Low-Complexity Combining Schemes in Dual-Hop AF Relaying Systems

Fakhreddine Gaaloul, Redha M. Radaydeh, and Mohamed-Slim Alouini
Division of Physical Sciences and Engineering
King Abdullah University of Science and Technology (KAUST)
Thuwal, Mekkah Province, Kingdom of Saudi Arabia
Email: {gaaloul.fakhreddine, redha.radaydeh, slim.alouini}@kaust.edu.sa

***Abstract*—This paper investigates the performance of different low-complexity combining schemes in the context of dual-hop amplify-and-forward relaying networks. It is assumed that the relay uses single transmit (receive) antenna due to space limitation and to reduce the processing complexity. On the other hand, the transmitter and the receiver use antenna arrays to improve the overall diversity gain. However, this gain is achieved at the expense of increased processing complexity and power consumption. To this end, some combining schemes aiming at reducing the processing complexity and decreasing the number of active receive channels are investigated. Through the analysis, new formulas for the end-to-end signal-to-noise ratio statistics in slowly varying and frequency flat Rayleigh fading channels are derived, which are then used to present some performance measures. Numerical and simulation results are presented to clarify the trade-off between the achieved diversity gain and the receive processing complexity.**

I. Introduction

It is well-known that relaying techniques can improve the spectral efficiency of wireless systems and mitigate the severe effect of large-scale fading. Moreover, the use of multiple antennas schemes in relaying networks have also shown further improvements in system capacity and communication reliability [1], [2]. Various diversity combining schemes have been studied in the context of multiple-antenna relaying systems [3]–[9]. However, an important aspect of employing diversity methods in such scenarios is to maintain as low processing complexity and power consumption as possible while satisfying a target performance level. This paper investigates different combining schemes with reduced processing complexity for dual-hop relaying networks employing the simple amplify-and-forward (AF) relaying protocol. The performance of this protocol can be improved when the channel state information (CSI) of the preceding hop is available to control the relaying gain, and it may be useful when simple relaying is required and/or when the transmitted data is time-sensitive [10], [11].

For the dual-hop AF relaying system under consideration, the relaying station is assumed to use single antenna in each direction due to space limitation and processing power constraints. On the other hand, different low-complexity configurations are investigated assuming uncorrelated antenna arrays at the transmitter and the receiver. Specifically, the scheme proposed in [12] is adopted to considerably reduce the power drain from the battery while satisfying a target performance. Moreover, when the CSI of all transmit antennas is perfectly known to the transmitter, the optimal first hop performance is obtained using the maximum ratio transmission (MRT) technique [13]. Based on the aforementioned schemes that vary significantly in terms of complexity and power requirements, different scenarios are investigated with an aim to reduce the receive processing requirements. Specifically, the low-complexity combining is applied at the receiver independently of the scheme used at the transmitter, which can be in the form of MRT for optimal first hop signal-to-noise ratio (SNR). Through the analysis, new analytical formulas for the statistics of the end-to-end combined SNR are obtained in exact forms. These formulas can be used to study the performance of the considered diversity combining schemes for the general case of dissimilar average fading conditions over the two hops. Numerical and simulation comparisons between the combining schemes are provided to validate the analytical development and to clarify the trade-off between processing complexity and achieved end-to-end outage performance.

II. System Model

A dual-hop relayed system $S \to R \to D$ in a cooperative wireless network is considered, where node S acts as the data source (transmitter) with n_t antennas. Node S uses amplify and forward (AF) relaying for data through another node R to the destination D (receiver) having an antenna array of size n_r. It is assumed that there is no direct link between S and D due, for example, to the effect of severe large-scale fading. It is also assumed that S and D have perfect CSI of the S-to-R and R-to-D links, respectively. The fading channels are assumed to spatially uncorrelated, slowly varying and frequency flat, where each channel fading envelope follows Rayleigh distribution.

Define γ_{1i}, for $i = 1, 2, .., n_t$, as the instantaneous SNR on the ith transmit channel and γ_{2j}, for $j = 1, 2, .., n_r$, as the instantaneous SNR on the jth receive channel. The distributions of transmit and receive channels instantaneous SNRs can then be expressed as $f_{\gamma_{1i}}(x) = \frac{1}{\bar{\gamma}_1}\, e^{-\frac{x}{\bar{\gamma}_1}}$, and $f_{\gamma_{2j}}(x) = \frac{1}{\bar{\gamma}_2}\, e^{-\frac{x}{\bar{\gamma}_2}}$, respectively, where $\bar{\gamma}_1$ and $\bar{\gamma}_2$ are the average SNRs on the first hop and the second hop, respectively.

As mentioned previously, an AF CSI-assisted relaying scheme is adopted, in which the CSI of the first hop is used to control the relaying gain for fixed retransmitted signal power.

The end-to-end SNR of AF dual-hop relaying systems, denoted by γ_c, has a well-known form [3], [11], which generally can be expressed as $\gamma_c = \frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2+b}$, where γ_1 and γ_2 are the combined SNRs on the first and second hops, respectively, and b is a constant. When $b = 0$, the resulting form of γ_c neglects the effect of white noise at the relay station.

III. Statistics of SNR and Performance Analysis

This section derives the statistics of the end-to-end SNR for the system model described above. These statistics can be used to obtain analytical expressions for various performance measures, including outage probability performance using the cumulative distribution function (CDF) of the end-to-end SNR and average error probability based on the moment generating function (MGF) approach [14]. The CDF of the end-to-end SNR can be written as

$$\begin{aligned} F_{\gamma_c}(x) &= \Pr\left\{\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2+b} \le x\right\} \\ &= 1 - \int_x^{+\infty} \bar{F}_{\gamma_2}\left(\frac{x(t+b)}{t-x}\right) f_{\gamma_1}(t)dt, \end{aligned} \quad (1)$$

where $\bar{F}_{\gamma_c}(x)$ is the complementary cumulative distribution function (CCDF) of γ_c, $\bar{F}_{\gamma_2}(x) = 1 - F_{\gamma_2}(x)$ is the CCDF of γ_2, $F_{\gamma_2}(x)$ is the CDF of γ_2, and $f_{\gamma_1}(x)$ is the probability density function (PDF) of γ_1. Using the CCDF of γ_c, the MGF can be obtained as

$$M_{\gamma_c}(s) = 1 - s\int_0^{+\infty} e^{-sx}\bar{F}_{\gamma_c}(x)\, dx. \quad (2)$$

A. Statistics of The end-to-end SNR

In this subsection, we derive the CDF and the MGF of the end-to-end SNR when the low-complexity combining scheme is applied at the receiver and the MRT scheme is employed at the transmitter.

With the use of MRT, the transmitted signals over the n_t antennas are weighted to maximize the received SNR. The statistics of γ_1, assuming perfect estimation of CSI on the first hop and error-free feedback link, can be written as $f_{\gamma_1}(x) = \frac{x^{n_t-1}e^{-\frac{x}{\bar{\gamma}_1}}}{(n_t-1)!\ \bar{\gamma}_1^{n_t}}$ and $F_{\gamma_1}(x) = 1 - \sum_{i=0}^{n_t-1}\frac{(\frac{x}{\bar{\gamma}_1})^i}{i!}e^{-\frac{x}{\bar{\gamma}_1}}$ [13, eq. (22)].

The scheme in [12] is adopted on the second hop in which the receiver tries to perform just enough processing complexity and power consumption to raise the second hop SNR above a preselected threshold, which is denoted by γ_T. This scheme suggests two phases of operation that are based on statistically unordered receive SNRs for single antenna switching and then ordered receive SNRs with multiple-antenna combining. The CDF of the resulting γ_2 is given by

$$\begin{aligned} F_{\gamma_2}(x) &= F_{\Gamma_{Lc}}(x)(U(x) - U(x-\gamma_T)) \\ &+ \left(A(x) + \sum_{l=2}^{Lc} F_{\gamma_c}^{(l)}(x) + F_{\Gamma_{Lc}}(\gamma_T)\right) U(x-\gamma_T), \end{aligned} \quad (3)$$

where $U(x)$ is the unit step function, Lc is the maximum number of allowed active channels, $F_{\Gamma_l}(.)$ is the CDF of $\Gamma_l = \sum_{i=1}^{l}\gamma_{i:n_r}$, where $\gamma_{i:n_r}$ represents the SNR of the ith strongest receive channel. In (3), the term $A(x)$ is given by

$$A(x) = \frac{1-(F_{\gamma_{\varsigma_2}}(\gamma_T))^{n_r}}{1-F_{\gamma_{\varsigma_2}}(\gamma_T)}\left(F_{\gamma_{\varsigma_2}}(x) - F_{\gamma_{\varsigma_2}}(\gamma_T)\right), \quad (4)$$

and the term $F_{\gamma_{\varsigma_2}}(.)$ is the CDF of the receive SNR on each receive channel. In addition,

$$F_{\gamma_c}^{(l)}(x) = \begin{cases} g(x,l), & \gamma_T \le x < \frac{l}{l-1}\gamma_T \\ F_{\Gamma_{l-1}}(\gamma_T) - F_{\Gamma_l}(\gamma_T), & \frac{l}{l-1}\gamma_T \le x, \end{cases} \quad (5)$$

where $F_{\Gamma_i}(.)$ and $g(x,i)$ are given in [15, eqs. (17) and (30)] with the parameters L and $\bar{\gamma}$ therein are replaced by n_r and $\bar{\gamma}_2$, respectively.

The distribution of γ_2, defined as $f_{\gamma_2}(x) = dF_{\gamma_2}(x)/dx$, can then be expressed as

$$\begin{aligned} f_{\gamma_2}(x) &= f_{\Gamma_{Lc}}(x)(U(x) - U(x-\gamma_T) \\ &+ \Big(\sum_{i=2}^{Lc} h(x,i)(U(x-\gamma_T) - U(x - \frac{i}{i-1}\gamma_T)) \\ &+ \frac{1-(F_{\gamma_{\varsigma_2}}(\gamma_T))^{n_r}}{1-F_{\gamma_{\varsigma_2}}(\gamma_T)} f_{\gamma_{\varsigma_2}}(x)\Big) U(x-\gamma_T), \end{aligned} \quad (6)$$

where $h(x,i)$ and $f_{\Gamma_{Lc}}(x)$ are given in [15, eq. (32) and (33)] with L and $\bar{\gamma}$ therein are replaced by n_r and $\bar{\gamma}_2$, respectively.

1) CDF of the end-to-end SNR: Substituting the CDF of γ_1 presented above and the result in (6) into (1), the CCDF of the end-to-end SNR can be obtained as

$$\begin{aligned} \bar{F}_{\gamma_c}(x) = &\left(\mathcal{W}_1(x) + \frac{1-(F_{\gamma_{\varsigma_2}}(\gamma_T))^{n_r}}{1-F_{\gamma_{\varsigma_2}}(\gamma_T)}\mathcal{P}_2(x,1,0,\gamma_T)\right. \\ &\left. + \sum_{l=2}^{L_c}\mathcal{W}_2(x,l)\right)(U(x) - U(x-\gamma_T)) \\ &+ \left(\frac{1-(F_{\gamma_{\varsigma_2}}(\gamma_T))^{n_r}}{1-F_{\gamma_{\varsigma_2}}(\gamma_T)}\mathcal{P}_1(x,1,0)\right. \\ &\left. + \sum_{l=2}^{L_c}\mathcal{W}_3(x,l)\,\mathcal{U}\left(\frac{l}{l-1}\gamma_T - x\right)\right) U(x-\gamma_T), \end{aligned} \quad (7)$$

where the terms $\mathcal{W}_1(x)$, $\mathcal{W}_2(x,a)$ and $\mathcal{W}_3(x,b)$ are given in exact forms as shown in (8) on the the next page, in which the terms $\mathcal{P}_1(x,a,b)$ and $\mathcal{P}_2(x,a,b,c)$ are expressed as

$$\begin{aligned} \mathcal{P}_1(x,p,d) &= \sum_{i=0}^{n_t-1}\sum_{k=0}^{i}\sum_{e=0}^{d}\frac{2\binom{i}{k}\binom{e}{d}}{i!\bar{\gamma}_1^i\bar{\gamma}_2}\left(\frac{\bar{\gamma}_2}{\bar{\gamma}_1 p}\right)^{\frac{k+e-i+1}{2}} x^{d+\frac{k-e+i+1}{2}} \\ &\times (x+b)^{\frac{i-k+e+1}{2}} e^{-(\frac{1}{\bar{\gamma}_1}+\frac{p}{\bar{\gamma}_2})x} K_{k-i+e+1}\left(2\sqrt{\frac{x(x+b)p}{\bar{\gamma}_1\bar{\gamma}_2}}\right) \end{aligned} \quad (9a)$$

$$\begin{aligned} \mathcal{P}_2(x,p,d,r) &= \sum_{i=0}^{n_t-1}\sum_{k=0}^{i}\sum_{e=0}^{d}\frac{4\binom{i}{k}\binom{e}{d}}{i!\bar{\gamma}_1^i\bar{\gamma}_2}\left(\frac{\bar{\gamma}_2}{\bar{\gamma}_1 p}\right)^{\frac{k+e-i+1}{2}} \\ &\times x^{d+\frac{k-e+i+1}{2}}(x+b)^{\frac{i-k+e+1}{2}} e^{-\left(\frac{1}{\bar{\gamma}_1}+\frac{p}{\bar{\gamma}_2}\right)x} \\ &\times k^{+}_{k+e-i+1}\left(2\sqrt{\frac{x(x+b)p}{\bar{\gamma}_1\bar{\gamma}_2}}, \log\left((r-x)\sqrt{\frac{p\bar{\gamma}_1}{x(x+b)\bar{\gamma}_2}}\right)\right), \end{aligned} \quad (9b)$$

where $K_\nu(a)$ is the modified Bessel function of second kind of order ν and $k_\nu^+(a,b)$ is the incomplete modified Bessel

$$\mathcal{W}_1(x) = \int_x^{\gamma_T} f_{\Gamma_{L_c}}(t)\bar{F}_{\gamma_1}\left(\frac{x(t+b)}{t-x}\right)dt$$
$$= \frac{\binom{n_r}{Lc}}{(Lc-1)!\bar{\gamma}_2^{Lc-1}}\left[\mathcal{P}_1(x,1,Lc-1) - \mathcal{P}_2(x,1,Lc-1,\gamma_T)\right] + \binom{n_r}{Lc}\sum_{l=1}^{n_r-Lc}(-1)^{L_c+l-1}\binom{n_r-Lc}{l}$$
$$\times\left\{\mathcal{P}_1\left(x,\frac{l}{Lc}+1,0\right) - \mathcal{P}_2\left(x,\frac{l}{Lc}+1,0,\gamma_T\right) - \sum_{m=0}^{Lc-2}\frac{1}{m!}\left(-\frac{l}{Lc\bar{\gamma}_2}\right)^m\left[\mathcal{P}_1(x,1,m) - \mathcal{P}_2(x,1,m,\gamma_T)\right]\right\}, \tag{8a}$$
$$\mathcal{W}_2(x,l) = \int_{\gamma_T}^{\frac{l}{l-1}\gamma_T} h(t,l)\bar{F}_{\gamma_1}\left(\frac{x(t+b)}{t-x}\right)dt$$
$$= \frac{\binom{n_r}{l}}{(l-1)!\bar{\gamma}_2^{l-1}}\sum_{u=0}^{l-1}\binom{l-1}{u}(-1)^u(l\gamma_T)^{l-1-u}(l-1)^u\left[\mathcal{P}_2(x,1,u,\gamma_T) - \mathcal{P}_2\left(x,1,u,\frac{l}{l-1}\gamma_T\right)\right]$$
$$+\sum_{j=1}^{n_r-l}\frac{n_r!(-1)^{j-l+1}}{(n_r-l-j)!l!j!}\left(\frac{l}{j}\right)^{l-1}\left[\mathcal{P}_2\left(x,\frac{l+j}{l},0,\gamma_T\right) - \mathcal{P}_2\left(x,\frac{l+j}{l},0,\frac{l}{l-1}\gamma_T\right)\right.$$
$$\left.-e^{\frac{j\gamma_T}{\bar{\gamma}_2}}\sum_{k=0}^{l-2}\sum_{u=0}^{k}\frac{\binom{k}{u}j^k(-l\gamma_T)^{k-u}(l-1)^u}{k!l^k\bar{\gamma}_2^k}\left(\mathcal{P}_2\left(x,\frac{j(l-1)+(l+j)}{l},u,\gamma_T\right) - \mathcal{P}_2\left(x,\frac{j(l-1)+(l+j)}{l},u,\frac{l}{l-1}\gamma_T\right)\right)\right], \tag{8b}$$
$$\mathcal{W}_3(x,l) = \int_x^{\frac{l}{l-1}\gamma_T} h(t,l)\bar{F}_{\gamma_1}\left(\frac{x(t+b)}{t-x}\right)dt$$
$$= \frac{\binom{n_r}{l}}{(l-1)!\bar{\gamma}_2^{l-1}}\sum_{u=0}^{l-1}\binom{l-1}{u}(-1)^u(l\gamma_T)^{l-1-u}(l-1)^u\left[\mathcal{P}_1(x,1,u) - \mathcal{P}_2\left(x,1,u,\frac{l}{l-1}\gamma_T\right)\right]$$
$$+\sum_{j=1}^{n_r-l}\frac{n_r!(-1)^{j-l+1}}{(n_r-l-j)!l!j!}\left(\frac{l}{j}\right)^{l-1}\left[\mathcal{P}_1\left(x,\frac{l+j}{l},0\right) - \mathcal{P}_2\left(x,\frac{l+j}{l},0,\frac{l}{l-1}\gamma_T\right) - e^{\frac{j\gamma_T}{\bar{\gamma}_2}}\right.$$
$$\left.\times\sum_{k=0}^{l-2}\sum_{u=0}^{k}\frac{\binom{k}{u}j^k(-l\gamma_T)^{k-u}(l-1)^u}{k!l^k\bar{\gamma}_2^k}\left(\mathcal{P}_1\left(x,\frac{j(l-1)+(l+j)}{l},u\right) - \mathcal{P}_2\left(x,\frac{j(l-1)+(l+j)}{l},u,\frac{l}{l-1}\gamma_T\right)\right)\right]. \tag{8c}$$

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function of second kind of order ν. Details about the properties of $k_\nu^+(a,b)$ can be found in [16].

2) MGF of the end-to-end SNR: Substituting the CCDF of γ_c, which is presented in (7), into (2), the MGF of γ_c can be obtained as shown in (10), in which

$$\mathcal{V}_1(s,p,d,u) = \mathcal{Q}_1^l(s,p,d,u) - \mathcal{Q}_2(s,p,d,u,u) \tag{11a}$$
$$\mathcal{V}_2(s,p,d,u,v) = \mathcal{V}_1(s,p,d,v) - \mathcal{V}_1(s,p,d,u), \tag{11b}$$

where the terms $\mathcal{Q}_1^l(s,p,d,u)$, $\mathcal{Q}_1^u(s,p,d,u)$, and $\mathcal{Q}_2(s,p,d,r,u)$ are given by

$$\mathcal{Q}_1^l(s,p,d,u) = \int_0^u e^{-sx}\mathcal{P}_1(x,p,d)dx \tag{12a}$$
$$\mathcal{Q}_1^u(s,p,d,u) = \int_u^{+\infty} e^{-sx}\mathcal{P}_1(x,p,d)dx \tag{12b}$$
$$\mathcal{Q}_2(s,p,d,r,u) = \int_0^u e^{-sx}\mathcal{P}_2(x,p,d,r)dx. \tag{12c}$$

To the best of our knowledge, the integrals in (12) may not have exact solutions. In such case, numerical integration methods, such as Gauss-Chebechev, can be applied.

The use of Gauss-Chebechev method to evaluate the terms in (12) is outlined as follows. Define $f_1(x) = e^{-sx}\mathcal{P}_1(x,p,d)$ and $f_2(x) = e^{-sx}\mathcal{P}_2(x,p,d,r)$, the integrals $\int_{\gamma_T}^{+\infty} f_1(x)dx$ and $\int_0^{\gamma_T} f_2(x)dx$ (or $\int_0^{\gamma_T} f_1(x)dx$) can be obtained as

$$\int_{\gamma_T}^{+\infty} f_1(x)dx = \left(\frac{\pi - 2\arctan(\gamma_T)}{4}\right)\sum_{l=1}^{N}\omega_l\sqrt{1-x_l^2}$$
$$\times\left(1+(\varphi_1(x_l))^2\right) f_1(\varphi_1(x_l)), \tag{13}$$

and

$$\int_0^{\gamma_T} f_2(x)dx = \frac{\gamma_T}{2}\sum_{l=1}^{N}\omega_l\sqrt{1-x_l^2}\, f_2(\varphi_2(x_l)), \tag{14}$$

where $\varphi_1(t) = \tan(\frac{\pi-2\arctan(\gamma_T)}{4}t + \frac{\pi+2\arctan(\gamma_T)}{4})$, $\varphi_2(t) = \frac{\gamma_T}{2}(t+1)$, and $\omega_l = \frac{\pi}{N}$. The values of N and $\{x_l\}_{l=1}^N$ can be adjusted to achieve a desired accuracy in numerical computations [17]. It has been found that the use of $N = 100$ and $\{x_l = \cos(\frac{(2l-1)\pi}{200})\}_{l=1}^{100}$ can provide quite accurate numerical results for the average SNR values between 0 and 20 dB.

IV. Numerical and Simulation Results

In this section, some numerical and simulation results are presented to compare the outage performance and the average numbers of estimated and activated receive channels (paths), considering the effects of the transmit and receive arrays sizes (i.e. n_t and n_r, respectively), the maximum number of allowed active paths Lc, and the combined SNR threshold γ_T. Although the analysis is generally applicable for the case

$$
\begin{aligned}
M_{\gamma_c}(s) = {} & 1 - s\frac{1-(F_{\gamma_{\varsigma_2}}(\gamma_T))^{n_r}}{1-F_{\gamma_{\varsigma_2}}(\gamma_T)}\left[\mathcal{Q}_1^u(s,1,0,\gamma_T)+\mathcal{Q}_2(s,1,0,\gamma_T,\gamma_T)\right] + \frac{s\binom{n_r}{Lc}}{(Lc-1)!\bar{\gamma}_2^{Lc-1}}\mathcal{V}_1(s,1,Lc-1,\gamma_T) \\
& + s\binom{n_r}{Lc}\sum_{l=1}^{n_r-Lc}(-1)^{L_c+l-1}\binom{n_r-Lc}{l}\left[\mathcal{V}_1(s,\frac{l}{Lc}+1,0,\gamma_T) - \sum_{m=0}^{Lc-2}\frac{1}{m!}\left(-\frac{l}{Lc\bar{\gamma}_2}\right)^m \mathcal{V}_1(s,1,m,\gamma_T)\right] \\
& + s\sum_{l=2}^{Lc}\left\{\frac{\binom{n_r}{l}}{(l-1)!\bar{\gamma}_2^{l-1}}\sum_{u=0}^{l-1}\binom{l-1}{u}(-1)^u\left(l\gamma_T\right)^{l-1-u}(l-1)^u\mathcal{V}_2\left(s,1,u,\gamma_T,\frac{l}{l-1}\gamma_T\right)\right. \\
& + \sum_{j=1}^{n_r-l}\frac{n_r!(-1)^{j-l+1}}{(n_r-l-j)!l!j!}\left(\frac{l}{j}\right)^{l-1}\left[\mathcal{V}_2\left(s,\frac{l+j}{l},0,\gamma_T,\frac{l}{l-1}\gamma_T\right) - e^{\frac{j\gamma_T}{\bar{\gamma}_2}}\right. \\
& \left.\left.\times\sum_{k=0}^{l-2}\sum_{u=0}^{k}\frac{\binom{k}{u}j^k\left(-l\gamma_T\right)^{k-u}(l-1)^u}{k!l^k\bar{\gamma}_2^k}\mathcal{V}_2\left(s,\frac{j(l-1)+(l+j)}{l},u,\gamma_T,\frac{l}{l-1}\gamma_T\right)\right]\right\}. \qquad (10)
\end{aligned}
$$

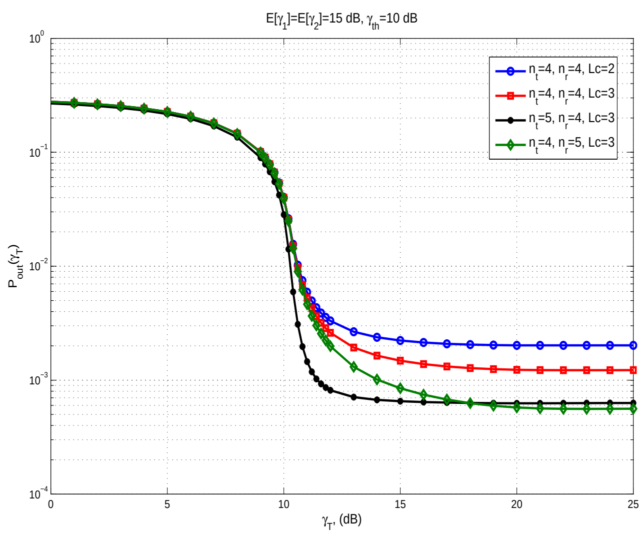

Fig. 1. Outage probability performance against γ_T for different combinations of n_t, n_r and Lc.

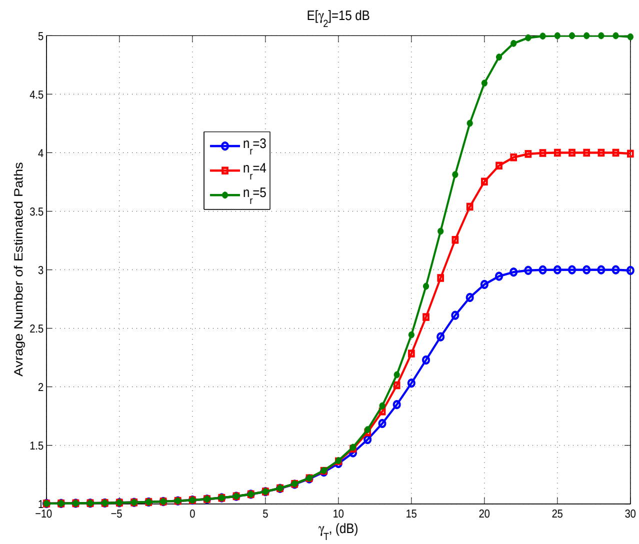

Fig. 2. Average number of estimated paths against γ_T for different combinations of n_r and Lc.

of dissimilar average conditions, the case of identical average fading conditions on the two hops is adopted herein for the sake of simplicity.

Fig. 1 shows the outage probability against γ_T for different values of n_t, n_r, and Lc and given values of $\bar{\gamma}_1 = \bar{\gamma}_2 = 15$ dB, and outage threshold $\gamma_{\text{th}} = 10$ dB assuming $b = 0$. It is seen that the increase in γ_T improves the outage performance. This can be explained by noting that the receiver tries to estimate and then combine as many paths as necessary to raise γ_2 above γ_T, which increases the end-to-end SNR and consequently improves the outage probability performance. For the same value of $Lc = 3$, the case when $n_t = 5$ and $n_r = 4$ outperforms that when $n_t = 4$ and $n_r = 4$ for all values of γ_T due to the increase in γ_1. Moreover, it seen that the case when $n_t = 5$ and $n_r = 4$ outperforms that when $n_t = 4$ and $n_r = 5$ as long as $\gamma_T < 18$ dB. For the values of $\gamma_T > 18$ dB, it appears that the selection of the most effective three receive antennas out of the five available ones compensate for the outage performance reduction due to the decrease in the number of transmit antennas.

Figs. 2 and 3 show the average number of estimated and combined (activated) paths, respectively, considering different values of n_r, and Lc and given values of $\bar{\gamma}_2 = 15$ dB and $b = 0$ (i.e. the cases in Fig. 1). It is seen that increasing γ_T requires estimating and activating more receive paths, as intuitively expected. For the system model under consideration, it is noted that the increase in n_t does not affect the average number of estimated and combined receive paths, but it improves the outage improvement due to the increase in γ_1. On the other hand, the increase in n_r improves the outage performance, but at the same time affect the average number of estimated receive paths, particularly when γ_T is relatively large (i.e.

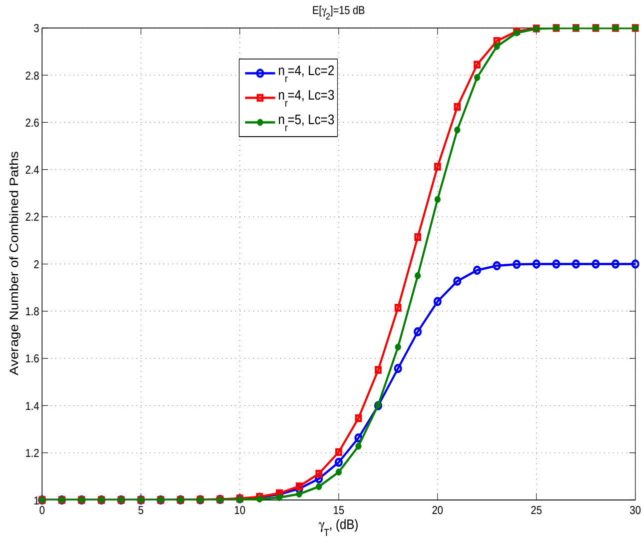

Fig. 3. Average number of combined paths against γ_T for different combinations of n_r and Lc.

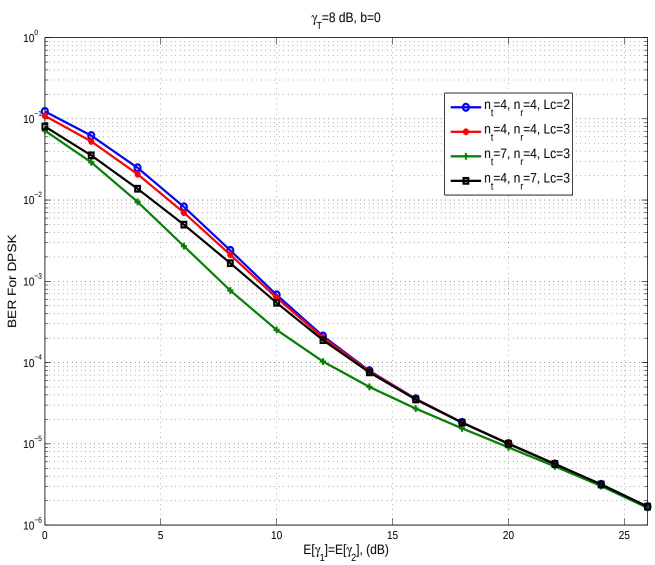

Fig. 4. Average BER of BDPSK scheme for different combinations of n_t, n_r and Lc.

above $\bar{\gamma}_2$). It is seen that the average number of activated paths decreases with the increase in n_r for a given value of Lc. However, the price to paid in this case is the increase in the average number of estimated paths. Further improvement in outage performance is also observed as Lc increases due to the increase in the number of possible active receive paths. For large values of γ_T, the average number of estimated paths is close to n_t, whereas the average number of active paths is around Lc. In this case, the identification of the suitable paths from an increasing set of estimated ones is expected to improve the system performance.

Fig 4 shows the average bit error rate (BER) of binary phase-shift keying (BDPSK), which obtained using $\overline{P}_e = 1/2M_{\gamma_c}(1)$, against the average SNR per each hop (i.e. $\bar{\gamma}_1 = \bar{\gamma}_2$) for different values of n_t, n_r, and Lc and given values of $\gamma_T = 8$ dB, $b = 0$. It is seen that the increase in the average SNR improves the average BER, as intuitively expected. Further improvement in average BER is observed as n_t, n_r, and/or Lc increase. For example, for the same values of $n_t = n_r = 4$, the case when $Lc = 3$ outperforms that when $Lc = 2$, particularly when the average SNR is relatively low. However, for average SNR above 22 dB, the increase in n_t, n_r, and/or Lc induce negligible effect on the average BER performance because the end-to-end SNR in this case is controlled by the weak second hop SNR as the first estimated path will be found sufficient to raise γ_2 above the threshold.

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