference received at the BS of interest of the macrocell. The theoretical bounds on the achievable capacity of the macrocell network in the considered HetSNet is given using (6.47) as [71]

$$C_l^\pm = w_l \int_0^\infty \int_{R_0}^{R_1} \frac{M_l^\pm (t) - M_{S_m,l,m}^\pm (t)}{t} e^{-\alpha t} f_r(r) dr dt, \quad (6.57)$$

where $M_{l,m}^\pm (t)$ is the lower and the upper bounds on the MGF of the bounded cumulative interference received at the BS of interest and $M_{S_m}(t)$ is the exact MGF of the desired signal. By assuming i.i.d interfering mobile users in $M - 1$ interfering macrocells, the MGF of the bounded cumulative interference $I_m^\pm$ can be expressed as

$$M_{l,m}^\pm (t) = \prod_{i=1,i\neq m}^{M} M_{\zeta_{l,i,B_m}} \left( \frac{P_m\zeta_{l,m,B_m}g_m^\beta_m (g_m + D_m^\pm)^{-\beta_m}}{(D_m^\pm)^{\alpha_m} \theta_l t} \right). \quad (6.58)$$

Here, we made use of the scaling property of the MGF, i.e. $M_{aX}(t) = M_X(at)$. Moreover, (6.58) is the generalized MGF, which can be used to derive the MGF of interfering signals over any type of fading channel knowing the MGF of the composite fading distribution. For Gamma fading channels, where $\zeta_{l,m,B_m}$ is a Gamma distributed random variable, the MGF $M_{l,m}^\pm (t)$ may be derived as

$$M_{l,m}^\pm (t) = \prod_{i=1,i\neq m}^{M} \left( 1 - \frac{P_m\zeta_{l,m,B_m}g_m^\beta_m (g_m + D_m^\pm)^{-\beta_m}}{(D_m^\pm)^{\alpha_m} \theta_l t} \right)^{-m_i}, \quad (6.59)$$

where the transformation from (6.58) to (6.59) can be understood from Appendix B.

Conditioned on the event $\zeta_{l,m,B_m} > C_m$, the received target signal power level at
the BS is $P_0$. Therefore, the distribution of macrocell signal $S_m$ can be expressed as

$$f_{S_m}(s_m | \zeta_{l_m,B_m} > C_m) = \delta(s_m - P_0). \quad (6.60)$$

Whereas, for the other event, i.e., condition $\zeta_{l_m,B_m} < C_m$ which implies $s_m < P_0$, the received signal power can be derived by RV transformation as:

$$f_{S_m}(s_m | \zeta_{l_m,B_m} < C_m) = \frac{C_m f_{\zeta_{l_m,B_m}}(s_m C_m / P_0)}{P_0 P(\zeta_{l_m,B_m} < C_m)}; \quad s_m < P_0. \quad (6.61)$$

Now by applying the total probability theorem we can write

$$f_{S_m}(s_m) = f_{S_m}(s_m | \zeta_{l_m,B_m} > C_m) P(\zeta_{l_m,B_m} > C_m) +$$

$$f_{S_m}(s_m | \zeta_{l_m,B_m} < C_m) P(\zeta_{l_m,B_m} < C_m). \quad (6.62)$$

For a given composite fading channel distribution, (6.62) is given as

$$f_{S_m}(s_m) = \frac{C_m U(P_0 - s_m)}{P_0} f_{\zeta_{l_m,B_m}}(s_m C_m / P_0) +$$

$$\delta(s_m - P_0) (1 - F_{\zeta_{l_m,B_m}}(C_m)), \quad (6.63)$$

where $f_{\zeta_{l_m,B_m}}(C_m s_m / P_0)$ and $F_{\zeta_{l_m,B_m}}(C_m)$ are the PDF and CDF of the respective random variables. Taking the Laplace transform of (6.63), the $M_{S_m}(t)$ is derived as

$$M_{S_m}(t) = \frac{C_m}{P_0} \int_0^{P_0} e^{-ts_m} f_{\zeta_{l_m,B_m}}(C_m s_m / P_0) \, ds_m +$$

$$e^{-tP_0} (1 - F_{\zeta_{l_m,B_m}}(C_m)). \quad (6.64)$$

The derived expression in (6.64) is generalized to any composite fading channel dis-
tribution. For Gamma fading channels, (6.64) can be transformed as

$$
\mathcal{M}_{S_m}(t) = \frac{C_m}{P_0} \int_0^{P_0} \frac{s_m^{m_d-1} e^{-\left(t + \frac{C_m}{\theta_d}\right)s_m}}{\Gamma(m_d) \theta_d} ds_m + e^{-t P_0} \left(1 - F_{\Theta_{m,B_m}}(C_m)\right).
$$

(6.65)

Finally, a closed-form expression for the indefinite integral in (6.65) can be derived using the identity as in [67],

$$
\int_0^u x^m e^{-\beta x^n} dx = \frac{\Gamma(v, \beta u^n)}{n \beta^v}, \text{ where } v = \frac{m+1}{n},
$$

as follows

$$
\mathcal{M}_{S_m}(t) = \left(\frac{C_m}{C_m + P_0 \theta_d t}\right)^{m_d} \left(\frac{\Gamma_1(m_d, (t + \frac{C_m}{\theta_d}P_0))}{\Gamma(m_d)}\right) + e^{-t P_0} \left(1 - \frac{\Gamma_1(m_d, 0)}{\Gamma(m_d)}\right),
$$

(6.66)

Moreover, the joint MGF $\mathcal{M}_{S_m,I_m}(t) = \mathcal{M}_{S_m}(t)\mathcal{M}_{I_m}(t)$.

**Bounds on the Ergodic Capacity of Small-cell Networks**

By assuming the worst and the best interfering mobile users in small-cell network, the SINR of the macrocell mobile user can be derived by substituting (6.53), (6.56) and (6.54) into (6.45) as

$$
\gamma_{f_n}^\pm = \frac{S_n}{I_n^\pm + \sigma^2} = \frac{P_{f_n,B_n}^n(\tilde{r}) \zeta_{f_n,B_n} \tilde{r}^{-\alpha_n} (g_n + \tilde{r} f_n,B_n)^{-\beta_n}}{\sum_{j=1, j \neq n}^N P_{f_j,B_n} n \zeta_{f_j,B_n} (d_{n}^\pm)^{-\alpha_n} (g_n + d_{n}^\pm)^{-\beta_n + \sigma^2}},
$$

where $S_n$ denote the desired signal power and $I_n$ denotes the bounded cumulative interference received at the BS of interest of a small-cell. Similarly, the analytical bounds on ergodic capacity of the small-cell network can be expressed as

$$
C_{f_n}^\pm = w_{f_n} \int_0^\infty \int_0^{R_n} \frac{\mathcal{M}_{I_n}^\pm(t) - \mathcal{M}_{S_n,I_n}^\pm(t)}{t} e^{-\sigma^2 f_{\tilde{r}}(\tilde{r})} d\tilde{r} dt,
$$

(6.67)
where $\mathcal{M}_{I_n}(t)$ is the lower and the upper bounds on the MGF of the bounded cumulative interference received at the BS of interest and $\mathcal{M}_{S_n}(t)$ is the exact MGF of the desired signal. Due to the mirror symmetry, half of the interferers are non identically distributed due to their varying locations, therefore the MGF of the bounded cumulative interference $I_n^{\pm}$ can be expressed as in (6.68) as

$$
\begin{align*}
\prod_{j=1,j \neq n}^{(N-1)/2} \left( \frac{\mathcal{M}_{\zeta_j, B_n} \left( \frac{P_n g_n^{\beta_n} (g_n + d_n^+) - \beta_n}{(d_n^+)^{\alpha_n}} \right)}{t} \right)^2, & \quad N \in \text{odd} \\
\mathcal{M}_{\zeta_j, B_n} \left( \frac{P_n g_n^{\beta_n} (g_n + (2R_m + R_n))^n - \beta_n}{(2R_m + R_n)^n} \right) \prod_{j=1,j \neq n}^{(N-2)/2} \left( \frac{\mathcal{M}_{\zeta_j, B_n} \left( \frac{P_n g_n^{\beta_n} (g_n + d_n^+) - \beta_n}{(d_n^+)^{\alpha_n}} \right)}{t} \right)^2, & \quad N \in \text{even}
\end{align*}
$$

(6.68)

Here, we made use of the scaling property of the MGF, i.e., $M_{aX}(t) = M_X(at)$. Note that (6.68) denotes the MGF of the bounded cumulative interference for any composite fading channels. The MGF of the signal power $S_n$ remains the same as in (6.66) and can be derived by simply replacing the notations.

The desired bounds on (6.37) can be expressed as

$$
\eta^{\pm} = \frac{4C^\pm}{\pi w_h R_u^2 (R_m + R_n)^2},
$$

(6.69)

where $C^\pm_h = C^\pm_m + C^\pm_n$.

### 6.4.5 Exact Area Green Efficiency of HetNets

The AGE of the HetSNet is defined as the aggregate power savings in the uplink per unit area. Mathematically, the AGE of the network is defined as follows

$$
\text{AGE} = \frac{P_m + P_n}{\pi (R_m + R_n)^2},
$$

(6.70)

where $P_m$ is the total power savings of $L$ macrocell mobile users and can be expressed as $P_m = \sum_{l=1}^L P_{m_l}$, where $P_{m_l}$ is the average power saving of $l$th macrocell mobile user.
Similarly, \( P_n \) is the total power savings of \( NF \) small-cell mobile users and can be calculated as 
\[
P_n = \sum_{n=1}^{N} \sum_{f=1}^{F} P_{fn},
\]
where \( P_{fn} \) is the power saving of the \( f^{th} \) mobile user located in \( n^{th} \) small-cell.

To calibrate the AGE of the energy efficient HetSNet, we guarantee the target signal power of \( P_0 \) at both small-cell and macrocell BSs. The fast PC allows power savings when a user is able to compensate its path loss, i.e., \( \zeta_{l_m,B_m} > C_m \), otherwise each user will transmit with its maximum transmission power and the power savings are almost zero. Therefore, we can define a RV which denotes the instantaneous uplink power saving per macrocell user or per macrocell frequency band, i.e.,
\[
T = \begin{cases} 
P_{\text{max}} - \frac{C_m P_{\text{max}}}{\zeta_{l_m,B_m}} & \zeta_{l_m,B_m} > C_m \\ 0 & \zeta_{l_m,B_m} < \text{otherwise} \end{cases}
\] (6.71)

The average power savings per frequency band conditioned on \( r \) are defined as
\[
\mathbb{E}[T|r] = \mathbb{E}[T|\zeta_{l_m,B_m} > C_m] P(\zeta_{l_m,B_m} > C_m). \tag{6.72}
\]

Here, we consider that each mobile user gets only one frequency band. Using the property of expected operator, \( \mathbb{E}[g(x)] = \int_{0}^{\infty} g(x) f(x) dx \), we can write (6.72) as
\[
\mathbb{E}[T|r] = \int_{C_m}^{\infty} P_{\text{max}} \left( 1 - \frac{C_m}{\zeta_{l_m,B_m}} \right) f_{\zeta_{l_m,B_m}}(\zeta_{l_m,B_m}) d\zeta_{l_m,B_m}. \tag{6.73}
\]

The integral expression in (6.73) can be decomposed and simplified as follows
\[
\mathbb{E}[T|r] = P_{\text{max}} \int_{C_m}^{\infty} f_{\zeta_{l_m,B_m}}(\zeta_{l_m,B_m}) d\zeta_{l_m,B_m} - P_{\text{max}} C_m \int_{C_m}^{\infty} \frac{f_{\zeta_{l_m,B_m}}(\zeta_{l_m,B_m})}{\zeta_{l_m,B_m}} d\zeta_{l_m,B_m}. \tag{6.74}
\]
Finally, (6.74) can be further solved and simplified as follows

$$\mathbb{E}[T|\tau] = P_{\text{max}}(1 - F_{\tau}(C_m)) - P_{\text{max}}C_m \int_{C_m}^{\infty} \frac{f_{\zeta_{l_m,B_m}}(\zeta_{l_m,B_m})}{\zeta_{l_m,B_m}} d\zeta_{l_m,B_m}. \quad (6.75)$$

Now for Gamma fading channels, (6.75) can be transformed as

$$\mathbb{E}[T|\tau] = P_{\text{max}} \left(1 - \frac{\Gamma_l(m_d, C_m)}{\Gamma(m_d)}\right) - P_{\text{max}}C_m \int_{C_m}^{\infty} \frac{e^{-\zeta_{l_m,B_m} \theta_d m_d}}{\Gamma(m_d) \theta_d^m} d\zeta_{l_m,B_m}, \quad (6.76)$$

using the identity [67], $\int_0^\infty x^m e^{-\beta x^n} dx = \frac{\Gamma_u(v, \beta u^n)}{u \beta^n}$, where $v = \frac{m+1}{n}$ and using properties of Gamma functions, (6.76) can be simplified to

$$\mathbb{E}[T|\tau] = P_{\text{max}} \left(\Gamma_u(m_d, C_m \theta_d) - C_m \Gamma_u(m_d - 1, C_m \theta_d)\right). \quad (6.77)$$

Similarly, for the small-cell network, the average power savings per frequency band conditioned on $\tilde{r}$ can be expressed as

$$\mathbb{E}[T|\tilde{r}] = P_{\text{max}} \left(\theta_d \Gamma_u(m_d, C_n \theta_d) - C_n \Gamma_u(m_d - 1, C_n \theta_d)\right). \quad (6.78)$$

Finally, by averaging (6.77) over the distribution of $r$, the average network power savings by a macrocell mobile user can be expressed as $P_{l_m} = \int_0^{R_1} \mathbb{E}[T|r] f_r(r) dr$. Similarly, by averaging (6.78) over the distribution of $\tilde{r}$, the average network power savings by a small-cell mobile user can be expressed as follows $P_{f_n} = \int_0^{R_n} \mathbb{E}[T|\tilde{r}] f_{\tilde{r}}(\tilde{r}) d\tilde{r}$. These expressions can be evaluated accurately using standard mathematical software packages such as MATHEMATICA.
6.5 Results and Discussions

In this section, we quantify the achievable ASE and AGE gain of the energy aware COE configuration compared to macro-only and UDC deployment through simulations. The simulation parameters are summarized in Table 1. The number of mobile users are considered to be adaptive with the coverage area, i.e., we ensure 0.001 mobile users per m$^2$. For sake of fair comparison, $N$ is taken to be same for UDC and COE configurations.

Fig. 6.2 quantifies the ASE of HetSNet as a function of macrocell radius over Gamma fading channels and compares the ASE of the proposed COE configuration with two types of networks namely (i) HetNets with UDC configuration where the small-cells are uniformly distributed and also receiving interferences from $N-1$ interfering small-cells and (ii) MoNet where the small-cells are not active. Note that SBS receive interference from $N-1$ small-cell interferers and MBS receive the interference from $M-1 = 6$ interferers. The mobile users in each of the macrocell and small-cell networks are transmitting (i) with the adaptive power scheme according to the fast
PC mechanism which is defined as (6.30) and (6.34) for macrocell and small-cell up-link, respectively, (see the red solid curve with square markers), and (ii) without PC, i.e., $P_{\text{max}}$ (see the red dashed dotted curve with square markers).

It can be seen clearly that the ASE of the HetNets can be increased significantly when the small-cells are active and complementing the macrocell BSs in comparison with the MoNet (compare the set of red curves including the solid curve with square markers, the dashed dotted curve with square markers and the black solid curve with circle markers with the set of the blue curves including the solid curve with diamond markers and the dashed dotted curve with diamond markers). It can also be clearly seen that the improvement is considerable under the proposed COE configuration in comparison with the traditional UDC configuration. This is due to the fact that the COE configuration restricts only the cell-edge mobile users to communicate with the small-cells which enhances the overall network ASE compared to the UDC configuration. More precisely, the UDC configuration allows the small-cell BSs to be deployed in the cell center which causes an under-utilization of the macro BS capabilities (existing infrastructure). Thus, the performance degradation due to the cell-edge mobile users still exists in the UDC configuration. Moreover, it is also illustrated that the ASE decreases with the application of PC scheme in HetSNet. However, the gain in ASE is still significant in comparison with the UDC configuration and MoNet since the effect of interferences is also reduced (compare the set of red curves with the black solid curve). Due to the weaker channel gains of the mobile users in the large macrocells, the degradation of ASE with $R_m$ is obvious.

Fig. 6.3 shows that the amount of power saved by the macrocell and small-cell mobile users who are transmitting with adaptive power over the fading channel. It can be observed that the HetSNet offers a significant improvement in power saving in comparison with the MoNet (compare the red solid curve with square markers and the black solid curve with circle markers with the blue solid curve with diamond
Figure 6.3: Average power savings by all users in the MoNet and two HetSNet configurations (i) Cell-on-edge (COE) (ii) Uniformly distributed cell (UDC), $\zeta_{l,m,B_m}, \zeta_{l,B_m} \sim \text{Gamma}(1,1)$, $\zeta_{f_n,B_n}, \zeta_{f_j,B_n} \sim \text{Gamma}(1,1)$. However, the performance of HetNets with COE configuration outperforms the performance of HetNets with UDC configuration where there are no edge mobile users which are transmitting with the maximum power which is contrary to the UDC configuration where the mobile users are located around the edge of the macrocells and are not necessarily transmitting with adaptive power. This is the reason why the power saving of the HetNets with COE configuration increases with the increase in macrocell radius. As an example, the power saving offered by HetNets COE configuration at $R_m = 200$ m is 154.5 kW which is much lesser than the saving that the network can achieve at $R_m = 600$ m and which is 600 kW.

ICI in both macrocell and small-cell networks reduces with a reduction in the transmit power (due to PC scheme) of the mobile users in the HetSNet. This effect is illustrated in Fig. 6.4 for (a) $R_m = 200$ m and (b) $R_m = 600$ m. Fig. 6.4(a) shows the ICI offered by the macrocell network and small-cell network in HetSNet with COE configuration with and without PC for $R_m = 200$ m. It can be seen clearly that the contribution of the small-cell network to the total interference is higher in comparison with the contribution of the macrocell. This is due to many factors including
Figure 6.4: Summary of Inter-cell-interference (ICI) reduction offered by HetSNet by employing power control mechanism in comparison with the HetSNet where the mobile users are transmitting with maximum power, i.e., $P_{\text{max}}$ for (a) $R_m = 200$ m; and (b) $R_m = 600$ m; $\zeta_{lm,B_m}, \zeta_{li,B_m} \sim \text{Gamma}(1, 1)$, $\zeta_{jn,B_n}, \zeta_{j,B_n} \sim \text{Gamma}(1, 1)$.

the large number of mobile users, small radius of small-cells and increased number of small-cells in the network. Therefore, the need of employing PC mechanism in HetSNet has become mandatory in order to control the astronomically increased amount of interferences produced by small-cells. It can be seen clearly that the significant re-
duction in interference can be achieved by employing a fast PC mechanism, where the mobile users in both macrocell and small-cell networks are transmitting with much reduced power and embrace a much lower interfering network. We also illustrated the aggregate ICI per channel of the HetNets, which is a combination of ICI offered by both macrocell and small-cell networks\(^2\). The improvement in aggregate ICI of the HetSNet can also be observed when PC is employed. Similar observations can be drawn from Fig. 6.4(b), where the reduction in ICI was shown for \(R_m = 600\) m. It can be seen clearly that the ICI offered by the macrocell network decreases significantly with the increase in macrocell radius which is self explanatory. However, the ICI offered by the small-cell network does not strongly depend on the macrocell radius. Therefore, the aggregate ICI of the HetNets strongly depends on the radius of the macrocell. Moreover, further reduction in the macrocell and small-cell ICI can be achieved by employing a PC mechanism and thereby reducing the aggregate ICI of the HetSNet.

Fig. 6.5 demonstrates the bounds on the ASE of HetNets with COE configuration when the interfering cells in each of the macrocell and small-cell networks are offering the worst (interference from mobile users which are located nearer to the desired mobile users) and the best (interfering mobile users are located far away from the desired mobile user) interference to the desired mobile users. The bounds provide insights on the gain in ASE of the desired mobile user suffering from the worst case or the best case interference configurations. We can observe that a variation in ASE slightly increases with the increase of \(R_m\) which is due to the fact that the worst case and the best case distance of interferers becomes larger with the increase of radius \(R_m\) and hence, it causes an increased deviation of the bounds from the achievable mean ASE. In addition, we can observe that the upper bound is quite tight which is the scenario in which the macrocell and small-cell interferer location is far from the

\(^2\)Here, the aggregate ICI per channel was calculated based on the weighted combination of the interferences from macrocell and small-cell networks.
desired cell center and the interferers in both of the macrocell and small-cell networks are transmitting with an average power $P_m$ and $P_n$, respectively; however, the lower bound on the ASE is slightly loose, which illustrates the worst case ASE degradation due to the strong interferences received from the mobile users located near to the reference cell. However, it can be seen that the significant improvement in ASE is still able to guarantee a considerable gain in comparison with the ASE of MoNet under the worst case interference scenario.

Fig. 6.6 illustrates the AGE of the HetSNet which shows the aggregate power saving in the uplink per unit macrocell area while ensuring a desired signal power received at the BS of interest as a function of (a) macrocell radius $R_m$; and (b) desired target signal power received at BS. The AGE of the HetNets with COE configuration is compared with the AGE of the HetSNet with UDC configuration and AGE of the MoNet. It is illustrated clearly that the significant gain in AGE of the HetSNet can be achieved by incorporating an adaptive PC mechanism by the mobile users located in both macrocell and small-cell networks. However, the performance of
Figure 6.6: Comparison of the AGE of MoNet with two HetSNet configurations (i) Cell-on-edge (COE) (ii) Uniformly distributed cell (UDC) as a function of (a) $R_m$; and (b) $P_0; \zeta_{i,m,B_m}, \zeta_i, B_m \sim \text{Gamma}(1, 1)$, $\zeta_{f_n,B_n}, \zeta_{f_j,B_n} \sim \text{Gamma}(1, 1)$.

Our proposed HetSNet configuration outperforms the performance of the deployment where small-cells are uniformly distributed since the deployment of small-cells at the macrocell edge mandates a reduction in the number of edge mobile users transmitting with the maximum power and thereby establishing a "greener" HetSNet. However,
the amount of power saving decreases with the increase in desired received signal power which is self explanatory as the mobile users in the small-cell network require relatively more transmission power to maintain higher desired signal power level. The AGE improvement is due to the fact that the number of energy efficient users increase in both UDC and COE deployments with the increase in macrocell radius. However, the energy efficiency of cell-edge users in COE is higher than in the UDC deployment.

6.6 Conclusion

In this chapter, we critically evaluated the energy and spectral efficiency of a macro-only and two HetNet deployments (i) cell-on-edge (COE) (ii) uniformly distributed small cell (UDC). In this context, we derive analytical expressions to compute the bounds on the ASE which provide useful insights and illustrate the effect of the worst and the best case interferences in the two tier network of HetNets. Later, we derive an exact analytical expression to compute the network AGE. It is shown that the interference and power consumption are reduced significantly by exploiting the PC in the uplink of the HetNets configuration while maintaining the high spectrum efficiency.
APPENDIX A - Simulation Parameters

<table>
<thead>
<tr>
<th>Simulation parameter</th>
<th>Small-cell</th>
<th>Macrocell</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transmit power ($P_{\text{max}}$)</td>
<td>0.8 W</td>
<td>0.8 W</td>
</tr>
<tr>
<td>Cell radius ($R_{(\cdot)}$)</td>
<td>50 m</td>
<td>200–600 m</td>
</tr>
<tr>
<td>Path-loss exponent ($\alpha_{(\cdot)}$)</td>
<td>1.8</td>
<td>2.0</td>
</tr>
<tr>
<td>Additional path-loss exponent ($\beta_{(\cdot)}$)</td>
<td>1.8</td>
<td>2.0</td>
</tr>
<tr>
<td>BS antenna height ($h_{rx}$)</td>
<td>12.5 m</td>
<td>25 m</td>
</tr>
<tr>
<td>Mobile antenna height ($h_{tx}$)</td>
<td>2 m</td>
<td>2 m</td>
</tr>
<tr>
<td>Reference distance ($R_{0}$)</td>
<td>-</td>
<td>1 m</td>
</tr>
<tr>
<td>Target Power Received ($P_{0}$)</td>
<td>0.8 $\mu$ W</td>
<td>0.8 $\mu$ W</td>
</tr>
<tr>
<td>Breakpoint distance ($g_{(\cdot)}$)</td>
<td>300 m</td>
<td>600 m</td>
</tr>
<tr>
<td>System Bandwidth ($w_{t}$)</td>
<td>20 MHz</td>
<td></td>
</tr>
<tr>
<td>Reuse Factor ($R_{u}$)</td>
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<td></td>
</tr>
<tr>
<td>Small-cell population factor (CPF)</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

APPENDIX B

Proof of Scaling Property for Gamma RV

If $x$ is a Gamma distributed random variable with PDF given by $f_{X}(x) = \frac{x^{m_{(\cdot)}-1} e^{-\frac{x}{\theta_{(\cdot)}}}}{\Gamma(m_{(\cdot)})\theta_{(\cdot)}}$, then MGF of $f_{X}(x)$ is expressed as $M_{X}(t)$. Similarly, the MGF of $f_{aX}(ax)$ may be expressed as $M_{aX}(t)$. Therefore, using scaling property of MGF, we have $M_{aX}(t) = M_{X}(at) = \int_{0}^{\infty} f_{X}(x)e^{atx}dx$. Finally, the MGF of scaled Gamma RV is derived as:

\[
\frac{1}{\Gamma(m_{(\cdot)})\theta_{(\cdot)}} \int_{0}^{\infty} x^{m_{(\cdot)}-1} e^{-\frac{x}{\theta_{(\cdot)}}} e^{atx} dx, \\
= \frac{1}{\Gamma(m_{(\cdot)})\theta_{(\cdot)}} \int_{0}^{\infty} x^{m_{(\cdot)}-1} e^{-x(\frac{1-a\theta_{(\cdot)}t}{\theta_{(\cdot)}})} dx, \\
= \left(\frac{1-a\theta_{(\cdot)}t}{\theta_{(\cdot)}}\right)^{-m_{(\cdot)}}, \\
=(1-a\theta_{(\cdot)}t)^{-m_{(\cdot)}}. \quad (B.79)
\]
APPENDIX C

Proof of (30) and (32)

Note that, in this section we denote $\zeta_{m,B_m}$ and $\zeta_{f_n,B_n}$ by $\zeta_{(\cdot)}$, $C_m$ and $C_n$ by $C_{(\cdot)}$, and $P_m$ and $P_n$ by $P_{(\cdot)}$. The average transmit power of a macrocell user or a small-cell mobile user, i.e., $P_m$ and $P_n$ can be written as

$$P_{(\cdot)} = \mathbb{E}_{\zeta_{(\cdot)}} [P_{tx}^{(\cdot)}] = \mathbb{E}_{\zeta_{(\cdot)}} \left[ \min \left( P_{max}, \frac{P_{max} C_{(\cdot)}}{\zeta_{(\cdot)}} \right) \right]. \quad (C.80)$$

More explicitly, we can write (C.80) as follows

$$P_{(\cdot)} = \begin{cases} P_{max} & \zeta_{(\cdot)} < C_{(\cdot)} \\ \mathbb{E}_{\zeta_{(\cdot)}} \left[ \frac{P_{max} C_{(\cdot)}}{\zeta_{(\cdot)}} \right] & \text{otherwise.} \end{cases} \quad (C.81)$$

$$= P_{max} F_{\zeta_{(\cdot)}} (C_{(\cdot)}) + P_{max} C_{(\cdot)} \mathbb{E}_{\zeta_{(\cdot)}} \left[ \frac{1}{\zeta_{(\cdot)}} \right] \zeta_{(\cdot)} \geq C_{(\cdot)}. \quad (C.82)$$

where

$$\mathbb{E}_{\zeta_{(\cdot)}} \left[ \frac{1}{\zeta_{(\cdot)}} \right] \zeta_{(\cdot)} \geq C_{(\cdot)} = \int_{C_{(\cdot)}}^{\infty} \frac{1}{\zeta_{(\cdot)}} f_{\zeta_{(\cdot)}} (\zeta_{(\cdot)}) d\zeta_{(\cdot)}. \quad (C.83)$$

For the Gamma composite fading channels, (C.83) is expressed as

$$\mathbb{E}_{\zeta_{(\cdot)}} \left[ \frac{1}{\zeta_{(\cdot)}} \right] \zeta_{(\cdot)} \geq C_{(\cdot)} = \int_{C_{(\cdot)}}^{\infty} \frac{\zeta_{(\cdot)}^{m_d-2} e^{-\zeta_{(\cdot)}/\theta_d}}{\Gamma(m_d) \theta_d^{m_d}} d\zeta_{(\cdot)}. \quad (C.84)$$
Finally, using the identity as in [67], \( \int_u^\infty x^m e^{-\beta x^n} dx = \frac{\Gamma_u(v, \beta u^n)}{n \beta v} \), where \( v = \frac{m+1}{n} \) and performing some algebraic manipulations, it follows that (C.84) is derived as

\[
\mathbb{E}_{\zeta(\cdot)} \left[ \frac{1}{\zeta(\cdot)} \right]_{\zeta(\cdot) \geq C_{\cdot}} = \frac{\Gamma_u(m_d - 1, \frac{C_{\cdot}}{\theta_d})}{\theta_d \Gamma(m_d)}. \tag{C.85}
\]

Substituting (C.85) in (C.82), we obtain (6.52) and (6.54).
Chapter 7

Concluding Remarks

7.1 Summary

Optimizing the network performance and calibrating the efficiency of various resource allocation schemes while ignoring the impact of ICI may be highly misleading for practical scenarios. Therefore, it is of immense importance for the system designers to develop reasonable bounds while calibrating the efficiency of a variety of resource allocation schemes available in the literature. In this thesis, we developed in Chapter 2 several centralized and distributed rate maximization resource allocation schemes and calibrated their efficiency with respect to the optimal solution by developing an upper bound and a lower bound to the optimal solution. Even though the developed schemes were less complex, the computational complexity of these schemes may become challenging in the practice with large number of users and subcarriers.

In this context, later we developed an analytical framework to model the uplink ICI considering (i) various coordinated and uncoordinated scheduling schemes in Chapter 3; (ii) conventional and modified power control mechanisms in Chapter 4; (iii) generalized composite fading models; and (iv) partial and fractional frequency reuse techniques in Chapter 5. The derived expressions are also useful to evaluate numerically important network performance metrics such as outage probability, ergodic capacity, and average fairness.
Finally, in Chapter 6 we investigated the impact of location-based and fast PC mechanism in the interference mitigation and energy efficiency of two tier Heterogeneous networks. In this context, we derived the analytical bounds on the spectral efficiency and exact energy efficiency expressions for two tier Heterogeneous networks.

### 7.2 Future Directions

The research topics and the mathematical framework covered in this thesis can be extended into several directions. Some of the areas are listed in what follows:

**Cross-Tier and Co-Tier Interference Modeling and Management in Heterogeneous Networks**

Heterogeneous networks emerges as the most influential trend to meet the challenges imposed on the spectral and energy efficiency of existing and upcoming wireless standards. Despite of the associated benefits, continuing with this trend may lead to overcrowded spectrum bands, increased interference which is highly in-deterministic due to the random locations and deployment density of small cells, their corresponding access methods (public or private), spectrum and power allocation strategies, and proximity with the neighboring macro-cell and small cells.

Numerous research studies in the literature are focused on illustrating and optimizing the spectral and energy efficiency of HetNets for specific scenarios through classical optimization techniques and algorithms. Some attention has been paid recently on the simulation [46, 47, 48] as well as theoretical modeling [49, 41, 42] of important network performance metrics in the downlink of heterogeneous cellular networks with a special focus on user association and access control policies.

It was shown in [46, 47, 48] that by employing public access the interference in two-tier networks can be successfully mitigated which results in the feasibility of
co-channel access. However, the implications of private and public access policies were not precisely quantified which include (i) increased handover frequency; (ii) limited resources for small cells which directly affects the performance of small cell subscribers. In [41, 42] the authors derived an optimal joint and disjoint spectrum allocation for both public and private access that has the potential to alleviate the downlink cross-tier interference.

Recently, a comprehensive analytical framework to derive the distribution of downlink SIR has been developed in [58, 49] for open access policies. The derived expressions are then utilized to evaluate the outage probability and the network area spectral efficiency. The work assumes no co-tier interference within a given cell. A theoretical framework for the uplink has also been proposed in [59] with a focus on different access policies. However, the framework stand on several major assumptions which include (i) no shadowing and fading (ii) no co-tier interference within the given cell, no co-tier as well as cross-tier interference from neighboring cells. (iii) perfect location-based power control is considered which is not necessarily the case for far-edge users, i.e., battery power constraint is ignored.

Due to the unplanned deployment of small cells, some small cells could reside close to a macro BS and their users will incur high uplink interference to the macro BS. Whereas, the same is also true when a macro-cell user is far from macro BS and resides in the vicinity of a small BS. To make matters worse, the small cell users may also increase their transmit power to maintain the receive SINR which leads to severe uplink interference to the macro-cells and small cells around them. Therefore, another possible interference source to small cell uplink can be the neighboring small cells as the unplanned deployment could place multiple small cells in proximity, possibly resulting in severe performance degradation without their proper coordination.

The modeling and management of cross-tier and co-tier interference is crucial and of paramount importance for successful operation of the HetNets. Motivated by the
recent initiatives on the deployment and standardization of HetNets, it is significantly interesting to statistically model the cross-tier and co-tier interference in HetNets such that the derived statistical models remain adaptive to the user and small cell deployment densities, various state-of-the-art spectrum and power allocation strategies. In general, the models available in this thesis are expected to be extendible in the theoretical performance analysis of HetNets. In addition, the conventional interference mitigation strategies such as slow, fast and fractional power control mechanisms, coordinated frequency reuse methods may also be worth incorporating factors while modeling the statistics of interference in HetNets.

**Fractional Power Control**

Power control plays an important role in balancing coverage and interference. However, the conventional power control suffers from high interference, whereas, the modified power control mechanisms suffers from reduced coverage at high desired signal power targets. Therefore, a hybrid approach in which cell edge users will transmit with reduced power instead of being silent can be introduced in cellular networks. An optimal fractional path loss compensation factor can be derived for different scenarios. Moreover, the application of all of these power control and scheduling mechanisms can be investigated in Heterogeneous networks.

**Soft Frequency Reuse Schemes**

Soft frequency reuse is a special type of universal frequency reuse, in which cell edge users are transmitting with high powers compared to cell center users. All resources are allocated in the cell-center of each cell whereas a small part of the resources is allocated to the edge users of a cell. Based on the theoretical approach developed in this thesis, interference models for soft frequency reuse with different scheduling schemes can be derived with the critical performance analysis in terms of spectral
Interference Modeling in Relay-assisted Multi-hop Cellular Networks

Multi-hop communication has been emerged mainly to overcome some limitations of cellular networks by increasing the network capacity and by reducing energy consumption which are amongst the most promising benefits of the multi-hop communications. Multi-hop transmission transform the single-hop path from a mobile user to the BS, into two or more hops, which reduces distance and in turn transmit power will be greatly reduced.

Various techniques have been considered in cellular networks to improve network coverage and throughput by simply allowing other devices to relay the traffic of a user that receives weak or no signal directly from its nearby BS. Clearly it will be useful in reduction of energy consumption and coverage holes. However, finding such relays can be a challenge. In this context, a probabilistic model can be developed which determines the probability that a user can find its relay and then analyze the average power consumptions of mobile users and their relays compared to the power consumption in case of direct transmission. As the number of relaying stations increases, it is highly probable that mobile users can find their relay and save their power.
Chapter 8

Published and Submitted Papers

Book Chapters


Published/Revised Journal Papers


hop transmission systems,” Revised for IEEE Transactions on Vehicular Technology.


**Submitted Journal Papers**


**Published Conference Papers**


Bibliography/References


